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Analytical solution of Natural convection between two horizontal concentric cylinders with partially insulated space

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ABSTRACT:An analytical solution for natural heat transfer in a cavity filled with air between two concentric cylinders with half insulated cavity and low Rayleigh number presented in the present work. Solution based on Perturbation method with two terms for both temperature and streamlines values. Variables taken in present work are; radii ratio, temperature difference between inner and outer tubes, and changing tilt angles. Solution of present work shows a good agreement with numerical solution of a near problem. Validity of solution led to show more results. Nusselt number show to increase as tilt angle increased, while local Nusselt number varies according to velocity pattern and temperature distribution.

KEYWORDS:Natural convection, low Rayleigh number, concentric cylinders.

Nomenclatures

Symbol	Definition	Units
a	Coefficients used in the solution	-
e	Coefficients used in the solution	-
g	Gravitational acceleration	Kg/m^2
p	Pressure	N/m^2
Pr	Prandtl number	-
r'	Dimentional radius notatio	m
r	Non-dimentional radius notation	-
R	Outer to inner radii ratio	-
Ra	Rayleigh number	-
T	Temperature	^oK
u	Fluid velocity componet	m/s
v	Fluid velocity component	m/s

Greek symbols

α	Fluid heat capacity	Js/K
θ	Angular coordinate	rad
ρ	Fluid density	kg/m^3
φ	Tilt angle of insulation	rad
ψ	Stream function	m^2/s

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I. INTRODUCTION

Natural convection is one of the important and common heat transfer modes in nature. No temperature difference between any surface and a fluid, does not contain a natural convection process even with presence of forced convection. For this reason, natural convection took a great attention from researchers and for its practical applications. One of the most important cases is natural convection inside an enclosure. This enclosure may take a form of rectangular [1, 2, 3, and 4], or between two eccentric cylinders [5, 6, 7, 8, 9, and10]. Convection may occur between two eccentric tubes with different cross sections [11, and13]. For all these cases, governing forces of buoyancy to dissipation of heat energy, as a ratio called Rayleigh number. As Reynolds number the most important parameter in forced convection, Rayleigh number is the parameter for natural convection. Ranges decides process and solution for natural convection. Rayleigh numbers may be small [5, 6, 10, and 12], medium [10], or great [1, 2, 3, 4, and 11]. For each range, a different solution procedure used according to mathematical and physical demands.

In present work, natural convection heat transfer between two concentric half insulated cylinders is studied. The reason behind solving such problem is due to wide application of the problem especially in reduction of heat loss from receiver tube in cylindrical trough collectors. When the gap between receiver and covering tube lost its vacuum status, natural convection plays the main role in loosing heat from receiver to glass cover then to ambient. The problem studied numerically by [5]. Perturbation method used here in order to distinguish the solution (solution divided into number of solutions connected by Rayleigh number raised to certain power), because of low Rayleigh number of problem. Method used in [6] for water at the range of (0-4 oC), and in [12] where air filled the space. Solution of present work should differ from [12], since symmetry of the problem is just a single case here. To authors' knowledge, no analytical solution demonstrated for present case. The problem shown in figure 1 below to demonstrate most important nomenclatures.

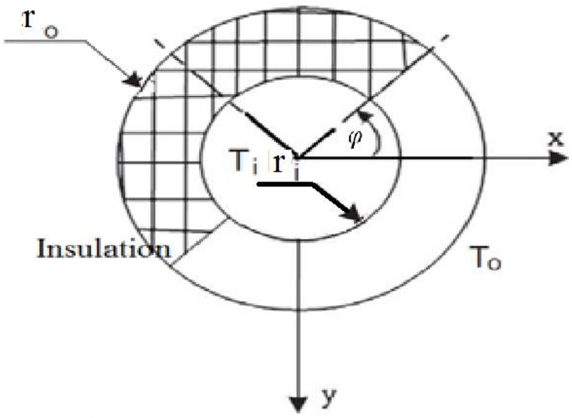

FIGURE 1 PROBLEM OF PRESENT WORK.

II. DIFFERENTIAL EQUATIONS

In present work, formulation of solution started with the differential equations. In each fluid flow in cylindrical coordinates, continuity equation, equation (1), along with momntum equations in r, equation (2), and θ axes, equation (3), as shown below used to fomulate the problem:

$$\frac{1}{r}\frac{\partial}{\partial r'}(r'u) + \frac{1}{r'}\frac{\partial}{\partial \theta}(v) = 0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r'} + \frac{v}{r'}\frac{\partial u}{\partial \theta} - \frac{v^2}{r'}\right) = -\frac{\partial P}{\partial r'} + \rho g_r + \mu\left[\left(\frac{1}{r'}\left(r'\frac{\partial u}{\partial r}\right)\right) - \frac{u}{r'^2} + \frac{1}{r'^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{2}{r'^2}\frac{\partial v}{\partial \theta}\right] \qquad (2)$$

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$$\rho\left(u\frac{\partial v}{\partial r'}+\frac{v}{r'}\frac{\partial v}{\partial \theta}-\frac{uv}{r'}\right)=-\frac{1}{r'}\frac{\partial P}{\partial \theta}+\rho g_\theta+\mu\left[\left(\frac{1}{r'}\left(r'\frac{\partial v}{\partial r'}\right)\right)-\frac{v}{r'^2}+\frac{1}{r'^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2}{r'^2}\frac{\partial u}{\partial \theta}\right](3)$$

Heat transfer equation, also used in the formulation as below:

$$\nabla^2 T=\frac{1}{r'}\left(\frac{\partial T}{\partial r'}r'u-\frac{\partial T}{\partial \theta}v\right)\quad(4)$$

