# HEAT TRANSFER

# انتقال الحرارة

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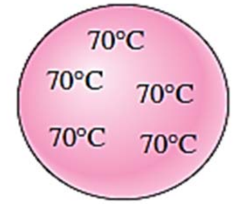
## CHAPTER FOUR TRANSIENT HEAT CONDUCTION

## 4–1 LUMPED SYSTEM ANALYSIS

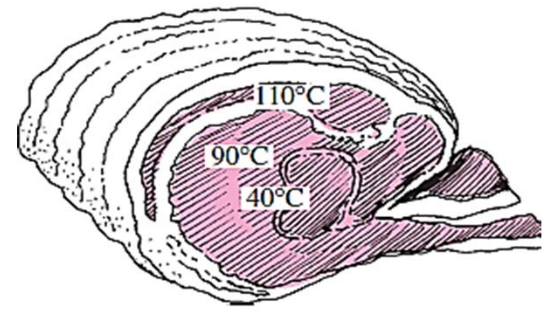
Consider a small hot copper ball coming out of an oven (Fig. 4–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time. Thus the temperature of the ball remains uniform at all times, and we can talk about the temperature of the ball with no reference to a specific location.

Now let us go to the other extreme and consider a large roast in an oven. If you have done any roasting, you must have noticed that the temperature distribution within the roast is not even close to being uniform. You can easily verify this by taking the roast out before it is completely done and cutting it in half. You will see that the outer parts of the roast are well done while the center part is barely warm. Thus, lumped system analysis is not applicable in this case. Before presenting a criterion about applicability of lumped system analysis, we develop the formulation associated with it.

Consider a body of arbitrary shape of mass $m$, volume $V$, surface area $A_s$, density $\rho$, and specific heat $C_p$ initially at a uniform temperature $T_i$ (Fig. 4–2). At time $t = 0$, the body is placed into a medium at temperature $T_\infty$, and heat transfer takes place between the body and its environment, with a heat transfer coefficient $h$. For the sake of discussion, we will assume that $T_\infty > T_i$, but the analysis is equally valid for the opposite case. We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, $T = T(t)$.




(*a*) Copper ball




(*b*) Roast beef

**FIGURE 4–1**
A small copper ball can be modeled as a lumped system, but a roast beef cannot.

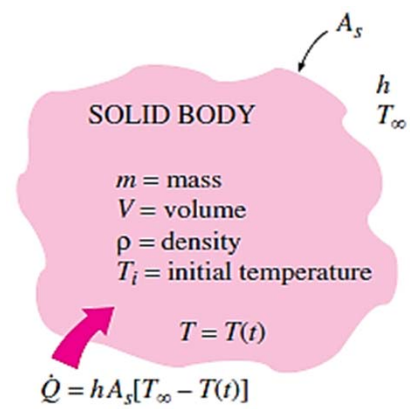



**FIGURE 4–2**

During a differential time interval $dt$, the temperature of the body rises by a differential amount $dT$. An energy balance of the solid for the time interval $dt$ can be expressed as

$$\begin{pmatrix}\text{Heat transfer into the body}\\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in the}\\ \text{energy of the body}\\ \text{during } dt\end{pmatrix} \qquad \text{or}$$

$$hA_s(T_\infty - T)\,dt = mC_p\,dT \tag{4-1}$$

Noting that $m = \rho V$ and $dT = d(T - T_\infty)$ since $T_\infty$ = constant, Eq. 4–1 can be rearranged as

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V C_p}\,dt \tag{4-2}$$

Integrating from $t = 0$, at which $T = T_i$, to any time $t$, at which $T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V C_p}\,t \tag{4-3}$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \tag{4-4}$$

where

$$b = \frac{hA_s}{\rho V C_p} \qquad (1/\text{s}) \tag{4-5}$$

is a positive quantity whose dimension is (time)$^{-1}$. The reciprocal of $b$ has time unit (usually s), and is called the **time constant.** Equation 4–4 is plotted in Fig. 4–3 for different values of $b$.

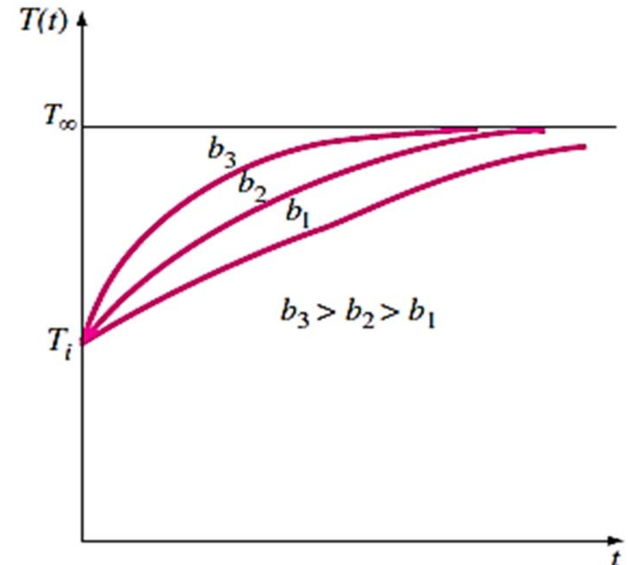



**FIGURE 4–3**
The temperature of a lumped system approaches the environment temperature as time gets larger.

Once the temperature $T(t)$ at time $t$ is available from Eq. 4–4, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \qquad \text{(W)} \qquad \text{(4-6)}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to $t$ is simply the change in the energy content of the body:

$$Q = mC_p[T(t) - T_i] \qquad \text{(kJ)} \qquad \text{(4-7)}$$

The amount of heat transfer reaches its *upper limit* when the body reaches the surrounding temperature $T_\infty$. Therefore, the *maximum* heat transfer between the body and its surroundings is (Fig. 4–4)

$$Q_{\max} = mC_p(T_\infty - T_i) \qquad \text{(kJ)} \qquad \text{(4-8)}$$

We could also obtain this equation by substituting the $T(t)$ relation from Eq. 4–4 into the $\dot{Q}(t)$ relation in Eq. 4–6 and integrating it from $t = 0$ to $t \to \infty$.

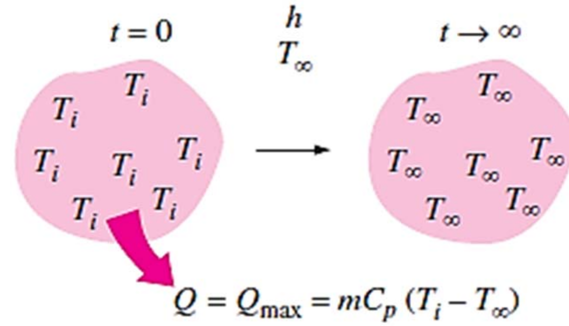



FIGURE 4–4
Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

## Criteria for Lumped System Analysis

The lumped system analysis certainly provides great convenience in heat transfer analysis, and naturally we would like to know when it is appropriate to use it. The first step in establishing a criterion for the applicability of the lumped system analysis is to define a **characteristic length** as

$$L_c = \frac{V}{A_s}$$

and a **Biot number** Bi as

$$\text{Bi} = \frac{hL_c}{k} \qquad \text{(4-9)}$$

It can also be expressed as (Fig. 4–5)

$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}} \quad \text{or} \quad \text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

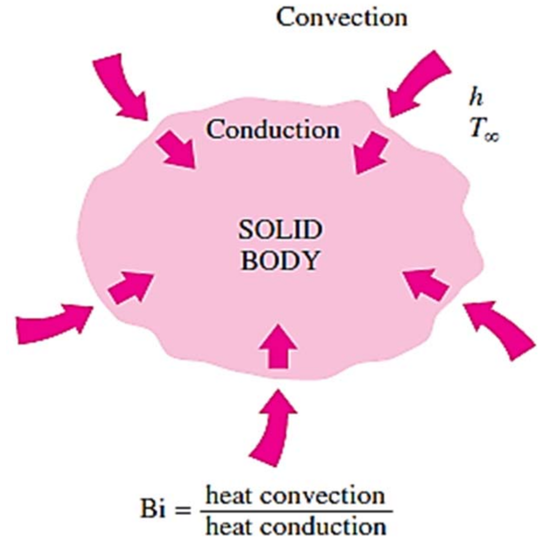
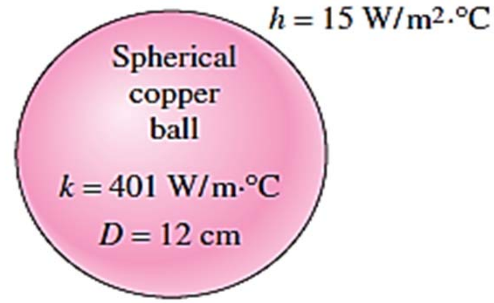



FIGURE 4–5

When this criterion is satisfied, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other even for well-rounded geometries such as a spherical ball. Thus, when Bi $< 0.1$, the variation of temperature with location within the body will be slight and can reasonably be approximated as being uniform.

The first step in the application of lumped system analysis is the calculation of the *Biot number,* and the assessment of the applicability of this approach. One may still wish to use lumped system analysis even when the criterion Bi $< 0.1$ is not satisfied, if high accuracy is not a major concern.

Note that the Biot number is the ratio of the *convection* at the surface to *conduction* within the body, and this number should be as small as possible for lumped system analysis to be applicable. Therefore, *small bodies* with *high thermal conductivity* are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, discussed earlier, is most likely to satisfy the criterion for lumped system analysis (Fig. 4–6).




$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02\ \mathrm{m}$$

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

**FIGURE 4–6**
Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

## *EXAMPLE 4–1* **Temperature Measurement by Thermocouples**

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere, as shown in Fig. 4–9. The properties of the junction are $k = 35$ W/m · °C, $\rho = 8500$ kg/m³, and $C_p = 320$ J/kg · °C, and the convection heat transfer coefficient between the junction and the gas is $h = 210$ W/m² · °C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

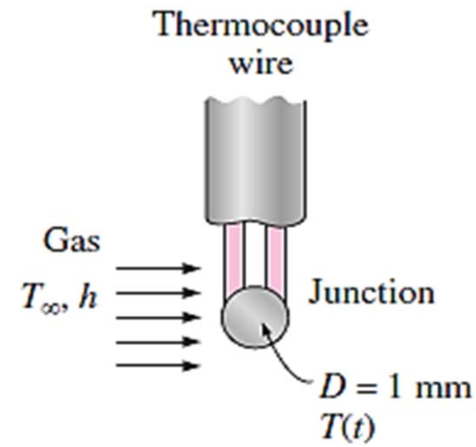



FIGURE 4–9

### SOLUTION

The characteristic length of the junction is

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = \frac{1}{6}(0.001 \text{ m}) = 1.67 \times 10^{-4} \text{ m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(210 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.67 \times 10^{-4} \text{ m})}{35 \text{ W/m} \cdot {}^\circ\text{C}} = 0.001 < 0.1$$

Therefore, lumped system analysis is applicable, and the error involved in this approximation is negligible.

In order to read 99 percent of the initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

For example, when $T_i = 0$°C and $T_\infty = 100$°C, a thermocouple is considered to have read 99 percent of this applied temperature difference when its reading indicates $T(t) = 99$°C.

The value of the exponent $b$ is

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{210 \text{ W/m}^2 \cdot {}^\circ\text{C}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg} \cdot {}^\circ\text{C})(1.67 \times 10^{-4} \text{ m})} = 0.462 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.462 \text{ s}^{-1})t} \longrightarrow t = 10 \text{ s}$$

Therefore, we must wait at least 10 s for the temperature of the thermocouple junction to approach within 1 percent of the initial junction-gas temperature difference.

## *EXAMPLE 4–2* Predicting the Time of Death

A person is found dead at 5 PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be $h = 8$ W/m² · °C. Modeling the body as a 30-cm-diameter, 1.70-m-long cylinder, estimate the time of death of that person (Fig. 4–10).

