Balancing of Masses



A car assembly line.

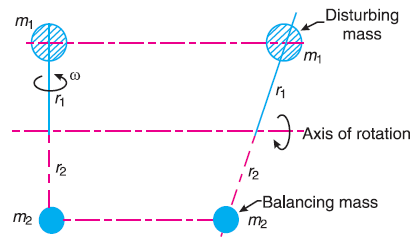
In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimize the loads on bearings and stresses in the various members, which causes dangerous vibrations when a machine is running.

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.
5. Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass *m*1 attached to a shaft rotating at ω rad/s as shown in Fig .1. Let *r*1 be the radius of rotation of the mass *m*1. We know that the centrifugal force exerted by the mass *m*1 on the shaft,

*F*Cl = *m*1. ω2. *r*1 …………….(1)



**Fig. 1.** Balancing of a single rotating mass by a single mass rotating in the same plane.

This centrifugal force produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (*m*2) may be attached in the same plane of rotation as that of disturbing mass (*m*1) such that the centrifugal forces due to the two masses are equal and opposite.

Let *r*2 = Radius of rotation of the balancing mass *m*2, centrifugal force due to mass *m*2,

*F*C2 = *m*2. ω2. *r*2 …………….(2)

Equating equations (1) and (2),

*m*1. ω2. *r*1 = *m*2. ω2. *r*2 or *m*1. *r*1 = *m*2. *r*2

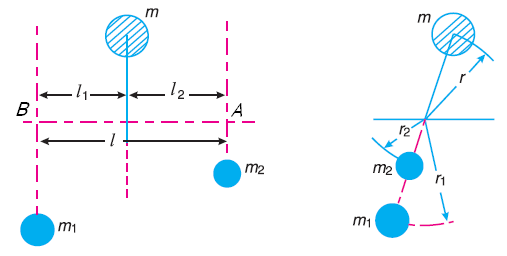
1. Balancing of a Single Rotating Mass By Two Masses Rotating in

Different Planes

In order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, There are two possibilities may arise while attaching the two balancing masses:

1. When the plane of the disturbing mass lies in between the planes of the two balancing masses

Consider a disturbing mass *m* balanced by two rotating masses *m*1 and *m*2 as shown in Fig. 2. Let *r*, *r*1 and *r*2 be the radii of rotation of the masses *m*, *m*1 and *m*2 respectively.



**Fig .2.** Balancing of a single rotating mass by two rotating masses in different planes when the

plane of single rotating mass lies in between the planes of two balancing masses.

The net force acting on the shaft must be equal to zero

*F*C = *F*Cl + *F*C2

*F*C : Centrifugal force exerted by the mass *m*

*F*C1 : Centrifugal force exerted by the mass *m*1

*F*C2 : Centrifugal force exerted by the mass *m*2

*m*. ω2. *r* = *m*1. ω2. *r*1 + *m*2. ω2. *r*2

*m*. *r* = *m*1. *r*1 + *m*2. *r*2

Now in order to find the magnitude of balancing force at the bearing *B* of a shaft, take moments about *A*. Therefore

*F*C1 × *l* = *F*C × *l*2 or *m*1. ω2. *r*1 × *l* = *m*. ω2. *r* × *l*2

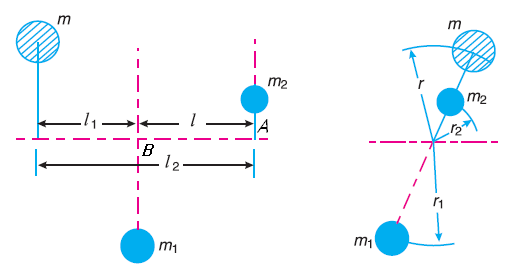
**∴** *m*1. *r*1 × *l* = *m*. *r* × *l*2

Similarly, in order to find the balancing force at the bearing *A* of a shaft, take moments about *B*. Therefore

*F*C2 × *l* = *F*C × *l*1 or *m*2. ω2. *r*2 × *l* = *m*. ω2. *r* × *l*1

**∴** *m*2. *r*2 × *l* = *m*. *r* × *l*1

1. When the plane of the disturbing mass lies on one end of the planes of the balancing masses



**Fig .3.** Balancing of a single rotating mass by two rotating masses in different planes, when the

plane of single rotating mass lies at one end of the planes of balancing masses.

This case shown in Fig .3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e*.