In order to combine (u) and (v) velocity components in one variable ψ (stream function), to simplify the solution by using following difinitions:

$$u=-\frac{1}{r'}\frac{\partial \psi}{\partial \theta}\quad(5)$$

$$v=\frac{\partial \psi}{\partial r'}\quad(6)$$

after partial differentiate equation (2) with respect to θ, and equation (3) with respet to r, just to eleimenate pressure gradient in the domain. A four degree differential equation obtained:

$$\nabla^4\psi=\frac{1}{Pr}\frac{1}{r}\left(\frac{\partial \psi}{\partial \theta}\frac{\partial \nabla^2\psi}{\partial r}-\frac{\partial \psi}{\partial r}\frac{\partial \nabla^2\psi}{\partial \theta}\right)+Ra\left(\frac{cos\theta}{r}\frac{\partial T}{\partial \theta}+sin\theta\frac{\partial T}{\partial r}\right)\quad(7)$$

And heat transfer equation become:

$$\nabla^2 T=\frac{1}{r}\left(\frac{\partial \psi}{\partial \theta}\frac{\partial T}{\partial r}-\frac{\partial \psi}{\partial r}\frac{\partial T}{\partial \theta}\right)\quad(8)$$

Where:

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\quad(9)$$

It should be noted that above equations are nondimentioalized by using the following definitions:

1- $T=\frac{T(r.\theta)-T_o}{T_i-T_o}$ (10)
2- $r=\frac{r'}{r_i}$ (11)
3- $R=\frac{r_o}{r_i}$ (12)
4- $Ra=\frac{g\beta r_i^3(T_i-T_o)^2}{\nu\alpha}$ (13)
5- $Pr=\frac{\nu}{\alpha}$ (14)

The boundary conditions of the problem is as follow:

1- $T(1.\theta)=1$ (15)
2- $T(R.\theta)=0$ (16)
3- $\frac{\partial T}{\partial \theta}=0\ at\ \theta=\varphi.and\ \theta=\varphi+\pi$ (17)
4- $\psi(r.\theta)=\frac{\partial \psi}{\partial r}=\frac{\partial \psi}{\partial \theta}=0\ at\ r=1.r=R.\theta=\varphi.and\ \theta=\varphi+\pi$ (18)

In equation (7), the left side is the effect of viscous stresses, while the bouyancy effect is represented by the first term in the right side of equation (7). The convection acceleration is represented by the second term of the right side. In equation (8), the left side is the conduction heat transfer, while the right side is the convection heat transfer.
The boundary conditions assure that there is no flow across the boundaries, while the inner and outer tubes maintain at uniform constant temperatures. The insulation works to prevent heat transfer in the angular direction across a half circle.

III. PERTURBATION SOLUTION

In order to solve both differential equations 7 and 8, a power series solution is adopted. The power series selected here based on the Rayleigh number, where for small ranges of that number it is suitable to use such series [5, and 6]. Since present work focousing on natural heat transfer between two concentric cylinders with the space filed with air, Rayleigh number for the conditions of pesent work should satisfies low Rayleugh number. Solution for both differential equations 7 and 8 can be show as follows:

$$T(r.\theta)=T_1+RaT_2\quad(19)$$

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$\psi(r.\theta) = Ra\psi_1 + Ra^2\psi_2$ (20)

Note that only two terms of series are taken for simplicity, and that higher terms have less effects due to very small coefficients related to equations of temperature and stream functions. Substitution of equations 19 and 20 in differential equations 7 and 8, and due to rearranging and sorting equations, the following differential equations resulted:

$\nabla^2 T_1 = 0$ (21)

$\nabla^4\psi_1 = sin\theta\frac{\partial T_1}{\partial r} + \frac{cos\theta}{r}\frac{\partial T_1}{\partial \theta}$ (22)

$\nabla^2 T_2 = \frac{1}{r}\left(\frac{\partial T_1}{\partial r}\frac{\partial \psi_1}{\partial \theta} - \frac{\partial T_1}{\partial \theta}\frac{\partial \psi_1}{\partial r}\right)$ (23)

$\nabla^4\psi_2 = sin\theta\frac{\partial T_2}{\partial r} + \frac{cos\theta}{r}\frac{\partial T_2}{\partial \theta} + \frac{1}{Pr}\frac{1}{r}\left(\frac{\partial \nabla^2\psi_{1_1}}{\partial r}\frac{\partial \psi_1}{\partial \theta} - \frac{\partial \nabla^2\psi_1}{\partial \theta}\frac{\partial \psi_1}{\partial r}\right)$ (24)

The boundary conditions are:

$T_1 = 1 \quad at \quad r = 1$ (25)

$T_1 = 0 \quad at \quad r = R$ (26)

$T_2 = 0 \quad at \quad r = 1$ (27)

$T_1 = 0 \quad at \quad r = R$ (28)

$\psi_1 = \psi_2 = \frac{\partial \psi_1}{\partial r} = \frac{\partial \psi_2}{\partial r} = \frac{\partial \psi_1}{\partial \theta} = \frac{\partial \psi_2}{\partial \theta} = 0 \quad at\ r = 1 \quad and\ r = R$ (29)

$\psi_1 = \psi_2 = \frac{\partial \psi_1}{\partial r} = \frac{\partial \psi_2}{\partial r} = \frac{\partial \psi_1}{\partial \theta} = \frac{\partial \psi_2}{\partial \theta} = 0 \quad at\ \theta = \varphi \quad and\ \theta = \varphi + \pi$ (30)

IV. THE SOLUTION

Differential equations (21 to 24), subjected to boundary conditions (25 to 30) are solved respectively as shown below:

$T_1(r.\theta) = \left(a_1 r - \frac{a_2}{r}\right)\cos(\theta - \varphi)$ (31)

$\psi_1(r.\theta) = (a_3r^5 + a_4r^3 + a_5r^2 + a_6r + a_7r^2 lnr + a_8)\sin(\theta - \pi)$ (32)

