

Subject: Engineering Management

Weekly Hours :Theoretical: 2
Tutorial:

Experimental:

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<u>week</u>	<u>Contents</u>	
1.	Concepts and objectives of Engineering Management	مفاهيم وأهداف الإدارة الهندسية .1
2.	Technical and economic studies for project feasibility.	الدراسات الفنية والاقتصادية لجدوى المشاريع .2
3.	Plant performance appraisal.	تقييم أداء المصانع .3
4.	Administrative and production organization of industrial enterprises	التنظيم الإداري والإنتاجي لمنشآت صناعية .4
5.	Using operation research in production.	استخدام بحوث العمليات في الإنتاج .5
6.	Linear programming.	البرمجة الخطية .6
7.	Graphical method.	الطريقة البيانية .7
8.	Algebraic method.	الطريقة الجبرية .8
9.	Simplex method.	الطريقة المبسطة .9
10.	Allocation of resources.	تخصيص الموارد .10
11.	Quality Control and production inspection method.	السيطرة النوعية وطرق فحص الإنتاج .11
12.	Industrial costs and controllable cost techniques.	التكاليف الصناعية وأساليب السيطرة عليها .12
13.	Time measurement studies for production operations.	دراسة الوقت للعمليات الإنتاجية .13
14.	Introduction Time studies for production operations.	حساب الزمن للعمليات الإنتاجية .14
15.	Method Time studies for production operations.	طرق حساب الزمن للعمليات الإنتاجية .15
16.	Construction of technological paths.	رسم المسالك التكنولوجية .16
17.	Internal plant layout.	التخطيط الداخلي للمصنع .17
18.	Productivity, measurement method, techniques	الإنتاجية، طرق قياسها، أساليبها .18
19.	Maintenance.	الصيانة .19
20.	Replacement.	الاستبدال .20
21.	Introduction to Inventory Control	مقدمة السيطرة على المخزون .21
22.	Method of Inventory Control	طرق السيطرة على المخزون .22
23.	Type of Inventory Control	أنواع السيطرة على المخزون .23
24.	Selection of plant location.	اختيار موقع المصنع .24
25.	Introduction to Floating planning for raw materials.	تخطيط التداول للمواد الأولية .25
26.	Method of Floating planning for raw materials.	طرق تخطيط التداول للمواد الأولية .26
27.	Floating planning for semi-manufacture product.	تخطيط التداول للمواد النصف مصنعة .27
28.	Floating planning for manufacture product.	تخطيط التداول للمواد تامة الصنع .28
29.	Applications	تطبيقات .29
30.	Applications	تطبيقات .30

الموضوع : الهندسة الصناعية

مدرس المادة : د. نبيل جورج ناسي

عدد الوحدات : 4

الكتب المنهجية : (1) د. عادل عبد المالك " الهندسة الصناعية " - دار الكتب للطباعة والنشر - جامعة البصرة - الطبعة الأولى 2000 .
(2) د. خليل العاني ، د. إسماعيل إبراهيم القزاز ، د. عادل عبد المالك كوريال " إدارة الجودة الشاملة ومتطلبات الأيزو 2000:9001 " الطبعة الأولى 2001 ، مطبعة الأشقر- بغداد .

3) Hamdy A. Taha " Operations Research : an introduction " 6th edition (1997), Prentice-Hall.

4) Prem Kumar Gupta and D.S. Hira " Operations Research : an introduction " 2nd edition (1989) S. Chand & Company LTD, NewDelhi .

5) Charles E. Ebeling "An Introduction to Reliability and Maintainability Engineering " (1997) , McGraw-Hill.

الكتب المساعدة : (1) د. مازن بكر عادل وآخرون " بحوث العمليات للإدارة الهندسية " جامعة الموصل 1986 .

2) Phillips,D.T.;Ravindran,A.;Solberg ,J." Operations Research : Principles and Practice " (1976) John Wiley.

الساعات الإيسوعية : النظري : 2 : العملي : — مناقشة : — .

الإسبوع	الصفحة	الفصل	عنوان المحاضرة	مختصر المحاضرة
1	7 – 1	الأول	مفاهيم وأهداف الهندسة الصناعية.	تعريفية بمفهوم الهندسة الصناعية كفرع من الإدارة الهندسية وأهدافها .
2	12 – 8	الثاني	الدراسات الفنية والإقتصادية لجدوى المشاريع الصناعية.	تتضمن دراسة تكاليف الإنتاج (المتغيرة والثابتة) واستخدام النسبة المنوية لربحية المشروع وحجم وقيمة نقطة التعادل والنسبة المنوية لحد الأمان ومدة إطفاء (إسترداد) المشروع للراسمال المستثمر .
3	16 – 12	الثاني	تكاليف الإنتاج.	تحليل نقطة التعادل والعلاقة بين الكلفة والربح وحجم الإنتاج .
4	20 – 17	الثاني	الإنتاجية.	أنواع الإنتاجية وطرق قياسها (طريقة معامل التحويل والطريقة النقدية) وأساليب زيادتها.
5	25 – 21	الثاني	دراسة العمل .	فوائد دراسة العمل وخطواتها الرئيسية و قياس محتوى العمل وتحديد الوقت القياسي .
6	36 – 29	الثالث	التنظيم الإداري والتكنولوجي لمنشأة صناعية.	أنواع التنظيمات والمستويات الإدارية المناظرة و التنظيم التكنولوجي وأنواعه ودورة الإنتاج .
7	مناقشة تقارير تقدم من قبل الطلبة فيما يخص الباب الأول (الفصول الأول والثاني و الثالث) مع إختبار .			
8	46 – 38	الرابع	إستخدام بحوث العمليات في الإنتاج / البرمجة الخطية .	المستلزمات الأساسية للبرمجة الخطية وصيغها (العامة ، القانونية و القياسية) وكيفية التحويل بينهما وكيفية صياغة النموذج رياضياً و حل النموذج رياضياً .
9	52 – 47	الرابع		حل النموذج الرياضي بالطرق Simplex و M- technique و 2-phase .
10	58 – 56	الخامس	نموذج مشكلة النقل	تعريف المشكلة كحالة خاصة من نماذج البرمجة الخطية وكيفية إيجاد الحل الأولي بإستخدام طرق الركن الشمالي الغربي، الأقل كلفة ، VAM و RAM .
11	67 – 59	الخامس		إختبار وتحسين الحل الأولي بإحدى الطريقتين : Stepping stone أو Multipliers للوصول للحل الأمثل .
12	74 – 68	الخامس	مشكلة التخصيص	تعريف المشكلة كحالة خاصة من نماذج البرمجة الخطية وكيفية إيجاد الحل الأمثل في حالتها تعظيم الربح أو الإيراد أو تقليل الكلفة أو الزمن.

تعريف بالمخططات الشبكية وكيفية رسمها وإيجاد المسار الحرج <i>C.P.</i> والوقت الحرج وأسلوب تقييم ومراجعة البرامج <i>PERT</i> . تعجيل وتبطين المخططات الشبكية (إيجاد أقل زمن وكلفة ممكنين لتنفيذ المشروع) .	المخططات الشبكية	السادس	81 – 75	13
تعريف نماذج التتابع وإنجاز n من المهام على ماكينة واحدة ، وإيجاد أقصر وقت تشغيل <i>S.P.T.</i> وكذلك أطول وقت تشغيل <i>L.P.T.</i> ، وإنجاز n من المهام على ماكنتين ، وإنجاز n من المهام على ثلاثة مكائن .	نماذج التتابع	السادس	88 – 81	14
إنجاز n من المهام على m من المكائن، وإنجاز n من المهام على ماكنتين في ورشة ذات مسالك تكنولوجية مختلفة (عشوائية الإنسياب) .		السابع	94 - 89	15
دراسة كلف الصيانة والتشغيل لإيجاد وقت إستبدال الماكينة مع دراسة معدل الكلفة الفردية لإستبدال الوحدات العاطلة ومعدل الكلفة الجماعية لإستبدال جميع الوحدات لتحديد سياسة الإستبدال المثلى .	نماذج الصيانة والإستبدال	الثامن	99 – 94	16
نموذج الصيانة مع حل تمارين الفصل .		الثامن	104 – 100	17
الجودة و إدارة الجودة الشاملة (مرتكزاتها الأولية ومبادئها) .	إدارة الجودة الشاملة <i>TQM</i> والأيزو	التاسع	108 – 104	18
إدارة الجودة الشاملة (عناصرها و مراحل تطبيقها و فوائدها) .	<i>ISO</i> .	التاسع	114 – 110	19
مفهوم الأيزو <i>ISO 9000</i> (مواصفاته و أسلوب إنجاز الأعمال التقنية ومبادئه) .		التاسع	117 – 115	20
مفهوم الأيزو <i>ISO 9000</i> (فوائده و مجموعة مواصفاته و مراحل تطبيقه) .		التاسع	121 – 117	21
تعريف بالسيطرة النوعية وإسلوب الفحص الشامل والعينات ومخططات السيطرة النوعية للوسط الحسابي والمدى .	السيطرة النوعية	العاشر	126 – 121	22
مخططات السيطرة النوعية للانحراف المعياري ولنسبة الوحدات المعيبة .		العاشر	131 – 127	23
مستوى الجودة ، والفحص بالعينات (الأحادية والثنائية والمتعددة) .		العاشر	138 – 131	24
تعريف بالمعولية ، ودالة العطل والإحتمالات ومتوسط زمن العطل <i>MTTF</i> والتباين ، دالة نسبة المخاطرة .	المعولية	الحادي عشر	146 – 138	25
دالة المعولية الشرطية ودالة المعولية الأسية وتوزيع ويبل <i>Weibul</i> للمعولية .		الحادي عشر	152 – 149	26
ربط المنظومة على التوالي (في حالة التوزيع الأسي وتوزيع ويبل) ، الربط على التوازي (في حالة التوزيع الأسي) والربط المختلط (توازي وتوالي) .		الحادي عشر	155 – 152	27
فائض بمستويين عال وواطيء مع حل تمارين خارجية .		الحادي عشر	162 – 155	28
		الحادي عشر	170 – 162	29
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مناقشة تقارير تقدم من قبل الطلبة فيما يخص الباب الثالث (الفصل التاسع) مع إختبار .

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- <i>Linear Programming</i>	-	
- <i>Dynamic Programming</i>	-	
- <i>Sequencing</i>	-	
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- <i>Decision Making theory</i>	-	
- <i>Game theory</i>	-	
- <i>Queuing theory ()</i>	-	
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- <i>Network analysis</i>	-	
- <i>Input-Output analysis</i>	-	
- <i>Markov analysis</i>	-	
- <i>Benefit-Cost analysis</i>	-	
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- <i>Transportation models</i>	-	
- <i>Inventory models</i>	-	

-Replacement models

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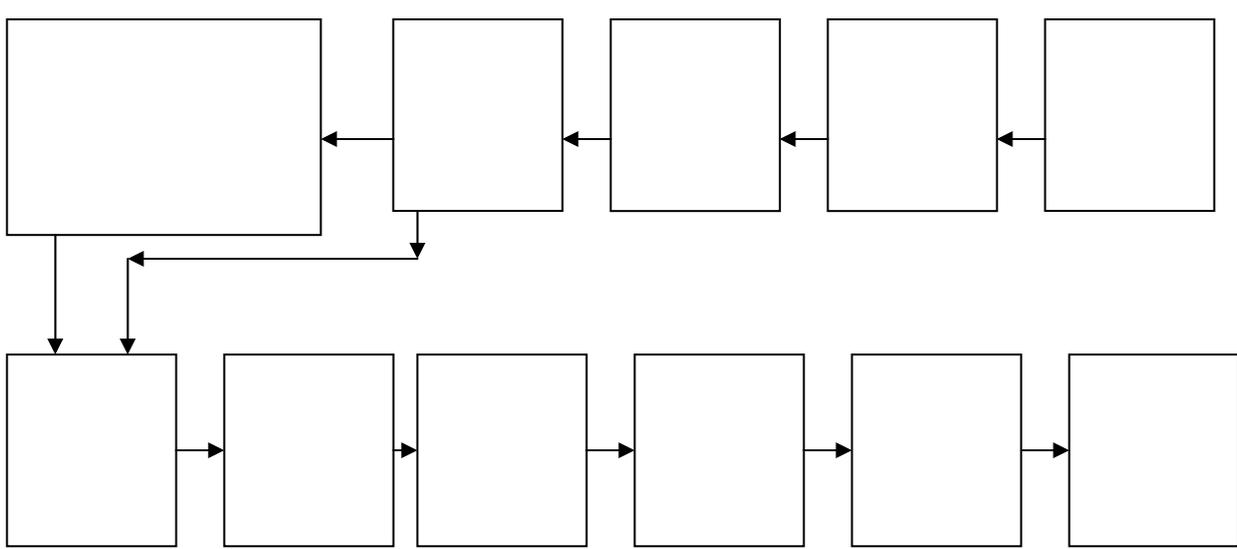
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& & 5850000 * 0.005 = 29250 & \\
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& & (5850000 + 2250000 + 585000) * 0.08 = 694800 & \\
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585000 + 292500 + 112500 + 45000 + 29250 + 694800 = 1759050 & & & \\
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4233000 - 697500 = 3535500 & : & & \\
\frac{2410950}{7685000} * 100\% = 31.4\% & : & -8 & \\
\frac{1759050}{1 - \frac{7080000}{11250000}} = 4745638 & : & -9 &
\end{array}$$

$$\frac{1759050}{225 - \frac{7080000}{50000}} = 21092 \quad :$$

$$\frac{11250000 - 4745638}{11250000} * 100\% = 58\% \quad :$$

$$\frac{8685000}{2410950 + 697500} = 2.79 \cong 3 \quad : (\quad) \quad -10$$

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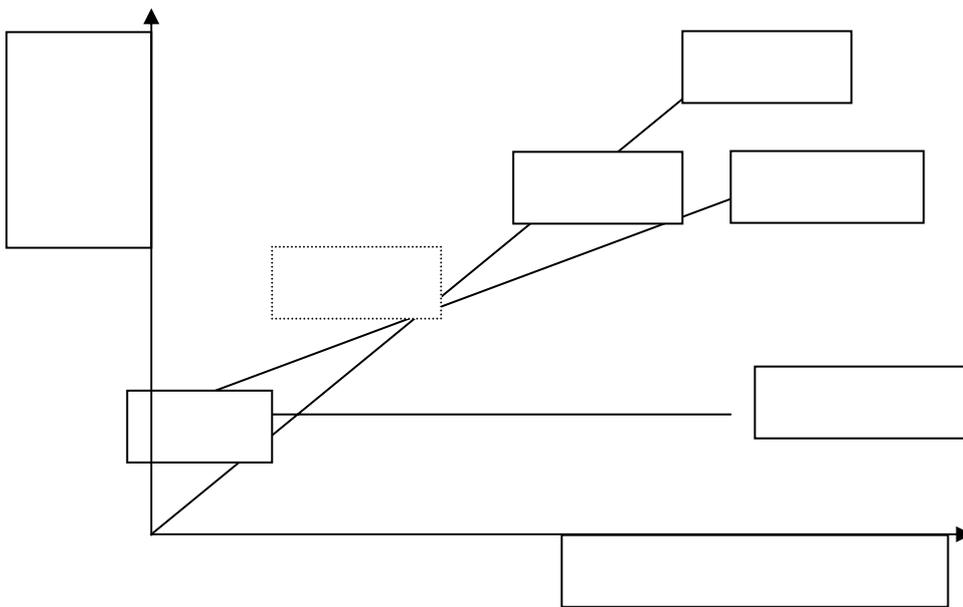
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 & \cdot \qquad \qquad \qquad \mathbf{1500 = 250 + 230 + 200 + 320 + 500 =} \\
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 \end{aligned}$$

$$P.O.E. = \frac{1500}{1 - \frac{3500}{7000}} = 3000$$

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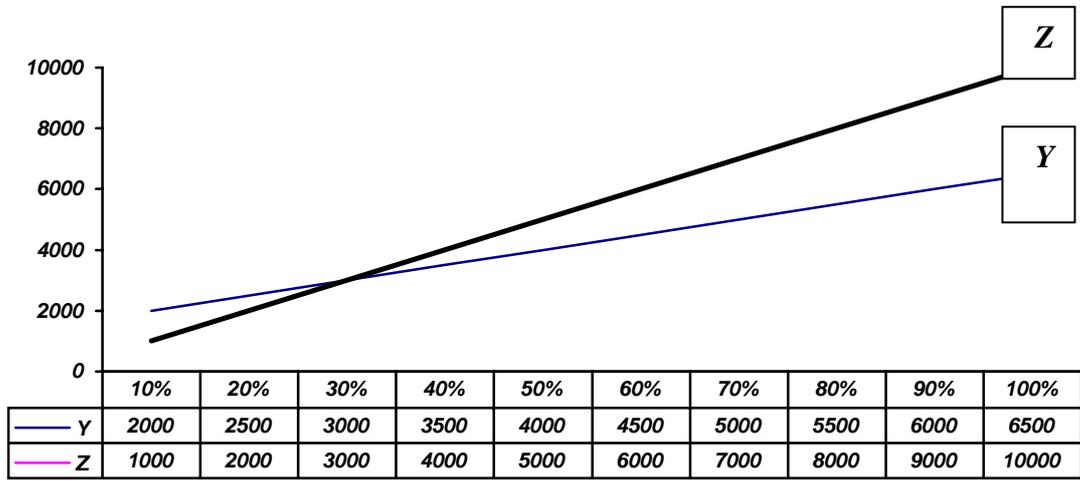
$$\frac{7000}{70} * 80 = 8000$$

$$\frac{3500}{70} * 80 = 4000$$

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10	500	1000
20	1000	2000
30	1500	3000
40	2000	4000
50	2500	5000
60	3000	6000
70	3500	7000
80	4000	8000
90	4500	9000
100	5000	10000

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$$L.O.S. = \frac{7000 - 3000}{7000} * 100\% = 57\%$$

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$$Profit = 9000 - (1500 + 4500) = 3000$$

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<i>Q</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>V.C.</i>	<i>18</i>	<i>30</i>	<i>40</i>	<i>55</i>	<i>70</i>	<i>90</i>	<i>120</i>	<i>150</i>

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<i>Q</i>	<i>P</i>	<i>T.R.</i>	<i>T.C.</i>	<i>M.R.</i>	<i>M.C.</i>	<i>V</i>
<i>1</i>	<i>20</i>	<i>20</i>	<i>18</i>	<i>---</i>	<i>---</i>	<i>2</i>
<i>2</i>	<i>20</i>	<i>40</i>	<i>30</i>	<i>20</i>	<i>12</i>	<i>10</i>
<i>3</i>	<i>20</i>	<i>60</i>	<i>40</i>	<i>20</i>	<i>10</i>	<i>20</i>
<i>4</i>	<i>20</i>	<i>80</i>	<i>55</i>	<i>20</i>	<i>15</i>	<i>25</i>
<i>5</i>	<i>20</i>	<i>100</i>	<i>70</i>	<i>20</i>	<i>15</i>	<i>30</i>
<i>6</i>	<i>20</i>	<i>120</i>	<i>90</i>	<i>20</i>	<i>20</i>	<i>30</i>
<i>7</i>	<i>20</i>	<i>140</i>	<i>120</i>	<i>20</i>	<i>30</i>	<i>20</i>
<i>8</i>	<i>20</i>	<i>160</i>	<i>150</i>	<i>20</i>	<i>30</i>	<i>10</i>

$$Q = 6$$

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$$\frac{1150 - 1000}{1000} * 100\% = 15\%$$

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$P :$

Q_i

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$$P = \frac{\sum_{i=1}^N Q_i \cdot C_i}{\sum_{i=1}^N T_i}$$

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$$40 =$$

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	A	B	C	D
()	750	400	300	1000
()	4	3	6	2

$C_1 = 1$:

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B

$$= (C_2) A \quad B$$

A

$$C_2 = \frac{3}{4} = 0.75, \quad C_3 = \frac{6}{4} = 1.5 \quad \text{and} \quad C_4 = \frac{2}{4} = 0.5 \quad :$$

$$\sum_{i=1}^4 Q_i \cdot C_i = 750 * 1 + 400 * 0.75 + 300 * 1.5 + 1000 * 0.5 = 2000 \quad \text{tons}$$

$$\sum_{i=1}^4 T_i = 40 * 250 * 1 * 8 = 80000 \quad \text{worker s.hours}$$

$$P = \frac{2000}{80000} = 0.025 \quad \text{tons / wor ker .hour}$$

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$$P_L = \frac{\sum_{i=1}^N Q_i \cdot P_i}{\sum_{i=1}^N W_i}$$

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X			
		1991	1992
A	200	50000	25000
B	250	40000	100000
C	1000	25000	25000

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Y			
		1991	1992
A	300	50000	60000
B	500	30000	50000

500

300

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$$P_L = \frac{\sum_{i=1}^N Q_i \cdot P_i}{\sum_{i=1}^N W_i}$$

$$P_{1991} = \frac{50000 * 200 + 40000 * 250 + 25000 * 1000 + 50000 * 300 + 30000 * 500}{500 * 300 * 7} = 71.43$$

$$P_{1992} = \frac{25000 * 200 + 100000 * 250 + 25000 * 1000 + 60000 * 300 + 50000 * 500}{500 * 300 * 7} = 93.33$$

. 1992 93.33 1991 71.43

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200

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	()										R	
	1	2	3	4	5	6	7	8	9	10		
A	50	60	55	55	55	55	50	50	60	60	55	10
B	36	34	25	25	30	25	25	30	30	30	29	11
C	125	115	115	115	115	120	120	125	125	125	120	10
D	40	35	36	36	36	35	35	35	36	36	36	5
E	30	30	30	25	25	35	35	28	32	30	30	10

:

$$55 + 29 + 120 + 36 + 30 = 270 \text{ seconds} = 4.5 \text{ minutes}$$

$$m = \left(\frac{\alpha \sqrt{n \sum_{i=1}^n X_i^2 - \left(\sum_{i=1}^n X_i \right)^2}}{\sum_{i=1}^n X_i} \right)^2$$

= m :

$$\alpha = 20 \quad \%90 \qquad \qquad \qquad = \alpha$$

= n

$$\alpha = 40 \quad \%95$$

= X_i

$$\sum_{i=1}^{10} X_i = 290 \quad \text{and} \quad \sum_{i=1}^{10} X_i^2 = 8552 \quad : \quad R = 11 \quad B$$

$$m = \left(\frac{20 * \sqrt{10 * 8552 - (290)^2}}{290} \right)^2 = 6.75 \cong 7 < n = 10$$

%100

*

%100 =

4.5 =

%20

$$4.5 * \frac{120}{100} = 5.4 \text{ min.} \quad :$$

:

: **%80**

$$4.5 * \frac{80}{100} = 3.6 \text{ min.}$$

:

%15

:

$$4.5 + 4.5 * \frac{15}{100} = 5.175 \text{ min.} \quad :$$

$$5.4 + 4.5 * \frac{15}{100} = 6.075 \text{ min.} \quad :$$

$$3.6 + 4.5 * \frac{15}{100} = 4.275 \text{ min.} \quad :$$

1250000

500

:

100000

%10

² **1750**

² **3500**

75

500

600

35000

:

100	1		500	1	
450	1		150	2	
180	10		200	1	
120	8		120	1	
90	5		100	1	

15240

()

%5 %10

%2 %5

%0.5

%8

0.200

()

(ans.: 191995 , 175082.5 , 38% , 402121 , 68% , 2.2 , yes)

-2

250 (8)

:

50000 -

-

5000 -

² 2000 -

(² 150)

(² 250)

(² 600)

² / 60

250000 -

:

75	5		450	1	
200	1		250	1	
100	1		150	2	
120	1		90	3	

7220

7000

1100

900 -

%5 %10 -

%2 %5 -

%0.5 -

%8 -

400 -

()

(ans.: 137220 , 129020 , 74% , 198 , 80% , 1.23 , yes)

5000 -3

:

	%100	%50
150	950	1650
150	150	350
50	50 ()	2400
	. 3600	

(ans.: 2500)

60 \$ -4

40 \$

()

250000 \$

(ans.: 750000 , 12500)

16 \$ -5

. 500000 \$

14 \$

%5

(ans.: 4000000 , 250000 ; 4000000 , 238095)

:

300 -6

8

33

	X_1	X_2	X_3	X_4
	500	350	200	600
	4	3	6	2

X_1 ()

(ans.: 0.0172) . ()

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1000

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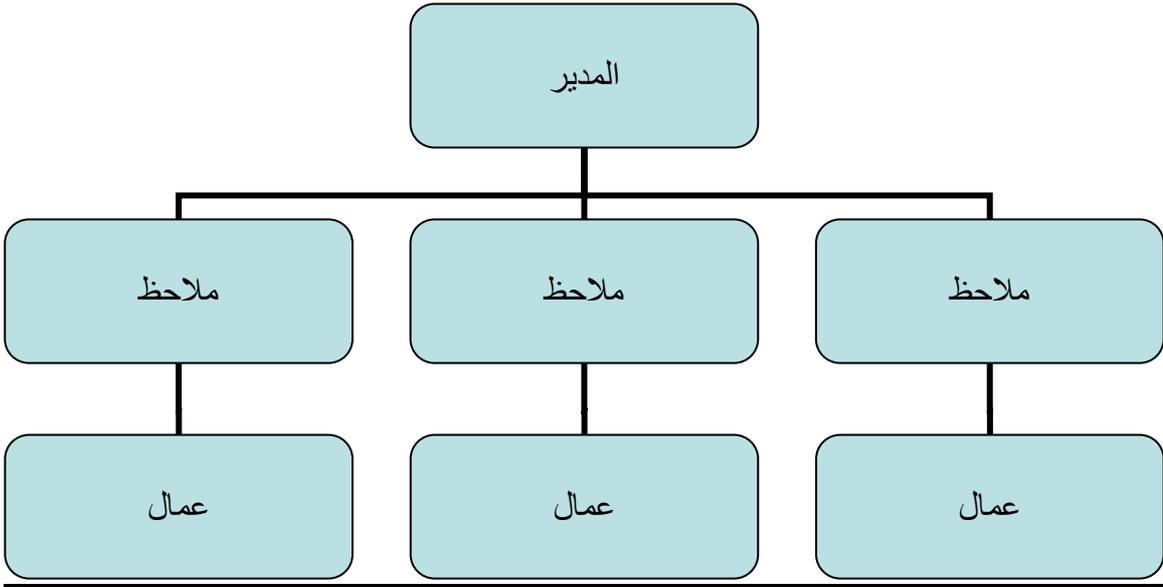
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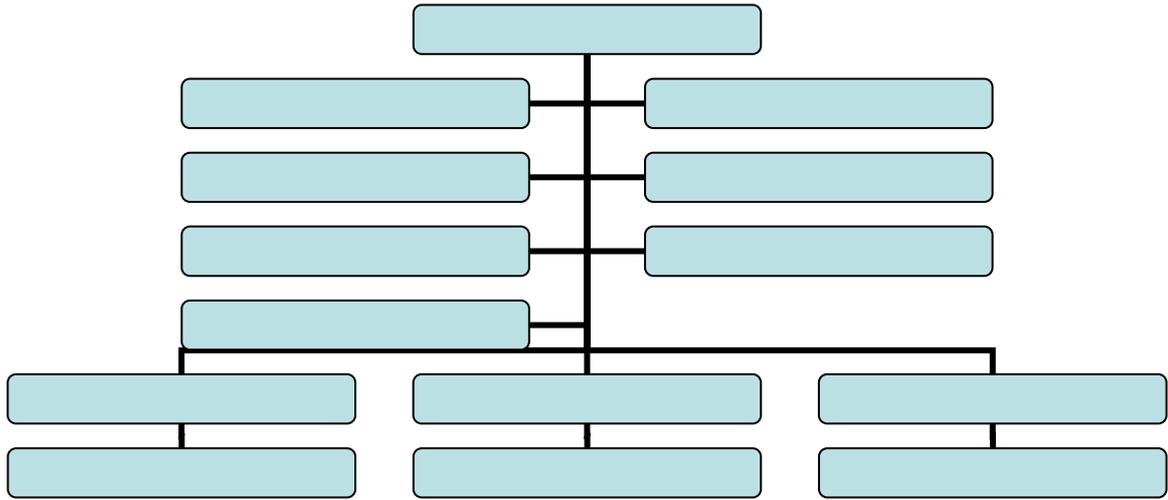
:(_____) _____ -1