### SOLUTION

*Properties* The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37 + 25)/2 = 31°C; $k = 0.617$ W/m · °C, $\rho = 996$ kg/m³, and $C_p = 4178$ J/kg · °C

The characteristic length of the body is

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.15\text{ m})^2(1.7\text{ m})}{2\pi(0.15\text{ m})(1.7\text{ m}) + 2\pi(0.15\text{ m})^2} = 0.0689\text{ m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(8\text{ W/m}^2\cdot{}^\circ\text{C})(0.0689\text{ m})}{0.617\text{ W/m}\cdot{}^\circ\text{C}} = 0.89 > 0.1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a "rough" estimate of the time of death. The exponent $b$ in this case is

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{8\text{ W/m}^2\cdot{}^\circ\text{C}}{(996\text{ kg/m}^3)(4178\text{ J/kg}\cdot{}^\circ\text{C})(0.0689\text{ m})}$$
$$= 2.79 \times 10^{-5}\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25 - 20}{37 - 20} = e^{-(2.79\times10^{-5}\text{ s}^{-1})t} \longrightarrow t = 43{,}860\text{ s} = 12.2\text{ h}$$

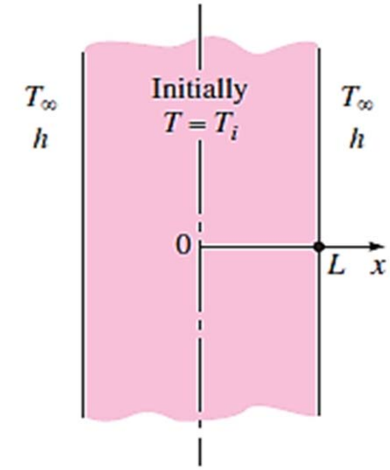
Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is 5 AM.

FIGURE 4–10

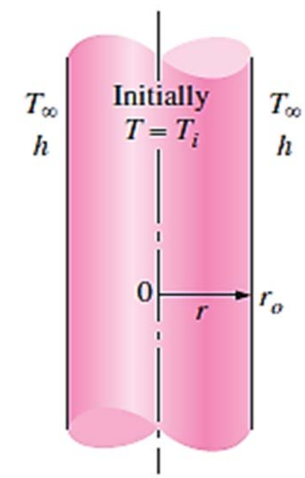
## 4–2 TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

In Section, 4–1, we considered bodies in which the variation of temperature within the body was negligible; that is, bodies that remain nearly *isothermal* during a process. Relatively *small* bodies of *highly conductive* materials approximate this behavior. In general, however, the temperature within a body will change from point to point as well as with time. In this section, we consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.

Consider a plane wall of thickness $2L$, a long cylinder of radius $r_o$, and a sphere of radius $r_o$ initially at a *uniform temperature* $T_i$, as shown in Fig. 4–11. At time $t = 0$, each geometry is placed in a large medium that is at a constant temperature $T_\infty$ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a *uniform* and *constant* heat transfer coefficient $h$. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its *center plane* ($x = 0$), the cylinder is symmetric about its *centerline* ($r = 0$), and the sphere is symmetric about its *center point* ($r = 0$). We neglect *radiation* heat transfer between these bodies and their surrounding surfaces, or incorporate the radiation effect into the convection heat transfer coefficient $h$.

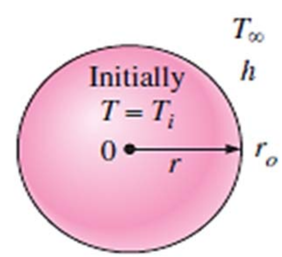



(*a*) A large plane wall




(*b*) A long cylinder

**FIGURE 4–11**
Schematic of the simple geometries in which heat transfer is one-dimensional.




(*c*) A sphere

The one-dimensional transient heat conduction problem just described can be solved exactly for any of the three geometries, but the solution involves infinite series, which are difficult to deal with. However, the terms in the solutions converge rapidly with increasing time, and for $\tau > 0.2$, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent. We are usually interested in the solution for times with $\tau > 0.2$, and thus it is very convenient to express the solution using this **one-term approximation,** given as

Plane wall:
$$\theta(x, t)_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2 \tag{4-10}$$

Cylinder:
$$\theta(r, t)_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2 \tag{4-11}$$

Sphere:
$$\theta(r, t)_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2 \tag{4-12}$$

where the constants $A_1$ and $\lambda_1$ are functions of the Bi number only, and their values are listed in Table 4–1 against the Bi number for all three geometries. The function $J_0$ is the zeroth-order Bessel function of the first kind, whose value can be determined from Table 4–2. Noting that $\cos(0) = J_0(0) = 1$ and the limit of $(\sin x)/x$ is also 1, these relations simplify to the next ones at the center of a plane wall, cylinder, or sphere:

Center of plane wall ($x = 0$):
$$\theta_{0,\text{ wall}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4-13}$$

Center of cylinder ($r = 0$):
$$\theta_{0,\text{ cyl}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4-14}$$

Center of sphere ($r = 0$):
$$\theta_{0,\text{ sph}} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4-15}$$

**TABLE 4–1**

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = $hL/k$ for a plane wall of thickness $2L$, and Bi = $hr_o/k$ for a cylinder or sphere of radius $r_o$)

| | *Plane Wall* | | *Cylinder* | | *Sphere* | |
|---|---|---|---|---|---|---|
| **Bi** | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ |
| 0.01 | 0.0998 | 1.0017 | 0.1412 | 1.0025 | 0.1730 | 1.0030 |
| 0.02 | 0.1410 | 1.0033 | 0.1995 | 1.0050 | 0.2445 | 1.0060 |
| 0.04 | 0.1987 | 1.0066 | 0.2814 | 1.0099 | 0.3450 | 1.0120 |
| 0.06 | 0.2425 | 1.0098 | 0.3438 | 1.0148 | 0.4217 | 1.0179 |
| 0.08 | 0.2791 | 1.0130 | 0.3960 | 1.0197 | 0.4860 | 1.0239 |
| 0.1 | 0.3111 | 1.0161 | 0.4417 | 1.0246 | 0.5423 | 1.0298 |
| 0.2 | 0.4328 | 1.0311 | 0.6170 | 1.0483 | 0.7593 | 1.0592 |
| 0.3 | 0.5218 | 1.0450 | 0.7465 | 1.0712 | 0.9208 | 1.0880 |
| 0.4 | 0.5932 | 1.0580 | 0.8516 | 1.0931 | 1.0528 | 1.1164 |
| 0.5 | 0.6533 | 1.0701 | 0.9408 | 1.1143 | 1.1656 | 1.1441 |
| 0.6 | 0.7051 | 1.0814 | 1.0184 | 1.1345 | 1.2644 | 1.1713 |
| 0.7 | 0.7506 | 1.0918 | 1.0873 | 1.1539 | 1.3525 | 1.1978 |
| 0.8 | 0.7910 | 1.1016 | 1.1490 | 1.1724 | 1.4320 | 1.2236 |
| 0.9 | 0.8274 | 1.1107 | 1.2048 | 1.1902 | 1.5044 | 1.2488 |
| 1.0 | 0.8603 | 1.1191 | 1.2558 | 1.2071 | 1.5708 | 1.2732 |
| 2.0 | 1.0769 | 1.1785 | 1.5995 | 1.3384 | 2.0288 | 1.4793 |
| 3.0 | 1.1925 | 1.2102 | 1.7887 | 1.4191 | 2.2889 | 1.6227 |
| 4.0 | 1.2646 | 1.2287 | 1.9081 | 1.4698 | 2.4556 | 1.7202 |
| 5.0 | 1.3138 | 1.2403 | 1.9898 | 1.5029 | 2.5704 | 1.7870 |
| 6.0 | 1.3496 | 1.2479 | 2.0490 | 1.5253 | 2.6537 | 1.8338 |
| 7.0 | 1.3766 | 1.2532 | 2.0937 | 1.5411 | 2.7165 | 1.8673 |
| 8.0 | 1.3978 | 1.2570 | 2.1286 | 1.5526 | 2.7654 | 1.8920 |
| 9.0 | 1.4149 | 1.2598 | 2.1566 | 1.5611 | 2.8044 | 1.9106 |
| 10.0 | 1.4289 | 1.2620 | 2.1795 | 1.5677 | 2.8363 | 1.9249 |
| 20.0 | 1.4961 | 1.2699 | 2.2880 | 1.5919 | 2.9857 | 1.9781 |
| 30.0 | 1.5202 | 1.2717 | 2.3261 | 1.5973 | 3.0372 | 1.9898 |
| 40.0 | 1.5325 | 1.2723 | 2.3455 | 1.5993 | 3.0632 | 1.9942 |
| 50.0 | 1.5400 | 1.2727 | 2.3572 | 1.6002 | 3.0788 | 1.9962 |
| 100.0 | 1.5552 | 1.2731 | 2.3809 | 1.6015 | 3.1102 | 1.9990 |
| ∞ | 1.5708 | 1.2732 | 2.4048 | 1.6021 | 3.1416 | 2.0000 |

**TABLE 4–2**

The zeroth- and first-order Bessel functions of the first kind

| $\xi$ | $J_o(\xi)$ | $J_1(\xi)$ |
|---|---|---|
| 0.0 | 1.0000 | 0.0000 |
| 0.1 | 0.9975 | 0.0499 |
| 0.2 | 0.9900 | 0.0995 |
| 0.3 | 0.9776 | 0.1483 |
| 0.4 | 0.9604 | 0.1960 |
| 0.5 | 0.9385 | 0.2423 |
| 0.6 | 0.9120 | 0.2867 |
| 0.7 | 0.8812 | 0.3290 |
| 0.8 | 0.8463 | 0.3688 |
| 0.9 | 0.8075 | 0.4059 |
| 1.0 | 0.7652 | 0.4400 |
| 1.1 | 0.7196 | 0.4709 |
| 1.2 | 0.6711 | 0.4983 |
| 1.3 | 0.6201 | 0.5220 |
| 1.4 | 0.5669 | 0.5419 |
| 1.5 | 0.5118 | 0.5579 |
| 1.6 | 0.4554 | 0.5699 |
| 1.7 | 0.3980 | 0.5778 |
| 1.8 | 0.3400 | 0.5815 |
| 1.9 | 0.2818 | 0.5812 |
| 2.0 | 0.2239 | 0.5767 |
| 2.1 | 0.1666 | 0.5683 |
| 2.2 | 0.1104 | 0.5560 |
| 2.3 | 0.0555 | 0.5399 |
| 2.4 | 0.0025 | 0.5202 |
| 2.6 | −0.0968 | −0.4708 |
| 2.8 | −0.1850 | −0.4097 |
| 3.0 | −0.2601 | −0.3391 |
| 3.2 | −0.3202 | −0.2613 |

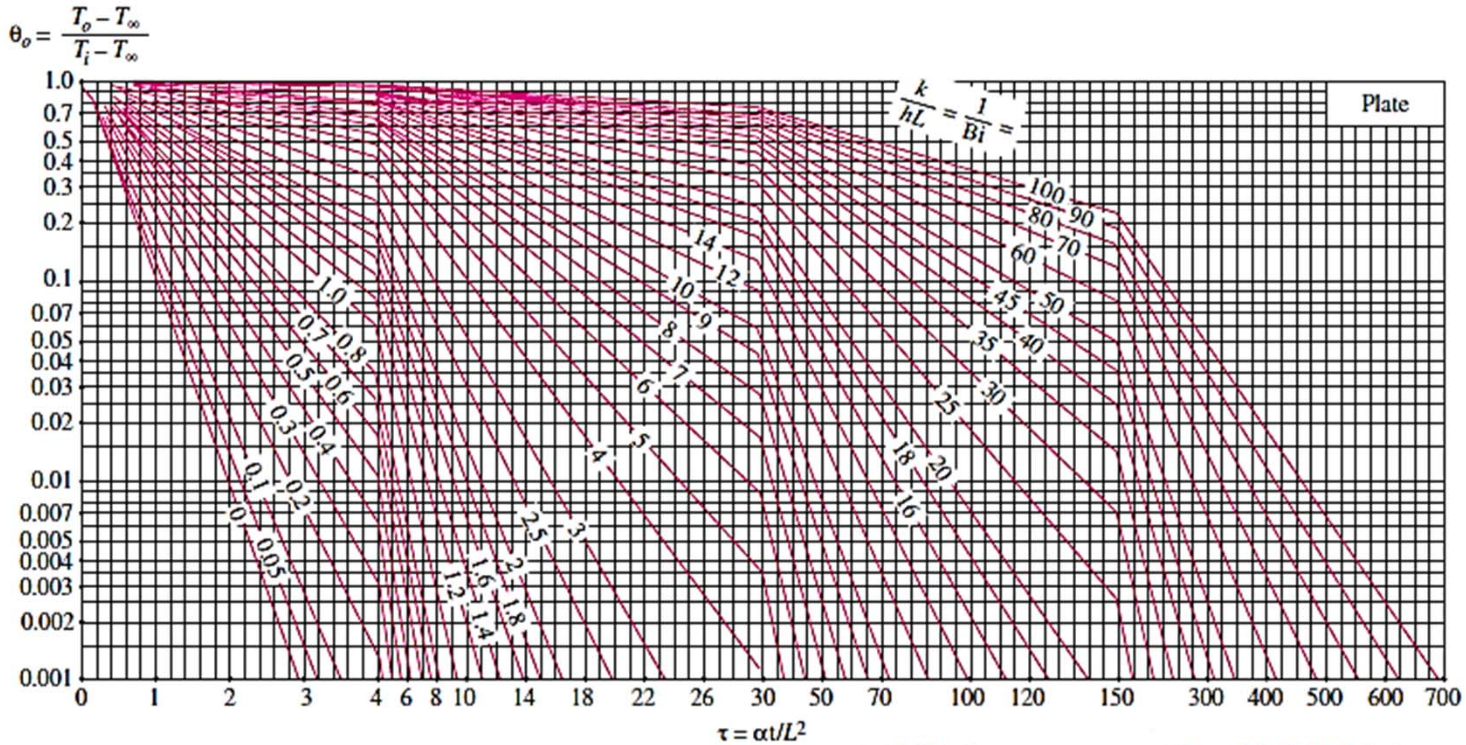
Note that the case $1/\text{Bi} = k/hL = 0$ corresponds to $h \to \infty$, which corresponds to the case of *specified surface temperature* $T_\infty$. That is, the case in which the surfaces of the body are suddenly brought to the temperature $T_\infty$ at $t = 0$ and kept at $T_\infty$ at all times can be handled by setting $h$ to infinity

