Session Objectives

- Aircraft Performance:
  - Basics of performance (steady state and accelerated)
  - Performance characteristics of aircraft for (Civil – passenger, cargo, Military- fighter, bomber)
  - Range, Endurance, Rate of climb, maximum Mach number
- Stability and Control
  - Basics of stability : CG location , AC, limits
  - Longitudinal, Lateral control

Performance Basics

Speeds : Maximum and Stall, Range, and Rate of Climb

Performance

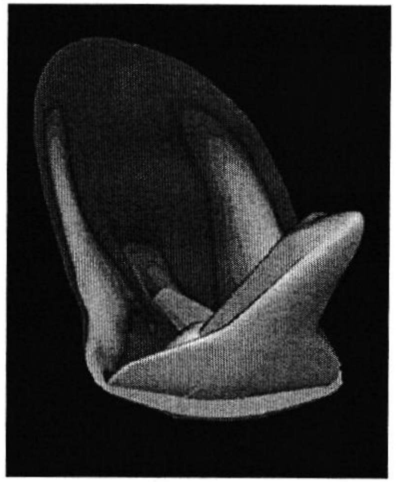
- Performance: A measure of how well a device does its job
- Airplane Performance Examples
  - Speed -> how fast/slow can it go?
  - Rate of Climb -> how fast can it go up?
  - Ceiling -> how high can it go?
  - Range -> how far can it go?
  - Endurance -> for how long can it fly?
  - Takeoff/Landing -> how much runway does it need?
  - Turning -> what is the minimum turn radius?

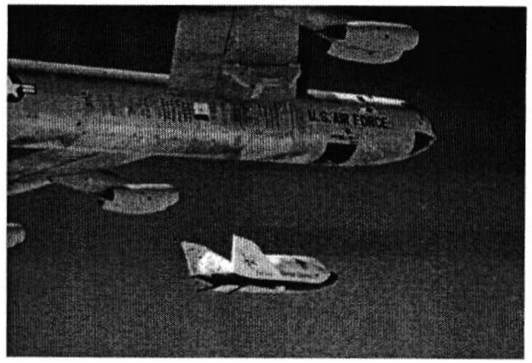
Performance

- Helicopter Performance Examples
  - Hover Capability -> how much weight can it lift vertically?
  - Speed -> how fast can it go?
  - Rate of Climb -> how fast can it go up?
  - Ceiling -> how high can it go?
  - Range -> how far can it go?
  - Endurance -> for how long can it fly?

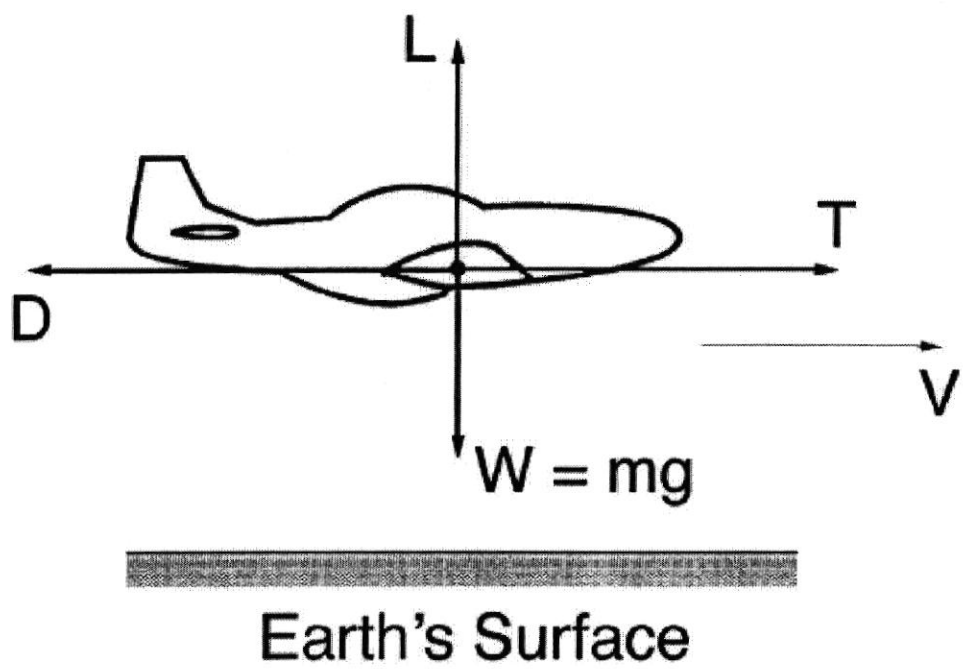
Performance

- Aircraft performance is determined by following
- Mathematical Modeling
  - Computational Fluid Dynamics
  - Classical aero/propulsive/mass analyses
- Ground Testing
  - Wind tunnel testing
  - Static engine testing
- Flight Testing





Steady level Flight



L - Lift
D - Drag
W - Weight
m - mass
g - acceleration of gravity
V - velocity, freestream airspeed (no wind)
Note: arrows not to scale!