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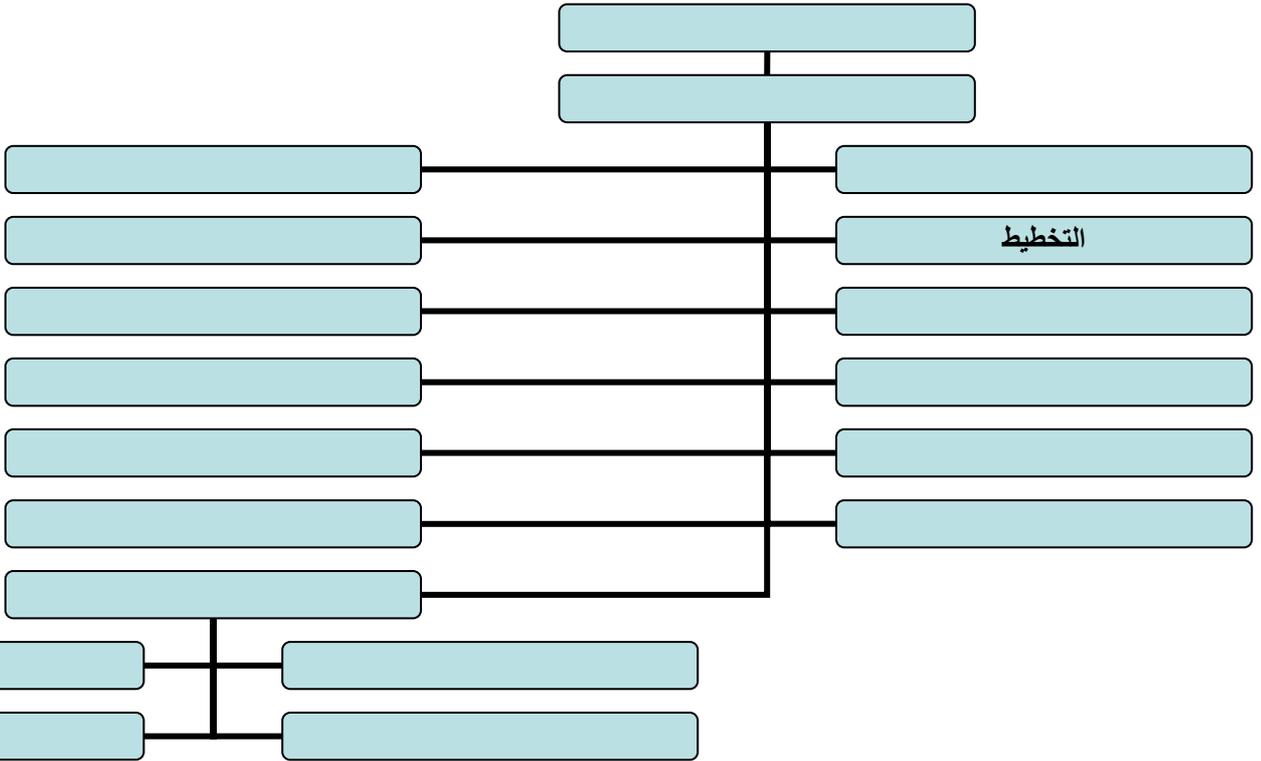
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12-10

6-1

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		1
72 = 12 * 6		7 = 6 + 1
432 = 12 * 6 * 6		43 = 6*6 + 7
2592 = 12 * 6 * 6 * 6		259 = 6*6*6 + 43
15552 = 12 * 6 * 6 * 6 * 6		1555 = 6*6*6*6 + 259
93312 = 12 * 6 * 6 * 6 * 6 * 6	9331 = 6*6*6*6*6 + 1555	

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: T_1 -1

: T_2 -2

$$T = T_1 + T_2 \quad : \quad T$$

$$\frac{T}{T_2} = 1$$

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-1-

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()	
7	1
4	2
8	3
5	4
6	5

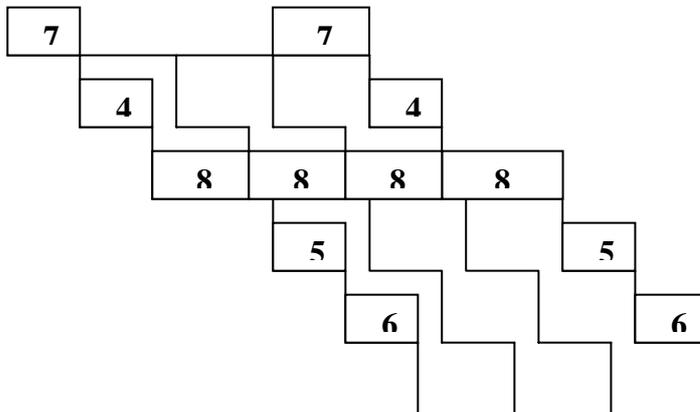
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$$(7 + 4 + 8 + 5 + 6) * 4 = 120 \text{ min.}$$

:



$$7 + 4 + 8 + 8 + 8 + 8 + 5 + 6 = 54 \text{ min.}$$

$$: T \quad : \quad -$$

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$$4 - 1 = 3 \quad :$$

$$T = 120 - (4 + 4 + 5 + 5) * 3 = 66$$

$$: \text{-----} -4-3$$

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Operations Research

[1] Hamdy A. Taha " *Operations Research : an introduction* " 6th edition (1997), Prentice-Hall.

[2] Prem Kumar Gupta and D.S. Hira " *Operations Research : an introduction* " 2nd edition (1989) S. Chand & Company LTD, NewDelhi .

Linear Programming

Operations research

1885

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-4

-5

Linear programming

1947

1873

Simplex method

	-1
	-2
	-3
	-4
	-5
	-1-4
	-1

max .or min . $Z = \sum_{j=1}^n C_j X_j$ *Objective function*

S.t. $\sum_{j=1}^n a_{ij} X_j \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} b_i$ *Constra int s*

$i = 1,2,\dots,m$

$j = 1,2,\dots,n$

- | | |
|---------------------------------|----------|
| | C_j |
| . <i>Decision variables</i> | X_j |
| . <i>Technical coefficients</i> | a_{ij} |
| . <i>Availability amounts</i> | b_i |

: *Canonical form* -2

max . $Z = \sum_{j=1}^n C_j X_j$ *Objective function*

S.t. $\sum_{j=1}^n a_{ij} X_j \leq b_i$ *Constra int s*

$X_j \geq 0$ *nonnegative constra int s*

- : . ($X_j \geq 0$) -1
- : . (\leq) -2
- : . *maximized* -3

maximized *minimized* -1

$$\max. Z = \min. (-Z) \quad : \quad (-1)$$

بضرب المتباينة في \leq \geq -2

$$\sum a_{ij} X_j \geq b_i \Leftrightarrow -\sum a_{ij} X_j \leq b_i \quad : \text{أي إن } (-1)$$

\leq (=) -3

$$\sum a_{ij} X_j = b_i \Leftrightarrow \left\{ \begin{array}{l} \sum a_{ij} X_j \leq b_i \\ -\sum a_{ij} X_j \leq -b_i \end{array} \right\}$$

absolute value -4

\leq

$$|\sum a_{ij} X_j| \leq b_i \Leftrightarrow \left\{ \begin{array}{l} \sum a_{ij} X_j \leq b_i \\ -\sum a_{ij} X_j \leq b_i \end{array} \right\}$$

$$\text{or } |\sum a_{ij} X_j| \geq b_i \Leftrightarrow \left\{ \begin{array}{l} -\sum a_{ij} X_j \leq -b_i \\ \sum a_{ij} X_j \leq -b_i \end{array} \right\}$$

unrestricted sign -5

:

$$X_i = X_i' - X_i'' \quad \text{and} \quad X_i', X_i'' \geq 0$$

Standard form -3

$$((=) \quad) \quad -1$$

$(X_j \geq 0)$ *nonnegative*

$$.(b_i \geq 0) \quad -2$$

$$. \max. \quad \min. \quad -3$$

:

$$\begin{array}{l} \sum a_{ij} X_j \leq b_i \Leftrightarrow \sum a_{ij} X_j + S_i = b_i \\ \sum a_{ij} X_j \geq b_i \Leftrightarrow \sum a_{ij} X_j - S_i = b_i \end{array}$$

)

Slack variables

S_i

$$.(S_i \geq 0)$$

:

1-

$$\begin{array}{l} \min. \quad Z = 2X_1 + 3X_2 + 5X_3 \\ \text{s.t.} \quad X_1 + X_2 - X_3 \geq -5 \\ \quad \quad -6X_1 + 7X_2 - 9X_3 = 15 \\ \quad \quad |19X_1 - 7X_2 + 5X_3| \leq 13 \\ \quad \quad X_1, X_2 \geq 0, X_3 \text{ unrestricted} \end{array}$$

$$X_3 = X_3' - X_3'' \quad : \quad : \underline{\quad}$$

$$: \quad : \quad -$$

min. $Z = -2X_1 - 3X_2 - 5(X_3' - X_3'')$

s.t. $-X_1 - X_2 + (X_3' - X_3'') \leq 5$

$$-6X_1 + 7X_2 - 9(X_3' - X_3'') \leq 15$$

$$6X_1 - 7X_2 + 9(X_3' - X_3'') \leq -15$$

$$19X_1 - 7X_2 + 5(X_3' - X_3'') \leq 13$$

$$-19X_1 + 7X_2 - 5(X_3' - X_3'') \leq 13$$

$$X_1, X_2, X_3', X_3'' \geq 0$$

max. $Z = 2X_1 + 3X_2 + 5(X_3' - X_3'')$

s.t. $-X_1 - X_2 + (X_3' - X_3'') + S_1 = 5$

$$-6X_1 + 7X_2 - 9(X_3' - X_3'') = 15$$

$$19X_1 - 7X_2 + 5(X_3' - X_3'') + S_3 = 13$$

$$-19X_1 + 7X_2 - 5(X_3' - X_3'') + S_4 = 13$$

$$X_1, X_2, X_3', X_3'', S_1, S_2, S_3, S_4 \geq 0$$

: Formulation of the model -2-4

C B A : 2-

I) ()

: () (

	()			
	A	B	C	
I	1	2	1	430
II	3	0	2	460
III	1	4	0	420
	3	2	5	

3)

6

(1 5

10

A

. 0.4

C B

C B A

 $X_3 \quad X_2 \quad X_1$

: —

$$\max . \quad Z = 3X_1 + 2X_2 + 5X_3$$

$$s.t. \quad X_1 + 2X_2 + X_3 \leq 430$$

$$3X_1 + 2X_3 \leq 460$$

$$X_1 + 4X_2 \leq 420$$

$$X_1, X_2, X_3 \geq 0$$

$$\max . \quad Z = 3X_1 + 2X_2 + 5X_3 + 6X_4$$

$$s.t. \quad X_1 + 2X_2 + X_3 + 3X_4 \leq 430$$

$$3X_1 + 2X_3 + 5X_4 \leq 460$$

$$X_1 + 4X_2 + X_4 = 420$$

$$X_1, X_2, X_3, X_4 \geq 0$$

$$430 - (X_1 + 2X_2 + X_3) + 460 - (3X_1 + 2X_3) + 420 - (X_1 + 4X_2) \leq 10$$

$$\rightarrow 5X_1 + 6X_2 + 3X_3 \geq 1300$$

لذا فالنموذج الرياضي سيكون :

$$\max . \quad Z = 3X_1 + 2X_2 + 5X_3$$

$$s.t. \quad X_1 + 2X_2 + X_3 \leq 430$$

$$3X_1 + 2X_3 \leq 460$$

$$X_1 + 4X_2 \leq 420$$

$$5X_1 + 6X_2 + 3X_3 \geq 1300$$

$$X_1, X_2, X_3 \geq 0$$

$$\frac{X_1}{X_2 + X_3} \geq 0.4 \Rightarrow X_1 - 0.4X_2 - 0.4X_3 \geq 0$$

$$\begin{aligned}
\text{max. } & Z = 3X_1 + 2X_2 + 5X_3 \\
\text{s.t. } & X_1 + 2X_2 + X_3 \leq 430 \\
& 3X_1 + 2X_3 \leq 460 \\
& X_1 + 4X_2 \leq 420 \\
& X_1 - 0.4X_2 - 0.4X_3 \geq 0 \\
& X_1, X_2, X_3 \geq 0
\end{aligned}$$

: _____ -3-4

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: Graphical method -1

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Feasible Solutions

Extreme Points

Region (F.S.R.)

Optimal values

. **Simplex method**

:

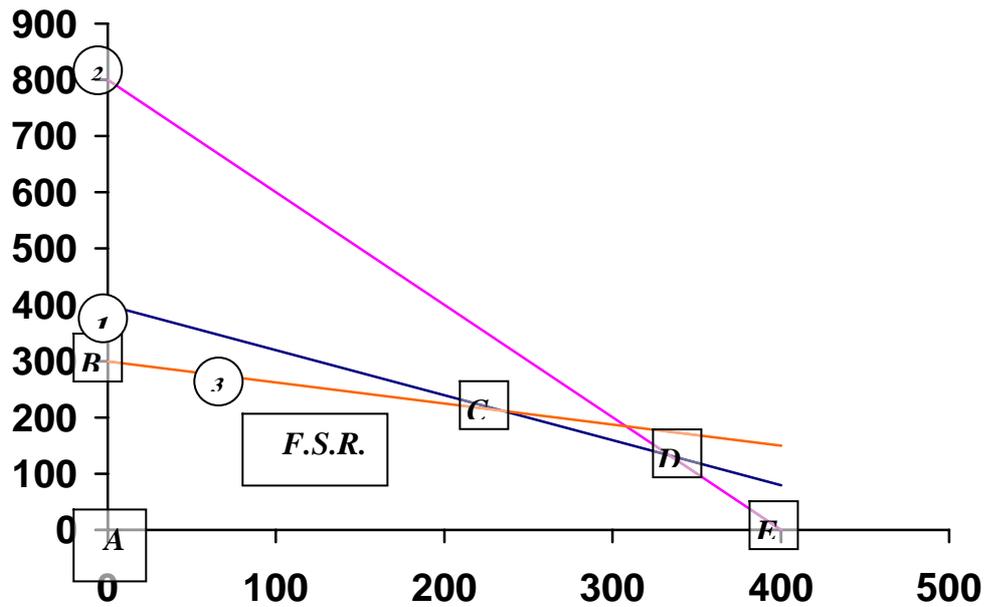
: 3-

$$\begin{aligned}
\text{max. } & Z = 120X + 100Y \\
\text{s.t. } & 2X + 2.5Y \leq 1000 \\
& 3X + 1.5Y \leq 1200 \\
& 1.5X + 4Y \leq 1200 \\
& X, Y \geq 0
\end{aligned}$$

: _____

1. $2X + 2.5Y = 1000$ if $X = 0$ then $Y = 400 \Rightarrow (0, 400)$
if $Y = 0$ then $X = 500 \Rightarrow (500, 0)$
2. $3X + 1.5Y = 1200$ if $X = 0$ then $Y = 800 \Rightarrow (0, 800)$
if $Y = 0$ then $X = 400 \Rightarrow (400, 0)$
3. $1.5X + 4Y = 1200$ if $X = 0$ then $Y = 300 \Rightarrow (0, 300)$
if $Y = 0$ then $X = 800 \Rightarrow (800, 0)$
4. $X = 0$
5. $Y = 0$

. (F.S.R.)



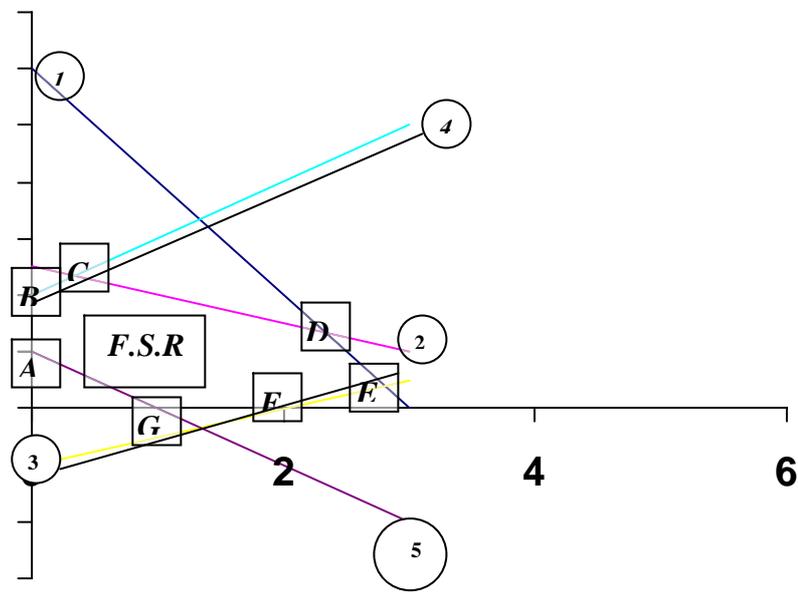
$$\begin{array}{ccccccc}
 & E & D & C & B & A \\
 C(4000/17, 3600/17) & \emptyset & 3 & 1 & \emptyset \\
 D(1000/3, 400/3) & \emptyset & 2 & 1 & \emptyset
 \end{array}$$

Points	$Z = 120X + 100Y$
$A(0,0)$	$Z = 0 + 0 = 0$
$B(0,300)$	$Z = 0 + 100 \cdot 300 = 30000$
$C(4000/17, 3600/17)$	$Z = 120 \cdot (4000/17) + 100 \cdot (3600/17) = 840000/17$
$D(1000/3, 400/3)$	$Z = 120 \cdot (1000/3) + 100 \cdot (400/3) = 160000/3 \rightarrow \text{max.}$
$E(400,0)$	$Z = 120 \cdot 400 + 0 = 48000$

$$\begin{array}{ccccccc}
 Z=160000/3 & & & & & & \\
 & i & Y=400/3 & X=1000/3 & \emptyset & \emptyset & \\
 & & & & \emptyset & \emptyset & \underline{4 \emptyset}
 \end{array}$$

$$\begin{array}{l}
 Z = 4X + 5Y \\
 \text{s.t. } 2X + Y \leq 6 \\
 X + 2Y \leq 5 \\
 X - 2Y \leq 2 \\
 -X + Y \leq 2 \\
 X + Y \geq 1 \\
 X, Y \geq 0
 \end{array}$$

1. $2X + Y = 6$ if $X = 0$ then $Y = 6 \Rightarrow (0,6)$
if $Y = 0$ then $X = 3 \Rightarrow (3,0)$
2. $X + 2Y = 5$ if $X = 0$ then $Y = 2.5 \Rightarrow (0,2.5)$
if $Y = 0$ then $X = 5 \Rightarrow (5,0)$
3. $X - 2Y = 2$ if $X = 0$ then $Y = -1 \Rightarrow (0,-1)$
if $Y = 0$ then $X = 2 \Rightarrow (2,0)$
4. $-X + Y = 2$ if $X = 0$ then $Y = 2 \Rightarrow (0,2)$
if $Y = 0$ then $X = -2 \Rightarrow (-2,0)$
5. $X + Y = 1$ if $X = 0$ then $Y = 1 \Rightarrow (0,1)$
if $Y = 0$ then $X = 1 \Rightarrow (1,0)$
6. $X = 0$
7. $Y = 0$



$C(1/3, 7/3)$	4	2	Ø
$D(7/3, 4/3)$	2	1	Ø
$E(14/5, 2/5)$	3	1	Ø

<i>Points</i>	$Z = 4X + 5Y$
$A(0, 1)$	$0 + 5 = 5$
$B(0, 2)$	$0 + 10 = 10$
$C(1/3, 7/3)$	$4/3 + 35/3 = 13$
$D(7/3, 4/3)$	$28/3 + 20/3 = 16 \rightarrow \text{max.}$
$E(14/5, 2/5)$	$56/5 + 10/5 = 66/5$
$F(2, 0)$	$8 + 0 = 8$
$G(1, 0)$	$4 + 0 = 4 \rightarrow \text{min.}$

$$Y = 4/3 \quad X = 7/3 \quad 16 \quad Z \quad \emptyset \quad \emptyset$$

$$Y = 0 \quad X = 1 \quad 4 \quad Z \quad \emptyset$$

$$\emptyset \quad \underline{5 \quad \emptyset}$$

$$\text{max. } Z = 2X + 4Y + 8$$

$$\text{s.t. } -X + 2Y \leq 2$$

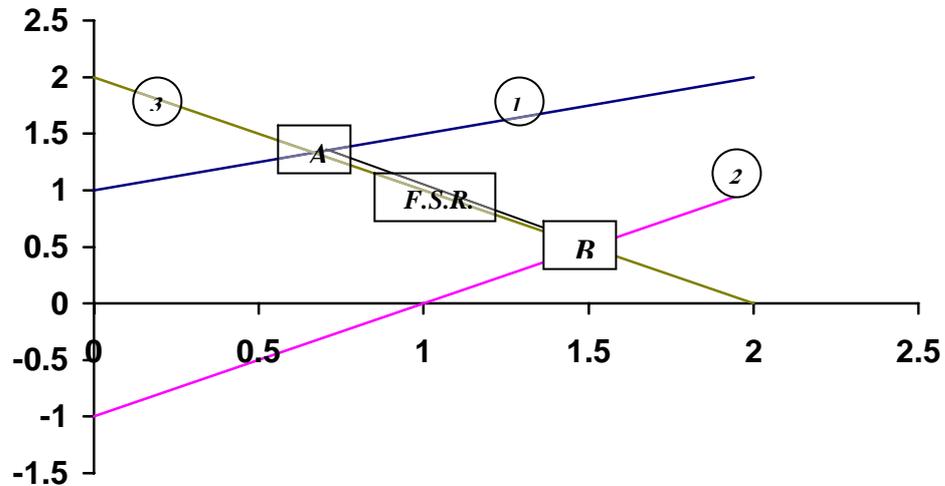
$$X - Y \leq 1$$

$$X + Y = 2$$

$$X, Y \geq 0$$

\emptyset

1. $-X + 2Y = 2$ if $X = 0$ then $Y = 1 \Rightarrow (0, 1)$
if $Y = 0$ then $X = -2 \Rightarrow (-2, 0)$
2. $X - Y = 1$ if $X = 0$ then $Y = -1 \Rightarrow (0, -1)$
if $Y = 0$ then $X = 1 \Rightarrow (1, 0)$
3. $X + Y = 2$ if $X = 0$ then $Y = 2 \Rightarrow (0, 2)$
if $Y = 0$ then $X = 2 \Rightarrow (2, 0)$
4. $X = 0$
5. $Y = 0$



AB

\emptyset

$$A(2/3, 4/3) \quad \acute{a} \quad 3 \quad 1 \quad \emptyset$$

$$B(3/2, 1/2) \quad \emptyset \quad 3 \quad 2 \quad \emptyset$$

Points	$Z = 2X + 4Y + 8$
$A(2/3, 4/3)$	$4/3 + 16/3 + 8 = 44/3 \rightarrow \text{max.}$
$B(3/2, 1/2)$	$3 + 2 + 8 = 13$

$$Z = 44/3$$

$$Y = 4/3 \quad X = 2/3 \quad \emptyset \quad \emptyset$$

: Simplex method -2

$$S_i > 0 \qquad S_i = 0$$

$$. S_i = 0$$

(\leq) بشرط $b_i \geq 0$ ، ماعدا قيد عدم السالبة إذ يبقى أكبر من أو يساوي (\geq).

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Starting Basic Feasible Solution(S.B.F.S.)

-2

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		C_1	C_2	\dots	C_n	0	0	\dots	0	0
B.C.	B.V.	X_1	X_2	\dots	X_n	S_1	S_2	\dots	S_m	R.H.S.
0	S_1	a_{11}	a_{12}	\dots	a_{1n}	1	0	\dots	0	b_1
0	S_2	a_{21}	a_{22}	\dots	a_{2n}	0	1	\dots	0	b_2
\vdots	\vdots	\vdots	\vdots	\dots	\vdots	\vdots	\vdots	\dots	\vdots	\vdots
0	S_m	a_{m1}	a_{m2}	\dots	a_{mn}	0	0	\dots	1	b_m
$Z_j - C_j$		$-C_1$	$-C_2$	\dots	$-C_n$	0	0	\dots	0	0

-3

$$(\qquad) Z_j - C_j$$

Optimality) min. max.

.(R.H.S. condition

Leaving Variable

Entering

R.H.S.

(Feasibility condition) Variable

. *Pivot element*

-4

$$Z_j - C_j$$

-5

min.

max.

. Simplex method

3- : 6-

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$$\begin{aligned} \text{max. } & Z = 120X + 100Y \\ \text{s.t. } & 2X + 2.5Y + S_1 = 1000 \\ & 3X + 1.5Y + S_2 = 1200 \\ & 1.5X + 4Y + S_3 = 1200 \\ & X, Y, S_1, S_2, S_3 \geq 0 \end{aligned}$$

		120	100	0	0	0	0	Ratio
B.C.	B.V.	X	Y	S ₁	S ₂	S ₃	R.H.S.	
0	S ₁	2	2.5	1	0	0	1000	500
←0	S ₂	3	1.5	0	1	0	1200	400 → min.
0	S ₃	1.5	4	0	0	1	1200	800
Z _j - C _j		-120 ↑	-100	0	0	0	0	
←0	S ₁	0	1.5	1	-2/3	0	200	133.3 → min.
120	X	1	0.5	0	1/3	0	400	800
0	S ₃	0	3.25	0	-0.5	1	600	184.6
Z _j - C _j		0	-40 ↑	0	40	0	48000	
100	Y	0	1	2/3	-4/9	0	400/3	
120	X	1	0	-1/3	5/9	0	1000/3	
0	S ₃	0	0	-13/6	17/18	1	500/3	
Z _j - C _j		0	0	80/3	200/9	0	160000/3	

max.

$$Z_j - C_j$$

$$Z = 160000/3$$

$$Y = 400/3 \quad X = 1000/3$$

. 3-

:

	X	$Z_j - C_j$					-120
$.3$	S_2	X	S_2	S_3	S_1	400	
		:					
-2^*	X	-2	-1	0	$-2/3$	0	-800
	S_1	2	2.5	1	0	0	1000
<hr/>							
	S_1	0	1.5	1	$-2/3$	0	200
-1.5^*	X	-1.5	-0.75	0	-0.5	0	-600
	S_3	1.5	4	0	0	1	1200
<hr/>							
	S_3	0	3.25	0	-0.5	1	600

(=) (≥) : _____

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Penalty method

:(M- technique) M - -1

: $b_i \geq 0$ بشرط (\leq)

Artificial variables (R_i)

(=) (≥)

: ()

$. S_i$ (\leq) -

S_i (\geq) -

$. R_i$

$. R_i$ (=) -

(- M) R_i

$. M$ $min.$ (+ M) $max.$

S_i

$$R_i \quad -$$

$$. (S.B.F.S.)$$

M

-

$$: \underline{7-}$$

$$\min. \quad Z = 5X_1 - 6X_2 - 7X_3$$

$$s.t. \quad X_1 + 5X_2 - 3X_3 \geq 15$$

$$5X_1 - 6X_2 + 10X_3 \leq 20$$

$$X_1 + X_2 + X_3 = 5$$

$$X_1, X_2, X_3 \geq 0$$

: _____

$$\min. \quad Z = 5X_1 - 6X_2 - 7X_3 + MR_1 + MR_2$$

$$s.t. \quad X_1 + 5X_2 - 3X_3 - S_1 + R_1 = 15$$

$$5X_1 - 6X_2 + 10X_3 + S_2 = 20$$

$$X_1 + X_2 + X_3 + R_2 = 5$$

$$X_1, X_2, X_3, S_1, S_2, R_1, R_2 \geq 0$$

		5	-6	-7	0	0	M	M	R.H.S.	Ratio
B.C.	B.V.	X ₁	X ₂	X ₃	S ₁	S ₂	R ₁	R ₂		
←M	R ₁	1	5	-3	-1	0	1	0	15	3min.
0	S ₂	5	-6	10	0	1	0	0	20	
M	R ₂	1	1	1	0	0	0	1	5	5
$Z_j - C_j$		2M-5	6M+6↑	-2M+7	-M	0	0	0	20M	
-6	X ₂	1/5	1	-3/5	-1/5	0	1/5	0	3	
0	S ₂	31/5	0	32/5	-6/5	1	6/5	0	38	5.9
←M	R ₃	4/5	0	8/5	1/5	0	-1/5	1	2	1.25min
$Z_j - C_j$		4/5M-31/5	0	8/5M+53/5↑	1/5M+6/5	0	-6/5M-6/5	0	2M-18	
-6	X ₂	1/2	1	0	-1/8	0	1/8	3/8	15/4	
0	S ₂	3	0	0	-2	1	2	-4	30	
-7	X ₃	1/2	0	1	1/8	0	-1/8	5/8	5/4	
$Z_j - C_j$		-23/2	0	0	-1/8	0	-M+1/8	-M-53/8	-125/4	

$$(Z_j - C_j)$$

$$X_3 = 5/4 \quad X_2 = 15/4 \quad X_1 = 0$$

$$. Z = -125/4$$

: Two- Phase technique -2

. (\leq)

:

: Phase - I -

.1

min. $R = \sum R_i$: .2

R_i

$R=0$.3

R_i

($R \neq 0$) .