$T_2(r.\theta) = \Big\{\Big[a_9r^6 + a_{10}r^5 + a_{11}r^3 + a_{12}r^2 + a_{13}r + a_{14}r^3lnr + a_{15}rlnr + \frac{a_{16}}{r^2} + \frac{a_{17}}{r}\Big]cos^2(\theta-\varphi) - \Big[a_{18}r^6 + a_{19}r^5 + a_{20}r^4 + a_{21}r^3 + a_{22}r^2 + a_{23}r + a_{24}r^3lnr + a_{25}rlnr + \frac{a_{26}}{r^2}\Big]sin^2(\theta-\varphi) + a_{27}lnr + a_{28}\Big\}cos2(\theta-\varphi)$(33)

$\psi_2(r.\theta) =$
$\Big\{[a_{29}r^7 + a_{30}r^5 + a_{31}r^4 + a_{32}r^3 + a_{33}r^2 + a_{34}r^4lnr + a_{35}r^2lnr + a_{36}r + a_{37}rlnr + a_{38}r(lnr)^2]cos^2(\theta-\varphi) - \Big[a_{39}r^7 + a_{40}r^6 + a_{41}r^4 + a_{42}r^3 + a_{43}r^2 + a_{44}r^4lnr + a_{45}r^2lnr + \frac{a_{46}}{r}\Big]sin^2(\theta-\varphi)\Big\}sin\theta + \frac{cos\theta}{r}\{[a_{47}r^9 + a_{48}r^8 + a_{49}r^7 + a_{50}r^5 + a_{51}r^4 + a_{52}r + a_{53}r^6lnr + a_{54}r^4lnr + a_{55}rlnr + a_{56}r^2(lnr)^2]cos^2(\theta-\varphi) + [a_{57}r^9 + a_{58}r^8 + a_{59}r^7 + a_{60}r^6 + a_{61}r^4 + a_{62}r^2 + a_{63}r + a_{64}r^6lnr + a_{65}r^4lnr + a_{66}rlnr]cos2(\theta-\varphi) + [a_{67}r^9 + a_{68}r^8 + a_{69}r^7 + a_{70}r^6 + a_{71}r^4 + a_{72}r + a_{73}r^6lnr + a_{74}r^4lnr]sin^2(\theta-\varphi) + a_{75}lnr + a_{76}\}sin2(\theta-\varphi) + \frac{1}{Pr}\frac{1}{r}[\{a_{77}r^9 + a_{78}r^8 + a_{79}r^7 + a_{80}r^6 + a_{81}r^5 + a_{82}r^4 + a_{83}r^3 + a_{84}r^7lnr + a_{85}r^5lnr\}sin2(\theta-\varphi) + \{a_{86}r^{10} + a_{87}r^8 + a_{88}r^7 + a_{89}r^6 + a_{90}r^5 + a_{91}r^4 + a_{92}r^3 + a_{93}r^2 + a_{94}r^7lnr + a_{95}r^4lnr + a_{96}r^3lnr + a_{97}r^2lnr\}sin2(\theta-\varphi)]$ (34)

It should be noted that the above solution of radius ratio and Prandtl number for each value of angular insulation position. The coefficients used in the solution arranged in Appendix-A.

In order to have local Nusselt number based on eaither of tubes (outer or inner), Nusselt can be defined as [6]:

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Vol. 7, Issue 1, January 2018

$Nu = -lnR\left[r\frac{\partial T}{\partial r}\right]_{r=R.1}$ (35)

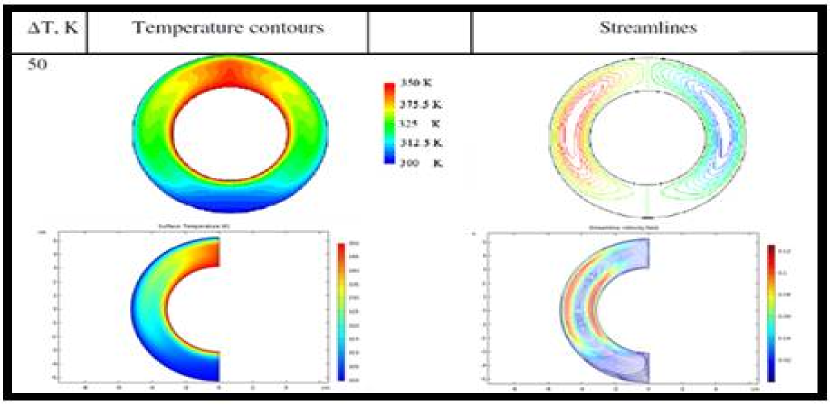
And average Nusselt number calculated from [6]:

$\overline{Nu} = \frac{1}{\pi}\int_0^{\pi} Nud\vartheta$ (36)

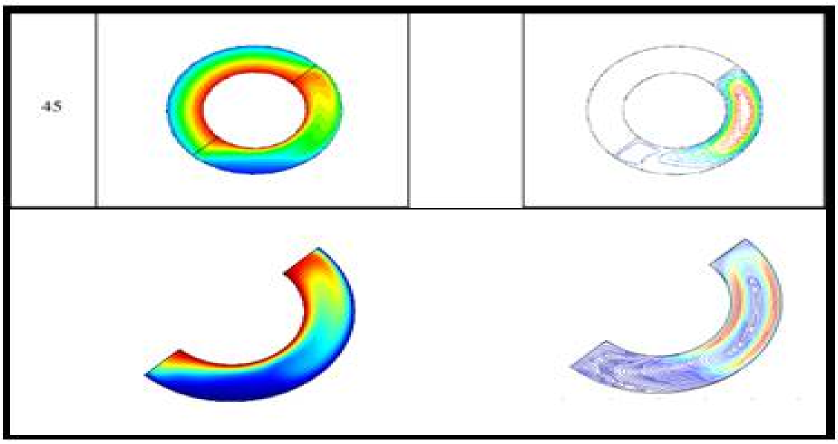
Note that T used in equations (35) and (36) based on the solution obtained in (19).