Graphics source: Anderson, Aircraft Performance and Design

Performance

Aerodynamic Models

The lift, L, and drag, D, of the airplane can be calculated from:

$$L = \frac{1}{2}\rho V^2 S C_L \qquad D = \frac{1}{2}\rho V^2 S C_D$$

Where:

ρ is the atmospheric density
V is the airspeed
S is the wing area
C_L is the lift coefficient
C_D is the drag coefficient

Performance

Aerodynamic Models - continued…

We will model the relationship between the lift and drag of an airplane through the drag polar:

$$C_D = C_{D,0} + KC_L^2$$

Where we know the values of $C_{D,0}$ and K for our airplane.

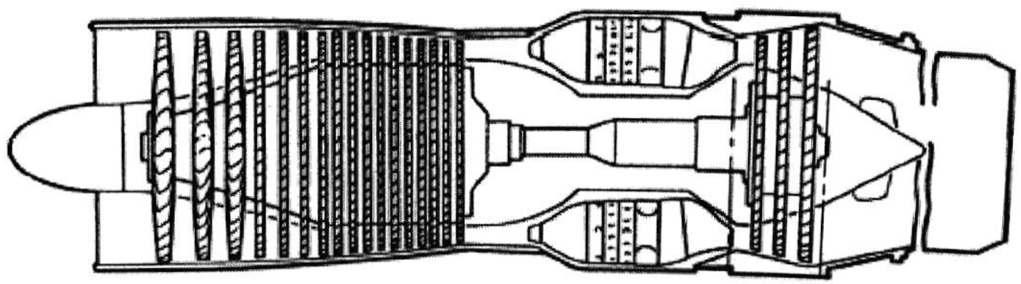
The lift coefficient, C_L, has a known maximum value:

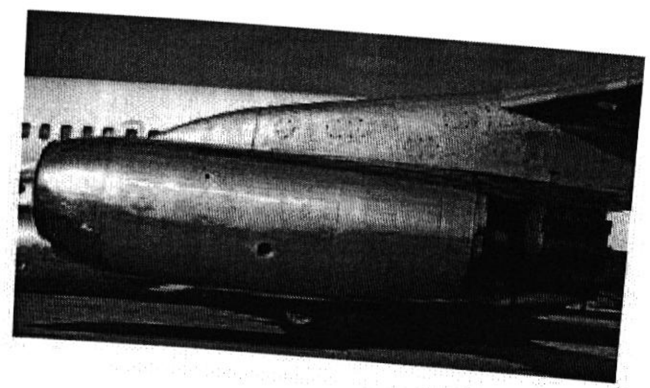
$$C_{L,Max}$$

Performance

Propulsion Models

In this introduction to performance lecture we will assume all our airplanes are using turbojet engines:





Performance

Propulsion Models - continued…

For a turbojet engine the maximum thrust, T_{Max}, does not change with airspeed, V, while flying at subsonic speeds.

For a turbojet engine the maximum thrust, T_{Max}, decreases with altitude as given by:

$$T_{Max} = T_{Max0}\left(\frac{\rho}{\rho_0}\right)$$

Where,

ρ_0 is the atmospheric density at sea level
T_{Max0} is the maximum thrust at sea level

Performance

Propulsion Models - continued…

How do we quantify the fuel consumption of a turbojet engine? We usually do so by calculating the thrust specific fuel consumption, c_t:

$$c_t = \text{thrust specific fuel consumption}$$

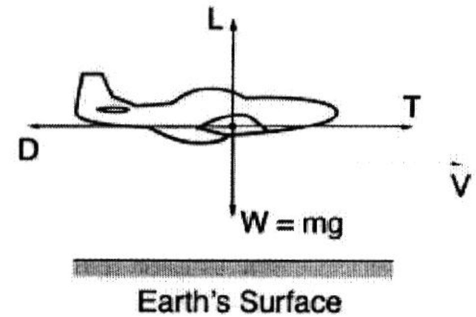
$$= \frac{(\text{weight of fuel consumed for a given time increment})}{(\text{thrust output}) \bullet (\text{time increment})}$$

$$= \frac{\dot{W}_{fuel}}{T}$$

In this lecture we will model c_t as follows:

At subsonic speeds c_t is constant. It does not vary with velocity or altitude.

Stall Speed



By definition, lift is perpendicular to the wing and V

By definition drag is parallel to V

No acceleration (steady), dV/dt = 0

Wings level

Horizontal flight parallel to Earth's surface (level)

Usually the magnitude of L and W are greater than D and T

Summing forces parallel to the flight direction:

$T - D = 0$ or equivalently $T = D$

Summing forces perpendicular to the flight direction:

$L - W = 0$ or equivalently $L = W$

Stall Speed

Let us determine how slow we can fly in steady level flight.
In other words, we want to determine the stall speed, V_{stall}.

$$L = \frac{1}{2}\rho V^2 S C_L = W$$

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

How do we minimize V for a given airplane?
By flying at $C_{L,Max}$!