*F*C + *F*C2 = *F*Cl or *m*. ω2. *r* + *m*2. ω2. *r*2 = *m*1. ω2. *r*1

**∴** *m*. *r* + *m*2. *r*2 = *m*1. *r*1

Now in order to find the magnitude of balancing force at the bearing *B* of a shaft, take moments about *A*. Therefore

*F*C1 × *l* = *F*C × *l*2 or *m*1. ω2. *r*1 × *l* = *m*. ω2. *r* × *l*2

**∴** *m*1. *r*1 × *l* = *m*. *r* × *l*2

Similarly, to find the balancing force at the bearing *A* of a shaft, take moments about *B*. Therefore

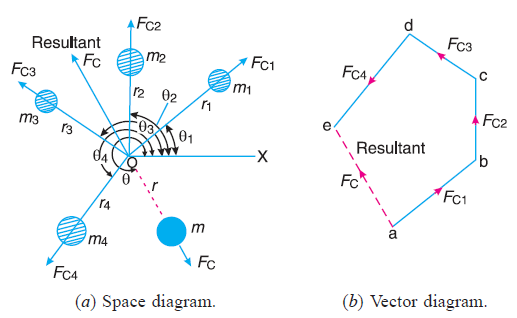
*F*C2 × *l* = *F*C × *l*1 or *m*2. ω2. *r*2 × *l* = *m*. ω2. *r* × *l*1

**∴** *m*2. *r*2 × *l* = *m*. *r* × *l*1

1. Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude *m*1, *m*2, *m*3 and *m*4 at distances of *r*1, *r*2, *r*3 and *r*4 from the axis of the rotating shaft. Let θ1, θ2,θ3 and θ4 be the angles of these masses with the horizontal line *OX*, as shown in Fig. 4 (*a*). Let these masses rotate about an axis through *O* and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below:



**Fig .4.** Balancing of several masses rotating in the same plane.

1. Analytical method

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e*.

Σ*H* = *m*1 . *r*1 cosθ1 + *m*2 . *r*2 cosθ2 + . . . . . .

Σ*V* = *m*1 . *r*1 sinθ1 + *m*2 . *r*2 sinθ2 + . . . . . .

Magnitude of the resultant centrifugal force,

*F*C =

If θ is the angle, which the resultant force makes with the horizontal, then

tan θ =

The balancing force is then equal to the resultant force, but in ***opposite direction*.** Now find out the magnitude of the balancing mass,

*F*C = *m*. *r*

where *m* = Balancing mass, and

*r* = Its radius of rotation.

1. Graphical method

The magnitude and position of the balancing mass may also be obtained graphically using Table: 1 and then draw:

1. Find the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
2. Draw the vector diagram with the obtained centrifugal forces to some suitable scale.
3. The closing side of polygon *ae* represents the resultant force in magnitude and direction, as shown in Fig. 4 (*b*).
4. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***
5. Find the magnitude of the balancing mass (*m*) at a given radius of rotation (*r*),

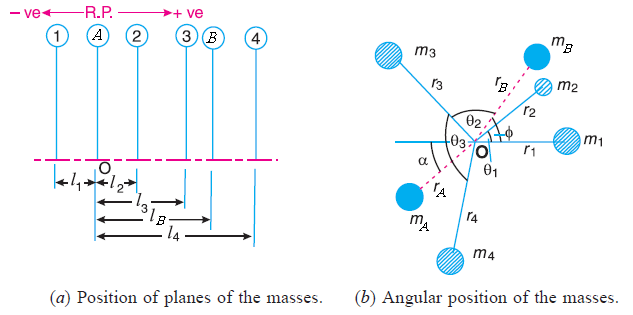
*m*.*r* = Resultant of *m*1.*r*1, *m*2.*r*2, *m*3.*r*3 and *m*4.*r*4

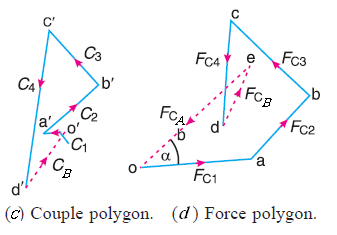
Table: 1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***No. of masses*** | ***Mass (m)*** | ***Radius (r)*** | ***Angle(θ)*** | ***Centrifugal force ÷ ω2 (m.r)*** |
| 1 | *m*1 | *r*1 | θ1 | *m*1 *r*1 |
| 2 | *m*2 | *r*2 | θ2 | *m*2*.r*2 |
| 3 | *m*3 | *r*3 | θ3 | *m*3*.r*3 |
| 4 | *m*4 | *r*4 | θ4 | *m*4*.r*4 |

1. Balancing of Several Masses Rotating in Different Planes

Let us consider four masses *m*1, *m*2, *m*3 and *m*4 revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. 5(*a*). The relative angular positions of these masses





**Fig. 5.** Balancing of several masses rotating in different planes.

are shown in Fig. 5(*b*). The magnitude of the balancing masses *m*A and *m*B in planes *A* and *B* may be obtained as discussed below:

1. Take one of the planes, say *A* as the reference plane (*R.P*.).
2. Tabulate the data as shown in Table: 2.
3. The couples about the reference plane must balance, *i.e*. the resultant couple must be zero.

+ *m*B *r*B *l*B = 0

+ *m*B *r*B *l*B = 0

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero*.*

+ *m*A *r*A = 0

+ *m*A *r*A = 0

Where  *n*: number of masses.