V. VALIDATION OF THE SOLUTION

For the knowladge of the author, no analyatical solution available for the problem. In order to validate the results of preset work, a comparison is done with a similar problem solved numerically by finite element method. In figure 2 below the data of [5] is shown and compared with that obtained by the above solution, where results of present work are drawn by the aid of COMSOL Multiphysics 5.3. A slight but acceptable difference (not to exceed 9%) is shown in the comparison, which is good agreement for a two term solution. Behavior of the results is the same. This lead to trust in solution obtained and make connfidance to have more results.



(a)



(b)

Figure 2 (a) Comparison between data of present work (lower line) and data of [5] (top line) for summetric case and temperature difference of 50 ^{o}K.

(b) Comparison between the streamlines and temperature data of [5] (upper line) and data of present work (lower line) for tilt angle of 45^{o} and temperature difference of 200 ^{o}K.

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VI. RESULTS AND DISCUSSIONS

The data obtained are applied in eq. (20) with six different tilt angles (20, 40, 50, 60, 80, and 90). The temperature difference applied is 50 ^{o}K. At 20^{o}. Streamlines (figure 3) show to have maximum flow values at small tilt angles. The largest values of tilt angles obtained, led to smallest values of stream flow. The reason behind this behavior is due to concentration of hot air in small tilt angle, where some hot air captured at the lower end of the pipe to form small circulation appeared slightly at 40^{o}. This circulation increased gradully as the tilt angle increment due to more hot air amount to trap in the lower end. As tilt angle reached 90^{o}, the flow shown to be symmetric in the two sides of the pipes cavity. This behavior should be followed by the values of velocity contours, which is shown in figure 4. Where the velocity values shown to be higher at the higher end, and lower at the lower end. In lower end thes values increased step by step as tilt angle increased to finally be symmetric at 90^{o}. In figure 5, temperature distribution over air filled cavity. At 20^{o} of tilt angle, high temperature values are shown to make a strip close to inner pipe. A circulate area of hot air near highr end of the cavity shown, since hot air is circulated there. Lower end of air filled cavity shown to have lowest values of temperature distribution, since cold air accumulate there due to high density. As tilt angle increased and hot air trapped at lower end cavity, a hot air circulation increased gradually to reach symmetry at 90^{o}. In the symmetry case, lower temperature values shown at the lower section of the middle of cavity. Figure (8) shows local Values of Nusselt number based on equation (35). Values shown are based on outer tube, where a symmetrical shape shown in titl angle 90^{o}, and values lost their symmetrical shape as tilt angle decreased. High values of Nusselt number obtained at ends of cavity due to high temperature difference between outer tube and inner air temperature. Still average Nusselt numbers, based on equation (36), show an increase in values as tilt angle increased. Low tilt angles shown to have lowest values of Nusselt number.

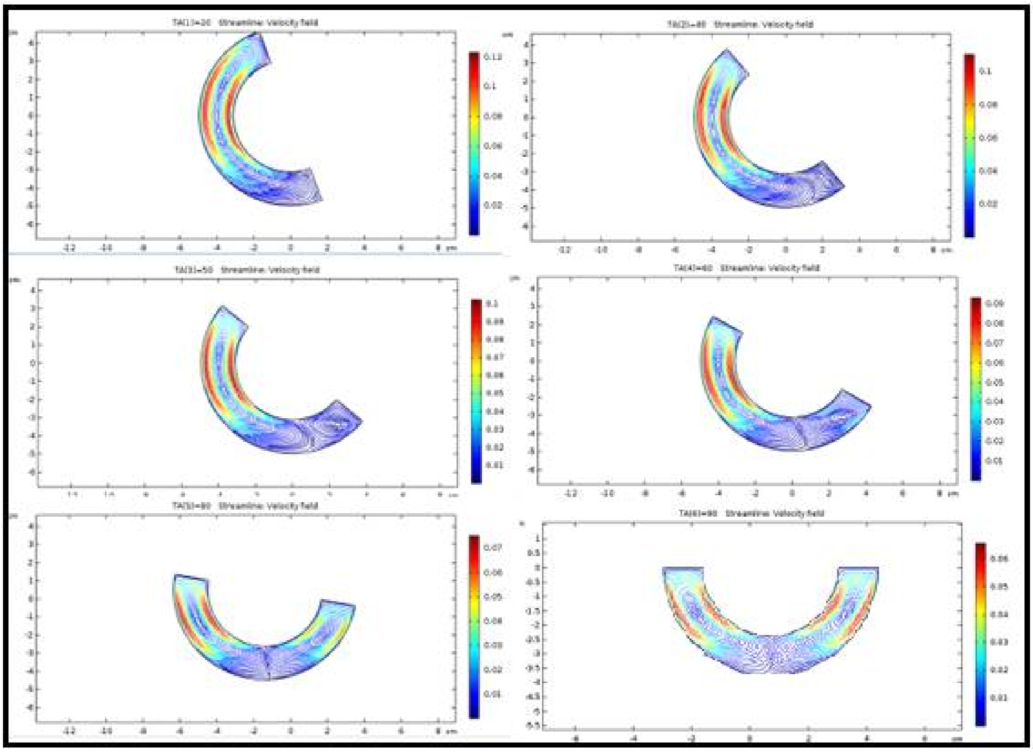

Figure 3 Streamlines of flow for tilt angles 20, 40, 50, 60, 80, and 90, with temperature difference of 50^{o}K.