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,Max}}}$$

Stall Speed : Example

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,Max}}}$$

How does the various parameters in this equation:

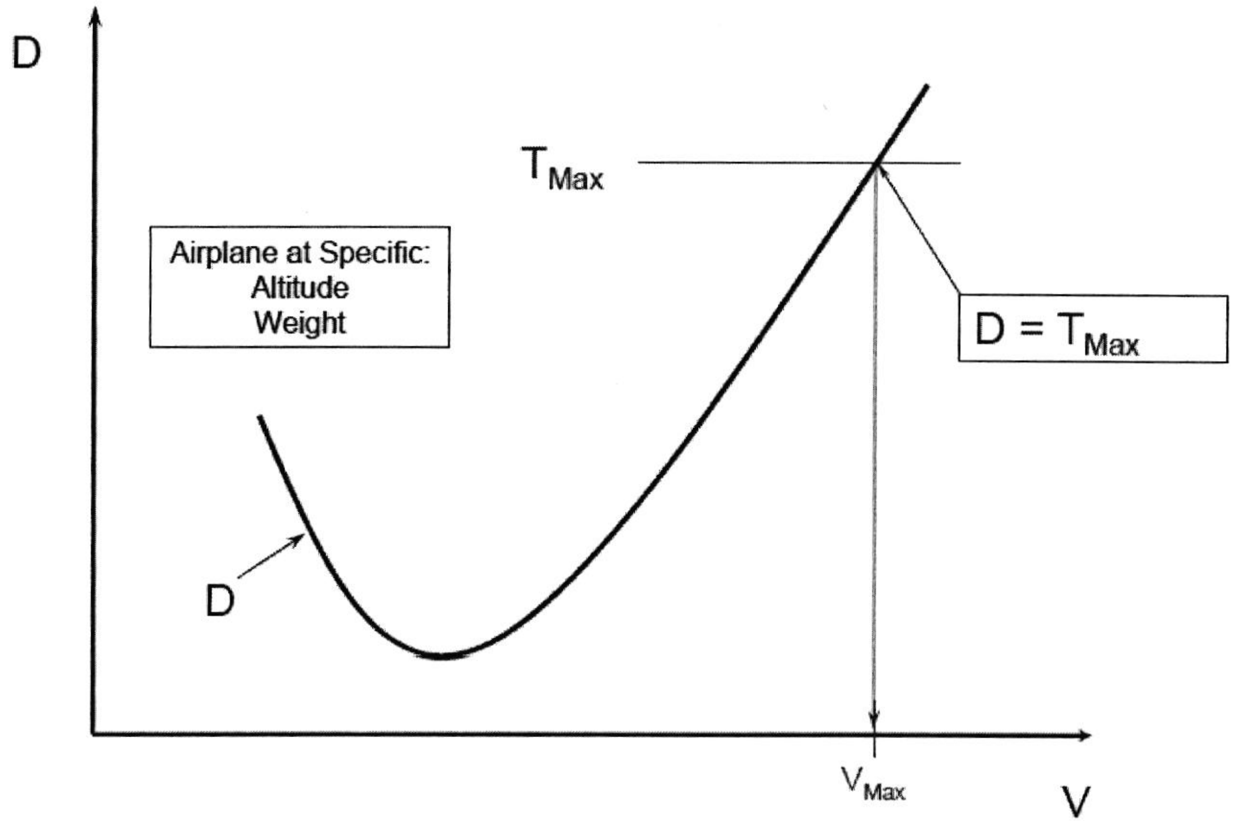
$$W,\ \rho,\ S,\ C_{L,Max}$$

affect V_{stall}?

Numerical Example: Motorglider

$m = 300$ kg $W = mg = 2{,}943$ N $\rho = 1.225\ kg/m^3$ (i.e., sea level) $S = 12.5\ m^2$ $C_{L,Max} = 1.5$	$V_{stall} = 16$ m/s

V_{max} : Maximum Speed



V_{max} : Maximum Speed

To determine the maximum speed of a turbojet airplane we start with the equation of motion:

$$T = D$$

But we agreed to model the thrust as:

$$T = T_{Max} = T_{Max0}\left(\frac{\rho}{\rho_0}\right)$$

and determine drag from:

$$D = \frac{1}{2}\rho V^2 S C_D$$

Substituting these equations into the first one gives us:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_D$$

V_{max} : Maximum Speed

$$C_D = C_{D,0} + K\frac{4W^2}{\left(\rho V^2 S\right)^2}$$

Which we can then plug in onto our equation relating thrust and drag:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_D$$

to get, after some rearranging:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_{D.0} + \frac{2KW^2}{\rho V^2 S}$$

Multiplying both sides of this equation by V^2 gives us:

V_{max} : Maximum Speed

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right)V^2 = \frac{1}{2}\rho V^4 S C_{D.0} + \frac{2KW^2}{\rho S}$$

Which, after some rearranging gives us:

$$\left[\frac{1}{2}\rho S C_{D.0}\right]V^4 - \left[T_{Max0}\left(\frac{\rho}{\rho_0}\right)\right]V^2 + \frac{2KW^2}{\rho S} = 0$$

