Chapter 2

Longitudinal stick–fixed static stability and control

Lecture 10

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Example 2.5

A sailplane has the following characteristics. $C_D = 0.02 + 0.025\, C_L^2$, $C_{L\alpha w} = 0.093$, $\alpha_{0Lw} = -4$, $i_w = 0$, a.c. location $= 0.24\bar{c}$, $S_t = S/7$, $l_t = 4\bar{c}$, $d\varepsilon/d\alpha = 0.4$, $C_{L\alpha t} = 0.05$ and $\eta = 0.9$. All the angles are in degrees. Neglect the contribution of fuselage. Find the c.g. location for which the equilibrium is reached with zero lift on the tail at the lift coefficient corresponding to the best guiding angle. Calculate the tail setting. Is the sailplane stable?

Solution:

The airplane prescribed in this exercise is a sailplane. A sailplane is a high performance glider. There is no power plant in a glider. Further, the contribution of fuselage is prescribed as negligible. Hence, the terms $(C_{mcg})_{f,n,p}$ and $(C_{m\alpha})_{f,n,p}$ are zero in the present case.

The given data is as follows.

$$C_D = C_{D0} + KC_L^2 = 0.02 + 0.025\, C_L^2$$

Wing: $C_{Lw} = 0.093\ (\alpha_w + 4)$, $C_{L\alpha w} = 0.093\ deg^{-1} = 5.329\ rad^{-1}$, $\alpha_{olw} = -4^0$,

$C_{mac} = -0.08$, $i_w = 0$, a.c. at $0.24\,\bar{c}$.

Tail: $S_t = S/7$, $d\varepsilon/d\alpha = 0.4$, $\eta = 0.9$, $C_{L\alpha t} = 0.05\ deg^{-1} = 2.865\ rad^{-1}$. For the best gliding angle (C_D/C_L) should be minimum.

This happens when $C_L = C_{Lmd}$ and $C_{Lmd} = \sqrt{C_{D0}/K}$

In the present case $C_{Lmd} = \sqrt{0.02/0.025} = 0.895$

When the airplane is flying at $C_L = C_{Lmd}$, the lift on tail is prescribe to be zero or $\alpha_t = 0$.

Now, $\alpha_t = \alpha_w - i_w - \varepsilon + i_t$,

At $C_L = C_{Lmd}$, $\alpha_w = (0.895/0.093) - 4 = 5.62^0$, $\varepsilon = \varepsilon_0 + (d\varepsilon/d\alpha)\,\alpha$;

$\varepsilon_0 = (d\varepsilon/d\alpha)(i_w - \alpha_{olw}) = 0.4\ (0 + 4) = 1.6^0$

At $C_L = C_{Lmd}$, $\varepsilon = 1.6 + 0.4\ (5.62 - 0) = 3.85^0$

Since, α_t is zero at $C_L = C_{Lmd,}$ gives the following result.

$0 = 5.62 - 3.85 + i_t$

or $i_t = -1.77^0$

To examine static stability, the quantity $(C_{m\alpha})_{stick\text{-}fix}$ is calculated. It is noted that:

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha$$

$$C_{mcg} = C_{mac} + C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{mcg})_{f,n,p} + C_{mcgt}$$

$$(C_{m\alpha})_{stick\text{-}fix} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - \eta V_H C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

To evaluate the expression for $C_{m\alpha}$ the quantity $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$ is needed. This can be obtained using the following steps.

When the airplane is flying at $C_L = C_{Lmd}$, the contributions of tail, fuselage, nacelle and power are zero. Hence, the expression for C_{mcg} reduces to

$$C_{mcg} = C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{mac}$$

For equilibrium C_{mcg} must be zero at C_{Lmd} i.e. :

$$0 = 0.895\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) - 0.08$$

$$\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} = \frac{0.08}{0.895} = 0.089$$

$$\text{or } \frac{x_{cg}}{\bar{c}} = 24 + 0.089 = 0.329$$

$$\text{Further, } V_H = \frac{S_t}{S}\frac{l_t}{\bar{c}} = \frac{4}{7}$$

$$\text{Finally, } C_{m\alpha} = 5.329(0.329-0.24) - 0.9\times\frac{4}{7}\times 2.865\times(1-0.4)$$

$$= 0.47428 - 0.88406 = -0.4098 \text{ rad}^{-1}$$

$C_{m\alpha}$ is negative and hence the sailplane is stable.