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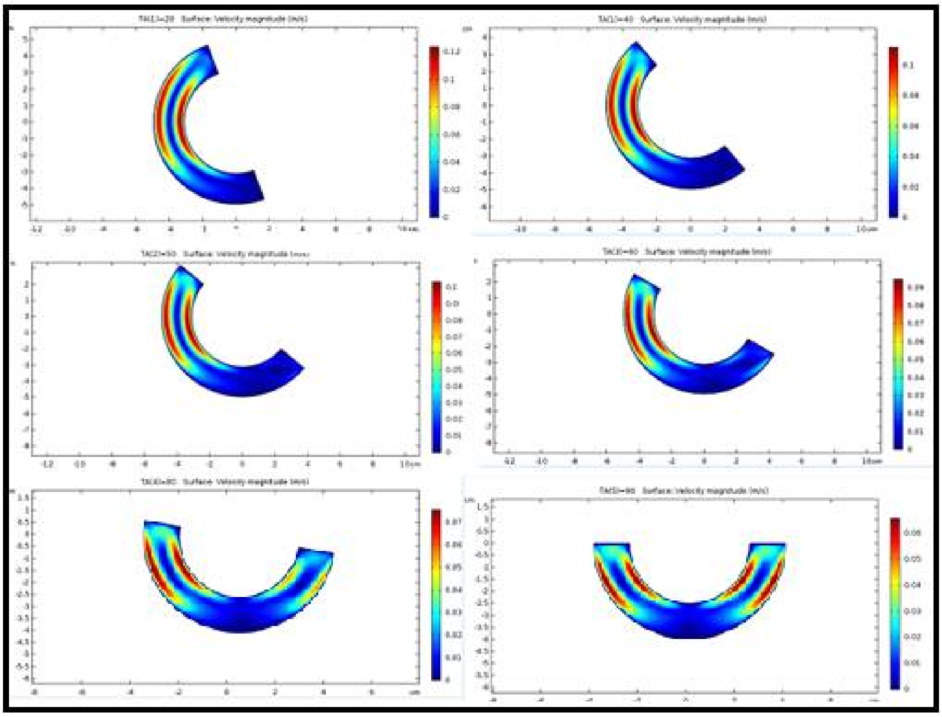

Figure 4 Velocity of flow for tilt angles 20, 40, 50, 60, 80, and 90, with temperature difference of 50^{o}K.

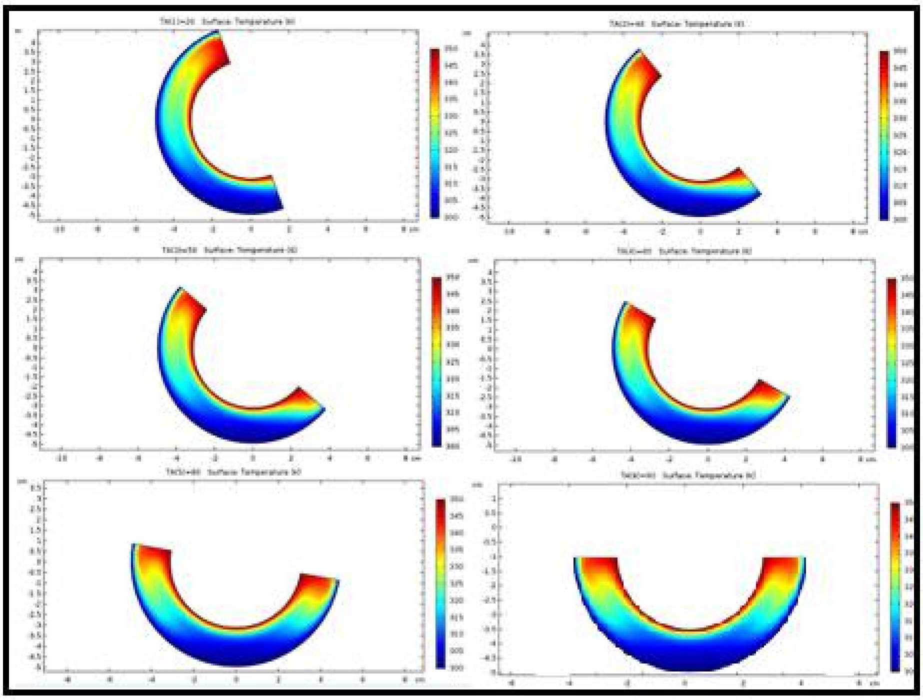

Figure 5 Temperature distribution for tilt angles 20, 40, 50, 60, 80, and 90, with temperature difference of 50^{o}K.

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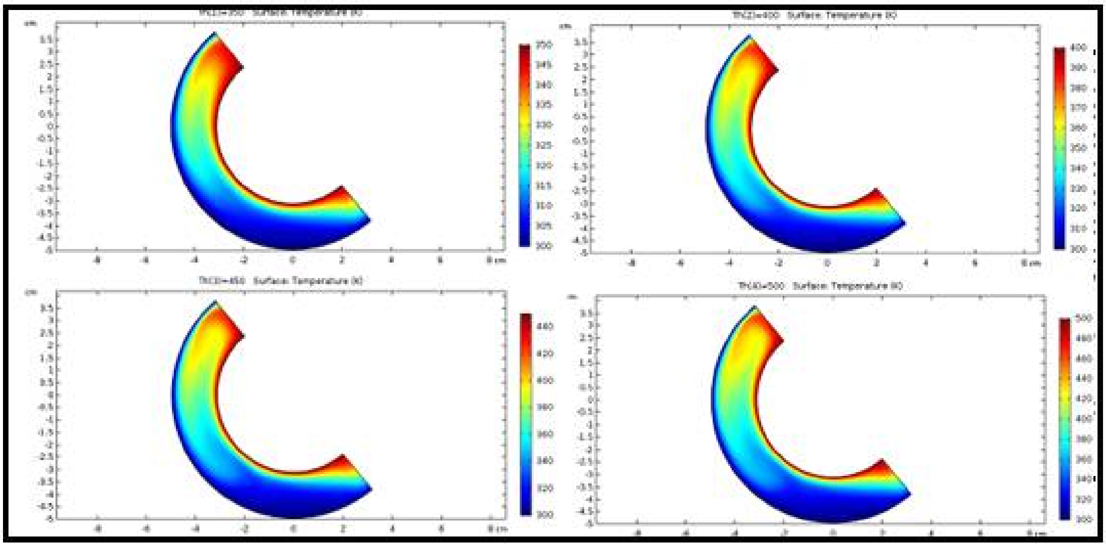

Figure 6 Temperature distribution for changing inside tube's temperatures of 350, 400, 450, and 500 ^{o}K with tilt angle of 40^{o}.