Notice that this is simply a quadratic equation in V^2 of the form:

$$a(V^2)^2 + bV^2 + c = 0$$

Where,

$$a = \left[\frac{1}{2}\rho S C_{D.0}\right] \qquad b = \left[-T_{Max0}\left(\frac{\rho}{\rho_0}\right)\right] \qquad c = \left[\frac{2KW^2}{\rho S}\right]$$

V_{max} : Maximum Speed

This quadratic equation has a solution of the form:

$$V^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$V = \sqrt{\frac{T_{Max0}\left(\frac{\rho}{\rho_0}\right) \pm \sqrt{T_{Max0}^2\left(\frac{\rho}{\rho_0}\right)^2 - 4C_{D,0}KW^2}}{\rho S C_{D,0}}}$$

Finally, we are interested in the maximum speed, V_{Max}, so we pick the positive root in the above equation to yield…

V_{max} : Maximum Speed

$$V_{Max} = \sqrt{\frac{T_{Max0}\left(\frac{\rho}{\rho_0}\right) + \sqrt{T^2_{Max0}\left(\frac{\rho}{\rho_0}\right)^2 - 4C_{D,0}KW^2}}{\rho S C_{D,0}}}$$

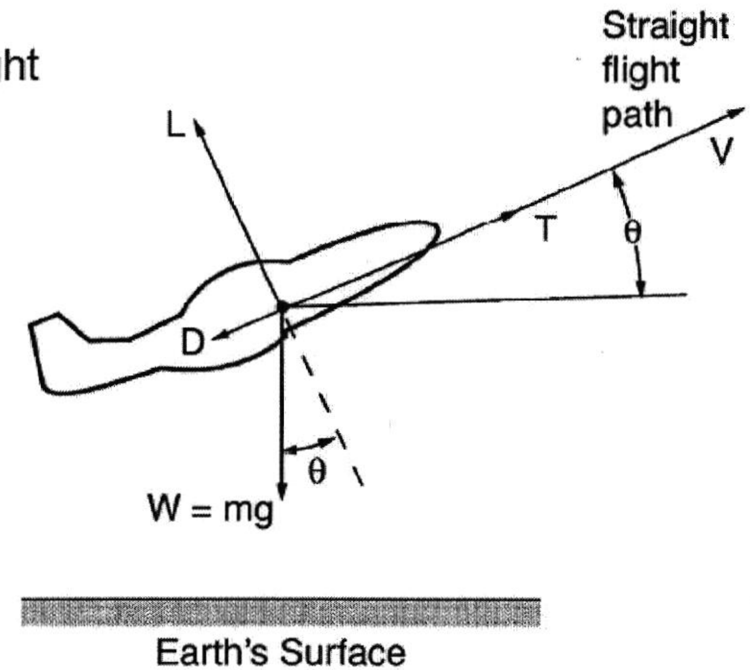
Numerical Example: Motorglider Powered by a Turbojet (Wow!)

$m = 300$ kg
$W = mg = 2{,}943$ N
$\rho = 0.6601$ kg/m^3 (i.e., 6,000 m altitude)
$\rho_0 = 1.225$ kg/m^3 (i.e., sea level)
$S = 12.5$ m^2
$C_{D,0} = 0.015$
$K = 0.020$
$T_{Max0} = 500$ N

$V_{Max} = 64.7$ m/s

Rate of Climb (ROC)

Steady Climbing Flight



θ - flight path angle wrt horizon (positive as shown)
Note: arrows not to scale!

Graphics source: Anderson, Aircraft Performance and Design

ROC

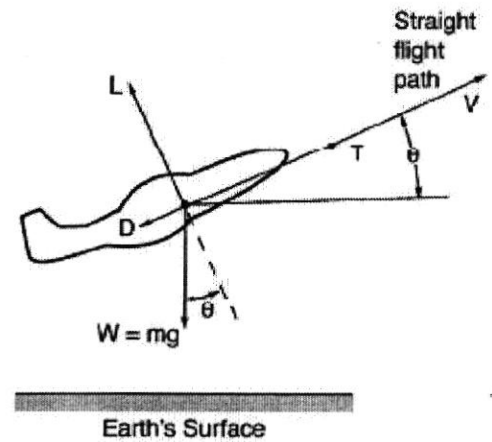
No acceleration (steady)

Wings level

Climbing flight at an angle θ to Earth's surface

Thrust parallel to velocity

Summing forces parallel to the flight direction:



$$T - D - W \sin\theta = 0$$

Summing forces perpendicular to the flight direction:

$$L - W \cos\theta = 0$$

Rate of Climb = ROC = $V \sin\theta$

Note that the same equations apply to steady descending flight!