: Phase-II -

R_i .1

.2

7- : 8-

Phase - I :

$$\begin{aligned}
 \text{min.} \quad & R = R_1 + R_2 \\
 \text{s.t.} \quad & X_1 + 5X_2 - 3X_3 - S_1 + R_1 = 15 \\
 & 5X_1 - 6X_2 + 10X_3 + S_2 = 20 \\
 & X_1 + X_2 + X_3 + R_2 = 5 \\
 & X_1, X_2, X_3, S_1, S_2, R_1, R_2 \geq 0
 \end{aligned}$$

		0	0	0	0	0	1	1		
B.C.	B.V.	X_1	X_2	X_3	S_1	S_2	R_1	R_2	R.H.S.	Ratio
$\leftarrow 1$	R_1	1	5	-3	-1	0	1	0	15	$3 \rightarrow \min.$
0	S_2	5	-6	10	0	1	0	0	20	
1	R_2	1	1	1	0	0	0	1	5	5
$R_j - C_j$		2	$6 \uparrow$	-2	-1	0	0	0	20	
0	X_2	1/5	1	-3/5	-1/5	0	1/5	0	3	
0	S_2	31/5	0	32/5	-6/5	1	6/5	0	38	5.9
$\leftarrow 1$	R_3	4/5	0	8/5	1/5	0	-1/5	1	2	$1.25 \rightarrow \min$
$R_j - C_j$		4/5	0	$8/5 \uparrow$	1/5	0	-6/5	0	2	
0	X_2	1/2	1	0	-1/8	0	1/8	3/8	15/4	
0	S_2	3	0	0	-2	1	2	-4	30	
0	X_3	1/2	0	1	1/8	0	-1/8	5/8	5/4	
$R_j - C_j$		0	0	0	0	0	-1	-1	0	

$R=0$ () R_2 R_1

: R_2 R_1

Phase-II :

		5	-6	-7	0	0		
B.C.	B.V.	X_1	X_2	X_3	S_1	S_2	R.H.S.	
-6	X_2	1/2	1	0	-1/8	0	15/4	
0	S_2	3	0	0	-2	1	30	
-7	X_3	1/2	0	1	1/8	0	5/4	
$Z_j - C_j$		-23/2	0	0	-1/8	0	-125/4	

min.

$X_3 = 5/4$ $X_2 = 15/4$ $X_1 = 0$:

$Z = -125/4$

-1

$$\begin{aligned}
 1) \quad & \text{max.} && Z = X_1 - 3X_2 \\
 & \text{s.t.} && -X_1 + 2X_2 \leq 5 \\
 & && X_1 + 3X_2 = 10 \\
 & && X_1, X_2 \text{ unrestricted in sign}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \text{min.} && Z = 3X_1 - 3X_2 + 7X_3 \\
 & \text{s.t.} && X_1 + X_2 + 3X_3 \leq 40 \\
 & && X_1 + 9X_2 - 7X_3 \geq 50 \\
 & && 2X_1 + 3X_2 = 20 \\
 & && |5X_2 + 8X_3| \leq 100 \\
 & && X_1, X_2 \geq 0, \quad X_3 \text{ unrest.}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \text{min.} && Z = -3X_1 + 4X_2 - 2X_3 + 5X_4 \\
 & \text{s.t.} && 4X_1 - X_2 + 2X_3 - X_4 = -2 \\
 & && X_1 + X_2 + 3X_3 - X_4 \leq 14 \\
 & && 2X_1 + 3X_2 - X_3 + 2X_4 \geq 2 \\
 & && X_1, X_2 \geq 0, \quad X_3 \leq 0, \quad X_4 \text{ unrest.}
 \end{aligned}$$

-2

:

<i>machines</i>	<i>Time per unit (hours/unit)</i>				<i>Cost (I.D./hour)</i>	<i>Availability hours</i>
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>		
<i>M₁</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>2</i>	<i>10</i>	<i>500</i>
<i>M₂</i>	<i>3</i>	<i>2</i>	<i>1</i>	<i>2</i>	<i>15</i>	<i>380</i>
<i>Sales price (I.D./unit)</i>	<i>65</i>	<i>70</i>	<i>55</i>	<i>45</i>		

:

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(/)

-3

:

<i>machines</i>	<i>Products</i>		<i>Operation Cost (I.D./day)</i>
	<i>I</i>	<i>II</i>	
<i>I</i>	<i>4</i>	<i>5</i>	<i>2000</i>
<i>II</i>	<i>6</i>	<i>3</i>	<i>2200</i>
<i>III</i>	<i>2</i>	<i>7</i>	<i>1800</i>
<i>IV</i>	<i>8</i>	<i>4</i>	<i>1600</i>

/ 60

/ 75

-4

500

150

5000

8000

250

(ans.: 125 , 250 , 2250000)

D , C , B , A

-5

:

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>Raw material-I</i>	<i>8</i>	<i>14</i>	<i>10</i>	<i>6</i>
<i>Raw material-II</i>	<i>2</i>	<i>4</i>	<i>7</i>	<i>6</i>
<i>Labor time (hours)</i>	<i>2</i>	<i>1</i>	<i>3</i>	<i>1</i>

RM-II

400 RM-I

800

4000 2000

150

1000

45000 63000 60000 40000

(ans.: 65 , 20 , 0 , 0 , 1210000)

-6

$$\begin{aligned}
 1) \quad & \max. \quad Z = 4X + 3Y \\
 & \text{s.t.} \quad 2X + 3Y \leq 6 \\
 & \quad \quad -3X + 2Y \leq 3 \\
 & \quad \quad 2Y \leq 5 \\
 & \quad \quad 2X + Y \leq 4 \\
 & \quad \quad X, Y \geq 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \max. \quad Z = 3X + 2Y \\
 & \text{s.t.} \quad |Y - X| \leq 2 \\
 & \quad \quad X + Y \geq 1 \\
 & \quad \quad X \leq 4 \\
 & \quad \quad Y \leq 3 \\
 & \quad \quad X, Y \geq 0
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \min. \quad Z = 8X + 5Y \\
 & \text{s.t.} \quad X + 2Y \leq 10 \\
 & \quad \quad X \geq 5 \\
 & \quad \quad Y \leq 2 \\
 & \quad \quad X, Y \geq 0
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \min. \quad Z = 2X + 3Y \\
 & \text{s.t.} \quad X + Y \leq 15 \\
 & \quad \quad X + 2Y \geq 10 \\
 & \quad \quad X, Y \geq 0
 \end{aligned}$$

(ans.: (X,Y,Z): 1)(3/2,1,9), 2)(4,3,18), 3) (5,0,40), 4) (0,5,15))

-7

$$\begin{aligned}
 1) \quad & \max. \quad Z = 2X_1 + X_2 - 3X_3 + 5X_4 \\
 & \text{s.t.} \quad X_1 + 7X_2 + 3X_3 + 7X_4 \leq 46 \\
 & \quad \quad 3X_1 - X_2 + X_3 + 2X_4 \leq 8 \\
 & \quad \quad 2X_1 + 3X_2 - X_3 + X_4 \leq 10 \\
 & \quad \quad X_1, X_2, X_3, X_4 \geq 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \min. \quad Z = X_1 - 3X_2 - 2X_3 \\
 & \quad \quad 3X_1 - X_2 + 2X_3 \leq 7 \\
 & \quad \quad -2X_1 + 4X_2 \leq 12 \\
 & \quad \quad -4X_1 + 3X_2 + 8X_3 \leq 10 \\
 & \quad \quad X_1, X_2, X_3 \geq 0
 \end{aligned}$$

(ans.: 1) (0,12/7,0,34/7; 26) , 2) (78/25,114/25,11/10; -319/25))

-8

X_6, X_5, X_4

: (S.B.F.S.)

$$\begin{aligned}
 \max. \quad & Z = 3X_1 + X_2 + 2X_3 \\
 \text{s.t.} \quad & 4X_1 + X_2 + 2X_3 + X_4 = 3 \\
 & 8X_1 + X_2 - 4X_3 + 2X_5 = 10 \\
 & 3X_1 - X_6 = 0 \\
 & X_1, X_2, X_3, X_4, X_5, X_6 \geq 0
 \end{aligned}$$

(ans.: (0,0,3/2,0,7/2,0;3))

-9

$$\begin{aligned}
 1) \quad & \min. \quad Z = 4X_1 + X_2 \\
 & \text{s.t.} \quad 3X_1 + X_2 = 3 \\
 & \quad \quad 4X_1 + 3X_2 \geq 6 \\
 & \quad \quad X_1 + 2X_2 \leq 3 \\
 & \quad \quad X_1, X_2 \geq 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \max. \quad Z = X_1 + 5X_2 + 3X_3 \\
 & \text{s.t.} \quad X_1 + 2X_2 + X_3 = 3 \\
 & \quad \quad 2X_1 - X_2 = 4 \\
 & \quad \quad X_1, X_2, X_3 \geq 0
 \end{aligned}$$

(ans.: 1) (3/5,6/5;18/5) , 2) (2,0,1;5)) .

X_3

: Transportation Model -1-5

)

() (

()

" 1941

" 1947 "

1951 1963 "

Modify Distribution method (MODI)

. 1954 *Stepping Stone*

Assignment problem 1955

1957

R.A.M. 1958 V.A.M.

. 1968

: The least cost transportation problem -2-1-5

		:	n	m	
	. i				S_i
	. j				D_j
. j	i	(i, j)			C_{ij}
	. j	i			X_{ij}
	j	i			
		:			

$$\begin{aligned} \min . \quad & Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n X_{ij} = a_i \\ & \sum_{i=1}^m X_{ij} = b_j \\ & X_{ij} \geq 0 \end{aligned}$$

:

unbalanced

-1

$$\cdot \sum_j b_j - \sum_i a_i$$

-2

$$\cdot \sum_i a_i - \sum_j b_j$$

:

S.B.F.S.

-1

-2

()

-3

S.B.F.S. -2-1-5

:

Northwest corner method

-1

) X_{11}

()

$$X_{11} = \min.(a_1, b_1)$$

(

()

()

Least cost method

-2

()

()

Vogel's Approximation Method (V.A.M.) -3

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.1 -

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.2

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.3

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)

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Russel's Approximation Method (R.A.M.) -4

)

:

(

(\bar{b}_j)

)

(\bar{a}_i)

)

-

$\Delta_{ij} = C_{ij} - \bar{a}_i - \bar{b}_j$:

-

Δ_{ij} ، ونعطي لمتغيرها أكبر كمية ممكنة والتي

-

تساوي $min.(a_i, b_j)$.

()

-

()

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.1 -

()

()

.2

: _____

: *Optimal Solution* -3-1-5

:

S.B.F.S.

m n $m+n-1$

:

: *Stepping Stone method* -1

\bar{C}_{ij}

\bar{C}_{ij}

(\quad)
 $)$

(\quad)

: *Multipliers method* -2
Modified Distribution method (MODI)

. *Duality theory*

V_j j U_i i
:
 $U_i + V_j = C_{ij}$
 X_{ij}

$(m+n)$ ($(m+n-1)$) $(m+n-1)$

$(U_1=0)$

: X_{pq} \bar{C}_{pq}
 $\bar{C}_{pq} = C_{pq} - (U_p + V_q)$

\bar{C}_{pq}

()

)

.

25 20 15 S_3, S_2, S_1 : 1-

15 12 10 8 C_4, C_3, C_2, C_1

()

: ()

	C_1	C_2	C_3	C_4
S_1	2	3	4	5
S_2	3	2	5	2
S_3	4	1	2	3

$(25+20+15=60)$:

C_5 $(8+10+12+15=45)$

$(60-45=15)$

: - S.B.F.S. -1

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3 7	4	5	0	15
S_2	3	2 3	5 12	2 5	0	20
S_3	4	1	2	3 10	0 15	25
Demand	8	10	12	15	15	60

:

$$T.T.C. = 2*8 + 3*7 + 2*3 + 5*12 + 2*5 + 3*10 + 0*15 = 143$$

:

	<i>C₁</i>	<i>C₂</i>	<i>C₃</i>	<i>C₄</i>	<i>C₅</i>	<i>Supply</i>
<i>S₁</i>	$\begin{array}{ c } \hline 2 \\ \hline 0 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 4 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline 15 \end{array}$	<i>15</i>
<i>S₂</i>	$\begin{array}{ c } \hline 3 \\ \hline 5 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 15 \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	<i>20</i>
<i>S₃</i>	$\begin{array}{ c } \hline 4 \\ \hline 3 \end{array}$	$\begin{array}{ c } \hline 1 \\ \hline 10 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 12 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	<i>25</i>
<i>Demand</i>	<i>8</i>	<i>10</i>	<i>12</i>	<i>15</i>	<i>15</i>	<i>60</i>

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: VAM

	<i>C₁</i>	<i>C₂</i>	<i>C₃</i>	<i>C₄</i>	<i>C₅</i>	<i>Supply</i>	<i>P.C.</i>
<i>S₁</i>	$\begin{array}{ c } \hline 2 \\ \hline 0 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 4 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline 15 \end{array}$	<i>15</i>	<u>2</u> 1 1 <u>3</u>
<i>S₂</i>	$\begin{array}{ c } \hline 3 \\ \hline 5 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 15 \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	<i>20</i>	2 0 0 1 1
<i>S₃</i>	$\begin{array}{ c } \hline 4 \\ \hline 3 \end{array}$	$\begin{array}{ c } \hline 1 \\ \hline 10 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 12 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	<i>25</i>	1 1 2 1 1
<i>Demand</i>	<i>8</i>	<i>10</i>	<i>12</i>	<i>15</i>	<i>15</i>	<i>60</i>	
<i>P.C.</i>	$\begin{array}{ c } \hline 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{ c } \hline 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{ c } \hline 2 \\ \underline{2} \\ \underline{2} \end{array}$	$\begin{array}{ c } \hline 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$		

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: RAM

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3	4	5	0 7	15
S_2	3	2	5	2 15	0 5	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

:

	C_1	C_2	C_3	C_4	C_5
S_1	-7	-5	-6	-5	-5
S_2	-6	-6	-5	-8	-5
S_3	-4	-6	-7	-6	-4

: C_4

X_{24}

	C_1	C_2	C_3	C_5
S_1	-6	-4	-5	-4
S_2	-6	-6	-5	-5
S_3	-4	-6	-7	-4

: C_3

X_{33}

	C_1	C_2	C_5
S_1	-5	-3	-3
S_2	-4	-4	-3
S_3	-4	-6	-4

: C_2

X_{32}

	C_1	C_5
S_1	-4	-2
S_2	-4	-3
S_3	-4	-4

: S_3

X_{35}

	C_1	C_5
S_1	-3	-2
S_2	-3	-3

X_{15} ,

S_2

X_{25}

X_{11}

$$T.T.C. = 2*8 + 0*7 + 2*15 + 0*5 + 1*10 + 2*12 + 0*3 = 80$$

$$RAM < (91) VAM \leq (91) < (143) \quad (80)$$

. VAM RAM
) VAM S.B.F.S.
 RAM

— (

No. of basic cells = $m+n-1 = 5+3-1=7$

: : Optimal solution -2
 : Stepping stone -

\bar{C}_{ij}

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 0	3	4	5	0 15	15
S_2	3 5	2	5	2 15	0	20
S_3	4 3	1 10	2 12	3	0	25
Demand	8	10	12	15	15	60

$$\begin{aligned}
 X_{12} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{12} = 3 - 1 + 4 - 2 = 4 \\
 X_{13} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{13} = 4 - 2 + 4 - 2 = 4 \\
 X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} & : \quad \bar{C}_{14} = 5 - 2 + 3 - 2 = 4 \\
 X_{22} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{22} = 2 - 1 + 4 - 3 = 2 \\
 X_{23} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{23} = 5 - 2 + 4 - 3 = 4 \\
 X_{25} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} & : \quad \bar{C}_{25} = 0 - 0 + 2 - 3 = -1 \\
 X_{34} \rightarrow X_{23} \rightarrow X_{21} \rightarrow X_{31} & : \quad \bar{C}_{34} = 3 - 2 + 3 - 4 = 0 \\
 X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{31} & : \quad \bar{C}_{35} = 0 - 0 + 2 - 4 = -2 \text{ most negative}
 \end{aligned}$$

entering variable

\bar{C}_{35}

leaving variable

$\cdot X_{35}$

$X_{ij} \quad X_{35}^+ \rightarrow X_{15}^- \rightarrow X_{11}^+ \rightarrow X_{31}^- :$

:

X_{31}

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 3	3	4	5	0 12	15
S_2	3 5	2	5	2 15	0	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$T.T.C. = 6 + 0 + 15 + 30 + 10 + 24 + 0 = 85$

$No. \text{ of basic cells} = 5 + 3 - 1 = 7$

$X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} \quad : \quad \bar{C}_{12} = 3 - 1 + 0 - 0 = 2$
 $X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} \quad : \quad \bar{C}_{13} = 4 - 2 + 0 - 0 = 2$
 $X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} \quad : \quad \bar{C}_{14} = 5 - 2 + 3 - 2 = 4$
 $X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} \quad : \quad \bar{C}_{22} = 2 - 1 + 0 - 0 + 2 - 3 = 0$
 $X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} \quad : \quad \bar{C}_{23} = 5 - 2 + 0 - 0 + 2 - 3 = 2$
 $X_{25} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15} \quad : \quad \bar{C}_{25} = 0 - 3 + 2 - 0 = -1 \quad \text{negative}$
 $X_{31} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35} \quad : \quad \bar{C}_{31} = 4 - 2 + 0 - 0 = 2$
 $X_{34} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35} \quad : \quad \bar{C}_{34} = 3 - 2 + 3 - 2 + 0 - 0 = 2$

:

X_{21}

X_{25}

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3	4	5	0 7	15
S_2	3	2	5	2 15	0 5	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$

$No. \text{ of basic cells} = 7$

$$\begin{aligned}
X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{12} = 3 - 1 + 0 - 0 = 2 \\
X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{13} = 4 - 2 + 0 - 0 = 2 \\
X_{14} \rightarrow X_{24} \rightarrow X_{25} \rightarrow X_{15} & : \bar{C}_{14} = 5 - 2 + 0 - 0 = 3 \\
X_{21} \rightarrow X_{25} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{21} = 3 - 0 + 0 - 2 = 1 \\
X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{22} = 2 - 1 + 0 - 0 = 1 \\
X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{23} = 5 - 2 + 0 - 0 = 3 \\
X_{31} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{31} = 4 - 0 + 0 - 2 = 2 \\
X_{34} \rightarrow X_{35} \rightarrow X_{25} \rightarrow X_{24} & : \bar{C}_{34} = 3 - 0 + 0 - 2 = 1
\end{aligned}$$

$$\begin{array}{ccc}
& & \bar{C}_{ij} \\
& & 8 \\
& & 15 \\
12 & 10 &
\end{array}$$

V_j, U_i

: Multipliers method

-2

$$U_i = 0 :$$

$$U_i + V_j = C_{ij} :$$

: VAM

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 0	3	4	5	0 15	15
S_2	3 5	2	5	2 15	0	20
S_3	4 3	1 10	2 12	3	0	25
Demand	8	10	12	15	15	60

T.T.C. = 91 and no. of basic cells = 7

$$\begin{aligned}
C_{11} = U_1 + V_1 = 2 & \xRightarrow{U_1=0} V_1 = 2 \\
C_{15} = U_1 + V_5 = 0 & \xRightarrow{U_1=0} V_5 = 0 \\
C_{21} = U_2 + V_1 = 3 & \xRightarrow{V_1=2} U_2 = 1 \\
C_{24} = U_2 + V_4 = 2 & \xRightarrow{U_2=1} V_4 = 1 \\
C_{31} = U_3 + V_1 = 4 & \xRightarrow{V_1=2} U_3 = 2 \\
C_{32} = U_3 + V_2 = 1 & \xRightarrow{U_3=2} V_2 = -1 \\
C_{33} = U_3 + V_3 = 2 & \xRightarrow{U_3=2} V_3 = 0
\end{aligned}$$

$$\begin{aligned} &: \quad \bar{C}_{ij} = C_{ij} - (U_i + V_j) \quad \bar{C}_{ij} \\ \bar{C}_{12} &= C_{12} - (U_1 + V_2) = 3 - (0 + (-1)) = 4 \\ \bar{C}_{13} &= C_{13} - (U_1 + V_3) = 4 - (0 + 0) = 4 \\ \bar{C}_{14} &= C_{14} - (U_1 + V_4) = 5 - (0 + 1) = 4 \\ \bar{C}_{22} &= C_{22} - (U_2 + V_2) = 2 - (0 - 1) = 2 \\ \bar{C}_{23} &= C_{23} - (U_2 + V_3) = 5 - (1 + 0) = 4 \\ \bar{C}_{25} &= C_{25} - (U_2 + V_5) = 0 - (1 + 0) = -1 \\ \bar{C}_{34} &= C_{34} - (U_3 + V_4) = 3 - (2 + 1) = 0 \\ \bar{C}_{35} &= C_{35} - (U_3 + V_5) = 0 - (2 + 0) = -2 \text{ most negative} \end{aligned}$$

$$\begin{aligned} &: \quad X_{35} \quad \bar{C}_{ij} \\ X_{35}^+ &\rightarrow X_{15}^- \rightarrow X_{11}^+ \rightarrow X_{31}^- \\ &: \quad X_{31} \end{aligned}$$

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 3	3	4	5	0 12	15
S_2	3 5	2	5	2 15	0	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$$T.T.C. = 6 + 0 + 15 + 30 + 10 + 24 + 0 = 85$$

$$\text{No. of basic cells} = m + n - 1 = 3 + 5 - 1 = 7$$

$$\bar{C}_{ij}$$

:

		$V_1=2$	$V_2=1$	$V_3=2$	$V_4=1$	$V_5=0$	Supply
		C_1	C_2	C_3	C_4	C_5	
$U_1=0$	S_1	2 3	3 2	4 2	5 4	0 12	15
$U_2=1$	S_2	3 5	2 0	5 2	2 15	0 -1	20
$U_3=0$	S_3	4 2	1 10	2 12	3 2	0 3	25
Demand		8	10	12	15	15	60

X_{21} \bar{C}_{ij} X_{25}

:

		$V_1=2$	$V_2=1$	$V_3=2$	$V_4=2$	$V_5=0$	Supply
		C_1	C_2	C_3	C_4	C_5	
$U_1=0$	S_1	2	3	4	5	0	15
		8	2	2	3	7	
$U_2=0$	S_2	3	2	5	2	0	20
		1	1	3	15	5	
$U_3=0$	S_3	4	1	2	3	0	25
		2	10	12	1	3	
Demand		8	10	12	15	15	60

$$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$$

) \bar{C}_{ij}

:

(

8

15

12 10

Jobs () n
 m (machines)

=)
 $(n < m)$

$(m - n)$ $m = n$ (
 $(n > m)$ $(n - m)$

: : minimized -
 .1

.2

.3

()

() .4

)

(2) .5

: maximized -

:

: 2-_____

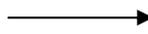
<i>jobs</i>	<i>machines</i>				
	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>
<i>J1</i>	10	11	4	2	8
<i>J2</i>	7	11	10	14	12
<i>J3</i>	5	6	9	12	14
<i>J4</i>	13	15	11	10	7

5 =

> 4 =

: _____

	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>
<i>J1</i>	10	11	4	2	8
<i>J2</i>	7	11	10	14	12
<i>J3</i>	5	6	9	12	14
<i>J4</i>	13	15	11	10	7
<i>J5</i>	0	0	0	0	0



	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>
<i>J1</i>	8	9	2	0	6
<i>J2</i>	0	4	3	7	5
<i>J3</i>	0	1	4	7	9
<i>J4</i>	6	8	4	3	0
<i>J5</i>	0	0	0	0	0

()

> 4 =

(1)

. 5 =

5 =

=

	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>M5</i>
<i>J1</i>	9	9	2	0	7
<i>J2</i>	0	3	2	6	5
<i>J3</i>	0	0	3	6	9
<i>J4</i>	6	7	3	2	0
<i>J5</i>	1	0	0	0	1

:

<i>Jobs</i>	<i>Machines</i>
<i>J1</i>	<i>M4</i>
<i>J2</i>	<i>M1</i>
<i>J3</i>	<i>M1 , M2</i>
<i>J4</i>	<i>M5</i>
<i>J5</i>	<i>M2 , M3 , M4</i>

<i>Jobs</i>	<i>Machines</i>			
	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>
<i>J1</i>	0	4	1	3
<i>J2</i>	0	2	1	3
<i>J3</i>	0	0	0	0
<i>J4</i>	2	0	0	0

1 $4 = (\quad) > 3 =$
:

<i>Jobs</i>	<i>Machines</i>			
	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>
<i>J1</i>	0	3	0	2
<i>J2</i>	0	1	0	2
<i>J3</i>	1	0	0	0
<i>J4</i>	3	0	0	0

$4 = (\quad) =$

:

<i>Jobs</i>	<i>Machines</i>
<i>J1</i>	<i>M1 , M3</i>
<i>J2</i>	<i>M1 , M3</i>
<i>J3</i>	<i>M2 , M3 , M4</i>
<i>J4</i>	<i>M2 , M3 , M4</i>

<i>Jobs</i>	<i>Mach.</i>	<i>profit</i>	<i>or</i>	<i>Jo.</i>	<i>Ma.</i>	<i>Pr.</i>	<i>or</i>	<i>Jo.</i>	<i>Ma.</i>	<i>Pr.</i>	<i>or</i>	<i>Jo.</i>	<i>Mach.</i>	<i>Pr.</i>
<i>J1</i>	<i>M1</i>	10		<i>J1</i>	<i>M1</i>	10		<i>J1</i>	<i>M3</i>	2		<i>J1</i>	<i>M3</i>	2
<i>J2</i>	<i>M3</i>	1	<i>J2</i>	<i>M3</i>	1	<i>J2</i>	<i>M1</i>	9	<i>J2</i>	<i>M1</i>	9			
<i>J3</i>	<i>M2</i>	5	<i>J3</i>	<i>M4</i>	5	<i>J3</i>	<i>M2</i>	5	<i>J3</i>	<i>M4</i>	5			
<i>J4</i>	<i>M4</i>	6	<i>J4</i>	<i>M2</i>	6	<i>J4</i>	<i>M4</i>	6	<i>J4</i>	<i>M2</i>	6			
Σ		22	Σ		22	Σ		22	Σ		22			

-1

a)

Sources	Destinations			Supply
	D1	D2	D3	
S1	1	2	6	7
S2	0	4	2	12
S3	3	1	5	11
Demand	10	10	10	30

b)

Sources	Destinations			Supply
	D1	D2	D3	
S1	5	1	8	12
S2	2	4	0	14
S3	3	6	7	4
Demand	9	10	11	30

c)

Sou.	Dest.			Sup.
	D1	D2	D3	
S1	5	1	7	10
S2	6	4	6	80
S3	3	2	2	15
Dem.	75	20	50	

Sou.	Dest.				Sup.
	D1	D2	D3	D4	
S1	10	20	5	7	10
S2	13	9	12	8	20
S3	4	15	7	9	30
S4	14	7	1	0	40
S5	3	12	5	19	50
Dem.	60	60	20	10	150

(ans. : a)(7,0,0,2,0,10,1,10,0;40) , b)(2,10,0,3,0,11,4,0,0;38) ,
 c)(0,10,0,35,10,35,0,0,15,40,0,0;500) ,
 d)(0,0,10,0,0,20,0,0,30,0,0,0,0,30,0,10,30,10,10,0;820))

M5 , M4 , M3 ,

W4 , W3 , W2 , W1

-2

30 60 40 70

. M2 , M1

50 60 30 20 40

Warehouses	Markets				
	M1	M2	M3	M4	M5
W1	7	6	5	4	2
W2	9	7	3	6	3
W3	8	8	7	3	1
W4	4	3	1	2	1

(ans.: (30,0,0,40,0,0,0,30,0,10,0,0,0,20,40,10,20,0,0,0;690))

: S4

D1

-3

Sources	Destinations			Supply
	D1	D2	D3	
S1	5	1	0	20
S2	3	2	4	10
S3	7	5	2	15
S4	9	6	0	15
Demand	5	10	15	

(ans.: (0,10,5,5,5,0,0,0,10,0,0,0,15,5,0,10,0;55))

-4

20 20

30 20 30 25

M4

. 25 10 30

:

. J4

machines	Jobs				
	J1	J2	J3	J4	J5
M1	10	2	3	15	9
M2	5	10	15	2	4
M3	15	5	14	7	15
M4	20	15	13	----	8

(ans.:(0,0,25,0,0,20,0,0,10,0,0,20,0,0,0,0,5,0,25;560))

-5

:

a)

Jobs	machines			
	M1	M2	M3	M4
J1	10	5	5	2
J2	9	8	4	3
J3	7	7	6	4
J4	8	7	5	5

b)

Jobs	Machines				
	M1	M2	M3	M4	M5
J1	3	8	2	10	3
J2	8	7	2	9	7
J3	6	4	2	7	5
J4	8	4	2	3	5
J5	9	10	6	9	10

(ans.:a) 1-2,2-4,3-1,4-3 or 1-4,2-3,3-1,4-2;20, b) 1-5 , 2-3 , 3-2 , 4-4 , 5-1 ;21)

-6

:

Jobs	Machines				
	M1	M2	M3	M4	M5
J1	3	9	2	3	7
J2	6	1	5	6	6
J3	9	4	7	10	3
J4	2	5	4	2	1
J5	9	6	2	4	6

(ans.:1-2 , 2-5 , 3-4 , 4-3 , 5-1 ;38)

P1

-7

:

M4

P3

M3

<i>Processes</i>	<i>machines</i>			
	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>
<i>P1</i>	5	5	---	2
<i>P2</i>	7	4	2	3
<i>P3</i>	9	3	5	---
<i>P4</i>	7	2	6	7

(ans.: 1-4, 2-3, 3-2, 4-1 ; 14)

:

-8

<i>Engineering</i>	<i>Lines</i>			
	<i>L1</i>	<i>L2</i>	<i>L3</i>	<i>L4</i>
<i>E1</i>	8	9	6	4
<i>E2</i>	5	7	7	8
<i>E3</i>	10	11	6	8
<i>E4</i>	3	9	5	7

:

. *L4*

E1

-

-

(ans.: a)1-4, 2-2, 3-3, 4-1;20, b)1-3, 2-2, 3-4, 4-1 ; 24 ; 4)

Network planning

: Critical Path -1-6

: Event -

: Activity -

Duration

: Path -

Critical Path (C.P.)