The temperature of the body changes from the initial temperature $T_i$ to the temperature of the surroundings $T_\infty$ at the end of the transient heat conduction process. Thus, the *maximum* amount of heat that a body can gain (or lose if $T_i > T_\infty$) is simply the *change* in the *energy content* of the body. That is,

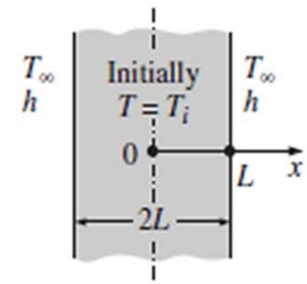
$$Q_{\max} = mC_p(T_\infty - T_i) = \rho VC_p(T_\infty - T_i) \qquad \text{(kJ)} \tag{4-16}$$

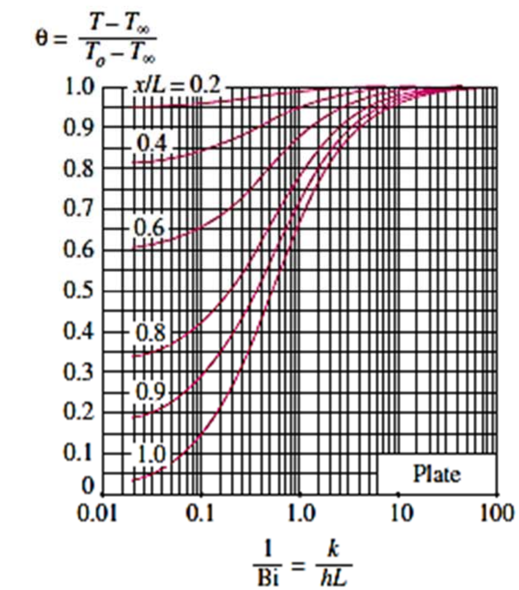
Once the Bi number is known, the above relations can be used to determine the temperature anywhere in the medium. The determination of the constants $A_1$ and $\lambda_1$ usually requires interpolation. For those who prefer reading charts to interpolating, the relations above are plotted and the one-term approximation solutions are presented in graphical form, known as the *transient temperature charts.* Note that the charts are sometimes difficult to read, and they are subject to reading errors. Therefore, the relations above should be preferred to the charts.

The transient temperature charts in Figs. 4–13, 4–14, and 4–15 for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called **Heisler charts.** They were supplemented in 1961 with transient heat transfer charts by H. Gröber. There are *three* charts associated with each geometry: the first chart is to determine the temperature $T_o$ at the *center* of the geometry at a given time $t$. The second chart is to determine the temperature at *other locations* at the same time in terms of $T_o$. The third chart is to determine the total amount of *heat transfer* up to the time $t$. These plots are valid for $\tau > 0.2$.

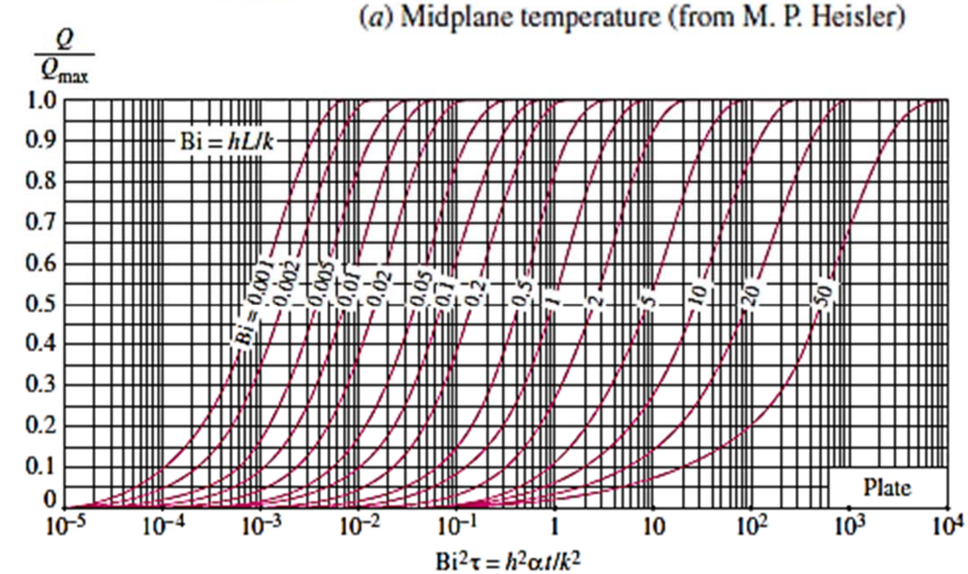



(*a*) Midplane temperature (from M. P. Heisler)



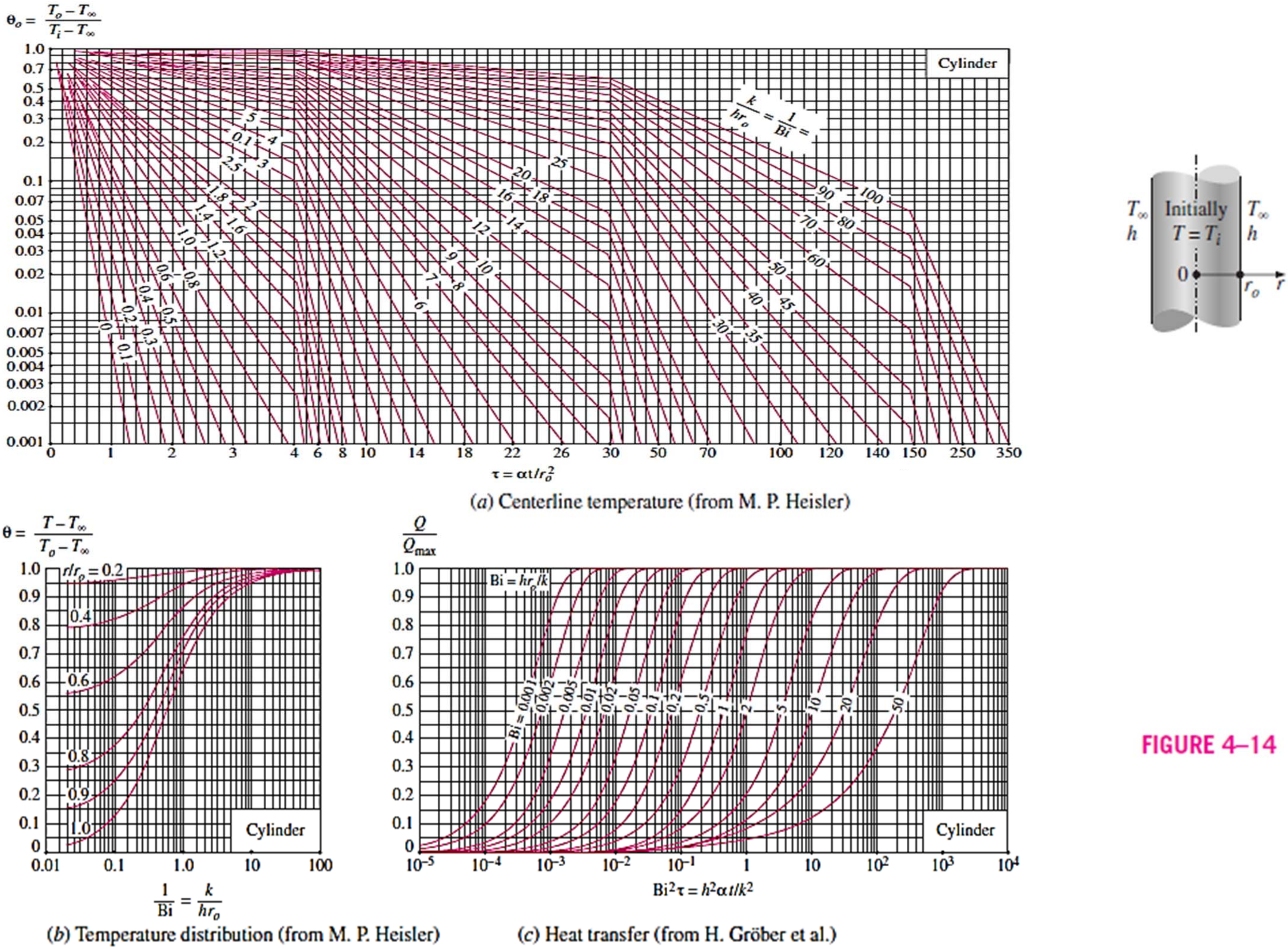




(*b*) Temperature distribution (from M. P. Heisler)



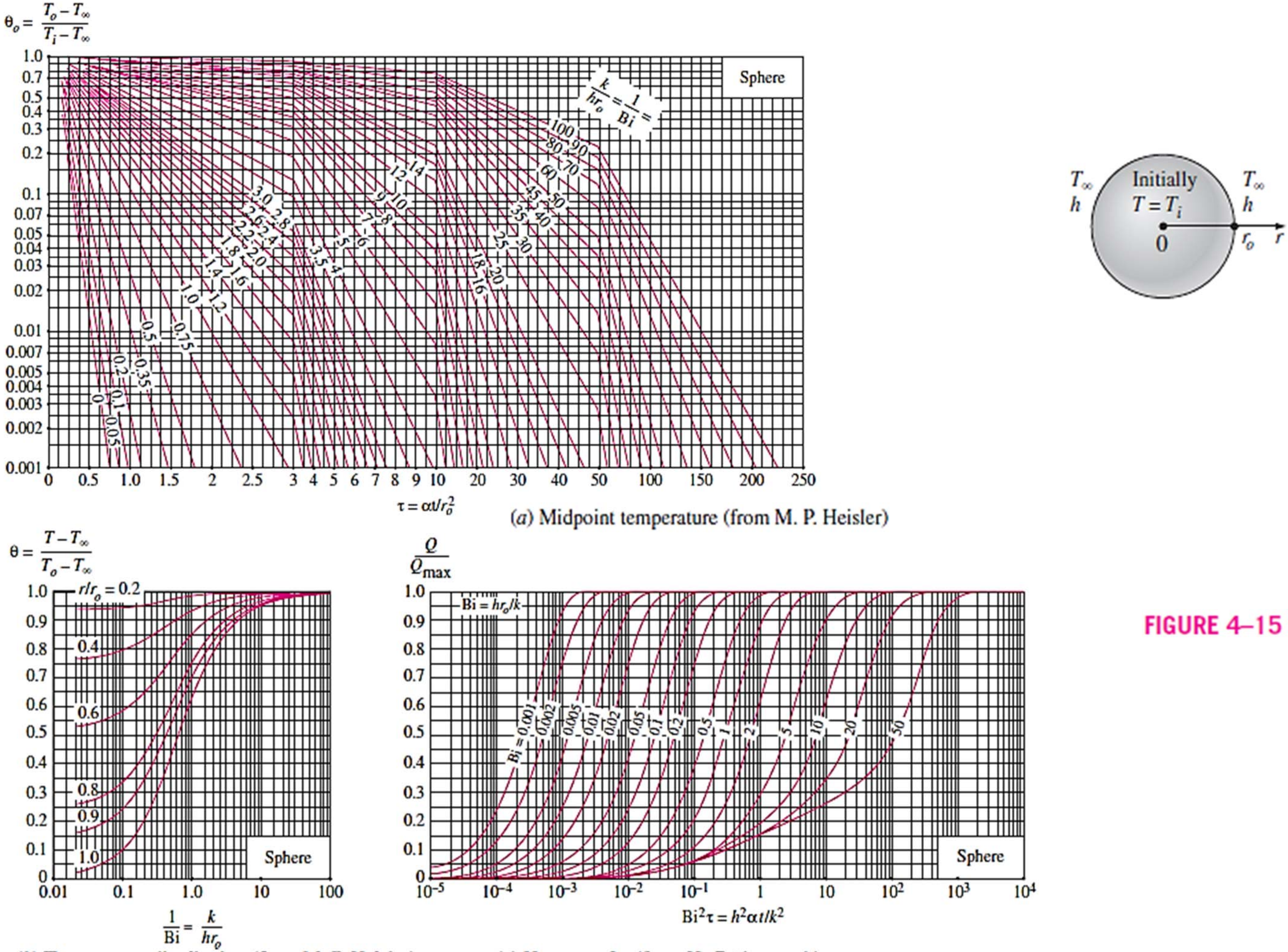

(*c*) Heat transfer (from H. Gröber et al.)