Example 2.6

The contribution of wing fuselage combination to the moment about the c.g. of an airplane is given below.

C_L	0.28	0.488	0.696	0.9
$(C_{mcg})_{w,f}$	- 0.0216	- 0.006	0.0064	0.0156

(i) If the wing loading is 850 N/m^2, find the flight velocity at sea level when the airplane is in trim with zero lift on the tail. (ii) Investigate the stability of the airplane with the following additional data: $C_{L\alpha w}$ = 0.08 deg^{-1}, $C_{L\alpha t}$ = 0.072 deg^{-1}, $d\varepsilon/d\alpha$ = 0.45 , $l_t = 2.9\bar{c}$, S_t = S/7, η=1.0. Assume the contributions of power to C_{mcg} and $C_{m\alpha}$ to be negligible.

Solution:

i) To answer the first part, the value of C_L at which the airplane is in trim with zero lift on tail needs to be obtained. In this case:

$$C_{mcg} = (C_{mcg})_{w,f} = 0$$

The prescribed variation of $(C_{mcg})_{w,f}$ with C_L is slightly non-linear. Hence, the given data are plotted and the value of C_L at which $(C_{mcg})_{w,f}$ is zero is obtained from the plot. The plot is shown in Fig. E2.6. When $(C_m)_{w,f}$ is zero, C_L equals 0.585 .

In level flight: $L = W = \frac{1}{2}\rho V^2 S C_L$

or $V = \sqrt{2W/SC_L}$

Substituting various values, the desired velocity is:

$$V = \sqrt{\frac{2\times 850}{1.225\times 0.585}} = 48.7\,\text{m}\,\text{s}^{-1}$$

ii) To examine the static stability $C_{m\alpha}$ needs to be calculated.

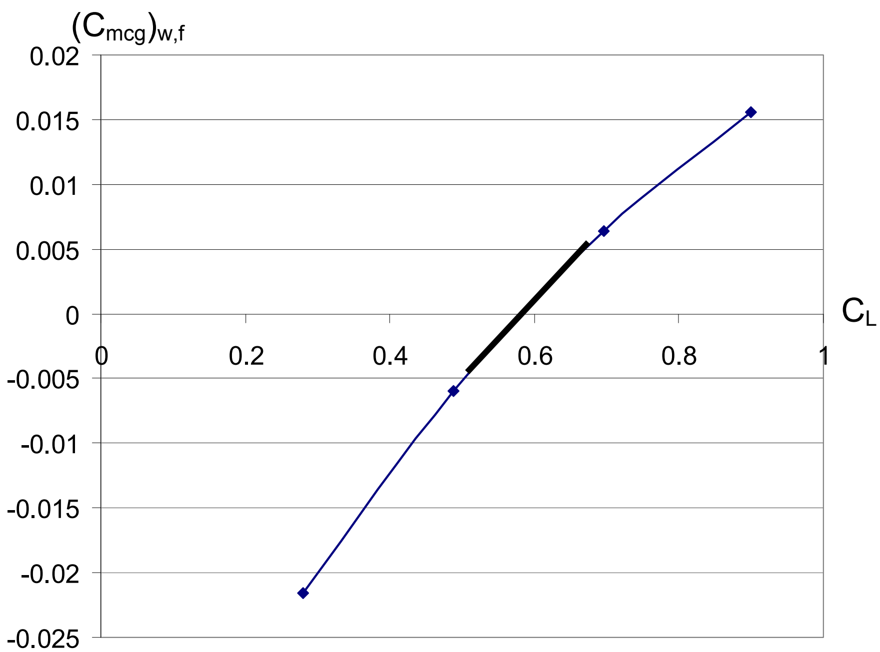

Fig.E2.6 $(C_{mcg})_{w,f}$ vs C_L

$$C_{m\alpha} = C_{L\alpha w}\left(\frac{dC_m}{dC_L}\right)_{w,f} - V_H\,\eta\,C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

$$C_{L\alpha w} = 0.08, C_{L\alpha t} = 0.072,\ V_H = \frac{1}{7}\times 2.9 = 0.414$$

$$\frac{d\varepsilon}{d\alpha} = 0.45$$

From graph in Fig.E2.6 at C_L = 0.585 we obtain, the slope of the curve as:

$(dC_m/dC_L)_{w,f} = 0.0615$

Hence, $C_{m\alpha} = 0.08 \times 57.3 \times 0.0615 - 0.414 \times 1 \times 0.072 \times 57.3 \times (1 - 0.45)$

$= 0.2819 - 0.9394 = - 0.6575 \ \text{rad}^{-1}$

Since, $C_{m\alpha}$ is negative the airplane is stable.