: *C.P.time*

Forward pass

Earliest time

$$ES_j = \max_i \{ES_i + D_{ij}\} \quad \forall (i, j) \text{ activities} \quad (ES_j)$$

$$D_{ij} \quad ES_1 = 0$$

∅

∅ ∅ ∅

Backward pass

Latest time (LC_i)

∅

$$LC_i = \min_j \{LC_j - D_{ij}\} \quad \forall (i, j) \text{ activities}$$

∅

$$LC_n = ES_n$$

(i, j)

∅

$$ES_j - ES_i = LC_j - LC_i = D_{ij}$$

∅

∅ **Free Float Time (F.F.)**

$$FF_{ij} = ES_j - ES_i - D_{ij}$$

1 ∅

a)

Act.	Pre-act.
A	----
B	----
C	A,B
D	A
E	C,D

b)

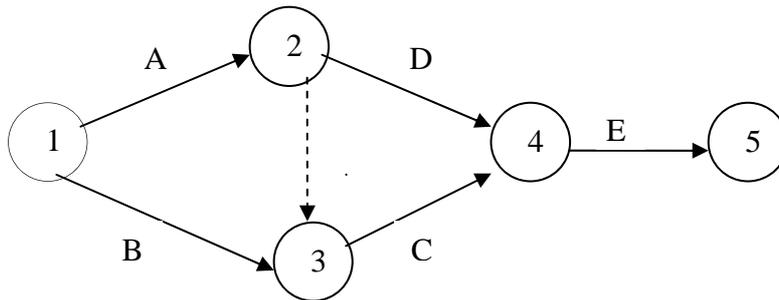
Act.	Pre-act.
A	----
B	A
C	A
D	B
E	B,C
F	D,E

c)

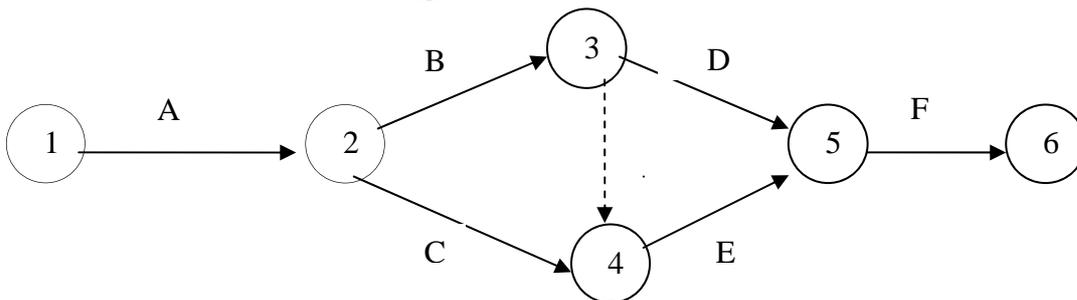
Act.	Pre-act.
A	----
B	----
C	A,B
D	A,B
E	B
F	D,E
G	C,F
H	D,E
I	G,H

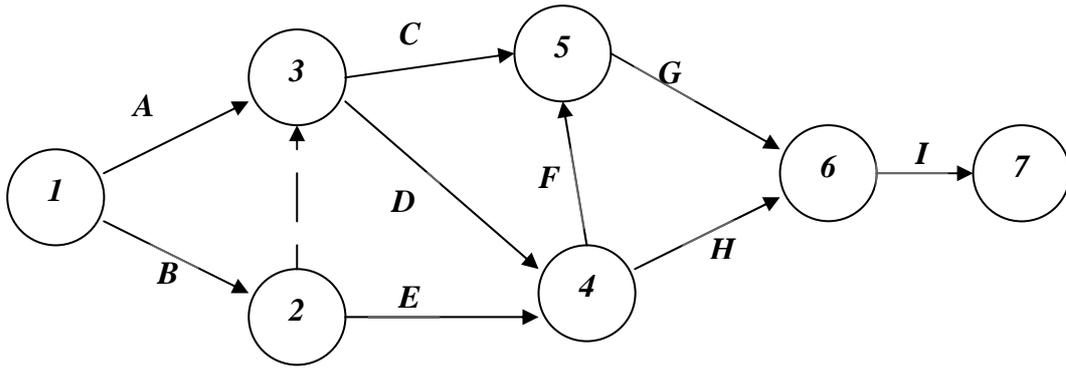
∅

a)



b)





2 ∅

∅

C, B, A

D

B, A

H, F, E

B

G

F, C

J, I

H, E

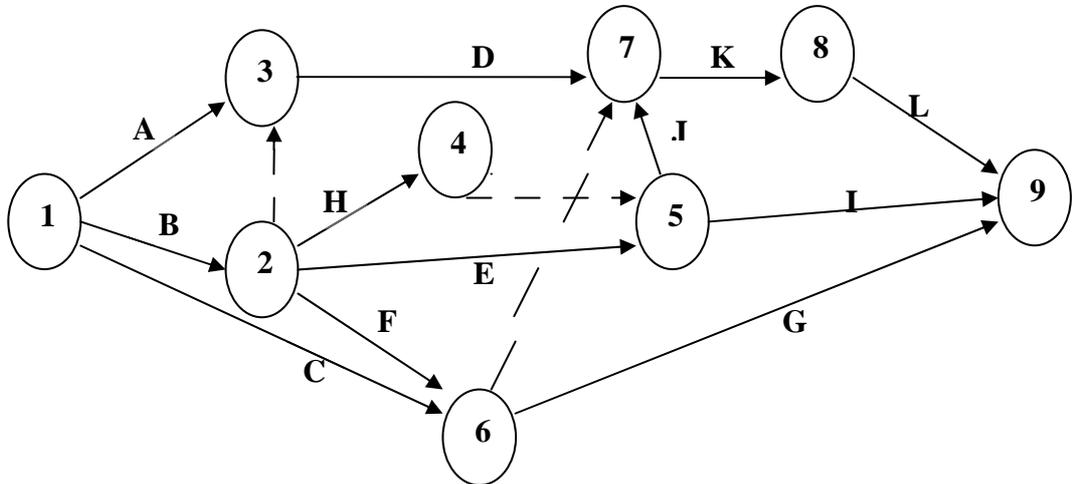
K

J, F, D, C

L

K

L, I, G



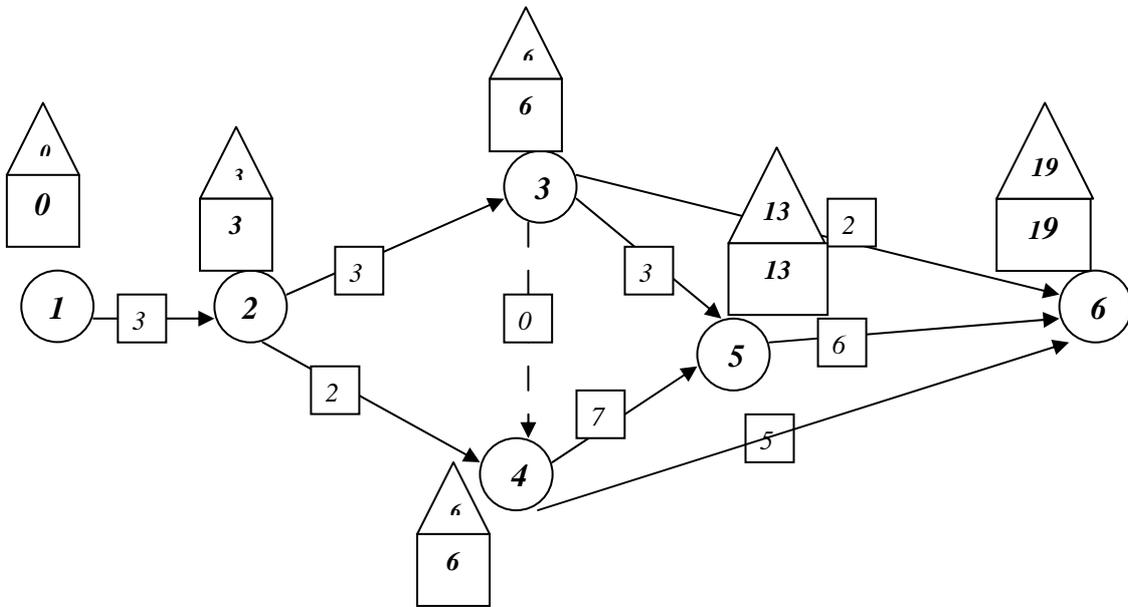
i

∅

∅

3 ∅

activity	1-2	2-3	2-4	3-4	3-5	3-6	4-5	4-6	5-6
D_{ij}	3	3	2	0	3	2	7	5	6



<i>Forward pass</i>	<i>Backward pass</i>
$ES_1 = 0$	$LC_6 = 19$
$ES_2 = 0 + 3 = 3$	$LC_5 = 19 - 6 = 13$
$ES_3 = 3 + 3 = 6$	$LC_4 = \min. \{ 13-7, 19-5 \} = 6$
$ES_4 = \max. \{ 3+2, 6+0 \} = 6$	$LC_3 = \min. \{ 6-0, 13-3, 19-2 \} = 6$
$ES_5 = \max. \{ 6+3, 6+7 \} = 13$	$LC_2 = \min. \{ 6-3, 6-2 \} = 3$
$ES_6 = \max. \{ 6+2, 6+5, 13+6 \} = 19$	$LC_1 = 3 - 3 = 0$

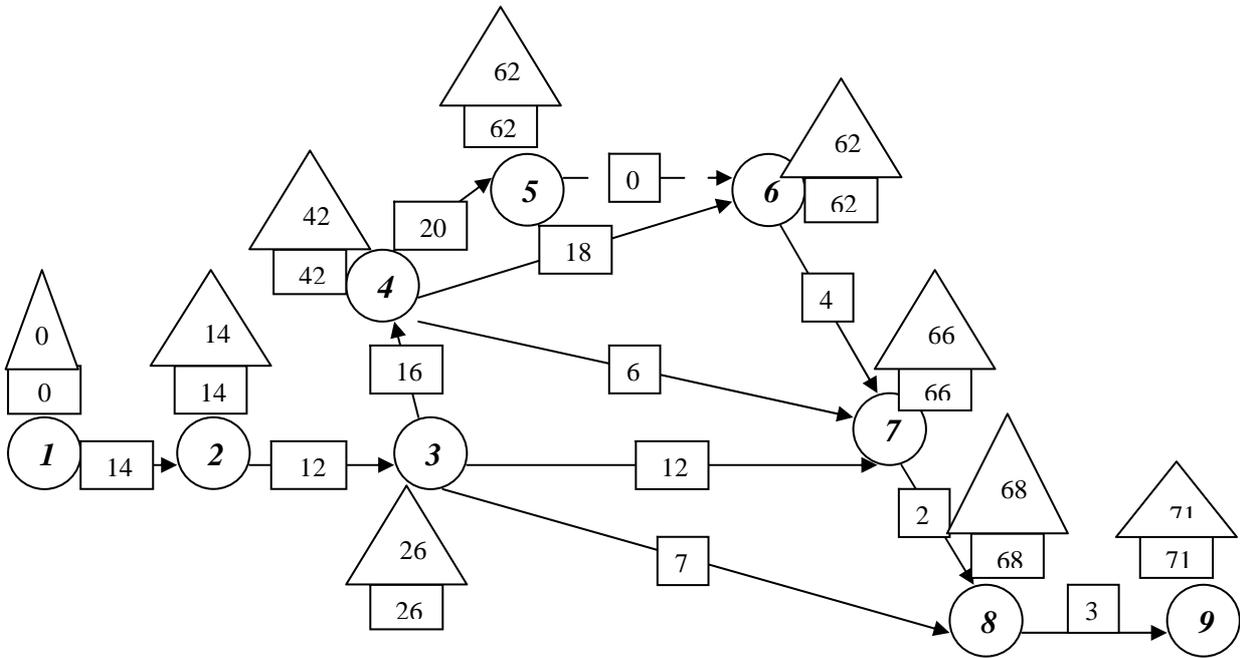
: 1-2-3-4-5-6 :

. 19 Critical time (1,2), (2,3), (3,4), (4,5), (5,6)

:

: 4-

<i>activity</i>	<i>Preceding activity</i>	<i>Duration</i>
A	---	14
B	A	12
C	B	16
D	B	7
E	B	12
F	C	20
G	C	18
H	C	6
I	F, G	4
J	E, H, I	2
K	D, J	3



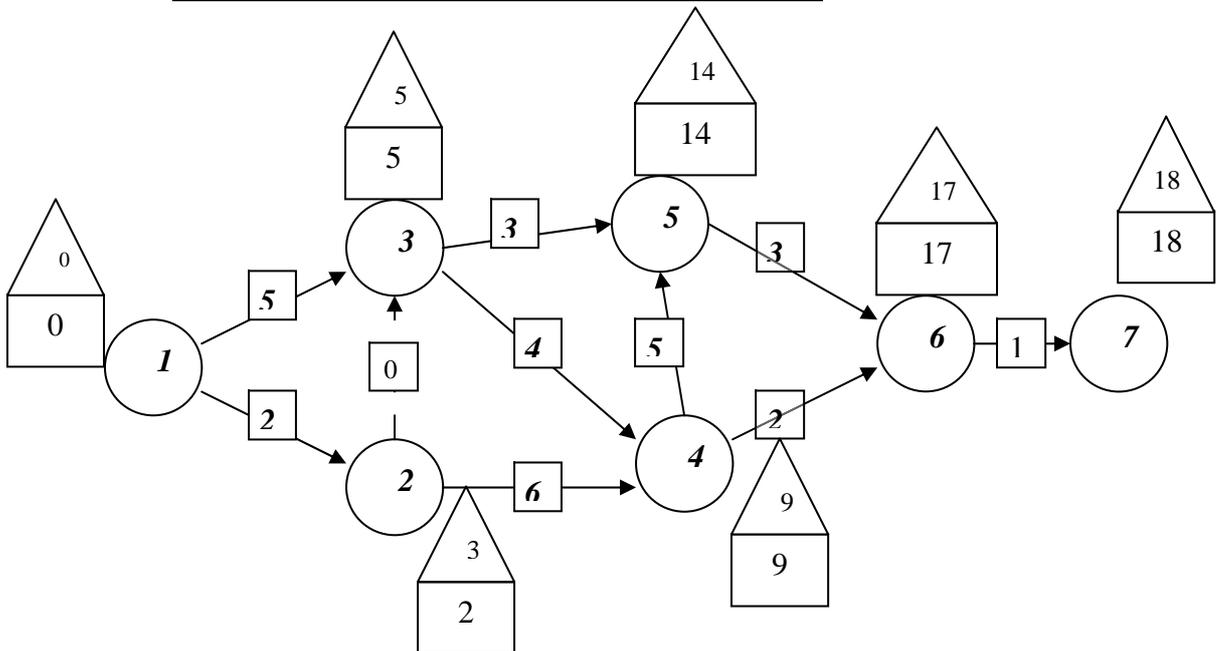
71

i $A \rightarrow B \rightarrow C \rightarrow F \rightarrow I \rightarrow J \rightarrow K$

\emptyset

5 0

<i>activity</i>	<i>Preceding activity</i>	<i>Duration</i>
<i>A</i>	---	<i>5</i>
<i>B</i>	---	<i>2</i>
<i>C</i>	<i>A, B</i>	<i>3</i>
<i>D</i>	<i>A, B</i>	<i>4</i>
<i>E</i>	<i>B</i>	<i>6</i>
<i>F</i>	<i>D, E</i>	<i>5</i>
<i>G</i>	<i>C, F</i>	<i>3</i>
<i>H</i>	<i>D, E</i>	<i>2</i>
<i>I</i>	<i>G, H</i>	<i>1</i>



. C.T.= 18

I, G, F, D, A :

Program Evaluation and Review

-2-6

: Technique (PERT)

PERT

:
 .
 .
 .
 .
 :

.
 .
 .
 .
 .
 .

Optimistic time (a) -
Pessimistic time (b) -
Most likely time (m) -

: (i , j) Expected time (\bar{D})

$$\bar{D} = \frac{a + b + 4m}{6}$$

: Variance (V)

$$V = \left(\frac{b - a}{6} \right)^2$$

$$Pr \left(Z \leq \frac{ST_i - CT_i}{\sqrt{V(\mu_i)}} \right)$$

ST_i

CT_i

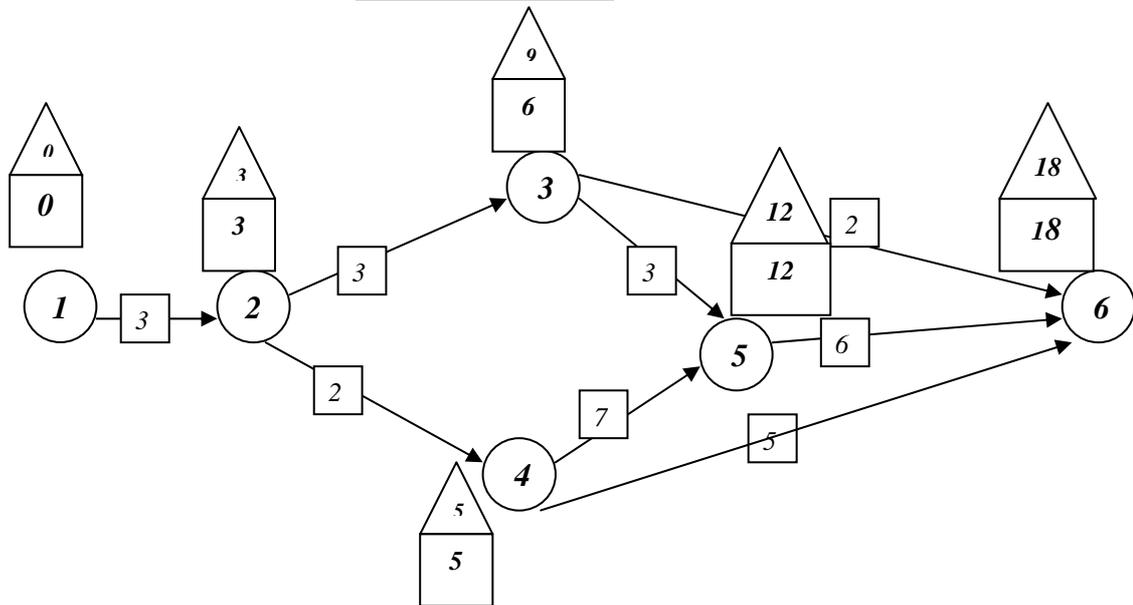
يمثل $V(\mu)$

: 6-

activity	a	b	m
1,2	2	8	2
2,3	1	11	1.5
2,4	0.5	7.5	1
3,5	1	7	2.5
3,6	1	3	2
4,5	6	8	7
4,6	3	11	4
5,6	4	8	6

activity	\bar{D}	V
1,2	3	1
2,3	3	---
2,4	2	1.36
3,5	3	----
3,6	2	----
4,5	7	0.11
4,6	5	----
5,6	6	0.44
	$V(\mu)$	2.91

: _____



.CT = 18 : (5,6), (4,5), (2,4), (1,2) :

$$Pr\left(Z_i \leq \frac{20 - 18}{\sqrt{2.91}}\right) = Pr(Z \leq 1.17) = 0.879$$

. %88 20

: _____ -3-6

:

. C_n D_n Normal () - -1
. C_c D_c Crash () - -2
:

. (CTN) -1
-2
. (CTC)

. $T = CTN - CTC$ -3
:

Slope -4

$$Slope = \frac{C_c - C_n}{D_n - D_c}$$

Free Float (FF) -5

$$FF_{ij} = ES_j - ES_i - D_{ij}$$

FF -

D_c D_n -

-6

+ =

-7

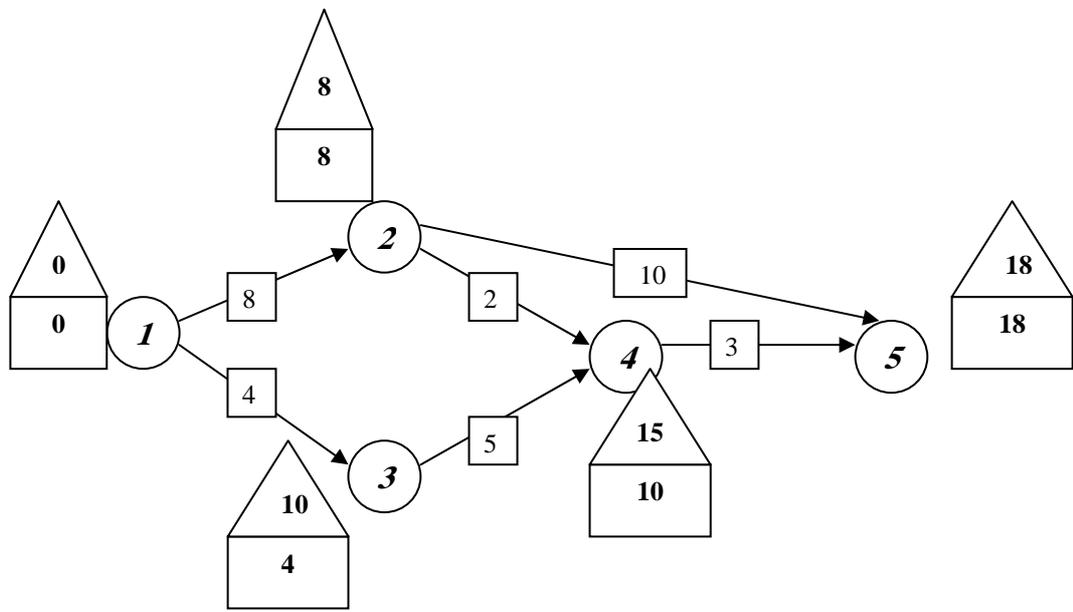
6 5 (2) (CTC)

() ()

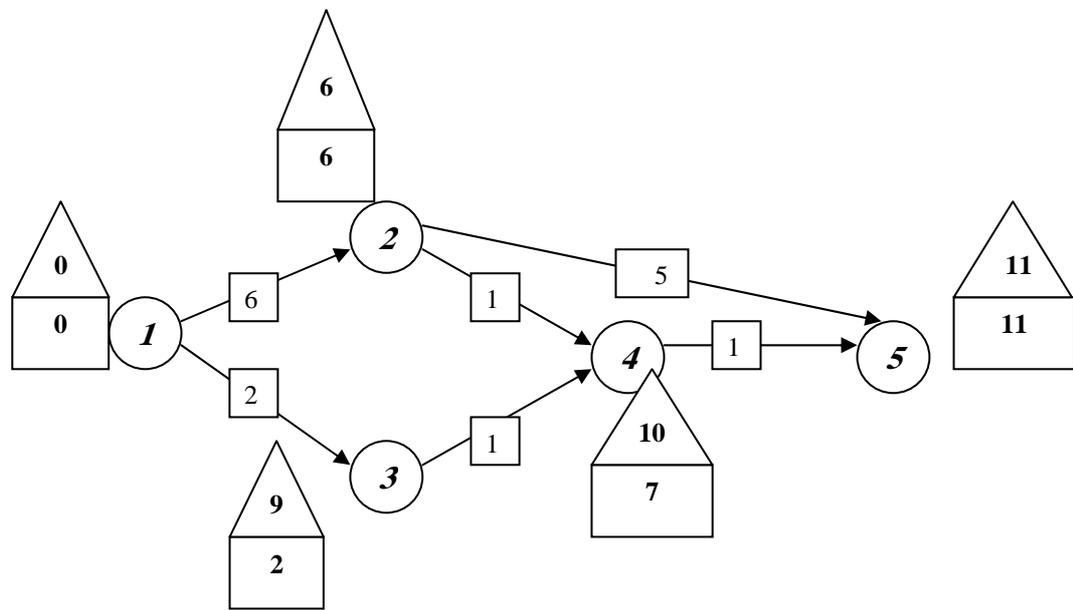
: 7-

activity	normal		Crash	
	D_n	C_n	D_c	C_c
1, 2	8	100	6	200
1, 3	4	150	2	350
2, 4	2	50	1	90
2, 5	10	100	5	400
3, 4	5	100	1	200
4, 5	3	80	1	100
Σ	---	580	---	1340

: _____



CTN = 18 , Total Cost = 580



CTC = 11 , Total Cost = 1340

CTN - CTC = 18 - 11 = 7

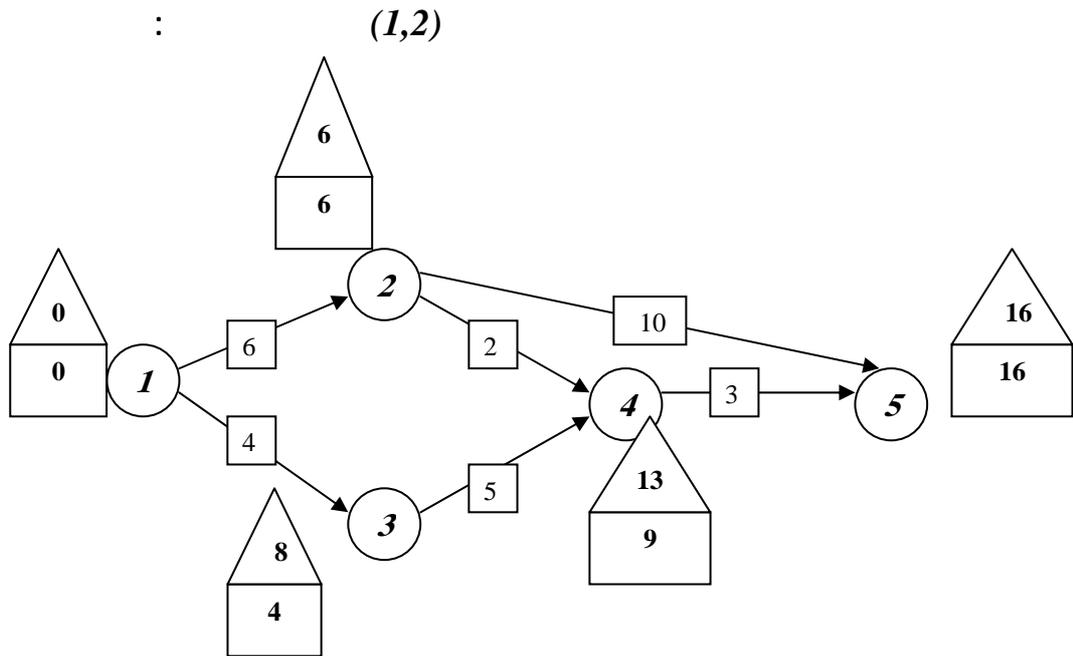
7

:

<i>activity</i>	<i>slope</i>	<i>F.F.</i>
<i>1, 2</i>	<i>50 *</i>	<i>-----</i>
<i>1, 3</i>	<i>100</i>	<i>4 - 0 - 4 = 0</i>
<i>2, 4</i>	<i>40</i>	<i>10 - 8 - 2 = 0</i>
<i>2, 5</i>	<i>60 *</i>	<i>-----</i>
<i>3, 4</i>	<i>25</i>	<i>10 - 4 - 5 = 1</i>
<i>4, 5</i>	<i>10</i>	<i>18 - 10 - 5 = 5 max.</i>

$\min. \{ 5, 8-6 \} : (\quad) (1,2)$

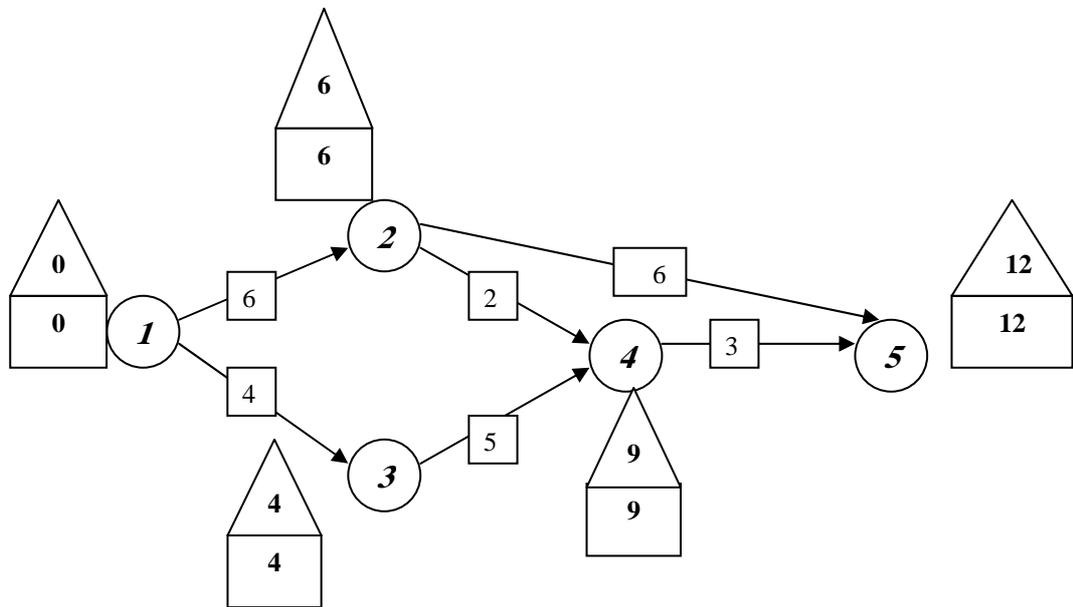
=2



Critical Path C.P. is : (1 , 2) , (2 , 5) and Total Cost T.C.= 580+ 2 * 50= 680

$\min. \{ 4 , 10-5 \} = 4 \quad (2,5)$

:



C.P. is : (1,2) , (2,5) and (1,3) , (3,4) , (4,5)

T.C. = 680 + 4 * 60 = 920

(2,5)

5

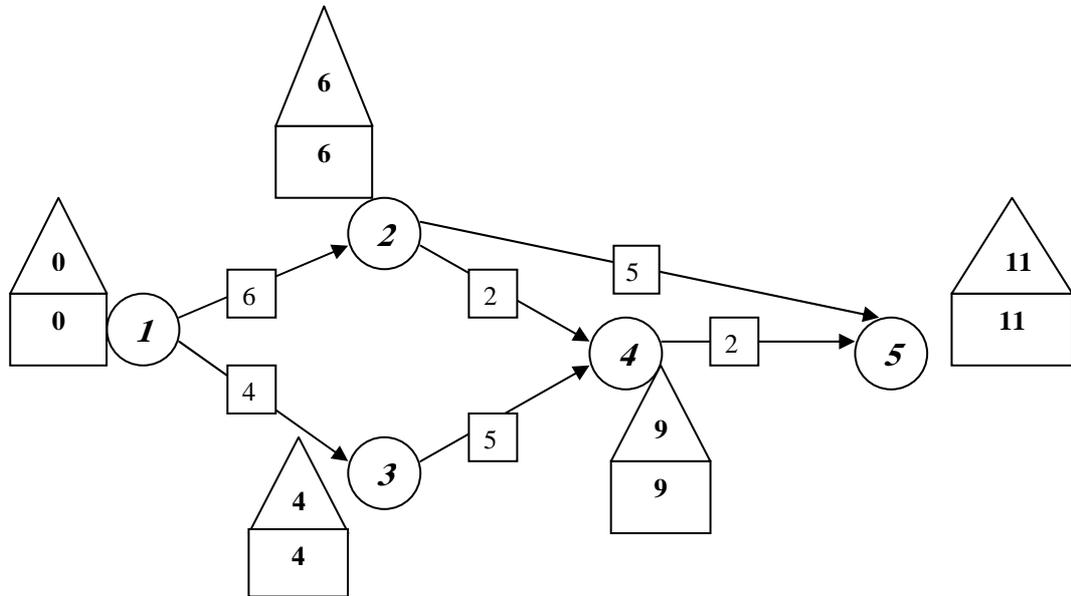
(2,5)

:

<i>activity</i>	<i>Slope</i>
(2,5), (1,3)	$60 + 100 = 160$
(2,5), (3,4)	$60 + 25 = 85$
(2,5), (4,5)	$60 + 10 = 70 \text{ min.}$

:

(4,5), (2,5)



C.P. is (1,2), (2,5) and (1,3), (3,4), (4,5)

T.C. = $920 + 1 * 70 = 990$

990 580

11 18

. 1340

-1

C, B, A -

A F, E, D -

. D, B G, I -

. G, C H -

. I L, K -

. H, E J -

. H, E F N, M -

. I, M O -

. O, L, J P -

. P, N, K -

-2

<i>activity</i>	<i>Pre. Act.</i>	<i>Duration</i>	<i>activity</i>	<i>Pre. Act.</i>	<i>Duration</i>
<i>R</i>	----	<i>24</i>	<i>D</i>	<i>C, B</i>	<i>6</i>
<i>E</i>	<i>R</i>	<i>16</i>	<i>C</i>	<i>A</i>	<i>8</i>
<i>H</i>	<i>G</i>	<i>16</i>	<i>B</i>	<i>A</i>	<i>16</i>
<i>N</i>	<i>P, Q, U, S</i>	<i>8</i>	<i>U</i>	<i>F</i>	<i>8</i>
<i>M</i>	<i>L, K</i>	<i>8</i>	<i>Q</i>	<i>E</i>	<i>12</i>
<i>K</i>	<i>H</i>	<i>16</i>	<i>A</i>	<i>R</i>	<i>16</i>
<i>P</i>	<i>E, D</i>	<i>36</i>	<i>F</i>	<i>R</i>	<i>40</i>
<i>S</i>	<i>T, M</i>	<i>16</i>	<i>G</i>	<i>R</i>	<i>24</i>
<i>L</i>	<i>H</i>	<i>24</i>	<i>T</i>	<i>G</i>	<i>4</i>

(ans.: *R, G, H, L, M, S, N*; 120)

-3

<i>activity</i>	<i>Pre. Act.</i>	<i>Duration</i>	<i>Activity</i>	<i>Pre. Act.</i>	<i>Duration</i>
<i>A</i>	----	<i>10</i>	<i>J</i>	<i>F</i>	<i>5</i>
<i>B</i>	----	<i>28</i>	<i>K</i>	<i>E, G, H</i>	<i>1</i>
<i>C</i>	<i>A</i>	<i>2</i>	<i>L</i>	<i>I, J</i>	<i>6</i>
<i>D</i>	<i>C</i>	<i>1</i>	<i>M</i>	<i>J, L</i>	<i>2</i>
<i>E</i>	<i>D</i>	<i>2</i>	<i>N</i>	<i>K, M</i>	<i>1</i>
<i>F</i>	<i>D</i>	<i>30</i>	<i>O</i>	<i>K, M</i>	<i>4</i>
<i>G</i>	<i>D</i>	<i>45</i>	<i>P</i>	<i>N</i>	<i>1</i>
<i>H</i>	<i>B, D</i>	<i>1</i>	<i>Q</i>	<i>N, O</i>	<i>1</i>
<i>I</i>	<i>E, H</i>	<i>6</i>	<i>R</i>	<i>P, Q</i>	<i>1</i>

(ans.: *A, C, D, G, K, O, Q, R*; 65)

: 1.9

<i>Act.</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
<i>Pre.act.</i>	---	---	<i>A</i>	<i>A,B</i>	<i>A,B</i>	<i>C,D</i>	<i>A</i>	<i>C,D,G</i>	<i>E,F,H</i>
<i>Duration</i>	5	7	6	8	7	5	6	9	10

(ans.: 99.6%)

:

<i>Act.</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
<i>Pre.act.</i>	---	---	<i>A,B</i>	<i>A,B</i>	<i>B</i>	<i>D,E</i>	<i>C,F</i>	<i>D,E</i>	<i>G,H</i>
<i>Duration</i>	6	5	7	8	4	6	7	4	<i>X</i>

Z = 1.5

I *X* - :

. 2 = 34

25 *H* -

16

6 50000

7 5 *C* -

25000

. 30000

(ans.: a) 4 ; b) 600000 ; c)- 300000)

:

<i>Act.</i>	<i>(a,b,m)</i>		<i>Act.</i>	<i>(a,b,m)</i>
1, 2	5, 7, 6		3, 6	3, 5, 4
1, 4	1, 5, 3		4, 6	4, 9, 8
1, 5	2, 6, 4		4, 7	4, 8, 6
2, 3	4, 6, 5		5, 6	9, 14, 10
2, 5	6, 10, 8		5, 7	4, 8, 6
2, 6	8, 13, 9		6, 7	3, 5, 4
3, 4	5, 10, 9			

34

(ans.: 98.9%)

:

<i>Act.</i>	<i>Normal</i>		<i>crash</i>		<i>Act.</i>	<i>normal</i>		<i>crash</i>	
	<i>D_n</i>	<i>C_n</i>	<i>D_c</i>	<i>C_c</i>		<i>D_n</i>	<i>C_n</i>	<i>D_c</i>	<i>C_c</i>
1, 2	4	100	1	400	3, 7	14	120	12	140
1, 4	9	120	6	180	4, 5	15	500	10	750
1, 3	8	400	5	640	4, 7	10	200	6	220
1, 6	3	20	1	60	5, 6	11	160	8	240
2, 3	5	60	3	100	5, 7	8	70	5	110
2, 5	9	210	7	270	6, 7	10	100	2	180
3, 4	12	400	8	800	∑	---	2460	--	4090

(ans.: 33 ; 3750)

:

<i>Act.</i>	<i>normal</i>		<i>crash</i>	
	<i>D_n</i>	<i>C_n</i>	<i>D_c</i>	<i>C_c</i>
1, 2	5	1000	4	1400
1, 3	9	2000	7	3000
2, 3	7	2500	4	3400
2, 4	9	2800	7	3400
3, 5	5	2500	2	4600
3, 6	11	4000	7	7200
4, 6	6	3000	4	4200
5, 6	8	800	6	1400
∑	--	18600	--	28600

(ans.: 16 ; 24600)

Sequencing models ^[2]

Sequencing models ()

($m = 1, 2, 3, \dots$)
idle time ()

m

- 1
- 2
- 3
- 4
- 5
- 6
- 7

: Processing n jobs through 1 machine n -1-7

n

Shortest processing time (S.p.t.)

Largest processing time (L.p.t.)

()

: 1-

<i>Jobs</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>Time</i>	<i>8</i>	<i>6</i>	<i>2</i>	<i>7</i>	<i>10</i>	<i>4</i>

Lpt

(*Spt*

(

sequence	jobs	time	Processing	
			Start	Finish
1	C	2	0	2
2	F	4	2	6
3	B	6	6	12
4	D	7	12	19
5	A	8	19	27
6	E	10	27	37
Σ				103

$Spt = 103/6 = 17.16 \text{ hrs.}$

Sequence	jobs	time	Processing	
			Start	Finish
1	E	10	0	10
2	A	8	10	18
3	D	7	18	25
4	B	6	25	31
5	F	4	31	35
6	C	2	35	37
Σ				156

$Lpt = 156/6 = 26 \text{ hrs}$

W_i

t_i

: ()

Jobs	A	B	C	D	E	F
Time t_i	10	6	5	4	2	8
Weight W_i	5	10	5	1	3	5

: \bar{t}

\bar{t}	Jobs
$10/5 = 2$	A
$6/10 = 0.6$	B
$5/5 = 1$	C
$4/1 = 4$	D
$2/3 = 0.67$	E
$8/5 = 1.6$	F

. B - E - C - F - A - D :

:Processing n jobs through 2 machines n -2-7

:

-1

)

-2

(

-3

)

-4

-5

-6

-7

idle time ()

B A B , A : 3-

:

()

<i>Jobs</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
<i>Mach. A</i>	<i>3</i>	<i>12</i>	<i>5</i>	<i>2</i>	<i>9</i>	<i>11</i>
<i>Mach. B</i>	<i>8</i>	<i>10</i>	<i>9</i>	<i>6</i>	<i>3</i>	<i>1</i>

1	2	3	4	5	6
2 <u>3</u>	12	3 <u>5</u>	1 <u>2</u>	9	11
8	4 <u>10</u>	9	6	5 <u>3</u>	6 <u>1</u>

The optimal sequencing is : 4 – 1 – 3 – 2 – 5 – 6

jobs	Mach. A			Mach. B			
	Time	Start	Finish	time	Start	Finish	Idle
4	2	0	2	6	2	8	2
1	3	2	5	8	8	16	0
3	5	5	10	9	16	25	0
2	12	10	22	10	25	35	0
5	9	22	31	3	35	38	0
6	11	31	42	1	42	43	4
Σ							6

. 43

43 - 42 = 1 hr. : A

6 hrs. : B

: () B A : 4-

jobs	1	2	3	4	5	6	7
Mach. A	3	12	15	6	10	11	9
Mach. B	8	10	10	6	12	1	3

:

	1	2	3	4	5	6	7
1	<u>3</u>	12	15	2 <u>6</u>	3 <u>10</u>	11	9
	8	5 <u>10</u>	4 <u>10</u>	6	12	7 <u>1</u>	6 <u>3</u>

: _____

The optimal sequencing is : 1 - 4 - 5 - 3 - 2 - 7 - 6

jobs	Mach. A			Mach. B			
	time	Start	finish	time	Start	finish	idle
1	3	0	3	8	3	11	3
4	6	3	9	6	11	17	0
5	10	9	19	12	19	31	2
3	15	19	34	10	34	44	3
2	12	34	46	10	46	56	2
7	9	46	55	3	56	59	0
6	11	55	66	1	66	67	7
Σ							17

. 67

. 67 - 66 = 1 hr. : A

. 17 hrs : B

: Processing n jobs through 3 machines n -3-7

:

أكبر وقت على الماكنة الثانية . -

أكبر وقت على الماكنة الثانية . -

:

: **G, H** .1

$$H_i = B_i + C_i, \quad G_i = A_i + B_i$$

. **H, G** .2

.3

.

.4

: **B**

- ()

+

. **ABC** **C, B, A** : 5-

: ()

Jobs	1	2	3	4	5	6
Mach. A	3	12	5	2	9	11
Mach. B	8	6	4	6	3	1
Mach. C	13	14	9	12	8	13

≤ : : _____

:

$$H_i = B_i + C_i, \quad G_i = A_i + B_i$$

jobs	1	2	3	4	5	6
Mach. G	<u>11</u> 3	<u>18</u> 5	<u>9</u> 2	<u>8</u> 1	12	<u>12</u> 4
Mach. H	21	20	13	18	<u>11</u> 6	14

The optimal sequencing is : 4 - 3 - 1 - 6 - 2 - 5

jobs	Mach. A			Mach. B				Mach. C				
	t.	S.	F.	t.	S.	F.	I.	t.	S.	F.	I.	
4	2	0	2	6	2	8	2	12	8	20	8	
3	5	2	7	4	8	12	0	9	20	29	0	
1	3	7	10	8	12	20	0	13	29	42	0	
6	11	10	21	1	21	22	1	13	42	55	0	
2	12	21	33	6	33	39	11	14	55	69	0	
5	9	33	42	3	42	45	3	8	69	77	0	
Σ							17					8

. 77 hrs. :

. 77 - 42 = 35 hrs. : A

. 77 - 45 + 17 = 49 hrs : B

. 8 hrs : C

: Processing n jobs through m machines m n -4-7

$m \geq 4$

m

n

:

\leq

\leq

m

()

:

H, G

$$G_i = M_1 + M_2 + \dots + M_{m-1}, \quad H_i = M_2 + M_3 + \dots + M_m$$

: ()

ABCDE

: 6-

jobs	machines				
	A	B	C	D	E
1	7	5	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

:

: _____

$$\text{Min. } \{ E \} = 6 \geq \text{max. } \{ B, C, D \} = 6$$

: H, G

$$G_i = A_i + B_i + C_i + D_i \quad \text{and} \quad H_i = B_i + C_i + D_i + E_i$$

	G	H
1	17	19

2	<u>21</u> 3	25
3	<u>20</u> 2	23
4	16	<u>14</u> 4

The optimal sequencing is : 1 - 3 - 2 - 4

Job	Mach. A			Mach. B				Mach. C				Mach. D				Mach. E					
	t.	S.	F.	t.	S.	F.	I.														
1	7	0	7	5	7	12	7	2	12	14	12	3	14	17	14	9	17	26	17		
3	5	7	12	4	12	16	0	5	16	21	2	6	21	27	4	8	27	35	1		
2	6	12	18	6	18	24	2	4	24	28	3	5	28	33	1	10	35	45	0		
4	8	18	26	3	26	29	2	3	29	32	1	2	33	35	0	6	45	51	0		
Σ							11					18					19				

. 51 hrs. :

. 51 - 26 = 15 hrs. : A

. 51 - 29 + 11 = 33 hrs. : B

. 51 - 32 + 18 = 37 hrs. : C

. 51 - 35 + 19 = 35 hrs. : D

. 18 hrs. : E

() : _____

:

: () ABCD : 7-_____

job	machines			
	A	B	C	D
1	58	14	14	48
2	30	10	18	32
3	28	12	16	44
4	64	16	12	42

: _____

Min. { A } = 28 ≥ max { B , C } = 18 and min. { D } = 32 ≥ max { B , C } = 18

:

$B_1 + C_1 = B_2 + C_2 = B_3 + C_3 = B_4 + C_4 = 28$

: D A

job	machines	
	A	D
1	58	<u>48</u> 3
2	<u>30</u> 2	32
3	<u>28</u> 1	44
4	64	<u>42</u> 4

The optimal sequencing is : 3 - 2 - 1 - 4

job	Mach. A			Mach. B				Mach. C				Mach. D					
	t.	S.	F.	t.	S.	F.	I.	t.	S.	F.	I.	t.	S.	F.	I.		
3	28	0	28	12	28	40	28	16	40	56	40	44	56	100	56		
2	30	28	58	10	58	68	18	18	68	86	12	32	100	132	0		
1	58	58	116	14	116	130	48	14	130	144	144	48	144	192	12		
4	64	116	180	16	180	196	50	12	196	208	52	42	208	250	16		
Σ							144					148					84

. 250 hrs. :

. 250 - 180 = 70 hrs. : A

. 250 - 196 + 144 = 198 hrs. : B

. 250 - 208 + 148 = 190 hrs. : C

. 84 hrs. : D

: () _____ n -5-7

:

. A -

. B -

. AB -

. BA -

:

: 8-_____

:

jobs		1	2	3	4	5	6	7	8	9	10
Operating order	1	A	A	A	A	B	B	B	B	B	A
	2	B	---	---	B	A	---	A	---	A	B
Operating Time	1	4	3	4	5	1	1	7	3	6	2
	2	6	---	---	2	2	---	8	---	7	4

:

$\{2, 3\}$ A : _____
 $\{6, 8\}$ B 3 2
 $\{1, 4, 10\}$ B A 8 6
 : 4 1 10

jobs	A	B
1	2 <u>4</u>	6
4	<u>5</u> 3	<u>2</u>
10	1 <u>2</u>	4

$\{5, 7, 9\}$ A B
 : 7 9 5

jobs	B	A
5	<u>1</u> 1	2
7	<u>7</u> 3	8
9	<u>6</u> 2	7

: BA A AB A
 10 - 1 - 4 - 2 - 3 - 5 - 9 - 7
 : AB B BA B
 5 - 9 - 7 - 6 - 8 - 10 - 1 - 4

Mach. A				Mach. B				job	Idle time
job	t.	S.	F.	job	t.	S.	F.	job	
10	2	0	2	5	1	0	1	1	$2 + 22 - 6 = 18$
1	4	2	6	9	6	1	7	2	11
4	5	6	11	7	7	7	14	3	14
2	3	11	14	6	1	14	15	4	$28 - 11 + 6 = 23$
3	4	14	18	8	3	15	18	5	$18 - 1 = 17$
5	2	18	20	10	4	18	22	6	14
9	7	20	27	1	6	22	28	7	$27 - 14 + 7 = 20$
7	8	27	35	4	2	28	30	8	15
								9	$20 - 7 + 1 = 14$
								10	$18 - 2 = 16$

. A
 . $35 - 30 = 5$: B

: () -1

<i>job</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>time</i>	<i>4</i>	<i>3</i>	<i>5</i>	<i>2</i>	<i>6</i>

(ans.: 10, 14)

. *Lpt* *Spt*

: () *B A* -2

<i>Job</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>Mach. A</i>	<i>5</i>	<i>1</i>	<i>9</i>	<i>3</i>	<i>10</i>
<i>Mach. B</i>	<i>2</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>4</i>

(ans.: 2-4-3-5-1, 2, 3)

: () *B A* -3

<i>Job</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>Mach. A</i>	<i>4</i>	<i>5</i>	<i>2</i>	<i>6</i>	<i>1</i>
<i>Mach. B</i>	<i>3</i>	<i>2</i>	<i>5</i>	<i>4</i>	<i>2</i>

(ans.: 5-3-4-1-2, 2, 4)

: () *C B A* -4

<i>Job</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>Mach. A</i>	<i>3</i>	<i>8</i>	<i>7</i>	<i>5</i>	<i>4</i>
<i>Mach. B</i>	<i>4</i>	<i>5</i>	<i>1</i>	<i>2</i>	<i>3</i>
<i>Mach. C</i>	<i>7</i>	<i>9</i>	<i>5</i>	<i>6</i>	<i>10</i>

(ans.: 4-1-5-2-3; 17, 29, 7)

: () *C B A* -5

<i>Job</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Mach. A</i>	<i>5</i>	<i>6</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>9</i>	<i>15</i>	<i>11</i>
<i>Mach. B</i>	<i>4</i>	<i>6</i>	<i>7</i>	<i>4</i>	<i>5</i>	<i>3</i>	<i>6</i>	<i>2</i>
<i>Mach. C</i>	<i>8</i>	<i>10</i>	<i>7</i>	<i>8</i>	<i>11</i>	<i>8</i>	<i>9</i>	<i>13</i>

(ans.: 4-1-3-5-2-8-7-6; 26, 44, 7)

: () C B A -6

Job	1	2	3	4	5
Mach. A	5	8	6	5	7
Mach. B	3	5	2	4	3
Mach. C	4	5	3	2	1

(ans.: 2-1-4-3-5 ; 35 ; 4 , 18 , 20)

. ABC -7

Job	1	2	3	4	5
Mach. A	8	9	5	6	15
Mach. B	4	5	1	2	3
Mach. C	3	8	7	7	7

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(

(ans.: a) 3-4-2-5-1 ; 50 , b) 7 , 35 , 18)

: () ABCDE -8

job	machines				
	A	B	C	D	E
1	13	9	5	8	19
2	12	11	6	2	8
3	15	4	10	2	16
4	17	9	12	11	10
5	20	10	4	7	17
6	16	6	5	9	13

(ans.:125,32,76 ,83,86,42) .

-9

Job		1	2	3	4	5	6	7	8	9	10	11	12
No. Process	1	A	A	B	A	B	B	B	B	A	A	A	A
	2	-	B	-	B	A	A	-	-	B	-	B	-
Operating time	1	15	3	12	4	4	8	4	8	2	8	5	7
	2	-	7	-	3	5	6	-	-	7	-	2	-

(ans.: mach.A(9-2-4-11-12-10-1-5-6);mach.B(5-6-7-8-3-9-2-4-11));55,0,0)

Replacement and Maintenance models ^[21]

: Replacement models -1-8

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C :

S

n

TC

$$T = \frac{TC}{n}$$

f(t)

:

. Resale Value

. t maintenance

$$TC = C + \int_0^n f(t)dt - S$$

$$T = \frac{TC}{n} = \frac{1}{n} \left(C - S + \int_0^n f(t) dt \right)$$

$$\frac{\partial T}{\partial n} = -\frac{1}{n^2} \left(C - S + \int_0^n f(t) dt \right) + \frac{1}{n} \frac{\partial}{\partial n} \int_0^n f(t) dt$$

$$\frac{\partial}{\partial n} \int_0^n f(t) dt = \frac{\partial}{\partial n} [F(t)]_0^n \quad \text{where} \quad \int f(t) dt = F(t) \quad :$$

$$= \frac{\partial}{\partial n} [F(n) - F(0)] \quad \text{since} \quad F(0) = 0$$

$$= \frac{\partial}{\partial n} F(n) = f(n)$$

$$\frac{\partial T}{\partial n} = -\frac{1}{n^2} \left(C - S + \int_0^n f(t) dt \right) + \frac{1}{n} f(n) = 0 \quad :$$

$$f(n) = \frac{1}{n} \left(C - S + \int_0^n f(t) dt \right)$$

$$\frac{\partial^2 T}{\partial n^2} = \frac{2}{n^3} \left(C - S + \int_0^n f(t) dt \right) + \frac{1}{n} f'(n) - \frac{2}{n^2} f(n) > 0 \Rightarrow \text{min.}$$

:

. Continuous t $g(n) = \frac{1}{n} \left(C - S + \int_0^n f(t) dt \right)$

. Discrete t $g(n) = \frac{1}{n} \left(C - S + \sum_{t=0}^n f(t) \right)$

:

8

: 1-

Year (t)	1	2	3	4	5	6	7	8
Maintenance f(t)	900	1200	1600	2100	2800	3700	4700	5900
Resale value (S)	4000	2000	1200	600	500	400	400	400

. 7000 \$

C = 7000 : _____

N	S	C - S	f(t)	∑f(t)	g(n)
1	4000	3000	900	900	3900
2	2000	5000	1200	2100	3550
3	1200	5800	1600	3700	3166.7
4	600	6400	2100	5800	3050
5	500	6500	2800	8600	3020 → mini.
6	400	6600	3700	12300	3150
7	400	6600	4700	17000	3371.4
8	400	6600	5900	22900	3687.5

200

9000

: 2-_____

2000

<i>N</i>	<i>C - S</i>	<i>f(t)</i>	$\sum f(t)$	<i>g(n)</i>
1	9000	200	200	9200
2	9000	2200	2400	5700
3	9000	4200	6600	5200 → <i>mini.</i>
4	9000	6200	12800	5450
5	9000	8200	21000	6100

Cost of individual replacement

Cost of grouped replacement

:

(*i*)

$$N_i = \sum_{j=1}^i N_{j-1} P_{i-j+1} = N_0 P_i + N_1 P_{i-1} + N_2 P_{i-2} + \dots + N_{i-1} P_1$$

: *Average life of items (AL)*

$$AL = \sum_{i=1}^n i P_i$$

: *Average failure per period (AF)* *n*

$$AF = \frac{N_0}{AL}$$

: *Cost of individual replacement (CIR)*

$$CIR = C_1 * AF$$

: *Average cost group replacement per period (i)*

$$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^i N_j}{i}$$

ACGR

CIR

i

ACGR_i

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i

P_i :

C₁

C_2 N_0 $i = 1, 2, 3, \dots, n$ n : 3-

:

End of week (i)	1	2	3	4	5	6	7	8	9	10	11
Prob.Of failure (P _i)	0.01	0.03	0.05	0.07	0.10	0.15	0.20	0.15	0.11	0.08	0.05