**FIGURE 4–13**



$\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty}$
Cylinder
$\frac{k}{hr_o} = \frac{1}{Bi} =$
$\tau = \alpha t/r_o^2$
(a) Centerline temperature (from M. P. Heisler)
$\theta = \frac{T - T_\infty}{T_o - T_\infty}$
$r/r_o = 0.2$
Cylinder
$\frac{1}{Bi} = \frac{k}{hr_o}$
(b) Temperature distribution (from M. P. Heisler)
$\frac{Q}{Q_{max}}$
$Bi = hr_o/k$
Cylinder
$Bi^2\tau = h^2\alpha t/k^2$
(c) Heat transfer (from H. Gröber et al.)
$T_\infty$
$h$
Initially
$T = T_i$
$r_o$
$r$


FIGURE 4–14




(*a*) Midpoint temperature (from M. P. Heisler)

(*b*) Temperature distribution (from M. P. Heisler)

(*c*) Heat transfer (from H. Gröber et al.)

**FIGURE 4–15**

The fraction of heat transfer can also be determined from these relations, which are based on the one-term approximations already discussed:

Plane wall: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\,\text{wall}} \frac{\sin \lambda_1}{\lambda_1} \tag{4-17}$$

Cylinder: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1} \tag{4-18}$$

Sphere: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3} \tag{4-19}$$

## *EXAMPLE 4–3* **Boiling Eggs**

An ordinary egg can be approximated as a 5-cm-diameter sphere (Fig. 4–19). The egg is initially at a uniform temperature of 5°C and is dropped into boiling water at 95°C. Taking the convection heat transfer coefficient to be $h = 1200\ \text{W/m}^2 \cdot °\text{C}$, determine how long it will take for the center of the egg to reach 70°C.

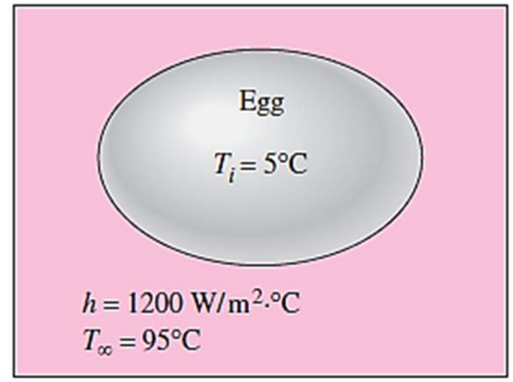



**FIGURE 4–19**

**SOLUTION**

***Properties*** The water content of eggs is about 74 percent, and thus the thermal conductivity and diffusivity of eggs can be approximated by those of water at the average temperature of (5 + 70)/2 = 37.5°C; $k = 0.627\ \text{W/m} \cdot °\text{C}$ and $\alpha = k/\rho C_p = 0.151 \times 10^{-6}\ \text{m}^2/\text{s}$

The Biot number for this problem is

$$\text{Bi} = \frac{hr_0}{k} = \frac{(1200\ \text{W/m}^2 \cdot °\text{C})(0.025\ \text{m})}{0.627\ \text{W/m} \cdot °\text{C}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients $\lambda_1$ and $A_1$ for a sphere corresponding to this Bi are, from Table 4–1,

$$\lambda_1 = 3.0753, \qquad A_1 = 1.9958$$

Substituting these and other values into Eq. 4–15 and solving for $\tau$ gives

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0753)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025\ \text{m})^2}{0.151 \times 10^{-6}\ \text{m}^2/\text{s}} = 865\ \text{s} \approx 14.4\ \text{min}$$

Therefore, it will take about 15 min for the center of the egg to be heated from 5°C to 70°C.

## *EXAMPLE 4–4* Heating of Large Brass Plates in an Oven

In a production facility, large brass plates of 4 cm thickness that are initially at a uniform temperature of 20°C are heated by passing them through an oven that is maintained at 500°C (Fig. 4–20). The plates remain in the oven for a period of 7 min. Taking the combined convection and radiation heat transfer coefficient to be $h = 120\ \mathrm{W/m^2 \cdot °C}$, determine the surface temperature of the plates when they come out of the oven.

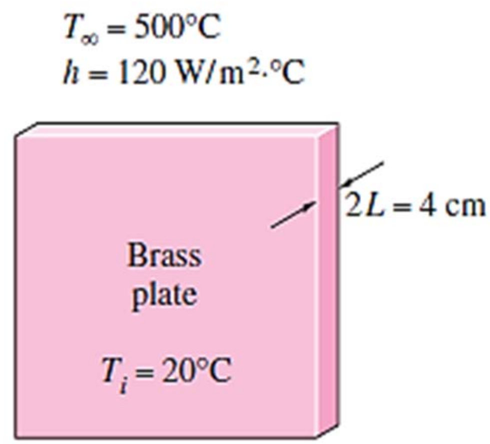



FIGURE 4–20

### SOLUTION

*Properties* The properties of brass at room temperature are $k = 110\ \mathrm{W/m \cdot °C}$, $\rho = 8530\ \mathrm{kg/m^3}$, $C_p = 380\ \mathrm{J/kg \cdot °C}$, and $\alpha = 33.9 \times 10^{-6}\ \mathrm{m^2/s}$
More accurate results are obtained by using properties at average temperature.

Noting that the half-thickness of the plate is $L = 0.02$ m, from Fig. 4–13 we have

$$\left.\begin{aligned} \frac{1}{\mathrm{Bi}} &= \frac{k}{hL} = \frac{100\ \mathrm{W/m \cdot °C}}{(120\ \mathrm{W/m^2 \cdot °C})(0.02\ \mathrm{m})} = 45.8 \\ \tau &= \frac{\alpha t}{L^2} = \frac{(33.9 \times 10^{-6}\ \mathrm{m^2/s})(7 \times 60\ \mathrm{s})}{(0.02\ \mathrm{m})^2} = 35.6 \end{aligned}\right\} \frac{T_o - T_\infty}{T_i - T_\infty} = 0.46 \quad \text{Also,}$$

$$\left.\begin{aligned} \frac{1}{\mathrm{Bi}} &= \frac{k}{hL} = 45.8 \\ \frac{x}{L} &= \frac{L}{L} = 1 \end{aligned}\right\} \frac{T - T_\infty}{T_o - T_\infty} = 0.99 \quad \text{Therefore,} \quad \frac{T - T_\infty}{T_i - T_\infty} = \frac{T - T_\infty}{T_o - T_\infty}\frac{T_o - T_\infty}{T_i - T_\infty} = 0.46 \times 0.99 = 0.455$$

and $$T = T_\infty + 0.455(T_i - T_\infty) = 500 + 0.455(20 - 500) = 282°\mathrm{C}$$

Therefore, the surface temperature of the plates will be 282°C when they leave the oven.

**EXAMPLE 4–5 Cooling of a Long Stainless Steel Cylindrical Shaft**

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C (Fig. 4–21). The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of $h = 80\ \text{W/m}^2\cdot°\text{C}$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

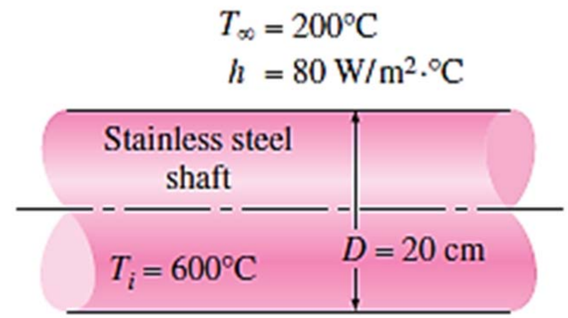



**FIGURE 4–21**

**SOLUTION**

*Properties* The properties of stainless steel 304 at room temperature are $k = 14.9\ \text{W/m}\cdot°\text{C}$, $\rho = 7900\ \text{kg/m}^3$, $C_p = 477\ \text{J/kg}\cdot°\text{C}$, and $\alpha = 3.95\times10^{-6}\ \text{m}^2\text{/s}$ More accurate results can be obtained by using properties at average temperature.

be determined from the Heisler charts. Noting that the radius of the shaft is $r_o = 0.1$ m, from Fig. 4–14 we have

$$\left.\begin{aligned}\frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{14.9\ \text{W/m}\cdot°\text{C}}{(80\ \text{W/m}^2\cdot°\text{C})(0.1\ \text{m})} = 1.86\\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.95\times10^{-6}\ \text{m}^2\text{/s})(45\times60\ \text{s})}{(0.1\ \text{m})^2} = 1.07\end{aligned}\right\}\ \frac{T_o - T_\infty}{T_i - T_\infty} = 0.40 \qquad \text{and}$$

$$T_o = T_\infty + 0.4(T_i - T_\infty) = 200 + 0.4(600 - 200) = 360°\text{C}$$

Therefore, the center temperature of the shaft will drop from 600°C to 360°C in 45 min.

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to its environment. Taking $L = 1$ m,

$$m = \rho V = \rho\pi r_o^2 L = (7900\ \text{kg/m}^3)\pi(0.1\ \text{m})^2(1\ \text{m}) = 248.2\ \text{kg}$$

$$Q_{\text{max}} = mC_p(T_\infty - T_i) = (248.2\ \text{kg})(0.477\ \text{kJ/kg}\cdot{}^\circ\text{C})(600 - 200)^\circ\text{C}$$
$$= 47{,}354\ \text{kJ}$$

The dimensionless heat transfer ratio is determined from Fig. 4–14*c* for a long cylinder to be

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{1.86} = 0.537 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.537)^2(1.07) = 0.309 \end{aligned}\right\} \frac{Q}{Q_{\text{max}}} = 0.62$$

Therefore, $Q = 0.62Q_{\text{max}} = 0.62 \times (47{,}354\ \text{kJ}) = 29{,}360\ \text{kJ}$

which is the total heat transfer from the shaft during the first 45 min of the cooling.

**ALTERNATIVE SOLUTION** We could also solve this problem using the one-term solution relation instead of the transient charts. First we find the Biot number

$$\text{Bi} = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2\cdot{}^\circ\text{C})(0.1\ \text{m})}{14.9\ \text{W/m}\cdot{}^\circ\text{C}} = 0.537$$

The coefficients $\lambda_1$ and $A_1$ for a cylinder corresponding to this Bi are determined from Table 4–1 to be $\lambda_1 = 0.970, \quad A_1 = 1.122$

Substituting these values into Eq. 4–14 gives $\theta_0 = \dfrac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = 1.122e^{-(0.970)^2(1.07)} = 0.41$

and thus $T_o = T_\infty + 0.41(T_i - T_\infty) = 200 + 0.41(600 - 200) = 364^\circ\text{C}$

The value of $J_1(\lambda_1)$ for $\lambda_1 = 0.970$ is determined from Table 4–2 to be 0.430. Then the fractional heat transfer is determined from Eq. 4–18 to be

$$\frac{Q}{Q_{\text{max}}} = 1 - 2\theta_0\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2 \times 0.41\frac{0.430}{0.970} = 0.636$$

and thus $Q = 0.636Q_{\text{max}} = 0.636 \times (47{,}354\ \text{kJ}) = 30{,}120\ \text{kJ}$

## 4–3 TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions, as shown in Fig. 4–22. This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface. The earth, for example, can be considered to be a semi-infinite medium in determining the variation of temperature near its surface. Also, a thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation.