2.12 Longitudinal control

An airplane is said to be trimmed at a given flight speed and altitude, when the moments are made zero by suitable deflection of control surfaces. For the longitudinal motion, the trim or C_{mcg} = 0 is achieved by suitable deflection of elevator. The convention regarding the elevator deflection is that a downward deflection of elevator is taken as positive (Fig.2.16b). For the conventional tail configuration, this deflection increases lift on tail and produces a negative moment about c.g..

Let ΔC_L and ΔC_{mcg} be the incremental lift and pitching moment due to the elevator deflection i.e.

$$\Delta C_L = \Delta C_{Lt} = C_{L\delta e}\,\delta_e \ ; \ C_{L\delta e} = \frac{\partial C_L}{\partial \delta_e} \qquad (2.72)$$

$$\Delta C_{mcg} = \Delta C_{mcgt} = C_{m\delta e} \times \delta_e \ ; \ C_{m\delta e} = \frac{\partial C_{mcg}}{\partial \delta_e} \qquad (2.73)$$

Hence, when the elevator is deflected, the lift coefficient (C_L) and moment coefficient about c.g. (C_{mcg}) for the airplane are :

$$C_L = C_{L\alpha}(\alpha - \alpha_{0L}) + C_{L\delta e}\,\delta_e \qquad (2.74)$$

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e \qquad (2.75)$$

Where C_L, $C_{L\alpha}$ and α_{0L} refer respectively to the lift coefficient, slope of the lift curve and zero lift angle of the airplane.

Note: In this section $C_{m\alpha}$ will mean $(C_{m\alpha})_{\text{stick-fixed}}$.

2.12.1 Elevator power ($C_{m\delta e}$)

The quantity $C_{m\delta e}$ is called elevator power. An expression for it has been hinted in Eq.(2.64). It can be derived as follows.

Let, $\Delta L_{\delta e}$ be the change in the airplane lift due to elevator deflection which is also the change in the lift of the horizontal tail i.e.

$$\Delta L_{\delta e} = (\Delta L_t)_{\delta e} = \frac{1}{2}\rho\, V_t^2\; S_t(\Delta C_{Lt})_{\delta e}$$

$$\Delta C_{L\delta e} = \frac{\Delta L_{\delta e}}{\frac{1}{2}\rho V^2 S} = \eta\frac{S_t}{S}(\Delta C_{Lt})_{\delta e} = \eta\frac{S_t}{S}\frac{\partial C_{Lt}}{\partial \delta_e}\delta_e \qquad (2.76)$$

Hence, $$\frac{\partial C_L}{\partial \delta_e} = C_{L\delta e} = \eta\frac{S_t}{S}\frac{\partial C_{Lt}}{\partial \delta_e} \qquad (2.77)$$

$$\Delta M_{\delta e} = \Delta L_{\delta e} l_t = \frac{1}{2}\rho V_t^2 S_t(\Delta C_{Lt})_{\delta e} l_t$$

$$\Delta C_{m\delta e} = \frac{\Delta M_{\delta e}}{\frac{1}{2}\rho V^2 S\bar{c}} = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V^2}\frac{S_t}{S}\frac{l_t}{\bar{c}}(\Delta C_{Lt})_{\delta e}$$

Or $$\Delta C_{m\delta e} = -V_H\;\eta\;(\Delta C_{Lt})_{\delta e} = -V_H\,\eta\frac{\partial C_{Lt}}{\partial \delta_e}\delta_e$$

Hence, $$\frac{\partial C_m}{\partial \delta_e} = C_{m\delta e} = -V_H\,\eta\frac{\partial C_{L_t}}{\partial \delta_e} = -V_H\,\eta\,\tau\, C_{L\alpha t};\; \tau = C_{L\delta e}/C_{L\alpha t} \qquad (2.78)$$

2.12.2 Control effectiveness parameter (τ)

The quantity 'τ' is called elevator effectiveness parameter. The value of τ depends on the geometrical parameters of the tail and the elevator. However, it mainly depends on (S_e / S_t) where S_e is the area of the elevator. References 1.12 and 2.2 give a detailed procedure for estimating it. However, Fig.2.32 can be used for an initial estimate.