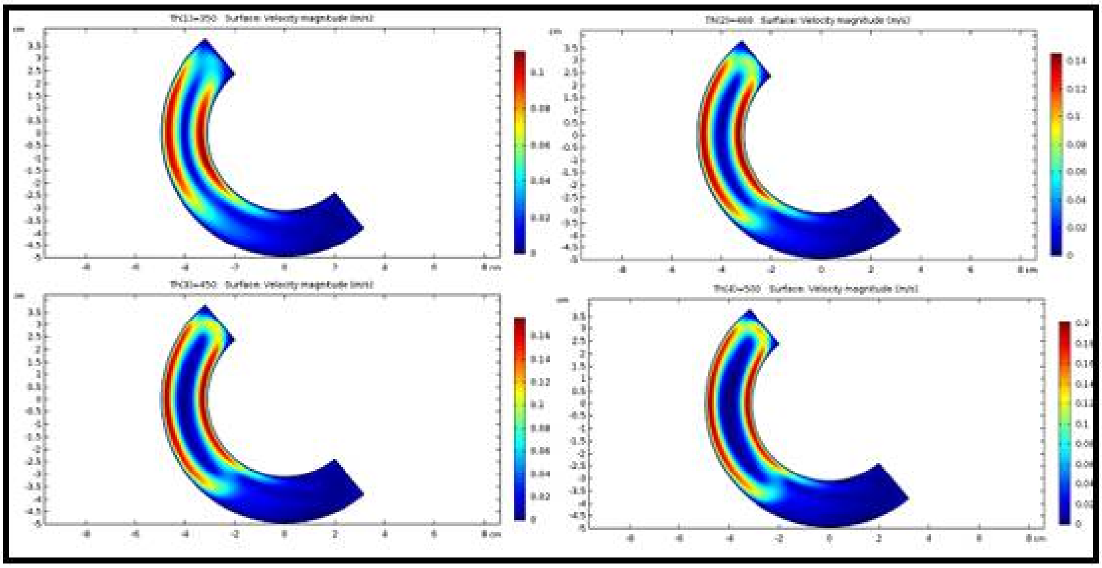

Figure 7 Velocity distribution contour for different inside tube's temperatures of 350, 400, 450, and 500 ^{o}K.

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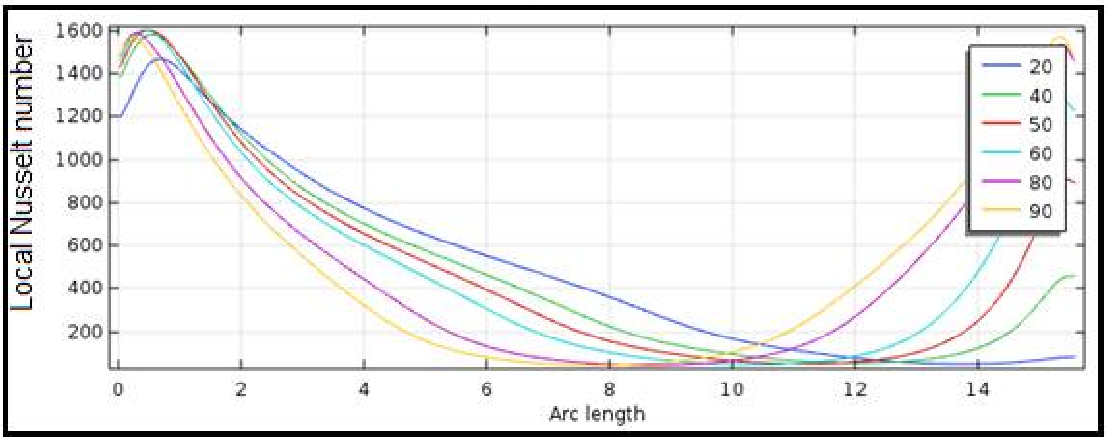

Figure 8 Local Nusselt number over arc length of tube with different tilt angle.

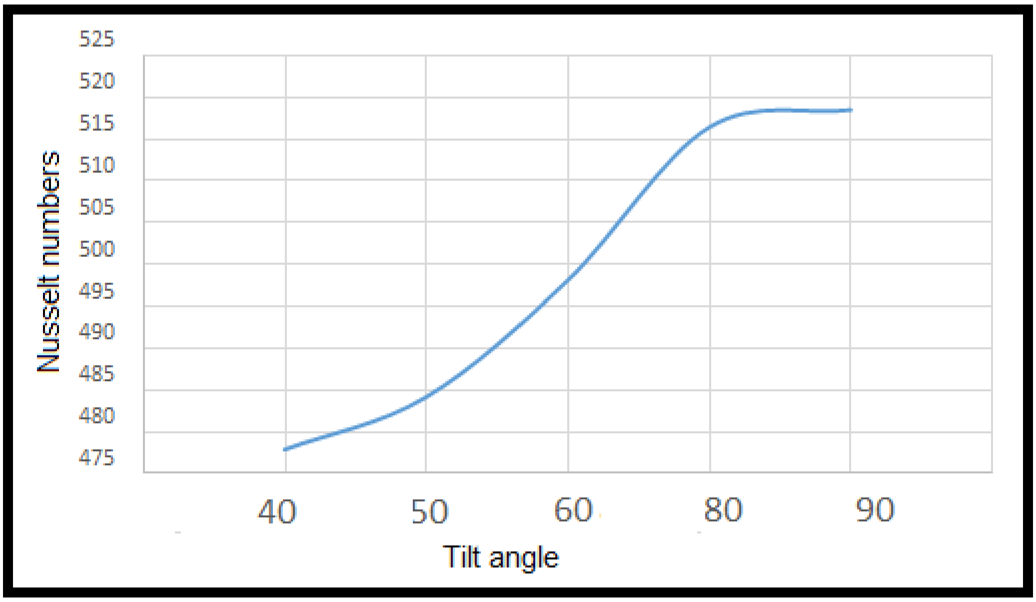

Figure 9 Average Nusselt Number for tube with different tilt angles.