Consider a semi-infinite solid that is at a uniform temperature $T_i$. At time $t = 0$, the surface of the solid at $x = 0$ is exposed to convection by a fluid at a constant temperature $T_\infty$, with a heat transfer coefficient $h$. This problem can be formulated as a partial differential equation, which can be solved analytically for the transient temperature distribution $T(x, t)$. The solution obtained is presented in Fig. 4–23 graphically for the *nondimensionalized temperature* defined as

$$1 - \theta(x, t) = 1 - \frac{T(x, t) - T_\infty}{T_i - T_\infty} = \frac{T(x, t) - T_i}{T_\infty - T_i} \qquad \text{(4-21)}$$

against the dimensionless variable $x/(2\sqrt{\alpha t})$ for various values of the parameter $h\sqrt{\alpha t}/k$.

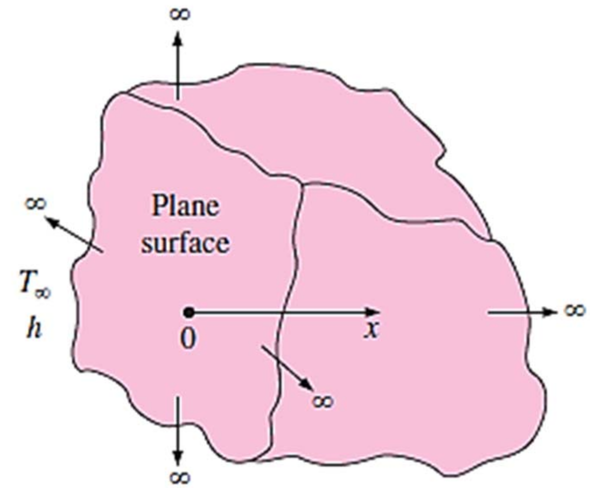



FIGURE 4–22

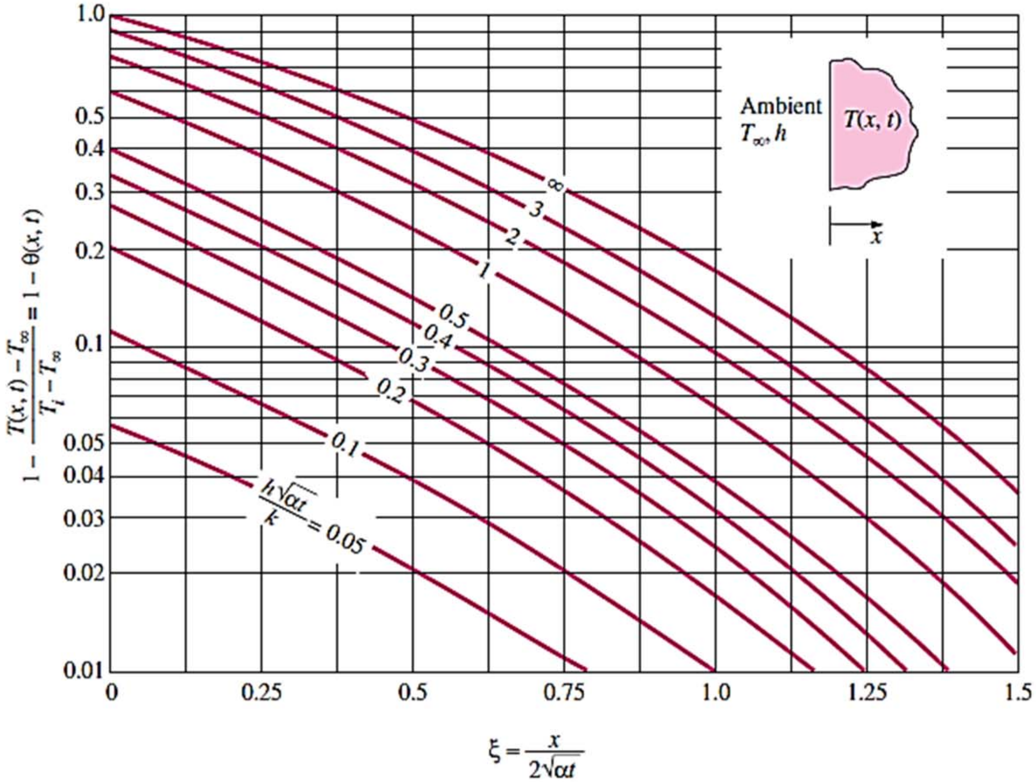



FIGURE 4–23

Note that the values on the vertical axis correspond to $x = 0$, and thus represent the surface temperature. The curve $h\sqrt{\alpha t}/k = \infty$ corresponds to $h \to \infty$, which corresponds to the case of *specified temperature* $T_\infty$ at the surface at $x = 0$. That is, the case in which the surface of the semi-infinite body is suddenly brought to temperature $T_\infty$ at $t = 0$ and kept at $T_\infty$ at all times can be handled by setting $h$ to infinity. The specified surface temperature case is closely approximated in practice when condensation or boiling takes place on the surface. For a *finite* heat transfer coefficient $h$, the surface temperature approaches the fluid temperature $T_\infty$ as the time $t$ approaches infinity.

The exact solution of the transient one-dimensional heat conduction problem in a semi-infinite medium that is initially at a uniform temperature of $T_i$ and is suddenly subjected to convection at time $t = 0$ has been obtained, and is expressed as

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right] \tag{4-22}$$

where the quantity erfc ($\xi$) is the **complementary error function,** defined as

$$\operatorname{erfc}(\xi) = 1 - \frac{2}{\sqrt{\pi}}\int_0^\xi e^{-u^2}\,du \tag{4-23}$$

Despite its simple appearance, the integral that appears in the above relation cannot be performed analytically. Therefore, it is evaluated numerically for different values of $\xi$, and the results are listed in Table 4–3. For the special case of $h \to \infty$, the surface temperature $T_s$ becomes equal to the fluid temperature $T_\infty$, and Eq. 4–22 reduces to

$$\frac{T(x, t) - T_i}{T_s - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{4-24}$$

**TABLE 4–3**

The complementary error function

| ξ | erfc (ξ) | ξ | erfc (ξ) | ξ | erfc (ξ) | ξ | erfc (ξ) | ξ | erfc (ξ) | ξ | erfc (ξ) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.00000 | 0.38 | 0.5910 | 0.76 | 0.2825 | 1.14 | 0.1069 | 1.52 | 0.03159 | 1.90 | 0.00721 |
| 0.02 | 0.9774 | 0.40 | 0.5716 | 0.78 | 0.2700 | 1.16 | 0.10090 | 1.54 | 0.02941 | 1.92 | 0.00662 |
| 0.04 | 0.9549 | 0.42 | 0.5525 | 0.80 | 0.2579 | 1.18 | 0.09516 | 1.56 | 0.02737 | 1.94 | 0.00608 |
| 0.06 | 0.9324 | 0.44 | 0.5338 | 0.82 | 0.2462 | 1.20 | 0.08969 | 1.58 | 0.02545 | 1.96 | 0.00557 |
| 0.08 | 0.9099 | 0.46 | 0.5153 | 0.84 | 0.2349 | 1.22 | 0.08447 | 1.60 | 0.02365 | 1.98 | 0.00511 |
| 0.10 | 0.8875 | 0.48 | 0.4973 | 0.86 | 0.2239 | 1.24 | 0.07950 | 1.62 | 0.02196 | 2.00 | 0.00468 |
| 0.12 | 0.8652 | 0.50 | 0.4795 | 0.88 | 0.2133 | 1.26 | 0.07476 | 1.64 | 0.02038 | 2.10 | 0.00298 |
| 0.14 | 0.8431 | 0.52 | 0.4621 | 0.90 | 0.2031 | 1.28 | 0.07027 | 1.66 | 0.01890 | 2.20 | 0.00186 |
| 0.16 | 0.8210 | 0.54 | 0.4451 | 0.92 | 0.1932 | 1.30 | 0.06599 | 1.68 | 0.01751 | 2.30 | 0.00114 |
| 0.18 | 0.7991 | 0.56 | 0.4284 | 0.94 | 0.1837 | 1.32 | 0.06194 | 1.70 | 0.01612 | 2.40 | 0.00069 |
| 0.20 | 0.7773 | 0.58 | 0.4121 | 0.96 | 0.1746 | 1.34 | 0.05809 | 1.72 | 0.01500 | 2.50 | 0.00041 |
| 0.22 | 0.7557 | 0.60 | 0.3961 | 0.98 | 0.1658 | 1.36 | 0.05444 | 1.74 | 0.01387 | 2.60 | 0.00024 |
| 0.24 | 0.7343 | 0.62 | 0.3806 | 1.00 | 0.1573 | 1.38 | 0.05098 | 1.76 | 0.01281 | 2.70 | 0.00013 |
| 0.26 | 0.7131 | 0.64 | 0.3654 | 1.02 | 0.1492 | 1.40 | 0.04772 | 1.78 | 0.01183 | 2.80 | 0.00008 |
| 0.28 | 0.6921 | 0.66 | 0.3506 | 1.04 | 0.1413 | 1.42 | 0.04462 | 1.80 | 0.01091 | 2.90 | 0.00004 |
| 0.30 | 0.6714 | 0.68 | 0.3362 | 1.06 | 0.1339 | 1.44 | 0.04170 | 1.82 | 0.01006 | 3.00 | 0.00002 |
| 0.32 | 0.6509 | 0.70 | 0.3222 | 1.08 | 0.1267 | 1.46 | 0.03895 | 1.84 | 0.00926 | 3.20 | 0.00001 |
| 0.34 | 0.6306 | 0.72 | 0.3086 | 1.10 | 0.1198 | 1.48 | 0.03635 | 1.86 | 0.00853 | 3.40 | 0.00000 |
| 0.36 | 0.6107 | 0.74 | 0.2953 | 1.12 | 0.1132 | 1.50 | 0.03390 | 1.88 | 0.00784 | 3.60 | 0.00000 |

## *EXAMPLE 4–6* Minimum Burial Depth of Water Pipes to Avoid Freezing

In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as an insulation to protect the water from subfreezing temperatures in winter.

The ground at a particular location is covered with snow pack at −10°C for a continuous period of three months, and the average soil properties at that location are $k = 0.4$ W/m · °C and $\alpha = 0.15 \times 10^{-6}$ m²/s (Fig. 4–24). Assuming an initial uniform temperature of 15°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

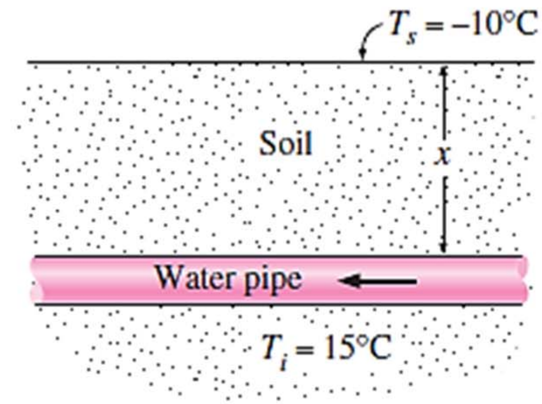



FIGURE 4–24

### SOLUTION

*Analysis* The temperature of the soil surrounding the pipes will be 0°C after three months in the case of minimum burial depth. Therefore, from Fig. 4–23, we have

$$\left.\begin{aligned} &\frac{h\sqrt{\alpha t}}{k} = \infty \qquad \text{(since } h \to \infty) \\ &1 - \frac{T(x,t) - T_\infty}{T_i - T_\infty} = 1 - \frac{0 - (-10)}{15 - (-10)} = 0.6 \end{aligned}\right\} \xi = \frac{x}{2\sqrt{\alpha t}} = 0.36$$

We note that

$$t = (90 \text{ days})(24 \text{ h/day})(3600 \text{ s/h}) = 7.78 \times 10^6 \text{ s}$$

and thus

$$x = 2\xi\sqrt{\alpha t} = 2 \times 0.36\sqrt{(0.15 \times 10^{-6} \text{ m}^2/\text{s})(7.78 \times 10^6 \text{ s})} = \mathbf{0.77\ m}$$

Therefore, the water pipes must be buried to a depth of at least 77 cm to avoid freezing under the specified harsh winter conditions.