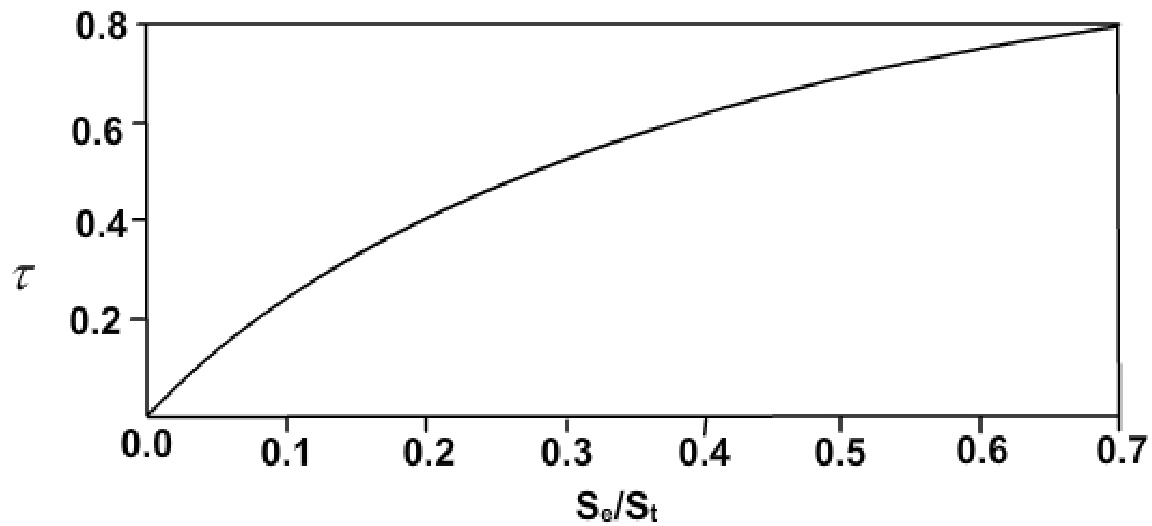

Fig.2.32 Control effectiveness parameter
(Reproduced from Refs.1.1, chapter 2 with permission from McGraw-Hill book company)

2.12.3 Elevator angle for trim

The following steps are followed to get the elevator angle for trim (δ_{etrim}).

From Eq. (2.75)

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e$$

For trim $C_{mcg} = 0$.

Hence,

$$0 = C_{m0} + C_{m\alpha}\,\alpha_{trim} + C_{m\delta e}\,\delta_{trim}$$

Or $$\delta_{trim} = \frac{-1}{C_{m\delta e}}\left[C_{m0} + C_{m\alpha}\,\alpha_{trim}\right] \qquad (2.79)$$

From Eq.(2.74)

$$C_{Ltrim} = C_{L\alpha}(\alpha_{trim} - \alpha_{0L}) + C_{L\delta e}\,\delta_{trim} \qquad (2.80)$$

Or $$\alpha_{trim} = \frac{1}{C_{L\alpha}}\{C_{Ltrim} - C_{L\delta e}\,\delta_{trim} + C_{L\alpha}\,\alpha_{0L}\}$$

Hence,

$$\delta_{trim} = \frac{-1}{C_{m\delta e}}[C_{m0} + \frac{C_{m\alpha}}{C_{L\alpha}}\{C_{Ltrim} - C_{L\delta e}\,\delta_{trim} + C_{L\alpha}\,\alpha_{0L}\}] \quad (2.81)$$

$$\text{Or} \quad C_{m\delta e}\delta_{trim} = -\frac{[C_{m0}\,C_{L\alpha} + C_{m\alpha}(C_{Ltrim} + C_{L\alpha}\alpha_{0L}) - C_{m\alpha}\,C_{L\delta e}\,\delta_{trim}]}{C_{L\alpha}}$$

$$\text{Simplifying,} \quad \delta_{trim} = -\frac{[C_{L\alpha}\,(C_{m0} + C_{m\alpha}\,\alpha_{0L}) + C_{m\alpha}\,C_{Ltrim}]}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]} \quad (2.82)$$

Differentiating with C_L ,

$$\frac{d\delta_{trim}}{dC_{Ltrim}} = -\frac{C_{m\alpha}}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]} \quad (2.83)$$

Following may be noted.