ROC

Starting with the sum of forces parallel to the flight direction:

$$T - D - W\sin\theta = 0$$

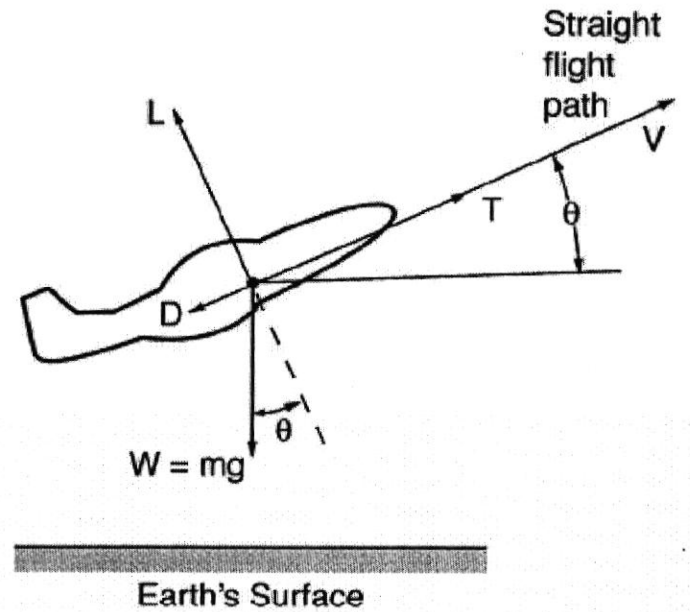
Multiply by V and re-arrange:

$$\frac{TV - DV}{W} = V\sin\theta = ROC$$

Note that:

$$T\,V = P_A = \text{Power Available}$$

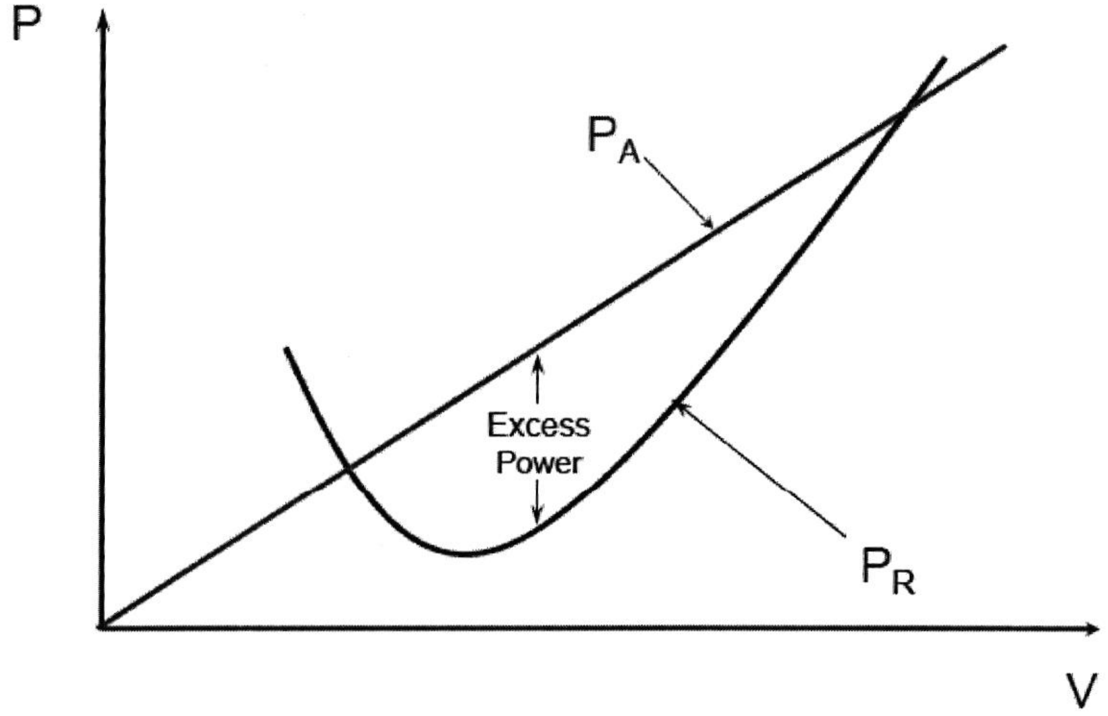
$$D\,V = P_R = \text{Power Required}$$



Thus,

$$ROC = \frac{\text{Power Available} - \text{Power Required}}{W} = \frac{\text{Excess Power}}{W}$$

ROC



$$ROC = \frac{TV - DV}{W} = \frac{P_A - P_R}{W} = \frac{\text{Power Available} - \text{Power Required}}{W} = \frac{\text{Excess Power}}{W}$$

Typical ROC numbers

Aircraft	Type	Sea level, R/C_{max}	Minimum time to climb
Mirage III	Interceptor	5.0 km/min	3.0 mt to 11 km
Mirage 2000	Interceptor	14.94 km/min	2.4 mt to 14.94 km
F-111	Fighter	10.94 km/min	N/A
F-4	Fighter	8.534 km/min	N/A
F-14	Interceptor	9.14 km/min	N/A
F-15	Fighter	15.24 km/min	N/A
F-16	Fighter	15.24 km/min	N/A

Ref : Performance Stability Dynamics and Control by Bandu Pamadi

Turn Performance

- Airplanes turn by tilting the Lift vector, to give :
  - Horizontal component, L * Sin (μ), 'μ' bank angle
  - Sharper the turn, larger the needed Lift Vector