**ALTERNATIVE SOLUTION** The solution of this problem could also be determined from Eq. 4–24:

$$\frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \longrightarrow \frac{0 - 15}{-10 - 15} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = 0.60$$

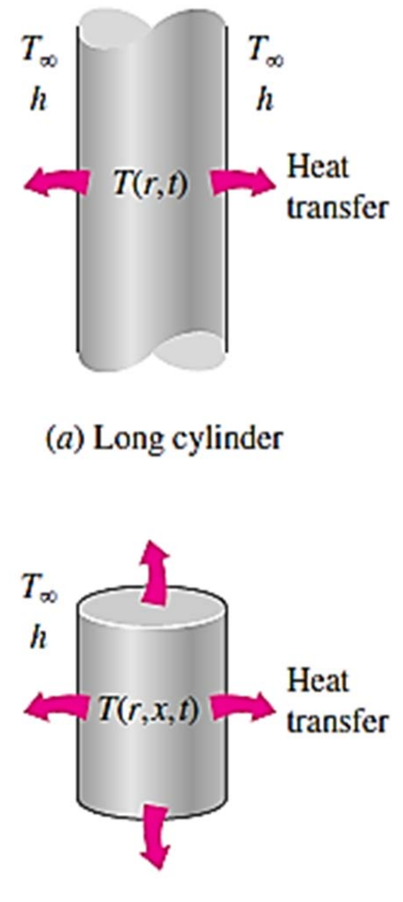
The argument that corresponds to this value of the complementary error function is determined from Table 4–3 to be $\xi = 0.37$. Therefore,

$$x = 2\xi\sqrt{\alpha t} = 2 \times 0.37\sqrt{(0.15 \times 10^{-6}\ \text{m}^2/\text{s})(7.78 \times 10^6\ \text{s})} = \mathbf{0.80\ m}$$

Again, the slight difference is due to the reading error of the chart.

## 4–4 TRANSIENT HEAT CONDUCTION IN MULTIDIMENSIONAL SYSTEMS

The transient temperature charts presented earlier can be used to determine the temperature distribution and heat transfer in *one-dimensional* heat conduction problems associated with a large plane wall, a long cylinder, a sphere, and a semi-infinite medium. Using a superposition approach called the **product solution,** these charts can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that *all* surfaces of the solid are subjected to convection to the *same* fluid at temperature $T_\infty$, with the *same* heat transfer coefficient $h$, and the body involves no heat generation (Fig. 4–25). The solution in such multidimensional geometries can be expressed as the *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.




(*b*) Short cylinder (two-dimensional)

**FIGURE 4–25**

Consider a *short cylinder* of height $a$ and radius $r_o$ initially at a uniform temperature $T_i$. There is no heat generation in the cylinder. At time $t = 0$, the cylinder is subjected to convection from all surfaces to a medium at temperature $T_\infty$ with a heat transfer coefficient $h$. The temperature within the cylinder will change with $x$ as well as $r$ and time $t$ since heat transfer will occur from the top and bottom of the cylinder as well as its side surfaces. That is, $T = T(r, x, t)$ and thus this is a two-dimensional transient heat conduction problem. When the properties are assumed to be constant, it can be shown that the solution of this two-dimensional problem can be expressed as

$$\left(\frac{T(r, x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}} \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}} \tag{4-25}$$

That is, the solution for the two-dimensional short cylinder of height $a$ and radius $r_o$ is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness $a$ and the long cylinder of radius $r_o$, which are the two geometries whose intersection is the short cylinder, as shown in Fig. 4–26. We generalize this as follows: *the solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.*

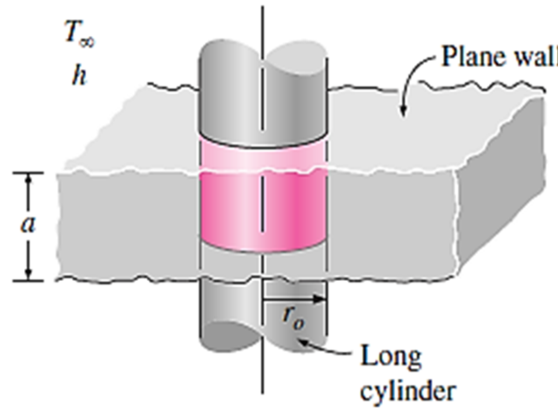



**FIGURE 4–26**

A short cylinder of radius $r_o$ and height $a$ is the *intersection* of a long cylinder of radius $r_o$ and a plane wall of thickness $a$.

For convenience, the one-dimensional solutions are denoted by

$$\theta_{\text{wall}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}}$$

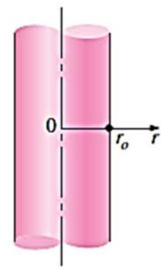
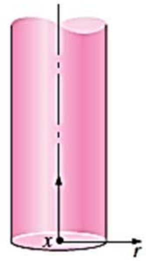
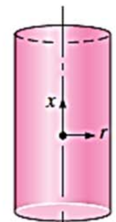
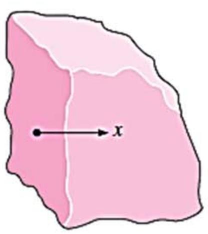
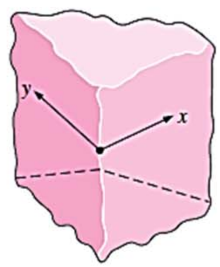
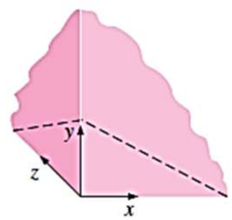
$$\theta_{\text{cyl}}(r, t) = \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}}$$

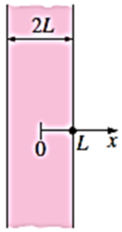
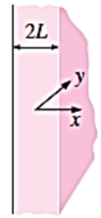
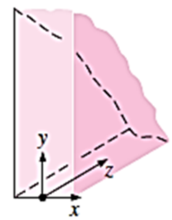
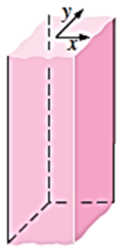
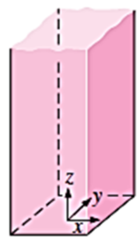
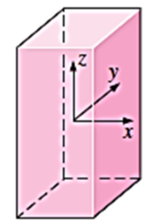
$$\theta_{\text{semi-inf}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\\text{solid}}} \tag{4-26}$$

The proper forms of the product solutions for some other geometries are given in Table 4–4. It is important to note that the $x$-coordinate is measured from the *surface* in a semi-infinite solid, and from the *midplane* in a plane wall. The radial distance $r$ is always measured from the centerline.

**TABLE 4–4**

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature $T_i$ and exposed to convection from all surfaces to a medium at $T_\infty$

| | | |
|---|---|---|
| <br>$\theta(r,t) = \theta_{cyl}(r,t)$<br>**Infinite cylinder** | <br>$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{semi\text{-}inf}(x,t)$<br>**Semi-infinite cylinder** | <br>$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{wall}(x,t)$<br>**Short cylinder** |
| <br>$\theta(x,t) = \theta_{semi\text{-}inf}(x,t)$<br>**Semi-infinite medium** | <br>$\theta(x,y,t) = \theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)$<br>**Quarter-infinite medium** | <br>$\theta(x,y,z,t) =$<br>$\theta_{semi\text{-}inf}(x,t)\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$<br>**Corner region of a large medium** |

| | | |
|---|---|---|
| <br><br>$\theta(x,t) = \theta_{wall}(x,t)$<br>**Infinite plate (or plane wall)** | <br><br>$\theta(x,y,t) = \theta_{wall}(x,t)\,\theta_{semi\text{-}inf}(y,t)$<br>**Semi-infinite plate** | <br><br>$\theta(x,y,z,t) =$<br>$\theta_{wall}(x,t)\,\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$<br>**Quarter-infinite plate** |
| <br><br>$\theta(x,y,t) = \theta_{wall}(x,t)\theta_{wall}(y,t)$<br>**Infinite rectangular bar** | <br><br>$\theta(x,y,z,t) =$<br>$\theta_{wall}(x,t)\,\theta_{wall}(y,t)\,\theta_{semi\text{-}inf}(z,t)$<br>**Semi-infinite rectangular bar** | <br><br>$\theta(x,y,z,t) =$<br>$\theta_{wall}(x,t)\,\theta_{wall}(y,t)\,\theta_{wall}(z,t)$<br>**Rectangular parallelepiped** |

transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{max}}\right)_{total,\,2D} = \left(\frac{Q}{Q_{max}}\right)_1 + \left(\frac{Q}{Q_{max}}\right)_2\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\left(\frac{Q}{Q_{max}}\right)_{total,\,3D} = \left(\frac{Q}{Q_{max}}\right)_1 + \left(\frac{Q}{Q_{max}}\right)_2\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right] + \left(\frac{Q}{Q_{max}}\right)_3\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right]\left[1-\left(\frac{Q}{Q_{max}}\right)_2\right]$$

## EXAMPLE 4–7 Cooling of a Short Brass Cylinder

A short brass cylinder of diameter $D = 10$ cm and height $H = 12$ cm is initially at a uniform temperature $T_i = 120°C$. The cylinder is now placed in atmospheric air at 25°C, where heat transfer takes place by convection, with a heat transfer coefficient of $h = 60$ W/m² · °C. Calculate the temperature at (*a*) the center of the cylinder and (*b*) the center of the top surface of the cylinder 15 min after the start of the cooling.

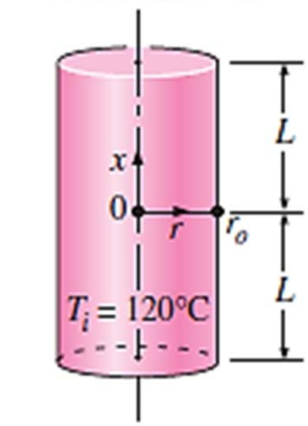



FIGURE 4–28

### SOLUTION

*Properties* The properties of brass at room temperature are $k = 110$ W/m · °C and $\alpha = 33.9 \times 10^{-6}$ m²/s

*Analysis* (*a*) This short cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = 5$ cm and a plane wall of thickness $2L = 12$ cm, as shown in Fig. 4–28. The dimensionless temperature at the center of the plane wall is determined from Figure 4–13*a* to be

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{L^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2/\text{s})(900\ \text{s})}{(0.06\ \text{m})^2} = 8.48 \\ \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110\ \text{W/m}\cdot°\text{C}}{(60\ \text{W/m}^2\cdot°\text{C})(0.06\ \text{m})} = 30.6 \end{aligned}\right\} \theta_{\text{wall}}(0, t) = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = 0.8$$

Similarly, at the center of the cylinder, we have

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2/\text{s})(900\ \text{s})}{(0.05\ \text{m})^2} = 12.2 \\ \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{110\ \text{W/m}\cdot°\text{C}}{(60\ \text{W/m}^2\cdot°\text{C})(0.05\ \text{m})} = 36.7 \end{aligned}\right\} \theta_{\text{cyl}}(0, t) = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = 0.5$$ Therefore,

$$\left(\frac{T(0, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{\text{wall}}(0, t) \times \theta_{\text{cyl}}(0, t) = 0.8 \times 0.5 = 0.4$$ and

$$T(0, 0, t) = T_\infty + 0.4(T_i - T_\infty) = 25 + 0.4(120 - 25) = 63°\text{C}$$

This is the temperature at the center of the short cylinder, which is also the center of both the long cylinder and the plate.

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder ($r = 0$), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to find the surface temperature of the wall. Noting that $x = L = 0.06$ m,

$$\left.\begin{aligned} \frac{x}{L} &= \frac{0.06\ \text{m}}{0.06\ \text{m}} = 1 \\ \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110\ \text{W/m}\cdot{}^\circ\text{C}}{(60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.06\ \text{m})} = 30.6 \end{aligned}\right\} \quad \frac{T(L,t)-T_\infty}{T_o-T_\infty} = 0.98$$

Then

$$\theta_{\text{wall}}(L,t) = \frac{T(L,t)-T_\infty}{T_i-T_\infty} = \left(\frac{T(L,t)-T_\infty}{T_o-T_\infty}\right)\left(\frac{T_o-T_\infty}{T_i-T_\infty}\right) = 0.98 \times 0.8 = 0.784$$

Therefore,

$$\left(\frac{T(L,0,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{\text{wall}}(L,t)\theta_{\text{cyl}}(0,t) = 0.784 \times 0.5 = 0.392$$

and

$$T(L,0,t) = T_\infty + 0.392(T_i - T_\infty) = 25 + 0.392(120 - 25) = 62.2^\circ\text{C}$$

which is the temperature at the center of the top surface of the cylinder.