(i) The quantities $C_{m\delta e}$, $C_{L\alpha}$ and $C_{L\delta e}$ depend on the airplane geometry. Further, for a given c.g. location, $C_{m\alpha}$ is also known and hence the term ($d\delta_{etrim}/dC_{Ltrim}$) in Eq.(2.83) is a constant. Hence, in the simplified analysis being followed here, it is observed that for a given c.g. location δ_{trim} is a linear function of C_L. Further the term $C_{m\alpha}\,C_{L\delta e}$ is much smaller than $C_{m\delta e}\,C_{L\alpha}$. Consequently, δ_{trim} as a function of C_L can be written as:

$$\delta_{trim} = \delta_{eoCL} - \frac{1}{C_{m\delta e}}(\frac{dC_m}{dC_L})_{stick\text{-}fixed}\,C_L \quad (2.84)$$

where,

$$\delta_{e0CL} = -\frac{C_{L\alpha}(C_{m0} + C_{m\alpha}\,\alpha_{0L})}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]}$$

Similarly, using Eq.(2.79), δ_{trim} as a function of α_{trim} can be expressed as:

$$\delta_{trim} = -\frac{C_{m0}}{C_{m\delta e}} - \frac{C_{m\alpha}}{C_{m\delta e}}\alpha_{trim} \quad (2.85)$$

Typical curves for the variations of δ_{trim} with C_L are shown in Fig.2.33 for different locations of c.g.. From Eq.(2.85) it is seen that at C_L = 0 the value of δ_{trim} is positive as C_{m0} is positive and $C_{m\delta e}$ is negative. For a stable airplane $(dC_m/dC_L)_{stick\text{-}fixed}$ is negative and hence the slope of δ_{trim} vrs C_L curve is negative.