VII. CONCLUSION

Prent work shows an elegant and useful analytical solution for natural convection between two concentric cylinders. The inner tube is the hot one, while outer tube is the cold one. Gab between two cylinders filled with air with half insuated area at different tilt angle. Resulta and according to solution show a relation between tilt angle and natural convection process occuring. As tilt angle increased heat transfer rate increased til reaching maximum values at tilt angle of 90^o. Lower values of heat transfer rates occurs at lower tilt angles.

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Appendix-A

The coefficients used in the solution of differential equations (21 to 24):

$a_1 = \frac{1}{1-R^2}$ (A-1)

$a_2 = \frac{R^2}{1-R^2}$ (A-2)

$a_3 = \frac{1}{45(1-R^2)}$ (A-3)

$a_4 = \frac{R^2}{9(1-R^2)}$ (A-4)

$a_5 = \frac{-2}{R+3}\left(\frac{a_1}{9} + \frac{a_2}{3}\right) - \frac{a_1}{5}(R^4 + R^3 + R^2 + R) + \frac{a_2}{9}(R^2 + R + 1) + \frac{e_1}{4}\left(\frac{R^2 lnR + R - 1}{R-1}\right)$ (A-5)

$a_6 = -\frac{a_1}{45}(R^4 + R^3 + R^2 + R) + \frac{a_2}{9}(R^2 + R + 1) + \frac{e_1}{4}\frac{R^2 lnR}{R-1} + \frac{a_5}{2}(R + 1)$ (A-6)

$a_7 = \frac{e_1}{4}$ (A-7)

$a_8 = a_1 + a_2 + a_5 + a_6$ (A-8)

$a_9 = \frac{e_4}{36}$ (A-9)

$a_{10} = \frac{e_5}{20}$ (A-10)

$a_{11} = \frac{3e_6-4e_7}{27}$ (A-11)

$a_{12} = \frac{e_8}{8}$ (A-12)

$a_{13} = e_9 - 2e_{10}$ (A-13)

$a_{14} = \frac{e_{11}}{9}$ (A-14)

$a_{15} = e_{10}$ (A-15)

$a_{16} = 2e_{12}$ (A-16)

$a_{17} = e_{13}$ (A-17)

$a_{18} = \frac{5e_{11}}{36}$ (A-18)

$a_{19} = \frac{e_{14}}{25}$ (A-19)

$a_{20} = \frac{e_{15}}{16}$ (A-20)

$a_{21} = \frac{3e_{16}-2a_{11}}{27}$ (A-21)

$a_{22} = \frac{e_{17}}{4}$ (A-22)

$a_{23} = e_{18}$ (A-23)

$a_{24} = 2e_{11}$ (A-24)

$a_{25} = -2e_{10}$ (A-25)

$a_{26} = -\frac{e_{12}}{2}$ (A-26)

$a_{27} = -e_{12}$ (A-27)

$a_{28} = e_{19}$ (A-28)

$a_{29} = \frac{e_4}{14406}$ (A-29)

$a_{30} = \frac{e_5}{2500}$ (A-30)

$a_{31} = \frac{3e_6-7e_3}{2304}$ (A-31)

$a_{32} = a_1 * a_8 + a_2 * a_5$ (A-32)

$a_{33} = -\frac{a_2 * a_6}{16}$ (A-33)

$a_{34} = \frac{a_1 * a_7}{768}$ (A-34)

$e_{35} = \frac{a_2 * a_7}{16}$ (A-35)

$a_{36} = -2a_2 * a_8$(A-36)

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$e_{37} = -a_2 * a_7$ (A-37)

$a_{38} = -a_{36}$ (A-38)

$a_{39} = \frac{5a_1 * a_3}{14406}$ (A-39)

$a_{40} = \frac{3a_1 * a_4}{6480}$ (A-40)

$a_{41} = \frac{a_1(a_7+a_4+a_6)-1.33a_1a_7}{1024}$ (A-41)

$a_{42} = \frac{0.66a_1 * a_7 - 9a_2 * a_4}{1728}$ (A-42)

$a_{43} = \frac{10a_2 * a_7 - a_2(a_7+2a_4)}{16}$ (A-43)

$a_{44} = \frac{2a_1 * a_7}{768}$ (A-44)

$a_{45} = \frac{-a_2 * a_7}{8}$ (A-45)

$a_{46} = \frac{e_{12}}{2}$ (A-46)

$a_{47} = \frac{e_4}{142884}$ (A-47)

$a_{48} = \frac{e_5}{57600}$ (A-48)

$a_{49} = \frac{3e_4 - 15e_7}{18144}$ (A-49)

$a_{50} = \frac{e_8}{900}$ (A-50)

$a_{51} = \frac{a_1(a_5+a_2)-6a_2 * a_7}{64}$ (A-51)

$a_{52} = \frac{a_2(8a_6-3a_8)}{2}$ (A-52)

$a_{53} = \frac{e_5}{57600}$ (A-53)

$a_{54} = \frac{a_2 * a_7}{64}$ (A-54)

$a_{55} = a_2 * a_8$ (A-55)

$a_{56} = \frac{a_{55}}{4}$ (A-56)

