Acceleration Diagram

In this lecture we shall discuss the acceleration of points in the mechanisms. The acceleration analysis plays a very important role in the development of machines and mechanisms.

1. Acceleration Diagram for a Link

Consider two points *A* and *B* on a rigid link as shown in Fig. 1 (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let *α* rad/s2 be the angular acceleration of the link *AB*.



**Fig.1.** Acceleration for a link.

The radial component of the acceleration of *B* with respect to *A*,

 *ar*BA = ω2 × *AB* = (from *B* → *A*)

Radial component of acceleration *ar*BA ⊥ *v*BA, In other words, *ar*BA // link *AB*.

The tangential component of the acceleration of *B* with respect to *A*,

*at*BA = *α* × *AB*

This tangential component of acceleration // *v*BA. In other words, *at*BA ⊥ link *AB*.

 *a*BA = *ar*BA + *at*BA (direction sum)

i. e. *a*BA =

1. **Acceleration of a Point on a Link**

Consider two points *A* and *B* on the rigid link, as shown in Fig. 2 (*a*). Let the acceleration of the point *A i.e. a*A is known in magnitude and direction and the direction of path of *B* is given. The acceleration of the point *B* is determined in magnitude and direction by drawing the acceleration diagram as discussed below.



**Fig. 2.** Acceleration of a point on a link.

1. From any point *o'*, draw vector *o'a'* parallel to the direction of absolute acceleration at point *A i.e. a*A *,* to some suitable scale, as shown in Fig. 2 (*b*).
2. The acceleration of *B* with respect to *A i.e. a*BA has two components *ar*BA and *at*BA. These two components are mutually perpendicular.
3. Draw vector *a'x* // link *AB*, vector *a'x* = *ar*BA =
4. From point *x*, draw vector *xb'*⊥link *AB* or vector *a'x* (because *at*BA ⊥ *ar*BA) and through *o'* draw a line parallel to the path of *B* to represent the absolute acceleration of *B i.e. a*B. The vectors *xb'* and *o'b'* intersect at *b'*. Now the values of *a*B and *at*BAmay be measured, to the scale.
5. By joining the points *a'* and *b'* we may determine the total acceleration of *B* with respect to *A i.e. a*BA. The vector *a' b'* is known as ***acceleration image*** of the link *AB*.
6. For any other point *C* on the link, draw triangle *a' b' c'* similar to triangle *ABC*. vector  *= a*CB, and vector *a'c'* = *a*CA*.*

 *a*CB and *a*CA will each have two components as follows :

1. *a*CB has two components; *ar*CB and *at*CB as shown by triangle *b'zc'* in Fig. 2 (*b*), in which *b'z* // *BC* and *zc'* ⊥ *b'z* or *BC.*
2. *a*CA has two components; *ar*CA and *at*CA as shown by triangle *a'yc'* in Fig. 2 (*b*), in which *a'y* // *AC* and *yc'* ⊥ *a' y* or *AC*.
3. Angular acceleration of the link *AB*,

*α*AB =

1. **Acceleration in the Slider Crank Mechanism**

A slider crank mechanism is shown in Fig. 3 (*a*). Let the crank *OB* makes an angle θ with horizontal and rotates in a clockwise direction about the fixed point *O* with uniform angular velocity ωBO rad/s.

*v*BO = *v*B = ωBO × *OB*

Radial acceleration of *B* (because *O* is a fixed point),

*ar*BO = *a*B = ω2BO × *OB* =

**Note:** A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration, *at*BO = 0.

 The acceleration diagram, as shown in Fig. 3(*b*), may now be drawn as discussed below:

1. Draw vector *o' b'* // *BO* represent *ar*BO= *a*B, to some suitable scale.
2. From point *b'*, draw vector *b'x* // *BA*,

vector *b'x* = *ar*BA =



**Fig. 3.** Acceleration in the slider crank mechanism.

1. From point *x*, draw vector *xa'* ⊥ *b'x* (or *AB*). The vector *xa'* represents *at*AB.
2. From *o'*, draw *o'a'* // *AO*, intersecting the vector *xa'* at *a'*.