**EXAMPLE 4–8** **Heat Transfer from a Short Cylinder**

Determine the total heat transfer from the short brass cylinder ($\rho$ = 8530 kg/m$^3$, $C_p$ = 0.380 kJ/kg · °C) discussed in Example 4–7.

**SOLUTION**

$$m = \rho V = \rho \pi r_o^2 L = (8530 \text{ kg/m}^3)\pi(0.05 \text{ m})^2(0.06 \text{ m}) = 4.02 \text{ kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (4.02 \text{ kg})(0.380 \text{ kJ/kg} \cdot {}^\circ\text{C})(120 - 25)^\circ\text{C} = 145.1 \text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries. For the plane wall, it is determined from Fig. 4–13*c* to be

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{30.6} = 0.0327 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.0327)^2(8.48) = 0.0091 \end{aligned}\right\} \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\ \text{wall}}} = 0.23$$

Similarly, for the cylinder, we have

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{36.7} = 0.0272 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.0272)^2(12.2) = 0.0090 \end{aligned}\right\} \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{infinite}\\ \text{cylinder}}} = 0.47$$

Then the heat transfer ratio for the short cylinder is,

$$\begin{aligned} \left(\frac{Q}{Q_{\max}}\right)_{\text{short cyl}} &= \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] \\ &= 0.23 + 0.47(1 - 0.23) = 0.592 \end{aligned}$$

Therefore, the total heat transfer from the cylinder during the first 15 min of cooling is

$$Q = 0.592 Q_{\max} = 0.592 \times (145.1 \text{ kJ}) = \mathbf{85.9 \text{ kJ}}$$

## *EXAMPLE 4–9* **Cooling of a Long Cylinder by Water**

A semi-infinite aluminum cylinder of diameter $D = 20$ cm is initially at a uniform temperature $T_i = 200°C$. The cylinder is now placed in water at 15°C where heat transfer takes place by convection, with a heat transfer coefficient of $h = 120$ W/m² · °C. Determine the temperature at the center of the cylinder 15 cm from the end surface 5 min after the start of the cooling.

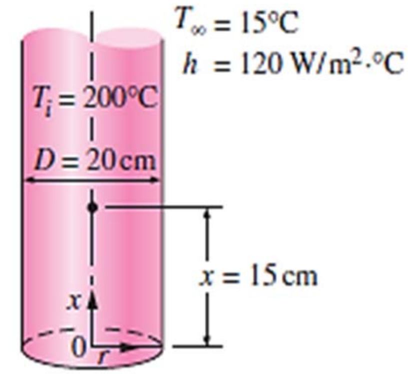



**FIGURE 4–29**

**SOLUTION**

*Properties* The properties of aluminum at room temperature are $k = 237$ W/m · °C and $\alpha = 9.71 \times 10^{-6}$ m²/s

First we consider the infinitely long cylinder and evaluate the Biot number:

$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{(120\ \mathrm{W/m^2 \cdot °C})(0.1\ \mathrm{m})}{237\ \mathrm{W/m \cdot °C}} = 0.05$$

The coefficients $\lambda_1$ and $A_1$ for a cylinder corresponding to this Bi are determined from Table 4–1 to be $\lambda_1 = 0.3126$ and $A_1 = 1.0124$. The Fourier number in this case is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71 \times 10^{-5}\ \mathrm{m^2/s})(5 \times 60\ \mathrm{s})}{(0.1\ \mathrm{m})^2} = 2.91 > 0.2$$

$$\theta_0 = \theta_{\mathrm{cyl}}(0, t) = A_1 e^{-\lambda_1^2 \tau} = 1.0124 e^{-(0.3126)^2(2.91)} = 0.762$$

The solution for the semi-infinite solid can be determined from

$$1 - \theta_{\text{semi-inf}}(x, t) = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\left[\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

First we determine the various quantities in parentheses:

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{0.15\ \mathrm{m}}{2\sqrt{(9.71 \times 10^{-5}\ \mathrm{m^2/s})(5 \times 60\ \mathrm{s})}} = 0.44$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(120\ \mathrm{W/m^2 \cdot °C})\sqrt{(9.71 \times 10^{-5}\ \mathrm{m^2/s})(300\ \mathrm{s})}}{237\ \mathrm{W/m \cdot °C}} = 0.086$$

$$\frac{hx}{k} = \frac{(120 \text{ W/m}^2 \cdot °\text{C})(0.15 \text{ m})}{237 \text{ W/m} \cdot °\text{C}} = 0.0759$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = (0.086)^2 = 0.0074$$

Substituting and evaluating the complementary error functions from Table 4–3,

$$\begin{aligned}\theta_{\text{semi-inf}}(x, t) &= 1 - \text{erfc}\,(0.44) + \exp\,(0.0759 + 0.0074)\ \text{erfc}\,(0.44 + 0.086) \\ &= 1 - 0.5338 + \exp\,(0.0833) \times 0.457 \\ &= 0.963\end{aligned}$$

Now we apply the product solution to get

$$\left(\frac{T(x, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = \theta_{\text{semi-inf}}(x, t)\theta_{\text{cyl}}(0, t) = 0.963 \times 0.762 = 0.734$$

and

$$T(x, 0, t) = T_\infty + 0.734(T_i - T_\infty) = 15 + 0.734(200 - 15) = \mathbf{151°C}$$

which is the temperature at the center of the cylinder 15 cm from the exposed bottom surface.

**PROBLEMS – 4**

**Lumped system applications and criteria**

4.1 A solid copper sphere of diameter 10cm, initially at a uniform temperature of 250°C, is suddenly immersed in a well-stirred fluid that is maintained at a uniform temperature of 50°C. The heat transfer coefficient between the sphere and the fluid is $200W/m^2.°C$. (a) Check whether lumped system analysis is applicable. (b) If it is applicable, determine the temperature of the copper block at times t=1min, t=2min, and t=5min after immersion in the cold fluid. [for copper k=386W/m.°C, $\rho=8954kg/m^3$, and C=383J/kg.°C.]

4.2 A solid iron sphere of diameter 5cm, initially at a uniform temperature of 700°C, is exposed to a cool air stream at 100°C. The heat coefficient between the air stream and the surface of the iron sphere is $80W/m^2.°C$. (a) Check whether lumped system analysis is applicable. (b) If applicable, determine the time required for the temperature of the sphere to reach 300°C. [For iron, k=60W/m.°C, $\rho=7800kg/m^3$, and C=460J/kg.°C.]
Answer: (b) 6.84min

4.3 A large aluminum plate of thickness 3cm is initially at a uniform temperature of 50°C. Suddenly it is subjected (both surfaces) to a cool air stream at 20°C. The heat transfer coefficient between the air stream and the surface is $50W/m^2.°C$. (a) Check whether lumped system analysis is applicable. (b) If applicable, determine the time required for the temperature of the plate to reach 40°C. [for aluminum, k=204W/m.°C, $\rho=2707kg/m^3$, and C=896J/kg.°C.]

4.4 A 3cm diameter aluminum sphere is initially at a temperature of 175°C. It is suddenly immersed in a well-stirred fluid at a temperature of 25°C The temperature of the sphere is lowered to 100°C in 42S. Calculate the heat transfer coefficient. Check whether lumped system analysis is applicable. [For aluminum, k=204W/m.°C, $\rho=2707kg/m^3$, and C=896J/kg.°C.]

4.5 A 6cm diameter potato at a uniform temperature of 80°C is taken out of the oven and suddenly exposed to ambient air at 20°C. If the heat coefficient between the air and the potato is $25W/m^2.°C$, determine the time required for the potato to reach 50°C.[for potato, k=7W/m.°C, $\rho=1300kg/m^3$, and C=4300J/kg.°C.] Answer: 26min

4.6 Consider an aluminum cube of side 3cm that is initially at a uniform temperature of 50°C. Suddenly all its surfaces are exposed to cool air at 20°C. The heat transfer coefficient between the air and the surfaces is $50W/m^2.°C$. Assuming that lumped system analysis is applicable, develop an expression for the temperature T(t) of the cube as a function of time and plot the temperature of the solid against time. [for aluminum, k=204W/m.°C, $\rho=2707kg/m^3$, and C=896J/kg.°C.]

4.7 Consider a copper block of sides 2cm x 2cm x 3cm, initially at a uniform temperature of 300°C, that is immersed in a fluid at 25°C. The heat transfer coefficient between the fluid and the surfaces is $80W/m^2.°C$. Calculate the time required for the cube to cool to 50°C. Check the validity of the lumped system analysis. [For copper, k=386W/m.°C, $\rho=8954kg/m^3$, and C=383J/kg.°C.] Answer 6.42min

4.8 A 0.1cm diameter long wooden stick at 15°C is suddenly exposed to 500°C gases with a surface heat transfer coefficient of $15W/m^2.°C$ between the stick and the gases. If the ignition temperature of the wood is 315°C, find the exposure time before possible ignition. [ For wood, k=0.14W/m.°C, $\rho=600kg/m^3$, and C=250J/kg.°C.] Answer: 2.41S

4.9 A short, cylindrical aluminum bar of 1cm diameter and 2cm height is initially at a uniform temperature of 150°C. Suddenly the surfaces are subjected to convective cooling with a heat transfer coefficient of $15W/m^2.°C$ into an ambient fluid at 30°C. Calculate the temperature of the cylinder 1min after the start of the cooling. [For aluminum, k=204W/m.°C, $\rho=2707kg/m^3$, and C=896J/kg.°C.] Answer: 129.7°C

**4.10** A thermocouple junction, approximated as a sphere of constantan, is to be used to measure the temperature of a gas. The heat transfer coefficient between the gas and the thermocouple is $400W/m^2.^oC$. Calculate the maximum allowable diameter of the junction if the thermocouple should measure 95 percent of the applied temperature difference in 5S.[ For constantan, $k=1.28W/m.^oC$, $\rho=1458kg/m^3$, and $C=410J/kg.^oC$.]

**4.11** A steel ball [$k=35W/m.^oC$, $\rho=7800kg/m^3$, and $C=460J/kg.^oC$.] 5.0cm in diameter and initially at a uniform temperature of $450^oC$ is suddenly placed in a controlled environment in which the temperature is maintained at $100^oC$. The convection heat transfer coefficient is $10W/m^2.^oC$. Calculate the time required for the ball attain a temperature of $150^oC$. Answer: 1.62hr

**4.12** A copper sphere having a diameter 3.0cm is initially at a uniform temperature of $50^oC$. It is suddenly exposed to an air stream of $10^oC$ with $h=15W/m^2.^oC$. How long does it take the sphere temperature to drop $25^oC$?

**Transient Temperature Heat Flow In A Semi-Infinite Solids**

**4.13** A thick stainless-steel slab [$\alpha=1.5x10^{-5}m^2/S$, and $k=60W/m.^oC$] is initially at a uniform temperature of $220^oC$ and maintained at that temperature. By treating the slab as a semi-infinite solid, determine the temperature at a depth 1cm from the surface and the heat flux at the surface 2min after the surface temperature lowered.

**4.14** A fireclay brick slab [$\alpha=5.4x10^{-7}m^2/S$, and $k=1W/m.^oC$] 10cm thick is initially at a uniform temperature of $350^oC$. suddenly one of its surfaces is subjected to convection with a heat transfer coefficient of $100W/m^2.^oC$ into an ambient at $40^oC$. Calculate the temperature at a depth 1cm from the surface 2min after start of cooling. Answer: $309.7^oC$

**4.15** A thick aluminum slab [$\alpha=8.4x10^{-5}m^2/S$, and $k=200W/m.^oC$] is initially at a uniform temperature of $20^oC$ suddenly one of its surfaces is raised to $100^oC$. Calculate the time required for the temperature at a depth 5cm from the surface to reach to $80^oC$.

Answer: 140.7S

**4.16** A thick stainless steel slab [$\alpha=1.6x10^{-5}m^2/S$, and $k=60W/m.^oC$] is initially at a uniform temperature of $100^oC$. One of its surfaces is suddenly lowered to $30^oC$. Determine the time required for the temperature at a depth 2m from the surface to reach $50^oC$.

**4.17** A thick bronze [$\alpha=.86x10^{-5}m^2/S$, and $k=26W/m.^oC$] is initially at a uniform temperature of $250^oC$. Suddenly one of its surfaces is exposed to convection cooling by a fluid at $25^oC$ Assuming that the heat transfer coefficient for convection between the fluid and the surface is $150W/m^2.^oC$, determine the temperature at a location 5cm from the surface 10min after the exposure.