$a_{57} = \frac{e_4 + 5a_1 * a_3}{93312}$ (A-57)

$a_{58} = \frac{e_5 + 3a_1 * a_4}{35000}$ (A-58)

$a_{59} = \frac{-5a_2 * a_3}{10752}$ (A-59)

$a_{60} = \frac{3e_6 - 9a_2 * a_4 - 21e_7}{2430}$ (A-60)

$a_{61} = \frac{-a_2(a_7+2a_4)+a_1 * a_8}{12}$ (A-61)

$a_{62} = 2a_{55}$ (A-62)

$a_{63} = -18a_2(a_6 + a_4)$ (A-63)

$a_{64} = \frac{a_7(a_1+a_2)}{2430}$ (A-64)

$a_{65} = \frac{-a_2 * a_7}{12}$ (A-65)

$a_{66} = 18a_2(a_6 - a_8)$ (A-66)

$a_{67} = \frac{5a_1 * a_3}{142884}$ (A-67)

$a_{68} = \frac{3a_1 * a_4}{57600}$ (A-68)

$a_{69} = \frac{-5a_1 * a_3}{7350}$ (A-69)

$a_{70} = \frac{-9a_2 * a_4 - 32a_1 * a_7}{15552}$ (A-70)

$a_{71} = \frac{a_2(8a_4-21a_7)}{64}$ (A-71)

$a_{72} = -2a_2 * a_6$ (A-72)

$a_{73} = \frac{a_1 * a_7}{2592}$ (A-73)

$a_{74} = \frac{-a_2 * a_7}{32}$ (A-74)

$a_{75} = 2a_{27}$ (A-75)

$a_{76} = 2a_{28}$ (A-76)

$a_{77} = \frac{a_1 * a_3}{15552}$ (A-77)

$a_{78} = \frac{a_1 * a_4 - 3a_2 * a_3}{6912}$ (A-78)

$a_{79} = \frac{4a_1(a_4-a_7)}{3675}$ (A-79)

$a_{80} = \frac{a_1 * a_6 - 3a_4 * a_7}{1728}$ (A-80)

$a_{81} = \frac{a_1 * a_8 + 9a_2 * a_7}{675}$ (A-81)

$a_{82} = \frac{-a_2 * a_5}{128}$ (A-82)

$a_{83} = -\frac{a_2 * a_8}{9}$ (A-83)

$a_{84} = \frac{a_1 * a_7}{3675}$ (A-84)

$a_{85} = -\frac{a_2 * a_7}{225}$ (A-85)

$a_{86} = \frac{a_1 * a_3}{180000}$ (A-86)

$a_{87} = \frac{-a_2 * a_9}{8192}$ (A-87)

$a_{88} = \frac{3a_1 * a_4 + 0.5a_3 * a_5 - 36a_3 * a_4}{43218}$ (A-88)

$a_{89} = \frac{a_1(8a_4-7a_7)}{23328}$ (A-89)

$a_{90} = \frac{a_1 * a_6 - 27a_2 * a_4}{11250}$ (A-90)

$a_{91} = \frac{0.5a_4 * a_5 + a_2(6a_7-15a_4)}{512}$ (A-91)

$a_{92} = \frac{a_7 + 8a_4 - a_2 * a_6 - 0.5a_4 * a_5 - 40a_4^2 + 3.5a_5 * a_7 + 4a_4 * a_7}{162}$ (A-92)

$a_{93} = \frac{a_6(a_5-2a_4)}{16}$ (A-93)

$a_{94} = \frac{a_3 * a_4}{4802}$ (A-94)

$a_{95} = \frac{a_2(3a_4-2a_7)}{512}$ (A-94)

$a_{96} = \frac{a_4(a_7+8a_4)+a_5 * a_7}{162}$ (A-95)

$a_{97} = \frac{a_4 * a_6}{32}$ (A-96)

$e_1 = \frac{e_2}{e_3}$ (A-97)

$e_2 = \frac{a_2}{3}R^2 + \frac{2R+1}{R+3}\left[\frac{a_1}{9} + \frac{a_2}{3} - \frac{a_1}{5}(R^4 + R^3 + R^2 + R) + \frac{a_2}{9}(R^2 + R + 1)\right]$ (A-98)

$e_3 = \frac{RlnR}{2} - \frac{R^3 lnR + R^2 - R}{2(R+3)(R-1)} - \frac{R}{4} - \frac{R^2 lnR}{4(R-1)} - \frac{R^2 lnR + R - 1}{8(R-1)}$ (A-99)

$e_4 = a_1 * a_3$ (A-100)

$e_5 = a_1 * a_6 + a_2 * a_4$ (A-101)

$e_6 = a_1 * a_5$ (A-102)

$e_7 = a_1 * 4a_3$ (A-103)

$e_8 = 2(a_1 * a_6 + a_2 * a_4)$ (A-104)

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$e_9 = a_2 * a_5 + a_1 * a_8$ (A-105)
$e_{10} = a_2 * a_7$ (A-106)
$e_{11} = a_1 * a_7$ (A-107)
$e_{12} = 2a_2 * a_6$ (A-108)
$e_{13} = a_2 * a_9$ (A-109)
$e_{14} = 5a_2 * a_9$ (A-110)
$e_{15} = -2a_2 * a_3$ (A-111)
$e_{16} = -3a_2 * a_4$ (A-112)
$e_{17} = a_1(a_7 + 2a_5 + a_6)$ (A-113)
$e_{18} = -(e_{10} + 2a_2 * a_5) - 2e_{16} * a_7$ (A-114)
$e_{19} = \frac{-1}{lnR}\left[a_9R^6 + a_{10}R^5 + a_{11}R^3 + a_{12}R^2 + a_{13}R + a_{14}R^3 lnR + a_{15}RlnR + \frac{a_{16}}{R^2} + \frac{a_{17}}{R}\right]$ (A-115)