Now the acceleration of the piston or the slider *A* (*a*A) and *at*AB may be measured to the scale.

1. *a*AB = vector *b'a'* = *b' x* + *x a'* (vectors sum)
2. The acceleration of any other point on *AB* such as *E* can be obtained by

 *=*

1. Angular acceleration of the connecting rod *AB*,

 *α*AB = (Clockwise about *B*)

**Example: 1**

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :

1. linear velocity and acceleration of the midpoint of the connecting rod, and
2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Solution:

1. Draw the mechanism, to some suitable scale, as shown in Fig.(*a*). let 100 mm = 1 cm in paper
2. Find  *v*BO *= v*B = ωBO × *OB*

*N*OB = 300 r.p.m. , ωBA = 2π × 300/60 = 31.42 rad/s

 *v*BO = *v*B = 31.42 × 150 = 4.713 m/s



1. Draw the velocity diagram, as shown in Fig. (*b*), since the point *O* is fixed. Now from point *o*, draw vector *ob* ⊥ *OB*, to some suitable scale.

let 4.713 m/s = 5 cm in paper

vector *ob = v*B

1. From point *b* draw vector *ba* ⊥ *BA* to represent *v*AB, and from point *o*, draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent *v*A. The vectors *ba* and *oa* intersect at *a*.

By measurement, we find that

*v*AB = vector *ab* = 3.6 × = 3.4 m/s

Angular velocity of the connecting rod = ωAB = = = 5.67 rad/s.

 (Anticlockwise about *B*) Ans.

*v*A = vector *oa* = 4.2 × = 4 m/s

1. Since the point *D* lies on *AB*,

 =

**Note:** Point *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

By measurement, we find that

 *v*D = vector *od* = 4.3 × = 4.1 m/s Ans.

Draw the acceleration diagram, as shown in Fig. (*c*)

1. *ar*BO = *a*B = = = 148.1 m/s2

 *ar*AB = = = 19.3 m/s2

Scale for acceleration diagram: let 148.1 m/s2 = 5 cm in paper

1. Draw vector *o'b'* // *BO* to represent *ar*BO or *a*B,

 vector *o'b'* = *ar*BO = *a*B = 5 cm (from *B* → *O*)

**Note:** Since the crank *OB* rotates at a constant speed, therefore *at*BO = 0

1. The acceleration of *A* with respect to *B (a*AB) has two components:

*a*AB = *ar*AB + *at*AB

from point *b'*, draw vector *b' x* // *AB* to represent *ar*AB= 19.3 m/s = 0.7 cm in paper (from *A* → *B*), and from point *x* draw vector *xa'* ⊥ vector *b'x* whose magnitude is yet unknown.

1. Now from *o'*, draw vector *o'a'* parallel to the path of motion of *A* (which is along *AO*) to represent *a*A *.* The vectors *xa'* and *o'a'* intersect at *a'*. Join *a' b'*.
2. Point *D* is the midpoint of *AB*, therefore *d'* is also midpoint of vector *b'a'*,

 =

Vector = 3.95 cm in paper

 *a*D = vector = 3.95 × = 117 m/s2 Ans.

 = 3.45 cm in paper

*at*AB = vector = 3.45 × = 103 m/s2

 αAB = = = 171.67 m/s2 (Clockwise about B)**Ans.**

**Example :2**

In the mechanism, as shown in Figure below, the crank *OA* rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks *B* and *D*. The dimensions of the various links are *OA* = 300 mm; *AB* = 1200 mm; *BC* = 450 mm and *CD* = 450 mm. For the given configuration, determine :

1. velocities of sliding at *B* and *D*,
2. Angular velocity of *CD*,
3. linear acceleration of *D*, and
4. angular acceleration of *CD*.