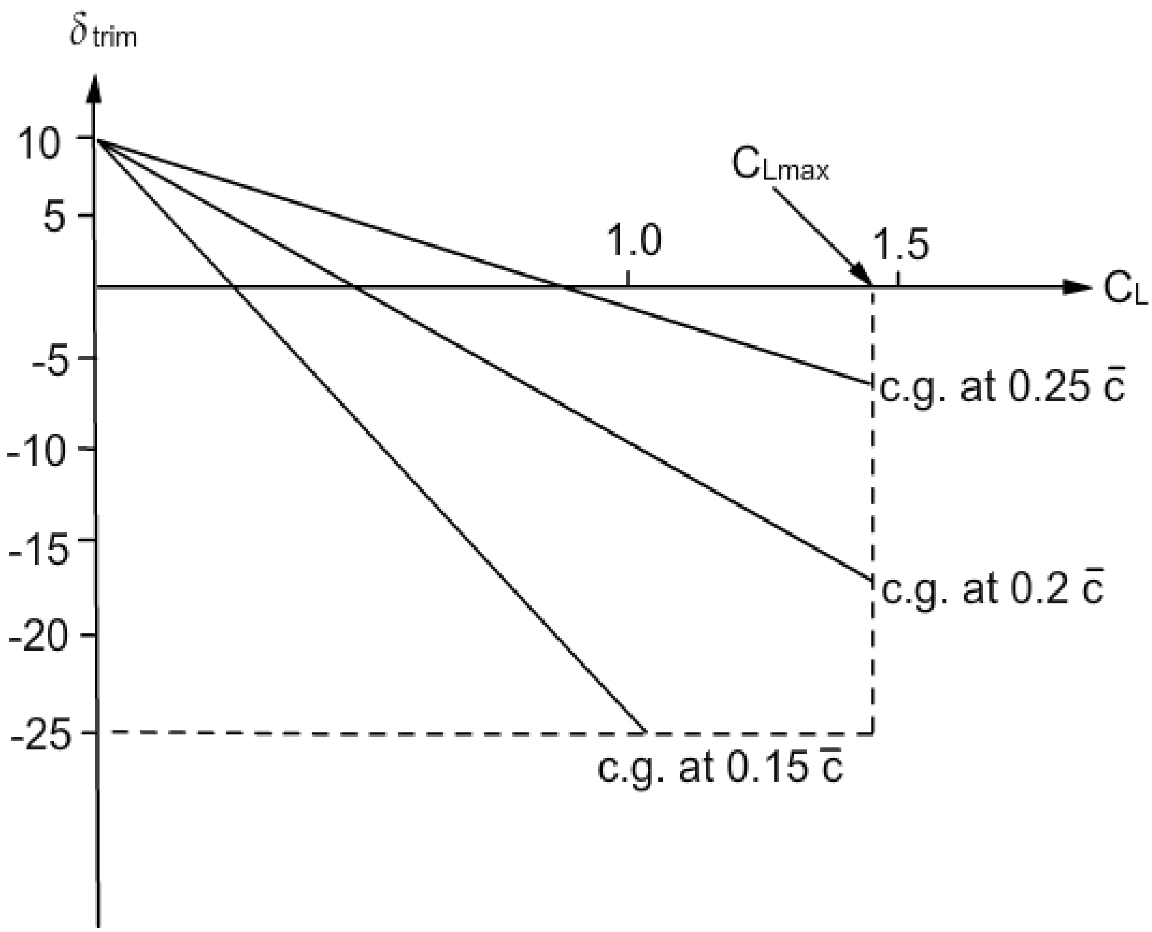

Fig.2.33 Elevator angle for trim

(ii) In light of the above analysis, consider a case when the airplane is trimmed at a chosen C_L by setting the elevator at corresponding $\delta_{trim.}$ Now, if the pilot wishes to fly at a lower speed which implies higher C_L, he would need to apply more negative elevator deflection or the incremental lift on the tail (ΔL_t) would be negative. This is what is implied when in section 2.4.1 it is mentioned that "… for achieving equilibrium with conventional tail configuration, the lift on the tail is generally in the downward direction". An alternate explanation is as follows.

When the pilot wishes to increase the angle of attack by $\Delta\alpha$, a statically stable airplane produces a moment $-\Delta M_{cg}$.To counterbalance this moment, the elevator must produce $+\Delta M_{cg}$. This requires $-\Delta L_t$ and inturn $-\Delta\delta_e$.

(iii) Military airplanes which are highly maneuverable, sometimes have the following features.

(a) An all movable tail in which the entire horizontal tail is rotated to achieve higher ΔM_{cg}. (b) Relaxed static stability wherein $C_{m\alpha}$ may have a small positive value. Such airplanes need automatic control (section 10.3). See section 6.1 of Ref.1.13 for further details.

2.12.4 Advantages and disadvantages of canard configuration

In light of the above discussion the advantages and disadvantages of the canard configuration can now be appreciated.

Advantages:

(a) The flow past canard is relatively free from wing or engine interference.

(b) For an airplane with $C_{m\alpha} < 0$, the lift on the horizontal stabilizer located behind the wing (i.e. conventional configuration) is negative when the angle of attack increases. Thus, for a conventional tail configuration, the wing is required to produce lift which is more than the weight of the airplane. If the surface for control of pitch, is ahead of the wing (canard), the lift on such horizontal control surface is positive and the lift produced by the wing equals the weight of the airplane minus the lift on canard. Thus, the wing size can be smaller in a canard configuration.

Disadvantages:

(a) The contribution of the canard to $C_{m\alpha}$ is positive i.e. destabilizing.

(b) As the wing, in this case, is located relatively aft, the c.g. of the airplane moves aft and consequently the moment arm for the vertical tail is small.

Topics for self study:

1. From Ref.2.3 study the airplanes with canard and obtain rough estimates of (S_t / S) and (l_t / c). Two examples of airplanes where canard is used are SAAB Viggen and X-29A.