 0.5 1.25 1000

: _____

$$N_0 = 1000$$

$$N_1 = N_0 * P_1 = 1000 * 0.01 = 10$$

$$N_2 = N_0 * P_2 + N_1 * P_1 = 1000 * 0.03 + 10 * 0.01 = 30.1$$

$$N_3 = N_0 * P_3 + N_1 * P_2 + N_2 * P_1 = 1000 * 0.05 + 10 * 0.03 + 30.1 * 0.01 = 50.6$$

$$N_4 = N_0 * P_4 + N_1 * P_3 + N_2 * P_2 + N_3 * P_1$$

$$= 1000 * 0.07 + 10 * 0.05 + 30.1 * 0.03 + 50.6 * 0.01 = 71.9$$

$$N_5 = N_0 * P_5 + N_1 * P_4 + N_2 * P_3 + N_3 * P_2 + N_4 * P_1$$

$$= 1000 * 0.10 + 10 * 0.07 + 30.1 * 0.05 + 50.6 * 0.03 + 71.9 * 0.01 = 104.4$$

$$N_6 = N_0 * P_6 + N_1 * P_5 + N_2 * P_4 + N_3 * P_3 + N_4 * P_2 + N_5 * P_1$$

$$= 1000 * 0.15 + 10 * 0.10 + 30.1 * 0.07 + 50.6 * 0.05 + 71.9 * 0.03 + 104.4 * 0.01 = 158.8$$

$$N_7 = N_0 * P_7 + N_1 * P_6 + N_2 * P_5 + N_3 * P_4 + N_4 * P_3 + N_5 * P_2 + N_6 * P_1$$

$$= 1000 * 0.2 + 10 * 0.15 + 30.1 * 0.1 + 50.6 * 0.07 + 71.9 * 0.05 + 104.4 * 0.03 + 158.8 * 0.01$$

$$= 216.4$$

$$N_8 = N_0 * P_8 + N_1 * P_7 + N_2 * P_6 + N_3 * P_5 + N_4 * P_4 + N_5 * P_3 + N_6 * P_2 + N_7 * P_1$$

$$= 1000 * 0.15 + 10 * 0.2 + 30.1 * 0.15 + 50.6 * 0.1 + 71.9 * 0.07 + 104.4 * 0.05 + 158.8 * 0.03$$

$$+ 216.4 * 0.01 = 178.8$$

$$N_9 = N_0 * P_9 + N_1 * P_8 + N_2 * P_7 + N_3 * P_6 + N_4 * P_5 + N_5 * P_4 + N_6 * P_3 + N_7 * P_2 + N_8 * P_1$$

$$= 1000 * 0.11 + 10 * 0.15 + 30.1 * 0.2 + 50.6 * 0.15 + 71.9 * 0.1 + 104.4 * 0.07 + 158.8 * 0.05$$

$$+ 216.4 * 0.03 + 178.8 * 0.01 = 155.8$$

$$N_{10} = N_0 * P_{10} + N_1 * P_9 + N_2 * P_8 + N_3 * P_7 + N_4 * P_6 + N_5 * P_5 + N_6 * P_4 + N_7 * P_3 + N_8 * P_2 + N_9 * P_1$$

$$= 1000 * 0.08 + 10 * 0.11 + 30.1 * 0.15 + 50.6 * 0.2 + 71.9 * 0.15 + 104.4 * 0.1 + 158.8$$

$$* 0.07 + 216.4 * 0.05 + 178.8 * 0.03 + 155.8 * 0.01 = 145.8$$

$$N_{11} = N_0 * P_{11} + N_1 * P_{10} + N_2 * P_9 + N_3 * P_8 + N_4 * P_7 + N_5 * P_6 + N_6 * P_5 + N_7 * P_4 + N_8 * P_3$$

$$+ N_9 * P_2 + N_{10} * P_1$$

$$= 1000 * 0.05 + 10 * 0.08 + 30.1 * 0.11 + 50.6 * 0.15 + 71.9 * 0.2 + 104.4 * 0.15 + 158.8$$

$$* 0.1 + 216.4 * 0.07 + 178.8 * 0.05 + 155.8 * 0.03 + 265.8 * 0.01 = 139.1$$

$$AL = \sum_{i=1}^{11} i * P_i = 1 * 0.01 + 2 * 0.03 + 3 * 0.05 + 4 * 0.07 + 5 * 0.1 + 6 * 0.15 + 7 * 0.2 + 8 * 0.15$$

$$+ 9 * 0.11 + 10 * 0.08 + 11 * 0.05 = 6.84$$

$$AF = \frac{N_0}{AL} = \frac{1000}{6.84} = 146.2 \quad \text{and} \quad CIR = C_1 * AF = 1.25 * 146.2 = 182.75$$

<i>End of week (i)</i>	$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^i N_j}{i}$
1	$\frac{1000 * 0.5 + 10 * 1.25}{1} = 512.5$
2	$\frac{1000 * 0.5 + (10 + 30.1) * 1.25}{2} = 275.06$
3	$\frac{1000 * 0.5 + (40.1 + 50.6) * 1.25}{3} = 204.46$
4	$\frac{1000 * 0.5 + (90.7 + 71.9) * 1.25}{4} = 175.81$
5	$\frac{1000 * 0.5 + (162.6 + 104.4) * 1.25}{5} = 166.75 \Rightarrow \min i.$
6	$\frac{1000 * 0.5 + (267 + 158.8) * 1.25}{6} = 172.04$
7	$\frac{1000 * 0.5 + (425.8 + 216.4) * 1.25}{7} = 186.11$
8	$\frac{1000 * 0.5 + (642.2 + 178.8) * 1.25}{8} = 190.78$
9	$\frac{1000 * 0.5 + (821 + 155.8) * 1.25}{9} = 191.22$
10	$\frac{1000 * 0.5 + (976.8 + 145.8) * 1.25}{10} = 190.33$
11	$\frac{1000 * 0.5 + (1122.6 + 139.1) * 1.25}{11} = 188.83$

.(CIR = 182.75)

> (ACGR₅=166.76)

: Maintenance Models -2-8

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: Cost of maintenance (CM) ()

$$CM = \frac{\text{Maintenance Cost per unit (MC)} * N_0}{\text{Expected life per unit (EL)}}$$

Keyboard

: 4-

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3

<i>End of year (i)</i>	1	2	3	4
<i>Prob. Of failure (P_i)</i>	0.20	0.25	0.42	0.13

45

48

4

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3.1

(())

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$$AL = \sum_{i=1}^4 i * P_i = 1 * 0.20 + 2 * 0.25 + 3 * 0.42 + 4 * 0.13 = 2.48$$

$$AF = \frac{N_0}{AL} = \frac{48}{2.48} = 19.355 \quad \text{and} \quad CIR = C_1 * AF = 3 * 19.355 = 58.065$$

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$$N_0 = 48$$

$$N_1 = N_0 * P_1 = 48 * 0.20 = 9.6$$

$$N_2 = N_0 * P_2 + N_1 * P_1 = 48 * 0.25 + 9.6 * 0.20 = 13.92$$

$$N_3 = N_0 * P_3 + N_1 * P_2 + N_2 * P_1 = 48 * 0.42 + 9.6 * 0.25 + 13.92 * 0.2 = 25.344$$

$$N_4 = N_0 * P_4 + N_1 * P_3 + N_2 * P_2 + N_3 * P_1 = 48 * 0.13 + 9.6 * 0.42 + 13.92 * 0.25 + 25.344 * 0.2 = 18.821$$

<i>End of year (i)</i>	$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^i N_j}{i}$
1	$\frac{45 + 9.6 * 3}{1} = 73.8$
2	$\frac{45 + (9.6 + 13.92) * 3}{2} = 57.78 \Rightarrow \min i.$
3	$\frac{45 + (23.52 + 25.344) * 3}{3} = 63.864$
4	$\frac{45 + (48.864 + 18.821) * 3}{4} = 62.014$

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$$CM = \frac{MC}{EL} * N_0 = \frac{4}{3.1} * 48 = 61.92$$

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57.78 = *58.065* = -

61.92 = -

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<i>Year</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
<i>Maintenance cost / year</i>	<i>200</i>	<i>450</i>	<i>680</i>	<i>850</i>	<i>1300</i>	<i>1600</i>
<i>Resale value</i>	<i>10000</i>	<i>8000</i>	<i>7000</i>	<i>5000</i>	<i>2000</i>	<i>1000</i>

15000

(ans.: 4)

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12200

<i>year</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Maintenance Cost</i>	<i>200</i>	<i>500</i>	<i>800</i>	<i>1200</i>	<i>1800</i>	<i>2500</i>	<i>3200</i>	<i>4000</i>

7500

5

500

400

(ans.:)

10000

B, A

-3

<i>Year</i>		<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>A</i>	<i>Resale</i>	<i>9</i>	<i>9</i>	<i>8</i>	<i>7</i>	<i>5</i>
	<i>Operating cost</i>	<i>1</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>3</i>
<i>B</i>	<i>Resale</i>	<i>8</i>	<i>7</i>	<i>7</i>	<i>6</i>	<i>6</i>
	<i>Operating cost</i>	<i>1</i>	<i>2</i>	<i>2</i>	<i>3</i>	<i>4</i>

B, A

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(ans.: a) 2, 3 ; b) A ; c) A)

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<i>End of week</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
<i>Prob. Of failure</i>	<i>0.09</i>	<i>0.25</i>	<i>0.49</i>	<i>0.85</i>	<i>0.97</i>	<i>1</i>

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(ans.: 2)

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<i>End of week</i>	<i>10</i>	<i>20</i>	<i>30</i>	<i>40</i>	<i>50</i>
<i>Prob. Of failure</i>	<i>0.05</i>	<i>0.15</i>	<i>0.35</i>	<i>0.65</i>	<i>1</i>

(ans.: 20)

2000

Quality Control and Reliability

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3- Charles E. Ebeling " *An introduction to Reliability and Maintainability Engineering* " (1997) , McGraw Hill companies , INC .

[1] ISO (T.Q.M.)

: Quality -1-9

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1914 : **1939** -3

: **Statistical Quality Control (SQC)** -4
1939

Control chart

: **Total Quality Control (TQC)** -5
1945

: **Quality Assurance (QA)** -6
1980

: **Total Quality Management (TQM)** -7
ISO:9000 **1987** **ISO**

: **Total Quality Management (TQM)** -2-9

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. <i>Pareto analysis</i>		-	
. <i>Fish bone chart</i>		-	
. <i>Run chart</i>		-	
. <i>Bar chart</i>		-	
. <i>Scatter chart</i>		-	
. <i>Flow chart</i>		-	
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- Preparation -1

- Planning -2

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ISO : *ISO 9000* -3-9

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- *Technical Committee (TC)* -1

- *Sub-Committee (SC)* -2

- *Task Group (TG)* -3

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- *Control of Records* -4

- *Auditing* -5

- *Corrective Action* -6

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	<i>- ISO 9004</i>	
	<i>- ISO 9000</i>	
<i>. ISO 9003</i>	<i>ISO 9002</i>	<i>ISO 9001</i>
	<i>ISO 8402</i>	
	<i>ISO 10011</i>	
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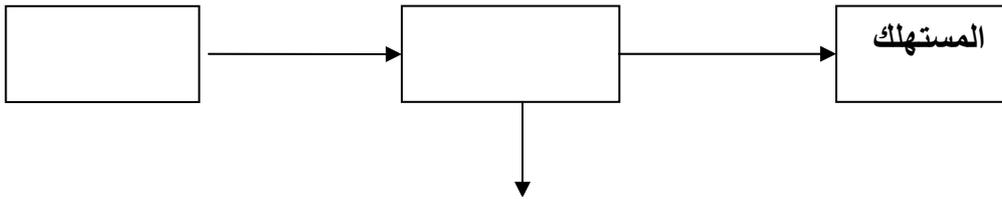
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Quality Control ^[2]

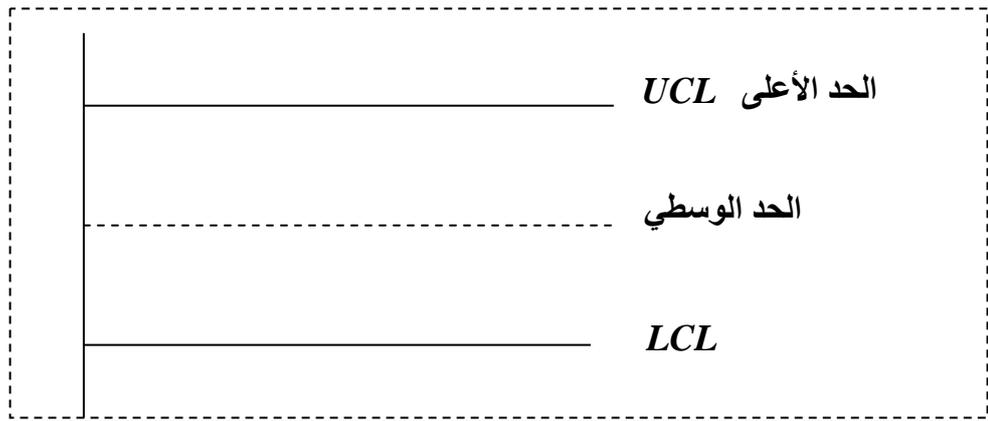
Quality



Quality Control

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Control Chart -1-10



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. R- Chart	-	
. σ - Chart	-	
:	:	-2
. ()	:	
.	:	
. P - Chart	-	
. C - Chart	-	
. U - Chart	-	

: \bar{X} - Chart -1-1-10

%99.7

%0.3

$$\bar{X}_i = \frac{\sum_{j=1}^m X_{ij}}{m}$$

\bar{X}_i

25

-

$$\bar{\bar{X}} = \frac{\sum_{i=1}^n \bar{X}_i}{n}$$

$j = 1, 2, \dots, m$ and $i = 1, 2, \dots, n$

m

n

X_{ij}

\bar{R}

-

$$\bar{R} = \frac{\sum_{i=1}^n R_i}{n} \text{ where } R_i = X_{iL} - X_{iS}$$

i

X_{iL}

i

X_{iS}

:

-

$$UCL(\bar{X}) = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A_2 \bar{R}$$

m

A_2

-

: R - Chart -2-1-10

%99.7

$$\bar{R} = \frac{\sum_{i=1}^n R_i}{n} \quad : \quad 25 \quad -$$

$$UCL(R) = D_4 \bar{R} \quad \text{and} \quad LCL(R) = D_3 \bar{R}$$

$$. m \quad D_4, D_3 \quad -$$

()

: σ - chart -3-1-10

:

:

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^m X_{ij}^2 - m \bar{X}_i^2}{m-1}} \quad \text{and} \quad \bar{\sigma} = \frac{\sum_{i=1}^n \sigma_i}{n}$$

:

$$UCL(\sigma) = B_2 \bar{\sigma} \quad \text{and} \quad LCL(\sigma) = B_1 \bar{\sigma}$$

()

: D_4, D_3, B_2, B_1, A_2

<i>m</i>	<i>A</i> ₂	<i>B</i> ₁	<i>B</i> ₂	<i>D</i> ₃	<i>D</i> ₄
2	1.880	0	3.267	0	3.268
3	1.023	0	2.568	0	2.574
4	0.729	0	2.266	0	2.282
5	0.577	0	2.-089	0	2.114
6	0.483	0.030	1.970	0	2.004
7	0.419	0.118	1.882	0.076	1.924
8	0.373	0.185	1.815	0.136	1.864
9	0.337	0.229	1.761	0.816	1.816
10	0.308	0.284	1.716	0.223	1.777

(mm)

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<i>no. of sample</i>	<i>X</i> ₁	<i>X</i> ₂	<i>X</i> ₃	<i>X</i> ₄	<i>no. of sample</i>	<i>X</i> ₁	<i>X</i> ₂	<i>X</i> ₃	<i>X</i> ₄
1	36	40	40	39	14	35	36	35	36
2	39	40	36	36	15	35	36	36	36
3	36	36	36	39	16	35	35	39	36
4	40	39	36	40	17	37	40	41	39
5	39	39	40	39	18	35	36	36	39
6	40	36	36	36	19	36	40	39	36
7	36	36	39	36	20	35	34	34	34
8	41	41	40	37	21	36	40	35	35
9	36	35	35	36	22	36	36	35	36
10	36	36	36	36	23	35	39	37	41
11	36	39	39	40	24	39	40	40	39
12	36	36	36	36	25	36	36	36	39
13	36	36	36	39					

:

*R*_{*i*}

\bar{X}_i

:

<i>no. of sample</i>	\bar{X}_i	<i>R</i> _{<i>i</i>}	<i>no. of sample</i>	\bar{X}_i	<i>R</i> _{<i>i</i>}
1	38.75	4	14	35.50	1
2	37.75	4	15	35.75	1
3	36.75	3	16	36.25	4
4	38.75	4	17	39.25	4
5	39.25	1	18	36.50	4
6	37.00	4	19	37.85	4
7	36.75	3	20	34.25	1
8	39.75	4	21	36.50	5
9	35.50	1	22	35.75	1
10	36.00	0	23	38.00	6
11	38.50	4	24	39.50	1
12	36.00	0	25	36.75	3
13	36.75	3	∑	929.25	70

: \bar{R} $\bar{\bar{X}}$

$$\bar{R} = \frac{\sum_{i=1}^n R_i}{n} = \frac{70}{25} = 2.8 \qquad \bar{\bar{X}} = \frac{\sum_{i=1}^n \bar{X}_i}{n} = \frac{929.25}{25} = 37.17$$

. $A_2 = 0.729$ with $m=4$ \bar{X} - Chart (

$$UCL(\bar{X}) = \bar{\bar{X}} + A_2 \bar{R} = 37.17 + 0.729 * 2.8 = 39.211$$

$$LCL(\bar{X}) = \bar{\bar{X}} - A_2 \bar{R} = 37.17 - 0.729 * 2.8 = 35.129$$

:

no. of sample	\bar{X}_i	R_i
5	39.25	1
8	39.75	4
17	39.25	4
20	34.25	1
24	39.50	1

$\bar{\bar{X}}_{new}$

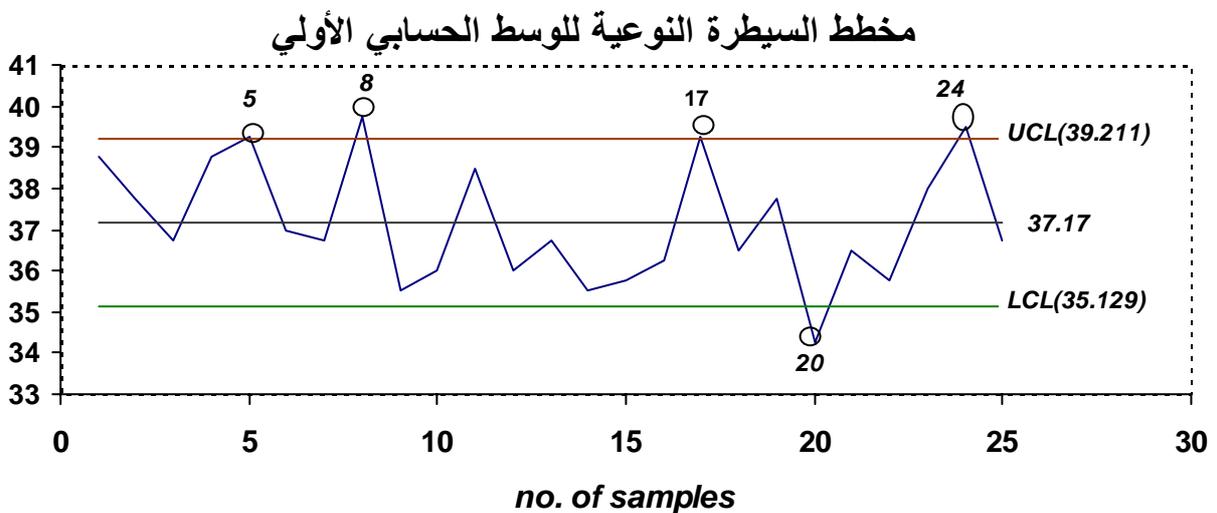
: \bar{R}_{new}

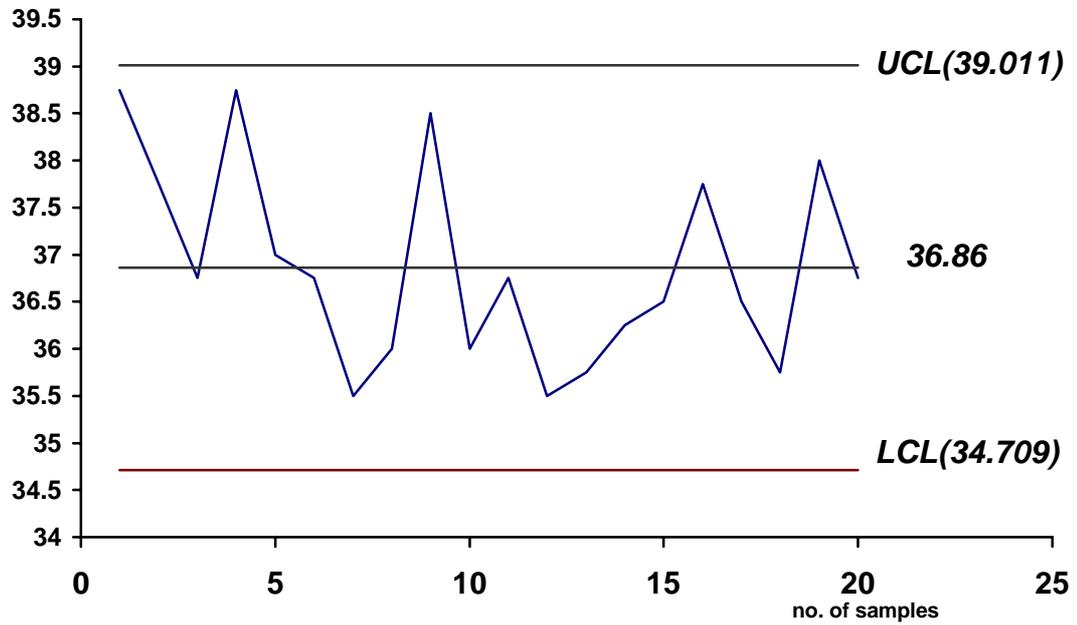
$$\bar{\bar{X}}_{new} = \frac{929.25 - 192}{25 - 5} = 36.86 \quad \text{and} \quad \bar{R}_{new} = \frac{70 - 11}{25 - 5} = 2.95$$

:

$$UCL(\bar{X})_{new} = \bar{\bar{X}}_{new} + A_2 \bar{R}_{new} = 36.86 + 0.729 * 2.95 = 39.011$$

$$LCL(\bar{X})_{new} = \bar{\bar{X}}_{new} - A_2 \bar{R}_{new} = 36.86 - 0.729 * 2.95 = 34.709$$

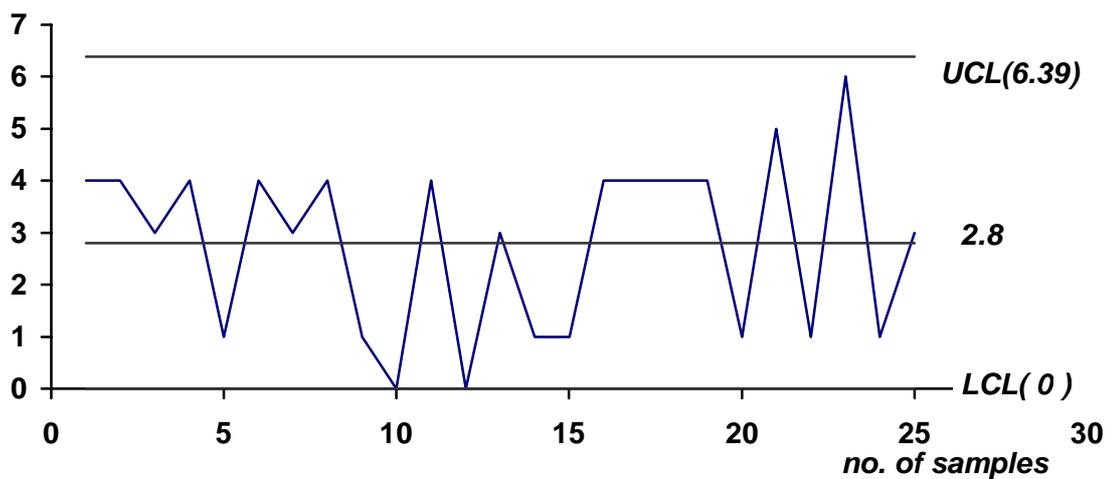




: *R-Chart* (

$$\bar{R} = 2.8 \quad , \quad D_3 = 0 \quad , \quad D_4 = 2.282$$

$$UCL(R) = D_4 \bar{R} = 2.282 * 2.8 = 6.39 \quad \text{and} \quad LCL(R) = D_3 \bar{R} = 0 * 2.8 = 0$$



R_i

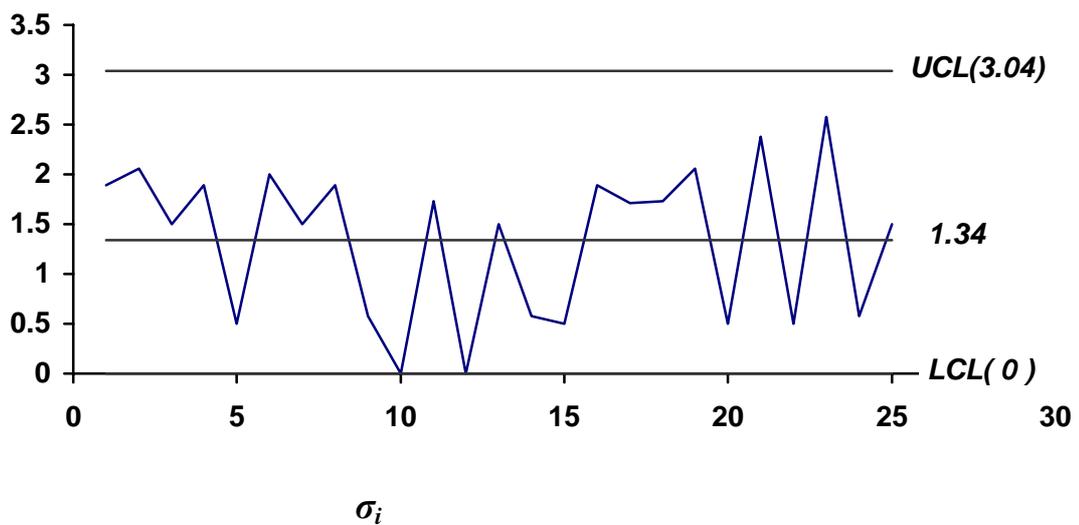
: σ - chart

no. of samples	σ_i	no. of samples	σ_i
1	1.89	14	0.58
2	2.06	15	0.50
3	1.50	16	1.89
4	1.89	17	1.71
5	0.50	18	1.73
6	2.00	19	2.06
7	1.50	20	0.50
8	1.89	21	2.38
9	0.58	22	0.50
10	0.00	23	2.58
11	1.73	24	0.58
12	0.00	25	1.50
13	1.50	Σ	33.55

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^m X_{ij}^2 - m \bar{X}_i^2}{m-1}} \Rightarrow \sigma_1 = \sqrt{\frac{36^2 + 40^2 + 40^2 + 39^2 - 4 * (38.75)^2}{4-1}} = 1.89 \dots etc.$$

$$\bar{\sigma} = \frac{33.55}{25} = 1.34$$

$$UCL(\sigma) = B_2 \times \bar{\sigma} = 2.266 \times 1.34 = 3.04 \quad \text{and} \quad LCL(\sigma) = B_1 \times \bar{\sigma} = 0 \times 1.34 = 0$$



$$UCL(\bar{P}) = \bar{P} + 3 * \sqrt{\frac{\bar{P}(1-\bar{P})}{m}} \quad \text{and} \quad LCL(\bar{P}) = \bar{P} - 3 * \sqrt{\frac{\bar{P}(1-\bar{P})}{m}}$$

$$\bar{P} = \frac{\sum_{i=1}^n P_i}{n}$$

P_i

- 100)

200

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: 2-

2, 3, 4, 0, 5, 2, 13, 2, 3, 10, 3, 0, 4, 2, 1, 4, 5, 3, 5, 4, 1, 2, 6, 2, 5

<i>n</i>	<i>defective</i>	P_i	<i>n</i>	<i>defective</i>	P_i
1	2	0.010	14	2	0.010
2	3	0.015	15	1	0.005
3	4	0.020	16	4	0.020
4	0	0.000	17	5	0.025
5	5	0.025	18	3	0.015
6	2	0.010	19	5	0.025
7	13	0.065	20	4	0.020
8	2	0.010	21	1	0.005
9	3	0.015	22	2	0.010
10	10	0.050	23	6	0.030
11	3	0.015	24	2	0.010
12	0	0.000	25	5	0.025
13	4	0.020	Σ	91	0.455

$$\bar{P} = \frac{\sum_{i=1}^n P_i}{n} = \frac{0.455}{25} = 0.0182$$

$$UCL(P) = \bar{P} + 3 * \sqrt{\frac{\bar{P}(1-\bar{P})}{m}} = 0.0182 + 3 * \sqrt{\frac{0.0182 * (1-0.0182)}{200}} = 0.0466$$