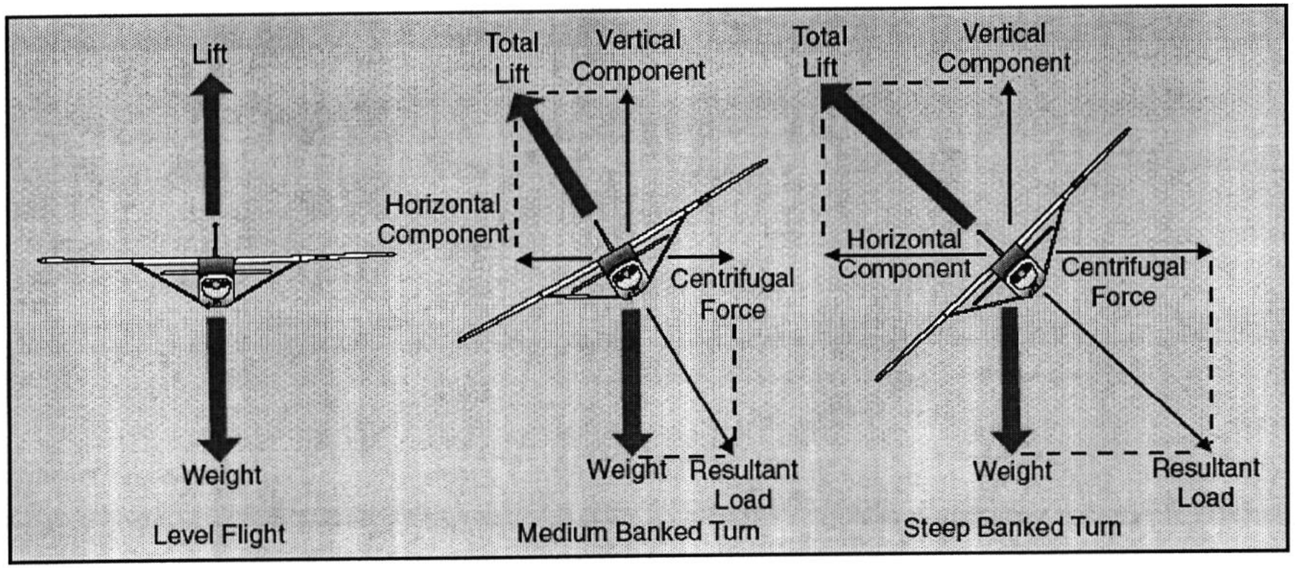


Figure 3-20. Forces during normal coordinated turn.