2.12.5 Limitations on forward moment of c.g. in free flight

As the c.g. moves forward, the airplane becomes more stable and hence requires larger elevator deflection for trim at a chosen C_L. It is seen that as C_L increases, more negative elevator deflection is required (Fig. 2.33). Further, each airplane has a value of C_{Lmax} which depends on the parameters of the wing. However, equilibrium at C_{Lmax} can be achieved only if the airplane can be trimmed at this lift coefficient. Further, the maximum elevator deflection is limited to approximately about 25^0(negative). Hence, there would be a forward c.g. location at which the maximum negative elevator deflection would be just able to permit trim at C_{Lmax}. This brings about a limitation on the forward movement of c.g. from control consideration. It may be recalled that the rearword movement of c.g. is limited by the stability consideration.

2.12.6 Limitations on forward movement of c.g in proximity of ground

As the airplane comes in to land, the lift coefficient is generally the highest. It is achieved using flaps and this makes C_{macw} more negative. Further, due to the proximity of ground, following changes in $C_{L\alpha w}$ and $(d\varepsilon / d\alpha)$ are observed.

(a) The slope of lift curve of the wing, i.e. $C_{L\alpha w}$ increases slightly. The actual amount of increase in $C_{L\alpha w}$ depends on the ratio of the height of the wing above the ground and its span (see Ref.1.7, chapter 5). There is no significant change in $C_{L\alpha t}$.

(b) The downwash due to wing decreases considerably (Fig.2.12) and consequently the tail contribution to stability ($C_{m\alpha t}$) becomes more negative (Eq.2.50) or the airplane becomes more stable.

The net effect is that the airplane requires more negative elevator deflection. This imposes further restrictions on the forward movement of c.g.. Figure 2.34 shows the restrictions on c.g. travel based on factors discussed so for. Additional restrictions on the movement of c.g. would be pointed out after discussions in chapters 3 and 4.

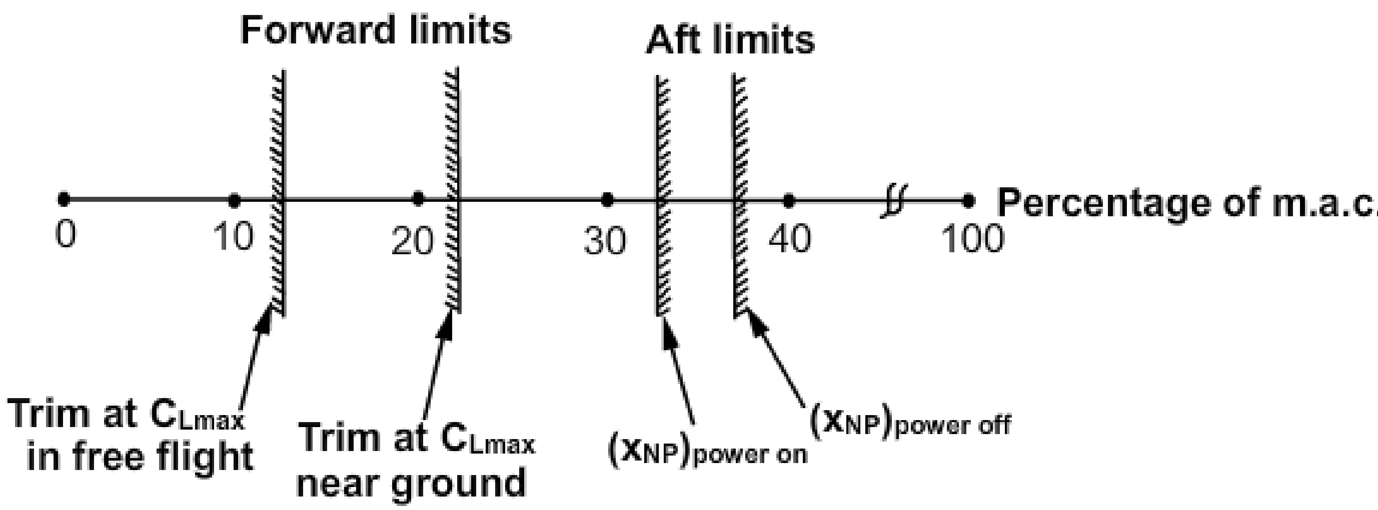

Fig.2.34 Restrictions on c.g. movement from stick fixed stability and control considerations (schematic)