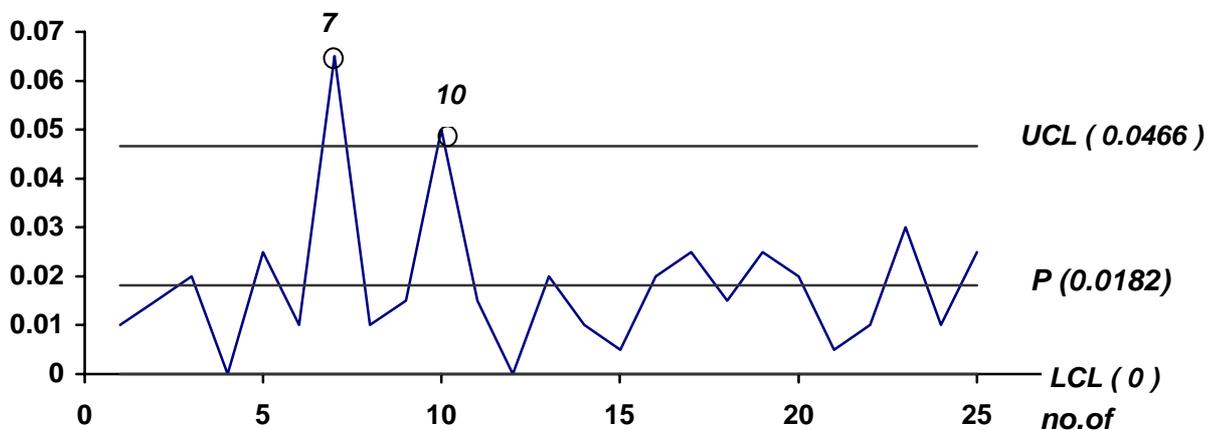
$$LCL(P) = \bar{P} - 3 * \sqrt{\frac{\bar{P}(1-\bar{P})}{m}} = 0.0182 - 3 * \sqrt{\frac{0.0182 * (1-0.0182)}{200}} = -0.010 \cong 0$$

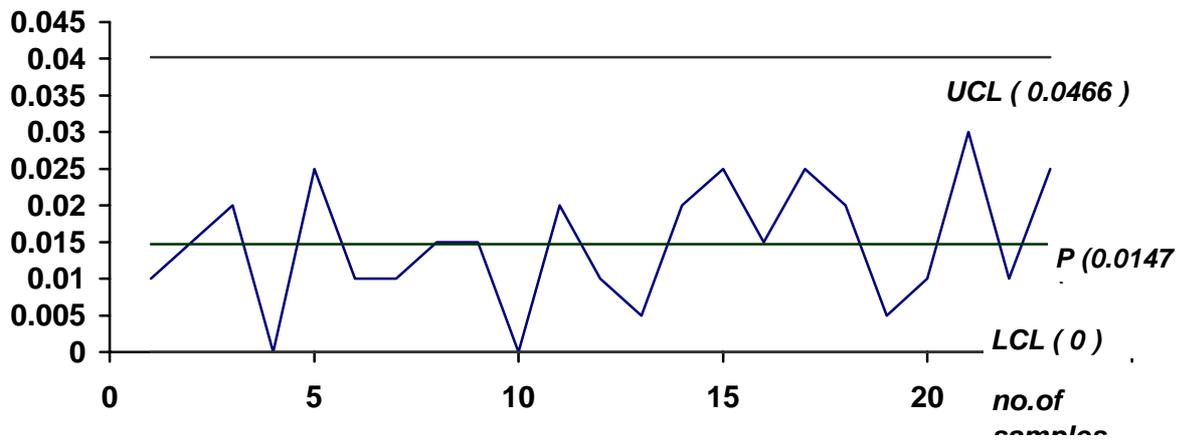
<i>n</i>	<i>Def.</i>	<i>P_i</i>
7	13	0.065
10	10	0.050
Σ	23	0.115

$$\bar{P}_{new} = \frac{0.455 - 0.115}{25 - 2} = 0.0147$$

$$UCL(P)_{new} = 0.0147 + 3 * \sqrt{\frac{0.0147 * (1-0.0147)}{200}} = 0.0402$$

$$LCL(P)_{new} = 0.0147 - 3 * \sqrt{\frac{0.0147 * (1-0.0147)}{200}} = -0.0108 \cong 0$$





: _____ -2-10

$$N_{\sigma} = \frac{T}{\sigma} \quad : \quad N_{\sigma} \quad \frac{3\sigma}{T} \leq 1 \quad :$$

:

N_σ	1/2 area	Def. %
0.00	0.500	100.0
0.25	0.401	80.2
0.50	0.309	61.8
0.75	0.227	45.4
1.00	0.159	30.8
1.25	0.106	21.2
1.50	0.067	13.4
1.75	0.040	8.0
2.00	0.023	4.6
2.25	0.012	2.4
2.50	0.006	1.2
2.75	0.003	0.6
3.00	0.001	0.2

:

: 3-

$$\sigma = 1.34 \quad 1-$$

:

$$T = 39.011 - 36.86 = 2.151 \quad :$$

$$\frac{3\sigma}{T} = \frac{3 * 1.34}{2.151} = 1.87 > 1$$

:

$$N_\sigma = \frac{T}{\sigma} = \frac{2.151}{1.34} = 1.6$$

1.6

:

%11

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d

d

d

Consumer's risk

Producer's risk

d_1 . d_2
)

d_1

$(1-P_1)$

P_1

d_2

(

:

P_2

:

: _____ -

n

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d

d

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d

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:

\cdot d_{22}
 \vdots
 \vdots
 $\cdot r$ d_r

n p : Binomial distribution
 x

\vdots
 $P(x) = C_x^n \cdot p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$

Where : $C_x^n = \frac{n!}{x!(n-x)!}, \quad n! = n(n-1)(n-2)\dots 2.1$

10

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0.03 0.01

0.025

.0.20

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= *P*

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+

$$P = P_1(0) + P_1(1) \cdot P_2(0) + P_1(1) \cdot P_2(1) + P_1(2) \cdot P_2(0)$$

.

x

*P*₁(*x*)

.

x

*P*₂(*x*)

:

$$P_1(x) = C_x^{10} p^x (1-p)^{10-x}, \quad x=0,1,2,\dots,10$$

$$P_1(0) = C_0^{10} p^0 (1-p)^{10-0} = (1-p)^{10}$$

$$P_1(1) = C_1^{10} p^1 (1-p)^{10-1} = 10p(1-p)^9$$

$$P_1(2) = C_2^{10} p^2 (1-p)^{10-2} = 45p^2(1-p)^8$$

$$P_2(x) = C_x^{20} p^x (1-p)^{20-x}, \quad x=0,1,2,\dots,20$$

$$P_2(0) = C_0^{20} p^0 (1-p)^{20-0} = (1-p)^{20}$$

$$P_2(1) = C_1^{20} p^1 (1-p)^{20-1} = 20p(1-p)^{19}$$

$$P(p) = (1-p)^{10} + 10p(1-p)^9(1-p)^{20} + 10p(1-p)^9 \cdot 20p(1-p)^{19} + 45p^2(1-p)^8(1-p)^{20}$$

$$P(p) = (1-p)^{10} [1 + 10p(1-p)^{18}(1 + 23.5p)]$$

:

p

<i>p</i>	0.01	0.03	0.05	0.10	0.15	0.20	0.25	0.30
<i>P</i>	0.998	0.955	0.857	0.524	0.269	0.129	0.062	0.029

: 0.025

$$1 - P(0.025) = 1 - (1 - 0.025)^{10} \{1 + 10 * 0.025 * (1 - 0.025)^{18} (1 + 23.5 * 0.025)\} = 0.03$$

. %3

. %12.9

0.20

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4 3 2 6 3 1 3 2 9 3 5 3 2 5 2 2 1 3 2 1

(ans. 0, 0.1533)

5

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<i>n</i>	<i>measurements</i>				
	X_1	X_2	X_3	X_4	X_5
1	1.04	1.01	0.98	1.02	1.00
2	1.02	0.97	0.96	1.01	1.02
3	1.01	1.07	0.99	1.03	1.00
4	0.98	0.97	1.02	0.98	0.98
5	0.99	1.03	0.98	1.02	1.01
6	1.02	0.95	1.04	1.02	0.95
7	1.00	0.99	1.01	1.02	1.01
8	0.99	1.02	1.00	1.04	1.09
9	1.03	1.04	0.99	1.02	0.94
10	1.02	0.98	1.00	0.99	1.02

. (:)

(ans. a) 0.9679, 1.0429, b) 0, 0.137)

-3

:

<i>Date</i>	<i>Sample size</i>	<i>No. of defectives</i>	<i>Date</i>	<i>Sample size</i>	<i>No. of defectives</i>
1	200	3	12	200	3
2	200	1	13	200	6
3	200	0	14	200	8
4	200	2	15	200	5
5	200	4	16	200	9
6	200	1	17	200	3
7	200	2	18	200	1
8	200	0	19	200	0
9	200	3	20	200	2
10	200	2	21	200	3
11	200	1	22	200	1

(ans. 0, 0.0332)

N	X_1	X_2	X_3	X_4	X_5	X_6
1	0.498	0.492	0.510	0.505	0.504	0.487
2	0.482	0.491	0.502	0.481	0.496	0.492
3	0.501	0.512	0.503	0.499	0.498	0.511
4	0.498	0.486	0.502	0.503	0.510	0.501
5	0.500	0.507	0.509	0.498	0.512	0.518
6	0.476	0.492	0.496	0.521	0.505	0.490
7	0.483	0.487	0.495	0.488	0.502	0.486
8	0.502	0.500	0.511	0.496	0.500	0.503
9	0.492	0.504	0.472	0.515	0.498	0.487
10	0.511	0.522	0.513	0.518	0.520	0.516
11	0.488	0.512	0.501	0.498	0.492	0.498
12	0.504	0.502	0.496	0.501	0.491	0.496
13	0.501	0.413	0.499	0.496	0.508	0.502
14	0.489	0.491	0.496	0.510	0.508	0.503
15	0.511	0.499	0.508	0.503	0.496	0.505

(ans. 0.0002 , 0.0158)

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-5

$$\sum \bar{X}_i = 160.25$$

$$\sum \sigma_i = 2.05$$

$$\sum R_i = 2.19$$

6.38 6.44 6.28 6.58 : ()

:

:

(((

(ans. a) yes , b) no , c) yes)

-Reliability ^[3]

() : Reliability

:

-1

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-2

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. () ()

Continuous Random Variable

T

:

Reliability function

$T \geq 0$

$$R(t) = \Pr(T \geq t) \quad \forall t \geq 0$$

where 1) $0 \leq R(t) \leq 1$

2) $R(0) = 1$

3) $\lim_{t \rightarrow \infty} R(t) = 0$

: t

$$F(t) = 1 - R(t) = \Pr(T < t)$$

where 1) $0 \leq F(t) \leq 1$

2) $F(0) = 0$

3) $\lim_{t \rightarrow \infty} F(t) = 1$

Cumulative distribution function (c.d.f.)

$F(t)$

. **Failure function**

: [a , b]

$$\Pr(a \leq T \leq b) = F(b) - F(a) = R(a) - R(b) = \int_a^b f(t) dt$$

Probability distribution function (p.d.f.)

$f(t)$

:

$$f(t) = \frac{d}{dt}F(t) = -\frac{d}{dt}R(t)$$

where 1) $f(t) \geq 0$
 2) $\int_0^t f(t) = 1$

$$F(t) = \int_0^t f(u)du \quad \text{and} \quad R(t) = \int_t^\infty f(u)du$$

T : Compressor

$$f(t) = \begin{cases} \frac{0.001}{(0.001t + 1)^2} & t \geq 0 \\ 0 & \text{o.w} \end{cases}$$

0.95

[10 , 100]

$$R(t) = \int_t^\infty \frac{0.001}{(0.001t + 1)^2} dt = -\left[\frac{1}{0.001t + 1} \right]_t^\infty = -\frac{1}{\infty} + \frac{1}{0.001t + 1} \Rightarrow R(t) = \frac{1}{0.001t + 1}$$

$$R(100) = \frac{1}{0.001 * 100 + 1} = 0.909$$

$$R(t) = \frac{1}{0.001t + 1} = 0.95 \Rightarrow t = \frac{1}{0.001} \left(\frac{1}{0.95} - 1 \right) = 52.6 \text{ hrs.}$$

$$Pr(10 \leq T \leq 100) = R(10) - R(100) = \frac{1}{0.001 * 10 + 1} - \frac{1}{0.001 * 100 + 1} = 0.081$$

: Mean Time of Failure (MTTF)

$$MTTF = E(t) = \int_0^\infty t \cdot f(t) dt = \int_0^\infty R(t) dt$$

Variance

$$\sigma^2 = V(t) = \int_0^\infty t^2 \cdot f(t) dt - (MTTF)^2$$

Standard deviation

-2-

$$f(t) = \begin{cases} 0.002 * e^{-0.002t} & t \geq 0 \\ 0 & o.w \end{cases}$$

MTTF

$$MTTF = \int_0^{\infty} t \cdot f(t) dt = \int_0^{\infty} 0.002t * e^{-0.002t} dt$$

$$MTTF = -\left[t * e^{-0.002t} + \int e^{-0.002t} \right]_0^{\infty} = \left[-t * e^{-0.002t} - \frac{1}{0.002} e^{-0.002t} \right]_0^{\infty} = 500 \text{ hrs.}$$

where $\lim_{t \rightarrow \infty} \frac{t}{e^{0.002t}} = 0$ by L'Hopital rule

$$\sigma^2 = \int_0^{\infty} t^2 f(t) dt - (MTTF)^2 = \int_0^{\infty} t^2 (0.002 * e^{-0.002t}) dt - (500)^2$$

$$\sigma^2 = \left[-t^2 e^{-0.002t} - \frac{2t}{0.002} e^{-0.002t} - \frac{2}{0.00004} e^{-0.002t} \right]_0^{\infty} - (500)^2 = 250000 \text{ hrs.}^2$$

By L'Hopital rule $\lim_{t \rightarrow \infty} \frac{t^2}{e^{0.002t}} = 0$ and $\lim_{t \rightarrow \infty} \frac{t}{e^{0.002t}} = 0$

$$\sigma = \sqrt{\sigma^2} = \sqrt{250000} = 500 \text{ hrs.}$$

Failure

: Hazard rate function

$t + \Delta t, t$

rate function

$$Pr(t \leq T \leq t + \Delta t / T \geq t) = \lambda(t) = \frac{f(t)}{R(t)} \Rightarrow R(t) = e^{-\int_0^t \lambda(t) dt}$$

$$\lambda(t) = 5 * 10^{-6} t$$

-3-

t
0.98

$$R(t) = e^{-\int_0^t \lambda(t) dt} \Rightarrow 0.98 = e^{-\int_0^t 5 \cdot 10^{-6} t dt}$$

$$0.98 = e^{-2.5 \cdot 10^{-6} t^2} \Rightarrow t = \sqrt{\frac{\ln 0.98}{-2.5 \cdot 10^{-6}}} \cong 90 \text{ hrs.}$$

-4-

$$R(t) = 1 - \frac{t^2}{a^2} \quad \text{where } 0 \leq t \leq a$$

: ()

a

p.d.f.

(

t

()

(

MTTF

(

$$f(t) = -\frac{d}{dt} R(t) = -\frac{d}{dt} \left(1 - \frac{t^2}{a^2} \right) = \frac{2t^2}{a^2}$$

(-)

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{2t}{a^2} \div \left(1 - \frac{t^2}{a^2} \right) = \frac{2t}{a^2 - t^2}$$

(

$$MTTF = \int_0^a R(t) dt = \int_0^a \left(1 - \frac{t^2}{a^2} \right) dt = \left[t - \frac{t^3}{3a^2} \right]_0^a = \frac{2}{3} a$$

(

: Conditional Reliability

: T_0

t

$$R(t / T_0) = \exp \left(- \int_{T_0}^{T_0+t} \lambda(t) dt \right) = \frac{R(T_0 + t)}{R(T_0)}$$

$$\lambda(t) = \frac{1}{2000} \left(\frac{t}{1000} \right)^{-0.5} \quad \text{where } t \text{ in years}$$

-5-

$$1) R(t) = 0.90 \quad \text{and} \quad 2) R(t / 0.5) = 0.90 \quad :$$

$$R(t) = e^{-\int_0^t \lambda(t) dt} = e^{-\int_0^t \frac{1}{2000} \left(\frac{t}{1000} \right)^{-0.5} dt} = e^{-\left(\frac{t}{1000} \right)^{0.5}} = 0.90$$

(1 -)

$$\Rightarrow t = 1000 * (\ln 0.90)^2 = 11.1 \text{ years}$$

$$R(t/0.5) = \frac{R(t+0.5)}{R(0.5)} = \frac{e^{-\left(\frac{t+0.5}{1000}\right)^{0.5}}}{e^{-\left(\frac{0.5}{1000}\right)^{0.5}}} = 0.90 \quad (2)$$

$$\Rightarrow t = 1000 \left[\left(\frac{0.5}{1000} \right)^{0.5} - \ln 0.90 \right]^2 - 0.5 = 15.813 \text{ years}$$

. $R(t/T_0)$ $\lambda(t) = \lambda t$ where $\lambda > 0$ **-6-**

$$R(t) = e^{-\int_0^t \lambda(t) dt} = e^{-\int_0^t \lambda t dt} = e^{-\frac{\lambda t^2}{2}}$$

$$R(t/T_0) = \frac{R(t+T_0)}{R(T_0)} = \frac{e^{-\frac{\lambda}{2}(t+T_0)^2}}{e^{-\frac{\lambda}{2}T_0^2}} = e^{-\frac{\lambda}{2}(t^2+2T_0t)}$$

. 4- $R(t/T_0)$ **-7-**

$$R(t/T_0) = \frac{R(t+T_0)}{R(T_0)} = \frac{1 - \frac{(t+T_0)^2}{a^2}}{1 - \frac{T_0^2}{a^2}} = \frac{a^2 - (t+T_0)^2}{a^2 - T_0^2}$$

: The Exponential Reliability function

exponential distribution

. *Constant Failure Rate (C.F.R.)*

: ($\lambda(t) = \lambda$ $t \geq 0$:)

$$R(t) = e^{-\lambda t} \quad , \quad F(t) = 1 - e^{-\lambda t}$$

$$f(t) = \lambda e^{-\lambda t} \quad , \quad MTTF = \frac{1}{\lambda}$$

$$\sigma^2 = \frac{1}{\lambda^2} \quad \text{and} \quad R(t/T_0) = R(t)$$

0.00034

Microwave transmitter **-8-**

:

$$\lambda(t) = 0.00034 \quad , \quad R(t) = e^{-0.00034t} = R(t/T_0)$$

$$F(t) = 1 - e^{-0.00034t} \quad , \quad f(t) = 0.00034 * e^{-0.00034t}$$

$$MTTF = \frac{1}{0.00034} = 2941.18hrs. \quad \text{and} \quad \sigma^2 = \frac{1}{(0.00034)^2} = 8650519hrs.^2$$

:

$$t = 30 * 24 = 720 \Rightarrow R(720) = e^{-0.00034*720} = 0.783$$

: Weibull distribution in reliability

:

$$\lambda(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \quad , \quad \theta, \beta > 0 \quad , \quad t \geq 0$$

where θ is scale parameter
 β is shape parameter

:

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta} \quad , \quad F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^\beta}$$

$$f(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^\beta} \quad , \quad MTTF = \theta \cdot \Gamma\left(\frac{1}{\beta} + 1\right)$$

$$\sigma^2 = \theta^2 \left[\Gamma\left(\frac{2}{\beta} + 1\right) - \left(\Gamma\left(\frac{1}{\beta} + 1\right)\right)^2 \right]$$

$$\text{and } R(t + T_0) = \exp\left[-\left(\frac{t + T_0}{\theta}\right)^\beta + \left(\frac{T_0}{\theta}\right)^\beta\right]$$

$$\text{where } \Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy = (\alpha - 1) \cdot \Gamma(\alpha - 1)$$

$\Gamma(\alpha)$

:

-9-

: Shape parameter (β) = 1 / 3 and Scale parameter (θ) = 16000

$$1) \lambda(t) = \frac{\frac{1}{3}}{16000} \left(\frac{t}{16000} \right)^{\frac{1}{3}-1} = 0.0132283 * t^{-\frac{2}{3}}$$

$$2) R(t) = e^{-\left(\frac{t}{16000}\right)^{\frac{1}{3}}} = e^{-0.0132283*t^{\frac{1}{3}}}$$

$$3) F(t) = 1 - e^{-0.0132283*t^{\frac{1}{3}}}$$

$$4) f(t) = \frac{\frac{1}{3}}{16000} \left(\frac{t}{16000} \right)^{\frac{1}{3}-1} e^{-\left(\frac{t}{16000}\right)^{\frac{1}{3}}} = 0.0132283 * t^{-\frac{2}{3}} e^{-0.0132283*t^{\frac{1}{3}}}$$

$$5) MTTF = 16000 * \Gamma\left(\frac{1}{3} + 1\right) = 16000 * \Gamma(4) = 16000 * 3! = 96000 \text{hrs.}$$

$$6) \sigma^2 = (16000)^2 \left[\Gamma\left(\frac{2}{3} + 1\right) - \left(\Gamma\left(\frac{1}{3} + 1\right) \right)^2 \right] = (16000)^2 \left[\Gamma(7) - (\Gamma(4))^2 \right]$$

$$= (16000)^2 [6! - (3!)^2] = 1.75104 * 10^{11} \Rightarrow \sigma = 418454.3$$

$$7) R(t/T_0) = \exp \left[- \left(\frac{t + T_0}{16000} \right)^{\frac{1}{3}} + \left(\frac{T_0}{16000} \right)^{\frac{1}{3}} \right]$$

$$: \quad R(t/10) = 0.90$$

$$0.90 = \exp \left[- \left(\frac{t + 10}{16000} \right)^{\frac{1}{3}} + \left(\frac{10}{16000} \right)^{\frac{1}{3}} \right]$$

$$\Rightarrow t = 16000 \left[\left(\frac{10}{16000} \right)^{\frac{1}{3}} - \ln 0.90 \right]^3 - 10 = 101.24 \text{hrs.}$$

: _____

. *Serial configuration* -1

. *Parallel configuration* -2

. *Combined series-parallel system* -3

: *Serial configuration* -1

:



: t System Reliability $R_S(t)$

$$R_S(t) = \prod_{i=1}^n R_i(t)$$

: exponential dist.

$$R_S(t) = \prod_{i=1}^n e^{-\lambda_i t} \Rightarrow R_S(t) = e^{-\lambda_s t} \text{ where } \lambda_s = \sum_{i=1}^n \lambda_i$$

: Weibull dist.

$$R_S(t) = \prod_{i=1}^n \exp\left[-\left(\frac{t}{\theta_i}\right)^{\beta_i}\right] \Rightarrow R_S(t) = \exp\left[-\sum_{i=1}^n \left(\frac{t}{\theta_i}\right)^{\beta_i}\right] \text{ and } \lambda(t) = \sum_{i=1}^n \frac{\beta_i}{\theta_i} \left(\frac{t}{\theta_i}\right)^{\beta_i-1}$$

-10-

MTTF

$R_S(100) = 0.95$

$$R_S(100) = e^{-100\lambda_s} = 0.95 \Rightarrow \lambda_s = \frac{\ln 0.95}{-100} = 0.0005129$$

$$\lambda = \frac{0.0005129}{4} = 0.000128$$

$$MTTF = \frac{1}{\lambda} = \frac{1}{0.000128} = 7812.5$$

-11-

Component	Scale parameter	Shape parameter
1	100	1.20
2	150	0.87
3	510	1.80
4	720	1.00

$t = 10$

$$R_S(t) = \exp \left[- \left(\left(\frac{t}{100} \right)^{1.20} + \left(\frac{t}{150} \right)^{0.87} + \left(\frac{t}{510} \right)^{1.80} + \left(\frac{t}{720} \right) \right) \right]$$

$$R_S(10) = \exp \left[- \left(\left(\frac{10}{100} \right)^{1.20} + \left(\frac{10}{150} \right)^{0.87} + \left(\frac{10}{510} \right)^{1.80} + \left(\frac{10}{720} \right) \right) \right] = 0.8415$$

β

n : _____

θ تكون (θ_i) ، وعليه فإن :

$$R(t) = \exp \left[- \left(\frac{t}{\theta} \right)^\beta \right] \quad \text{where} \quad \theta = \left[\sum_{i=1}^n \left(\frac{1}{\theta_i} \right)^\beta \right]^{-\frac{1}{\beta}}$$

Jet engine

-12-

:

$$\theta_5 = 9300 , \theta_4 = 4780 , \theta_3 = 5850 , \theta_2 = 7200 , \theta_1 = 3600 \quad \text{و} \quad \beta = 1.5$$

MTTF

- _____

$$\theta = \left[\left(\frac{1}{3600} \right)^{1.5} + \left(\frac{1}{7200} \right)^{1.5} + \left(\frac{1}{5850} \right)^{1.5} + \left(\frac{1}{4780} \right)^{1.5} + \left(\frac{1}{9300} \right)^{1.5} \right]^{-\frac{1}{1.5}} = 1842.7$$

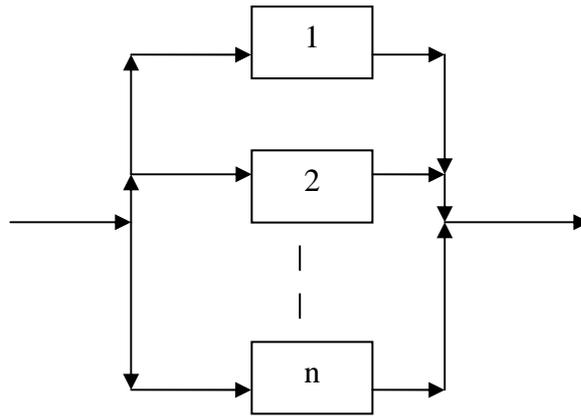
$$MTTF = \theta \cdot \Gamma \left(\frac{1}{\beta} + 1 \right) = 1842.7 * \Gamma \left(\frac{1}{1.5} + 1 \right) = 1842.7 * 0.9033 = 1664.5$$

$$R_S(t) = \exp \left[- \left(\frac{t}{\theta} \right)^\beta \right] = \exp \left[- \left(\frac{t}{1842.7} \right)^{1.5} \right] , \quad t \geq 0$$

: Parallel configuration -3

Redundante

:



$$: \quad t \quad R_S(t)$$

$$R_S(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

$$. \quad t \quad i \quad R_i(t)$$

:

$$R_S(t) = 1 - \prod_{i=1}^n (1 - e^{-\lambda_i t})$$

$$. \quad i \quad \lambda_i$$

-13-

$$R_S(t) = 1 - \prod_{i=1}^n (1 - e^{-\lambda_i t}) = 1 - (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t}) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

$$MTTF = \int_0^{\infty} R_S(t) dt = \int_0^{\infty} (e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}) dt = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

-14-

MTTF

$$R_S(1000) = 0.95$$

$$R_S(1000) = 1 - (1 - e^{-1000\lambda})(1 - e^{-1000\lambda}) \Rightarrow 0.95 = 2e^{-1000\lambda} - e^{-2000\lambda}$$

$$\text{Let } e^{-1000\lambda} = X \Rightarrow X^2 - 2X + 0.95 = 0$$

$$\text{either } X = 1.223606798 \Rightarrow \lambda = -0.000201802 \text{ neglected}$$

$$\text{or } X = 0.776393202 \Rightarrow \lambda = 0.000253096$$

$$MTTF = \frac{1}{\lambda} = \frac{1}{0.000253096} = 3951$$

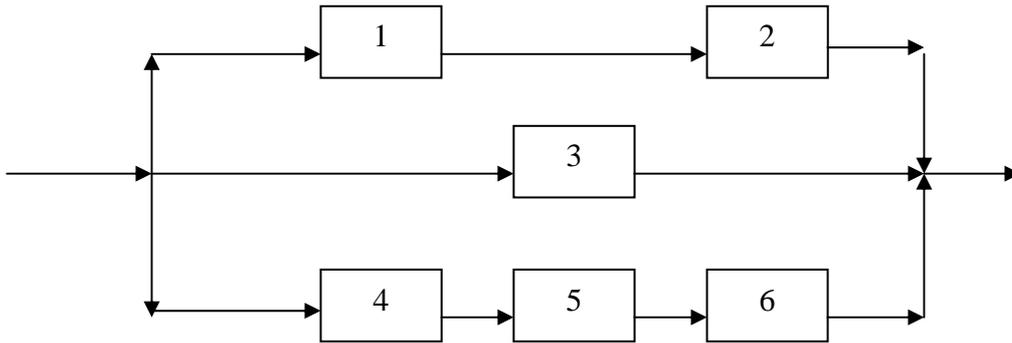
$$MTTF_S = \int_0^{\infty} R_S(t) dt = \frac{1}{\lambda} + \frac{1}{\lambda} - \frac{1}{\lambda + \lambda} = \frac{2}{0.000253096} - \frac{1}{2 * 0.000253096} = 5927$$