Turn Performance

$$T - D = 0$$

$$L \cos \mu - W = 0$$

$$L \sin \mu - \frac{WV^2}{gR} = 0$$

$$n = \frac{1}{\cos \mu}$$

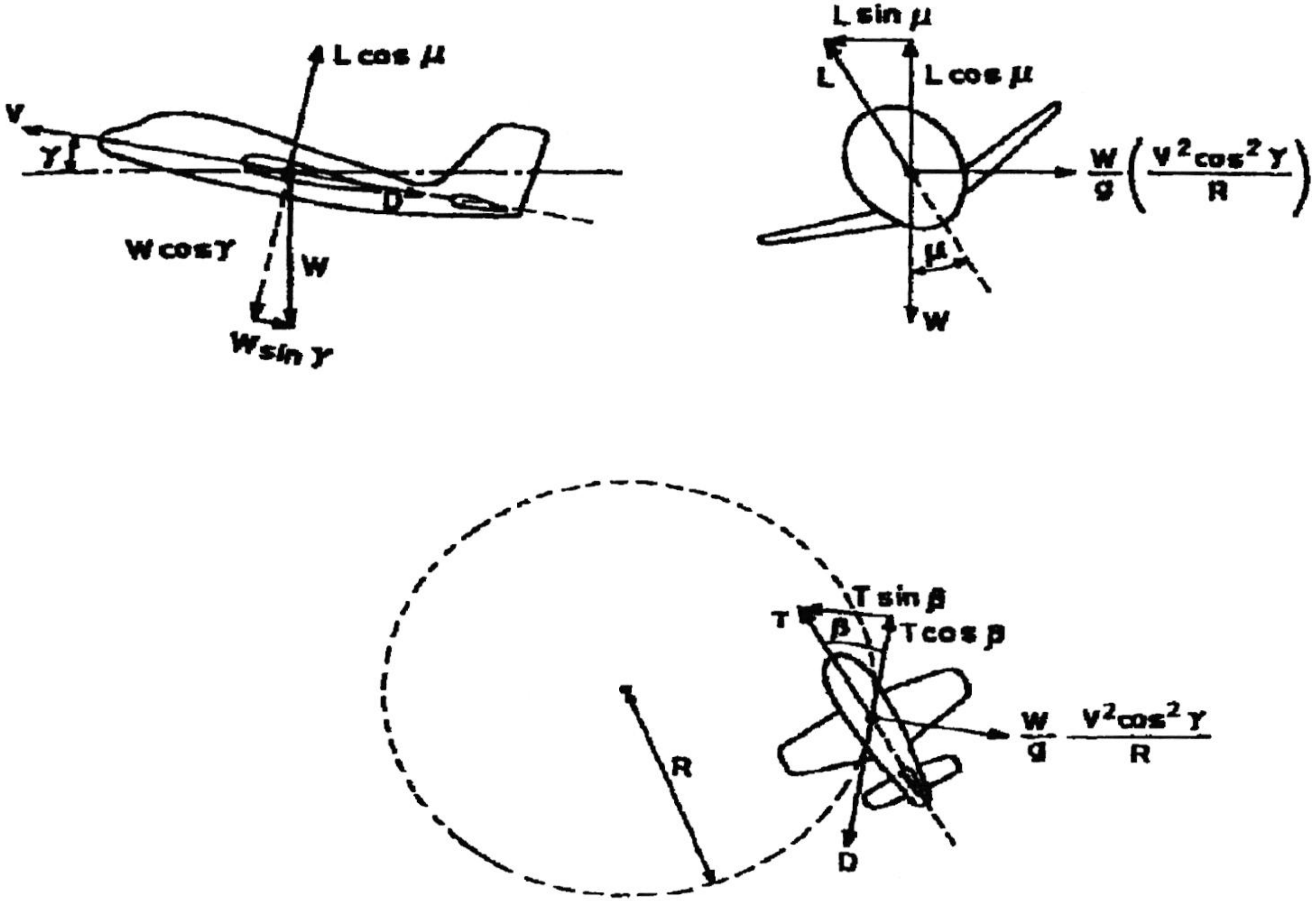
$$V = \sqrt{\frac{2nW}{\rho S C_L}}$$

$$= \sqrt{\frac{2W}{\rho S C_L \cos \mu}}$$

$$\tan \mu = \frac{V^2}{Rg}$$

$$R = \frac{V^2}{g \tan \mu}$$

Turn Performance



Range : How Far

Range - How far can we fly?

In this discussion we will try develop integral equations to calculate the range of an airplane during its cruise leg.

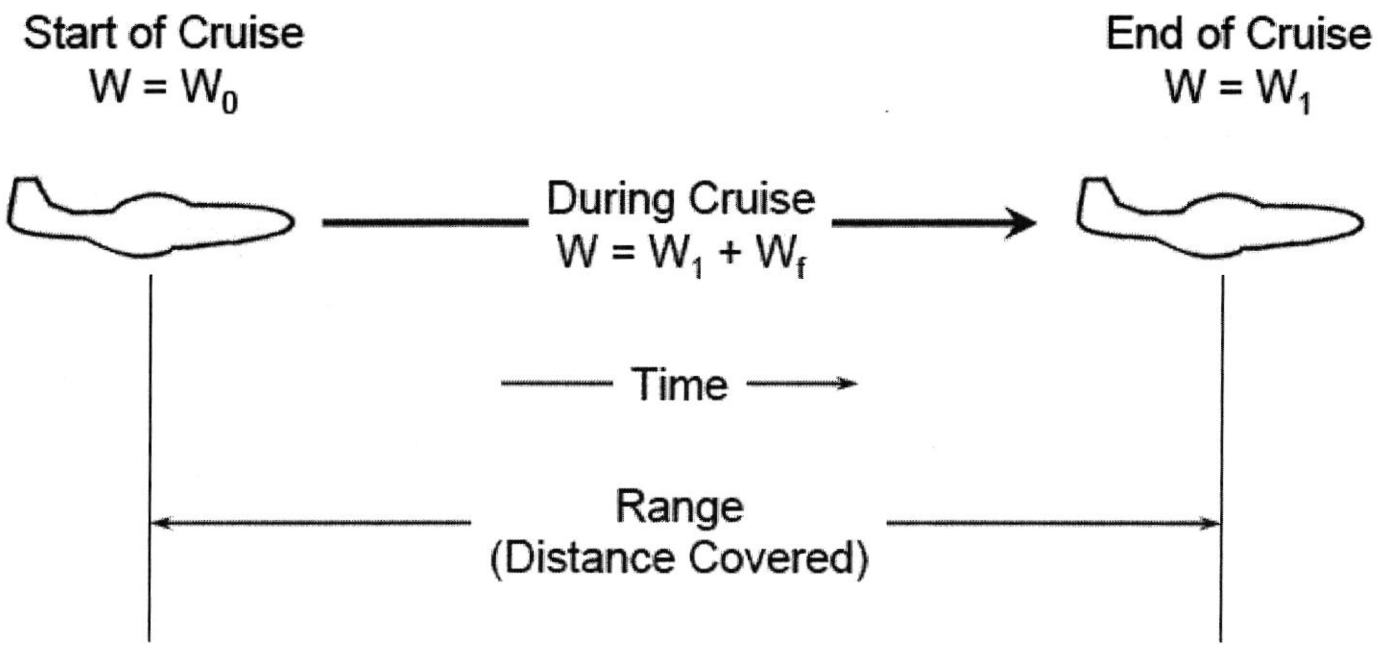
Range will be significantly affected by weight, so let us start by defining some weight quantities. Let:

- W_0 - Weight of the airplane at the start of cruise leg
- W_f - weight of fuel at a specific point in time; varies throughout the cruise leg
- W_1 - weight of the airplane at the end of the cruise leg
- W - weight of the airplane at a specific point in time; varies throughout the cruise leg

At any point during the cruise leg, then:

$$W = W_1 + W_f$$

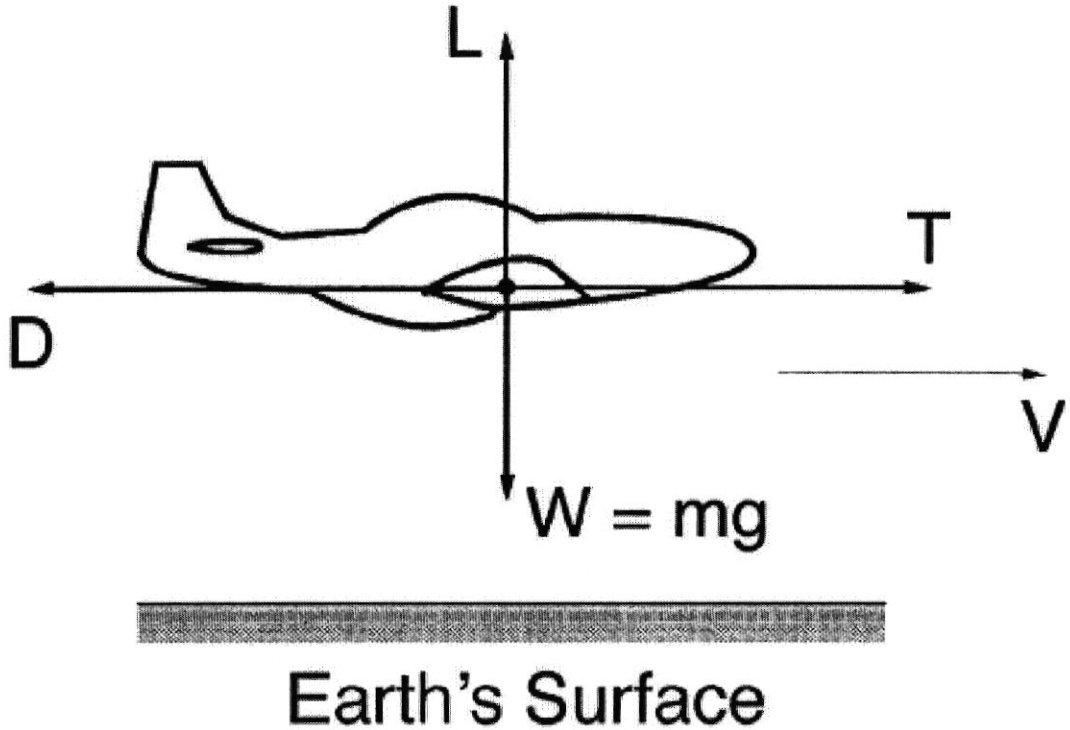
Range



Note: No assumption is being made about the flight altitude at this time - altitude during cruise *__may__* *or* *__may not__* *be constant!*

Range

Let us assume we are in level steady flight, and that our thrust vector is aligned with the airspeed:



T = D

L = W

Range

After a certain amount of calculus and manipulation we can write the following integral equation for the range, R, of a turbojet airplane:

$$R = \int_{W_1}^{W_0} \frac{V}{c_t} \frac{L}{D} \frac{dW}{W} = \int_{W_1}^{W_0} \frac{1}{c_t} \sqrt{\frac{2}{\rho S}} \frac{C_L^{1/2}}{C_D} \frac{dW}{\sqrt{W}}$$

Very few assumptions were made to derive this equation:

- Flight in no-wind conditions
- Steady level flight
  $L = W$
  $T = D$
- Thrust aligned with V

Range

We can make some further assumptions and that will simplify the integral and allow us to evaluate it explicitly:

- The thrust specific fuel consumption, c_t is constant
- We are flying at a constant altitude, thus ρ is constant
- The wing area, S, is constant (we hope so!)
- We are flying at a constant value of $\left(C_L^{1/2}/C_D\right)$

These assumptions allow us to move various quantities in front of the integral:

Range

$$R = \frac{1}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\int_{W_1}^{W_0}\frac{dW}{\sqrt{W}}$$

Now we can evaluate this integral explicitly:

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

This is one version of the range equation for a turbojet airplane.

Range

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

What is this equation telling us about obtaining maximum range for a turbojet airplane?

- For maximum range, fly at the value of C_L where $\left(C_L^{1/2}/C_D\right)$ is maximum.
- Fly at high altitude so that ρ is small (within limits…).
- Use an efficient engine with a low value of c_t.
- Carry lots of fuel.

Range

A similar equation can be derived for an airplane powered by a reciprocating engine/propeller combination:

$$R = \frac{\eta_{pr}}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$$

Where,

η_{pr} is the propeller efficiency
c is the power specific fuel consumption

This is one of the best known equations in aeronautics. It is known as the Breguet Range Equation.

What is this equation telling us about obtaining maximum range for a reciprocating engine/propeller airplane?