**4.18** A thick wood wall [$\alpha=.82x10^{-7}m^2/S$, and $k=0.15W/m.^oC$] is initially at a uniform temperature of $20^oC$. Suddenly one of its surfaces is raised to $80^oC$. Calculate the temperature at a distance 2cm from the surface 10min after the exposure.

**4.19** A thick concrete wall having a uniform temperature of $54^oC$ is suddenly subjected to an air stream at $10^oC$. The heat transfer coefficient is $2.6W/m^2.^oC$. Calculate the temperature in the concrete slab at depth 7cm after 30min.

**4.20** A very large slab of copper is initially at a temperature of $300^oC$. The surface temperature is suddenly lowered to $35^oC$. What is the temperature at a depth of 7.5cm 4min after the surface temperature is changed.

**4.21** On a summer day a concrete driveway may reach a temperature of $50^oC$. Suppose that a stream of water is directed on the driveway so that the surface temperature is suddenly lowered to $10^oC$. How long will it take to cool the concrete to $25^oC$ at a depth of 5cm from the surface?

**4.22** A semi-infinite slab of copper is exposed to a constant heat flux at the surface of $0.32 MW/m^2$. Assume that the slab is in a vacuum, so that there is no convection at the surface. What is the surface temperature after 5min if the initial temperature of the slab is 30°C? What is the temperature at a distance of 15 cm from the surface after 5min?

**4.23** A large slab of copper is initially at a uniform temperature of 100°C. Its surface temperature is suddenly lowered to 40°C. Calculate the heat-transfer rate through a plane and the temperature at the depth of 7.5cm from the surface 5S after the surface temperature is lowered.

**4.24** A large slab of aluminum at a uniform temperature of 25°C is suddenly exposed to a constant surface heat flux of $25 kW/m^2$. What is the temperature at a depth of 2.5cm after 2min. How long would take for the temperature to reach 150°C at the depth of 3cm?

**4.25** A piece of ceramic material [$k=0.8 W/m.°C$, $\rho=2700 kg/m^3$, $C=0.8 kJ/kg.°C$] is quite thick and initially at a uniform temperature of 25°C. The surface of the material is suddenly exposed to a constant heat flux of $1000 W/m^2$. Calculate the temperature at a depth of 1.5cm 5min after.

**4.26** A large slab of concrete is suddenly exposed to a constant radiant heat flux of $1000 W/m^2$ on one of its surfaces. The slab is initially in temperature at 25°C. calculate the temperature at a depth of 10cm in the slab after a time of 10h.

**4.27** A very thick plate of stainless steal (18% Cr, 8%Ni) at a uniform temperature of 300°C has its surface temperature suddenly lowered to 100°C. Calculate the time required for the temperature at a depth of 3cm to attain a value of 200°C.

**4.28** A thick wood slab [$\alpha=1.28\text{x}10^{-5} m^2/S$, and $k=0.17 W/m.°C$] that is initially at a uniform temperature of 25°C is exposed to hot gases at 550°C for a period of 6min. the heat transfer coefficient between the gases and the wood slab is $30 W/m^2.°C$. If the ignition temperature of the wood is 420°C, determine if the wood will ignite.

## Large Plane Wall

**4.29** A steel plate [$\alpha=1.2\text{x}10^{-5} m^2.S$, $k=45 W/m.°C$, $Cp=465 J/kg.°C$, $\rho=7833 kg/m^3$] thickness 6cm, initially at a uniform temperature of 250°C, is suddenly immersed in an oil bath at 30°C. The convective heat transfer coefficient between the fluid and the surface is $500 W/m^2.°C$. How long it take for the center-plane to cool to 140°C?

**4.30** A copper plate of thickness 4cm is initially at a uniform temperature of 25°C. Suddenly both of its surfaces are raised to 50°C. Calculate the centerline temperature 10min after the surface temperature is raised.

**4.31** A fireclay brick slab [[$\alpha=5.4\text{x}10^{-7} m^2.S$, $k=1 W/m.°C$] of thickness 6cm is initially at a uniform temperature of 400°C. Suddenly one of its surfaces is subjected to convection with a heat transfer of $100 W/m^2.°C$.into an ambient at 50°C. The other surface is insulated. Calculate the centerline temperature 1h after the start of cooling

**4.32** A large slab of aluminum has a thickness of 10cm and is initially uniform in temperature at 400°C. Suddenly it is exposed to a convection environment at 100°C with $h=1500 W/m^2.°C$. How long it take the centerline temperature to drop to 200°C.

**4.33** A horizontal copper plate 12cm thick is initially uniform in temperature at 250°C. The bottom surface of the plate is insulated. The top surface is suddenly exposed to a fluid at 50°C. After 6min the surface temperature has dropped to 150°C. Calculate the convection heat-transfer coefficient which causes this drop.

**4.34** A plate of stainless steel (18%Cr,85Ni) has a thickness of 3.0cm and is initially uniform in temperature at 500°C. The plate is suddenly exposed to a convection environment on both sides at 50°C with $h=150 W/m^2.°C$. Calculate the times for the centerline and face temperature to reach 100°C.

**4.35** In a meat processing plant, 2cm thick steaks [$k=0.5W/m.^oC$, and $\alpha=1x10^{-7}m^2/S$] that are initially at $30^oC$ are to be cooled by passing them through a refrigeration room at $-10^oC$. The heat transfer coefficient on both sides of the steaks is $10W/m^2.^oC$. If both surfaces of the steaks are to be cooled to $3^oC$, determine how long the steaks should be kept in the refrigeration room.

**Long Cylinder**

**4.36** A long steel shaft of radius 15cm[$\alpha=1.6x10^{-5}m^2.S$, $k=60W/m.^oC$, $Cp=465J/kg.^oC$, $\rho=7833kg/m^3$] is taken out of an oven at a uniform temperature of $500^oC$ and immersed in a well-stirred large of $25^oC$ coolant. The heat transfer coefficient between the shaft surface and the coolant is $200W/m^2.^oC$. Calculate the time required for the shaft center to reach $100^oC$ and the amount of heat transfer form the shaft at this time.

**4.37** A long steel bar of diameter 6cm is initially at a uniform temperature of $200^oC$. Suddenly the surface of the bar is exposed to an ambient at $20^oC$ with a heat transfer coefficient of $400W/m^2.^oC$. Calculate the center temperature 3min after the start of the cooling. Calculate the energy removed from the bar per meter length during this time period.

**4.38** A hot dog can be regarded as a solid having a shape in the form of a long solid cylinder. Consider a hot dog [$\alpha=1.6x10^{-7}m^2/S$, and $k=0.5W/m.^oC$] of diameter 2.4cm, initially at a uniform temperature of $5^oC$, dropped into boiling water at $100^oC$. The heat transfer coefficient between the water and the surface is $200W/m^2.^oC$. If the meat is considered cooked when its center temperature reaches $80^oC$, how long will it take for the centerline temperature to reach $80^oC$?

**4.39** A long pure copper rod of diameter 8cm is initially at a uniform temperature of $120^oC$. It is suddenly dropped into a coolant pool at $25^oC$. The heat transfer coefficient between the coolant and the rod is $400W/m^2.^oC$. Determine the center temperature of the rod 120S after exposure to the coolant. Calculate the energy removed from the rod per meter length during this time period.

**4.40** A long 35cm diameter cylindrical shaft made of stainless steel 304 [$k=15W/m.^oC$,$\rho=7900kg/m^3$, $Cp=477J/kg.^oC$, and $\alpha=3.95x10^{-6}m^2/S$] comes out of an oven at a uniform temperature of $500^oC$. The shaft is then allowed to cool slowly in a chamber at $200^oC$ with an average convection heat transfer coefficient of $h=75W/m^2.^oC$. Determine the temperature at the center of the shaft 30min after the start of the cooling process. Also determine the heat transfer per unit length of the shaft during this time period.

**4.41** A long cylindrical wood log [$k=0.16W/m.^oC$ and $\alpha=1.28x10^{-7}m^2/S$]is 12cm in diameter and is initially at a uniform temperature of $10^oC$. It is exposed to hot gases at $540^oC$ in a fireplace with a heat transfer coefficient of $12W/m^2.^oC$ on the surface. If the ignition temperature of the wood is $420^oC$, determine how long it will be before the log ignites.

**4.42** A long cupper bar of radius 8cm, is initially came out of oven at a uniform temperature of $600^oC$. suddenly exposed to environment at a temperature of $50^oC$, with a heat transfer coefficient between the bar and the fluid is $500W/m^2.^oC$. calculate the time for the center temperature and surface temperature that reach $325^oC$.

**Sphere**

**4.43** A solid aluminum sphere of diameter 10cm is initially at $250^oC$. Suddenly it is immersed in a well-stirred bath at $125^oC$. The heat transfer coefficient between the fluid and the sphere surface is $600W/m^2.^oC$. How long will it take for the center of the sphere to cool to $150^oC$?

**4.44** A solid aluminum sphere of diameter 8cm is initially at $120^oC$. Suddenly its surface is lowered to $20^oC$. Determine the center temperature of the sphere 5S lowering the surface temperature.

**4.45** An 9cm diameter potato, initially at a uniform temperature of 25°C, is suddenly dropped into boiling water at 100°C. The heat transfer coefficient between the water and the surface of the potato is 5000W/m$^2$.°C. Assume the thermal properties of potato to be [$\alpha$=1.5x10$^{-7}$m$^2$/S and k=7W/m.°C]. Determine the time required for the center temperature of the potato to reach 75°C.

**4.46** A copper ball [$\alpha$=1.1x10$^{-4}$m$^2$/S and k=380W/m.°C] 5cm in diameter is initially at a uniform temperature of 150°C. It is suddenly dropped into a coolant pool at 20°C. The heat transfer coefficient between the coolant and the ball is 400W/m$^2$.°C. Determine the center temperature of the ball 90S after exposure to the coolant. Calculate the energy removed from the ball during the time period.

**4.47** Consider an 8cm diameter orange that is initially at 15°C. A cold front moves in one night, and the ambient temperature suddenly drops to -6°C, with a heat transfer coefficient of 15W/m$^2$.°C. Using the properties of water for the orange and assuming the ambient condition to remain constant for 4h before the cold front moves out, determine if any part of the orange will freeze that night.

**4.48** A steel sphere 10cm in diameter is suddenly immersed in a tank of oil at 15°C. The initial temperature of the sphere is 225°C; h=4500W/m$^2$.°C. How long will it take the center of the sphere to cool to 120?

**Transient Heat Conduction In Multidimensional System**

**4.49** A long steel bar 5 by 10 cm is initially maintained at a uniform temperature of 300°C. It is suddenly subjected to a change such that the environment temperature is lowered to 25°C. Assuming a heat transfer coefficient to be 25W/m$^2$.°C, estimate the time required for the center temperature to reach 100°C.

**4.50** A steel bar 2.5cm square and 7.5cm long is initially at a temperature of 250°C. It is immersed in a tank of oil maintained at 30°C. The heat transfer coefficient is 600W/m$^2$.°C. Calculate the temperature in the center of the bar after 100S.

**4.51** A copper short bar 2.5cm-3cm-5cm is initially at a uniform temperature of 500°C . It is immersed in the tank of water maintained at 20°C. the heat transfer coefficient is 480W/m$^2$.°C. Calculate the temperature at the center of each face 2min after the starting of cooling.

**4.52** A cube of aluminum 12cm on each side is initially at a temperature of 400°C and is immersed in a fluid at 100°C. the heat transfer coefficient is 800W/m$^2$.°C. Calculate the temperature at the center of one face after 1.5min.

**4.53** A short concrete cylinder 20cm diameter and 30cm long is initially at 25°C. It is allowed to cool in an atmospheric environment in which the temperature is 0°C. Calculate the time required for the center temperature to reach 7°C if the heat transfer coefficient is 20W/m$^2$.°C.

**4.54** A semi-infinite aluminum cylinder of a diameter of 10cm is initially at uniform temperature of 500°C. it is exposed suddenly to air stream of a temperature 50°C. The heat transfer coefficient between the cylinder and the fluid is 2000W/m$^2$.°C. Determine the temperature at the center line and deep 5cm from the finite end 2min after start cooling.

**4.55** A 4.0-cm square bar of aluminum is initially at 450°C and is suddenly exposed to a convection environment at 100°C with h=1200W/m$^2$.°C. find the temperature of its face center 100S after the start of cooling.

**4.56** An cube of aluminum 12cm each side is initially at a temperature of 400°C. It is suddenly immersed in a tank of oil maintained at 80°C. The convection coefficient is 1200W/m$^2$.°C. Calculate the temperature at the center of one face and the mid point at one edge and one corner after a time of one min. also calculate the amount of heat transfer.