-Combined series- parallel system -3

:

) - Series-parallel system - -

:(



:

$$R_{S_1}(t) = \prod_{i=1}^2 R_i(t)$$

$$R_{S_2}(t) = R_3(t)$$

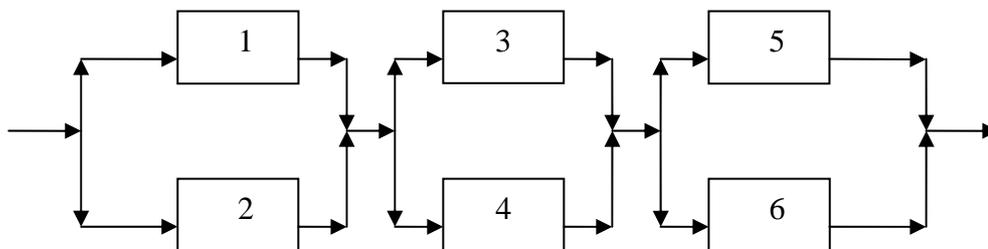
$$R_{S_3}(t) = \prod_{i=4}^6 R_i(t)$$

:

$$R_S(t) = 1 - \prod_{i=1}^3 (1 - R_{S_i}(t))$$

() - parallel-series system - -

:



:

$$R_{S_1}(t) = 1 - \prod_{i=1}^2 (1 - R_i(t))$$

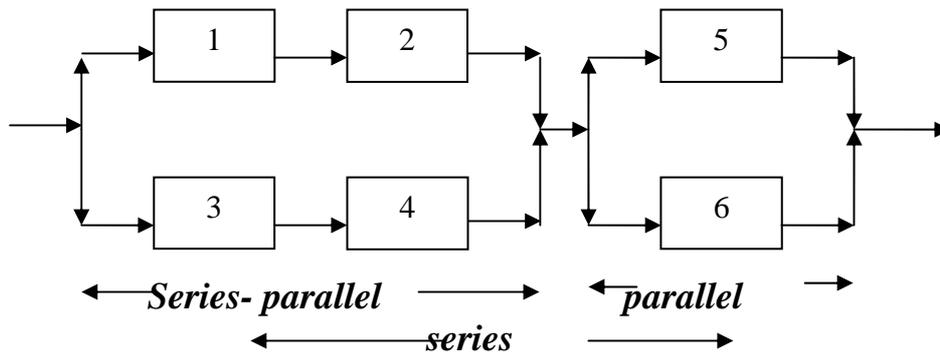
$$R_{S_2}(t) = 1 - \prod_{i=3}^4 (1 - R_i(t))$$

$$R_{S_3}(t) = 1 - \prod_{i=5}^6 (1 - R_i(t))$$

$$R_S(t) = \prod_{i=1}^3 R_{S_i}(t) \quad :$$

Mixed parallel-series and series-parallel - - -

:()



:

$$R_{S_{11}}(t) = \prod_{i=1}^2 R_i(t) \quad :$$

$$R_{S_{12}}(t) = \prod_{i=3}^4 R_i(t) \quad :$$

$$R_{S_1}(t) = 1 - \prod_{i=1}^2 (1 - R_{S_{1i}}(t)) \quad :$$

$$R_{S_2}(t) = 1 - \prod_{i=5}^6 (1 - R_i(t)) \quad :$$

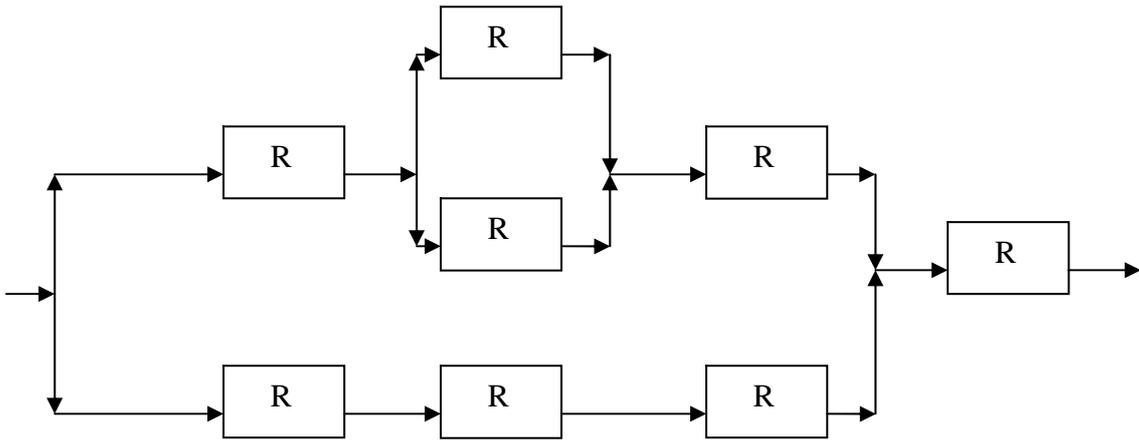
$$R_S(t) = \prod_{i=1}^2 R_{S_i}(t) \quad :$$

:

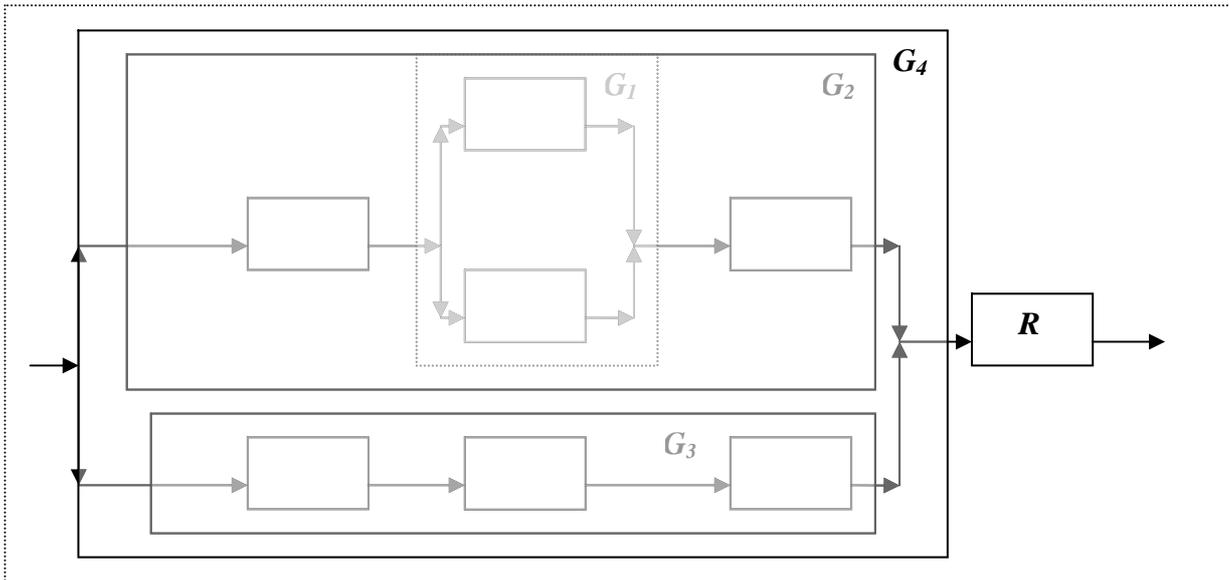
-15-

.(R)

. $R = 0.90$



Ø



$$R_{G1} = 1 - (1 - R)^2 = R(2 - R)$$

$$R_{G2} = R * R_{G1} * R = R^3(2 - R)$$

$$R_{G3} = R^3$$

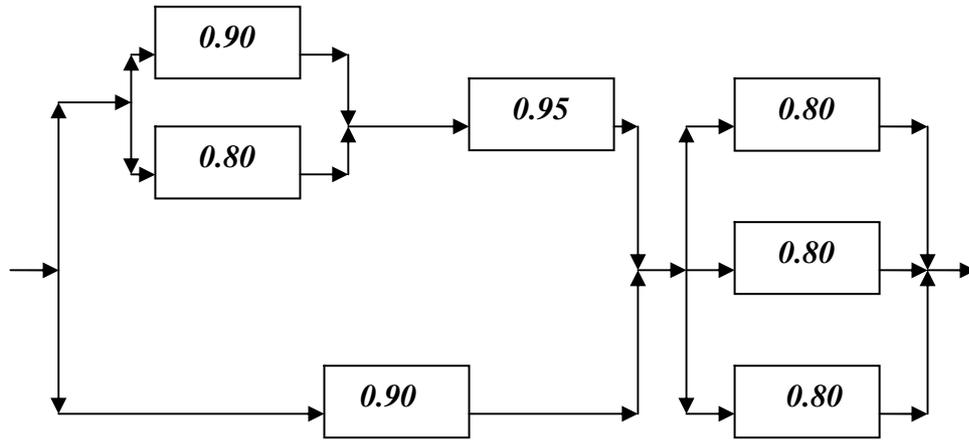
$$R_{G4} = 1 - (1 - R_{G2})(1 - R_{G3}) = R^3(R^4 - 2R^3 - R + 3)$$

$$R_S = R_{G4} * R = R^4(R^4 - 2R^3 - R + 3)$$

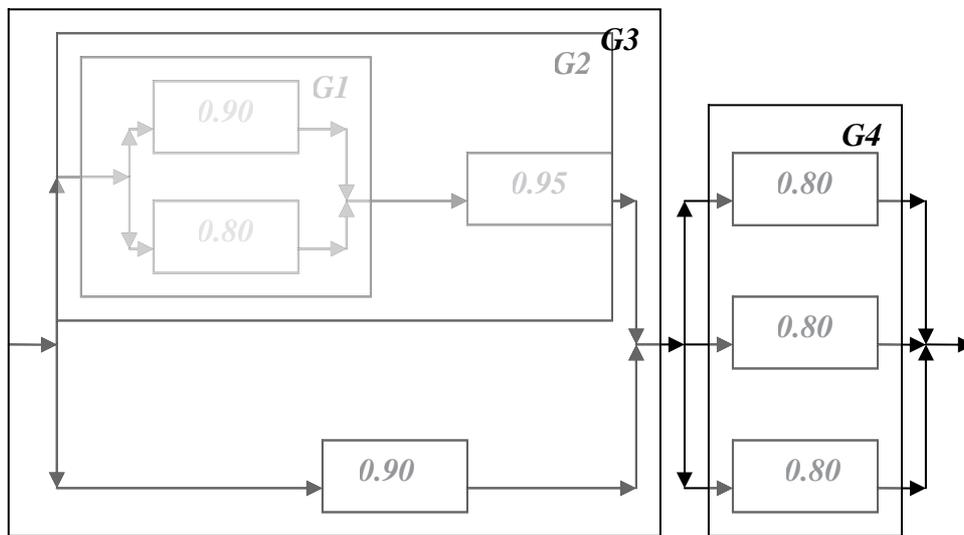
$$\text{at } R = 0.90 \Rightarrow R_S = (0.9)^4 [(0.9)^4 - 2(0.9)^3 - 0.9 + 3] = 0.8517$$

Ø

16 Ø



Ø



$$R_{G1} = 1 - (1 - 0.9)(1 - 0.8) = 0.98$$

$$R_{G2} = R_{G1} * 0.95 = 0.98 * 0.95 = 0.931$$

$$R_{G3} = 1 - (1 - 0.931)(1 - 0.90) = 0.9931$$

$$R_{G4} = 1 - (1 - 0.80)^3 = 0.992$$

$$R_S = R_{G3} * R_{G4} = 0.9931 * 0.992 = 0.9852$$

Ø

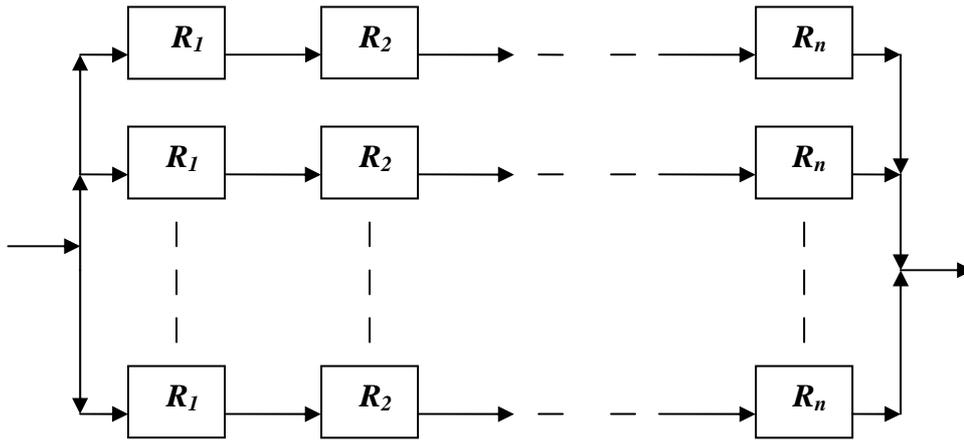
High-level and low-level redundancy

á

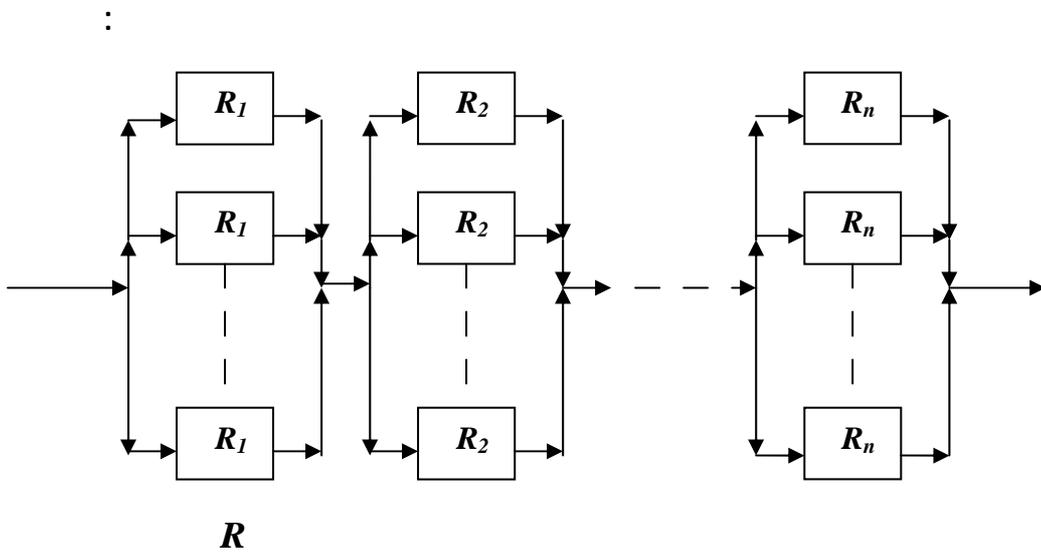
Ø

Ø

High-level redundancy á



- Low-level redundancy -



$$R_{high} = 1 - (1 - R \cdot R)(1 - R \cdot R) = 2R^2 - R^4$$

$$R_{low} = [1 - (1 - R)(1 - R)][1 - (1 - R)(1 - R)] = (2R - R^2)^2$$

$$R_{low} - R_{high} = (2R - R^2)^2 - 2R^2 - R^4 = 2R^2(R - 1)^2 \geq 0$$

$\cdot R_{high}$

R_{low}

:

$\cdot 0.80$

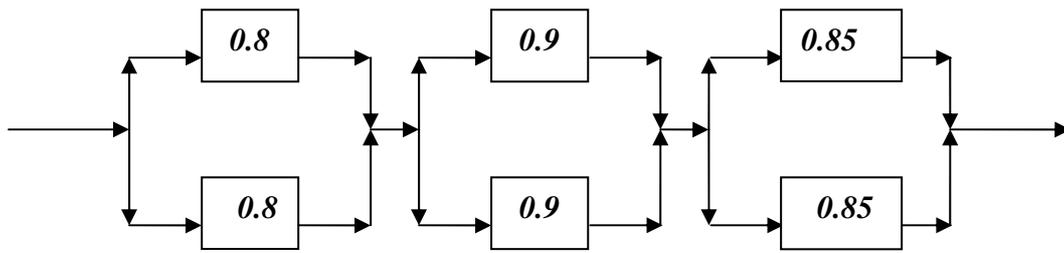
Power Supply

$\cdot 0.90$

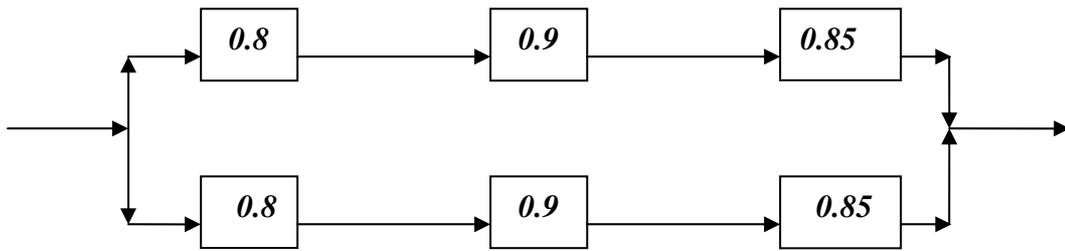
Receiver

$\cdot 0.85$

Amplifier



Low – level Redundancy



High – level Redundancy

$$R_{low} = [1 - (1 - 0.8)^2] [1 - (1 - 0.9)^2] [1 - (1 - 0.85)^2] = 0.929$$

$$R_{high} = 1 - (1 - 0.8 * 0.9 * 0.85)^2 = 0.849$$

$$R(t) = \frac{1}{0.001t + 1}, \quad t \geq 0, \quad t \text{ in hours}$$

: -1

1000 100

IFR DFR

(ans.: a) 0.909, 0.5 ; b) DFR)

$$f(t) = 0.01, \quad 0 \leq t \leq 100 \text{ days}$$

: p.d.f. -2

(((:

(ans.: a) 1-0.01t, b) 1/(100-t), c) 50 ; d) 28.868)

$$f(t) = \frac{3t^2}{10^9}, \quad 0 \leq t \leq 1000 \text{ hrs.}$$

: -3

100

. 0.99

(ans.: a) 0.001, b) 750 ; c) 215.44)

$$R(t) = e^{-\sqrt{0.001t}}, \quad t \geq 0$$

: -4

100

. 50 10

10 0.95

(ans.: a) 0.7289, c) 0.8651 ; d) 12.89)

: () -5

$$f(t) = \frac{200}{(t+10)^3}, \quad t \geq 0$$

CFR IFR DFR 0.95 -
 () -
 (ans. : a) $f(t) = 100 / (t + 10)^2$, 0.8264, b) 10, c) 0.2598 ; d) DFR)

: Uniform dist. p.d.f. -6
 $f(t) = \frac{1}{b}$, $0 \leq t \leq b$

. σ , MTTF, $\lambda(t)$, $R(t)$, $F(t)$:
 (ans. : $F(t) = t/b$, $R(t) = 1 - t/b$, $\lambda(t) = 1 / (1 - b)$, $MTTF = b/2$; $\sigma = b/2\sqrt{3}$)

: Fuel injection -7
 $R(t) = (t + 1)^{-\frac{3}{2}}$, $t \geq 0$ in years
 . 0.19

6
 (ans. : 0.2806 ; 0.0906)

: CFR -8
 . / $\lambda_3 = 0.0025$, $\lambda_2 = 0.015$, $\lambda_1 = 0.002$
 (ans. : a) $R(t) = \exp(-0.0195t)$, b) 51.282)

: MTTF = 1100 CFR -9
 . 200 -
 . 0.90 -

.() 200 -
 (ans.: a) 0.8338, b) 115.897, c) 0.9724)

. 1000 $\lambda = 0.0004$ CFR -10
 (ans.: 0.9608 ; 0.6703) 1000 100

Power unit -11
 : 0.95 5

CFR

-

(ans.: a)292.398 , 584.795 , 194.932 ; 97.466 , b) 0.9975 , 146.199)

-12

0.03125

0.95

(ans.: 4.69)

24

$\lambda(t)=0.00021$

Electronic Circuit Board

-13

(ans.: 0.9932)

1000

Weibull

-14

Shape parameter = 1.4 , Scale parameter = 550

(100)

(ans.: a) 0.9122, b) 501.47 , c) 359.9 , d)110.224) . 0.90

Rectifier

Power Supply

-15

$\theta_2 = 18.5$, $\theta_1 = 12$ مختلفة بحيث $\beta = 2.1$ وإن θ

. $\theta_3=21.5$. أوجد متوسط زمن العطل ، والعمر المصمم لوحدة تجهيز القدرة بمعولية 0.90 .
(ans.: 8.2627 ; 3.1948)

16- ما هو العدد الأعظم من المركبات المتماثلة والمستقلة التي لها توزيع ويبيل بالمعلمات :

Shape parameter = 1.3 , Scale parameter = 10000

100 0.95

(ans.: 21 ; 888.111)

:

-17

$$\lambda(t) = 0.003 \left(\frac{t}{500} \right)^{0.5} , t \geq 0 \text{ in hours}$$

50 -

. 0.90 -

. -

50 50 -

(ans.: a) 0.969 , b) 111.538 , c) 451.375 ; d) 0.9438)

-18

Pressure gauge

: *Shape parameter = 2.1 , Scale parameter = 12000*

. 5000 -

. -

. -

:

. 5000 (1

. (2

. (3

(ans.: a) 0.853 , b) 10628 , 5315.8 , c) 0.403 , d) 0.978 , 18777.638 , 0.356)

-19

Cyclone-365 ()

$$f(t) = \frac{1}{(t+1)^2}, \quad t \geq 0$$

. -

6

0.999 -

(ans.: a) 0.963 , b) 0.111)

-20

: 100 () ()

MTTF = 1000 *CFR* -

مرتبطة على التوالي مع مركبة لها $\theta = 10000$, $\beta = 2$ -

(ans.: b) . 0.00005 بنسبة عطل قدرها *CFR* توزيع أسي

-21

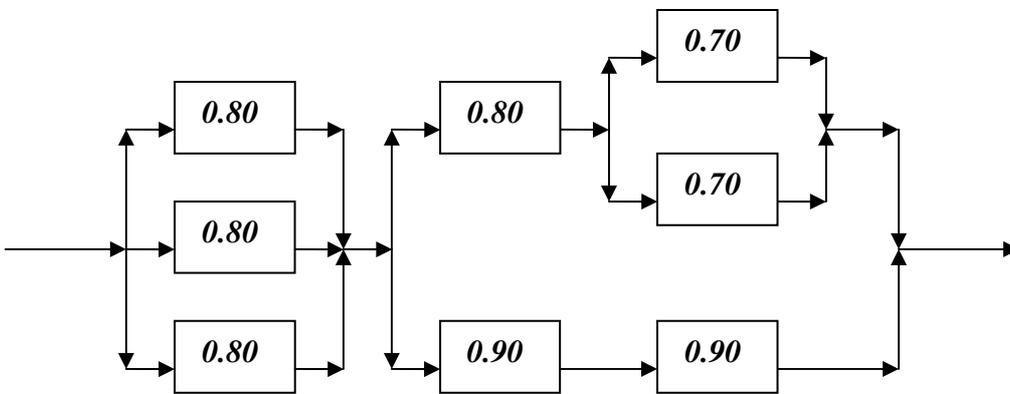
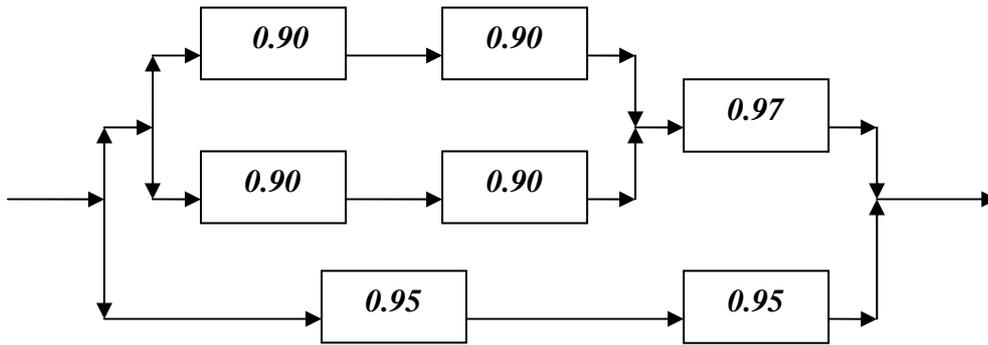
. $\beta = 0.80$

. 0.99 θ

(ans.: 5588.23)

:

-22

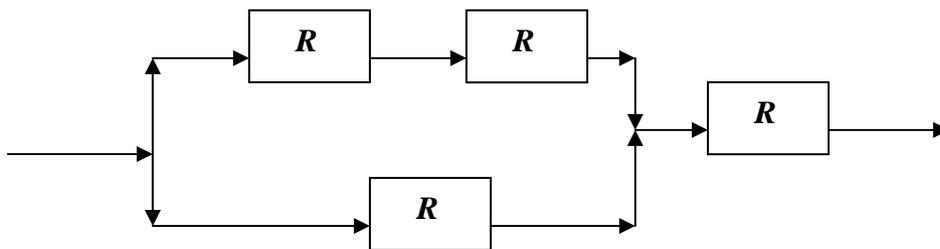


(ans.: a) 0.994 , b) 0.940)

: R

0.99

-23



(ans.: 0.99)

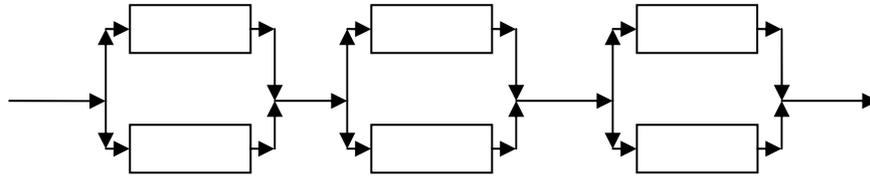
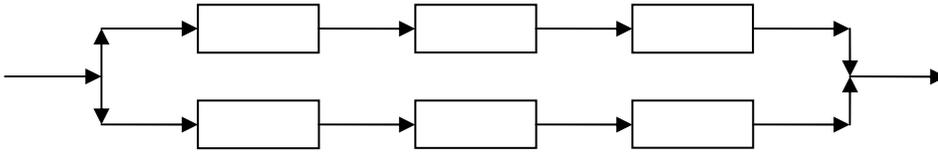
MTTF

-24

. 0.90

100

. CFR



(ans.: a) 787.4 , b) 486.6)

100

-25

:

. $\beta = 1.2$, $\theta = 840$

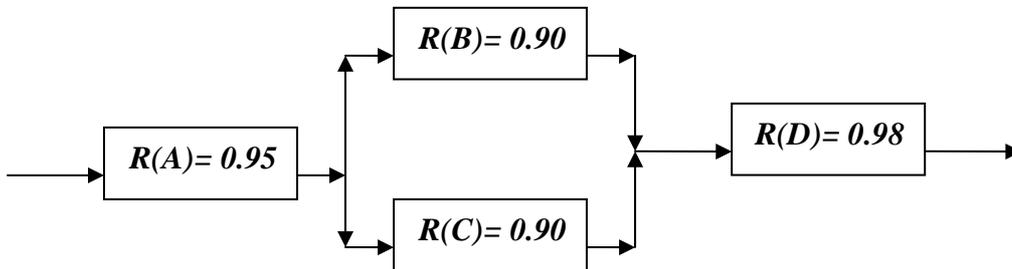
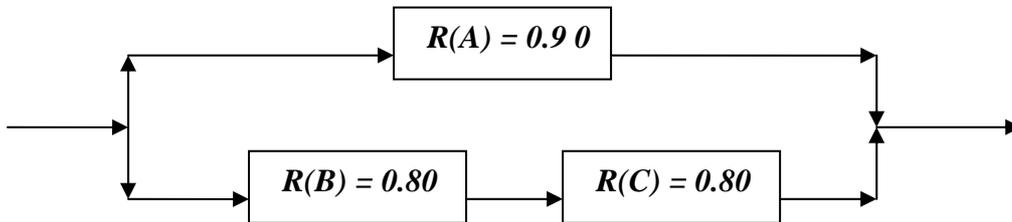
. $\lambda = 0.0001$ CFR

(ans.: a) 0.7919 , b) 0.9704)

:

Structure function

-26



(ans.: a) $R_S(t) = R(A) + R(B).R(C) - R(A).R(B).R(C); 0.964$,

b) $R_S(t) = R(A).R(D) [R(C) + R(B) - R(B).R(C)] ; 0.9217$)