Table: 2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***plane*** | ***Mass (m)*** | ***Radius (r)*** | ***Angle(θ)*** | ***Centrifugal force ÷ ω2 (m.r)*** |  | ***Distance from***  ***Plane A* (*l*)** | ***Couple ÷ ω2 (m.r.l)*** |
| 1 | *m*1 | *r*1 | θ1 | *m*1*.r*1 |  | *–l*1 | *–m*1*.r*1.*l*1 |
| *A(R.P.)* | *m*A | *rA* | θA | *m*A*.r*A |  | 0 | 0 |
| 2 | *m*2 | *r*2 | θ2 | *m*2*.r*2 |  | *l*2 | *m*2*.r*2.*l*2 |
| 3 | *m*3 | *r*3 | θ3 | *m*3*.r*3 |  | *l*3 | *m*3*.r*3.*l*3 |
| *B* | *m*B | *rB* | θB | *m*B*.r*B |  | *l*B | *m*B*.r*B.*l*B |
| 4 | *m*4 | *r*4 | θ4 | *m*4*.r*4 |  | *l*4 | *m*4*.r*4.*l*4 |

Graphical Representation

1. Draw moment polygon (assuming that the moment direction is toward the centrifugal force) as shown in Fig .5(*c)*.
2. Draw force polygon as shown in Fig .5(*d)*.

**Example: 1**

Four masses *m*1, *m*2, *m*3 and *m*4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

Solution:

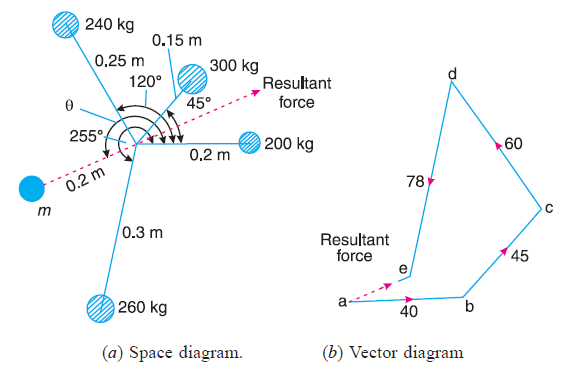
Let  *m* = Balancing mass, and

θ = The angle which the balancing mass makes with *m*1*.*

1. Analytical method

Σ*H* =

Σ*H* = 40 cos0 + 45 cos45 + 60 cos120 + 78 cos255 = 21.63 kg.m



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***No. of masses*** | ***Mass (kg)*** | ***Radius (m)*** | ***Angle(deg.)*** | ***Centrifugal force ÷ ω2 (kg.m)*** |
| 1 | 200 | 0.2 | 0 | 40 |
| 2 | 300 | 0.15 | 45 | 45 |
| 3 | 240 | 0.25 | 120 | 60 |
| 4 | 260 | 0.3 | 255 | 78 |
| 5 | *m* | 0.2 | θ | *m.r* |

Σ*V* =

Σ*V* = 40 sin0 + 45 sin45 + 60 sin120 + 78 sin255 = 8.43 kg.m

*F*C = = 23.21 kg.m

*F*C = *m.r*, 23.21 = 0.2 × *m* ∴  *m =* 116 kg **Ans.**

tan = = ∴ = 21.3°

Since is the angle of the resultant *R* from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

θ = 180° + 21.48° = 201.48° **Ans.**

1. Graphical method

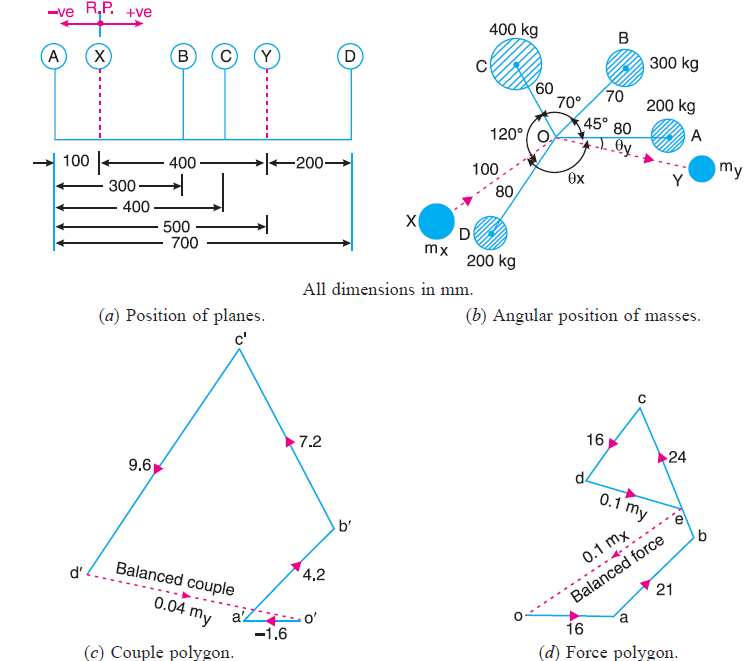
***Scale:*** let 10 kg.m = 1cm in paper.

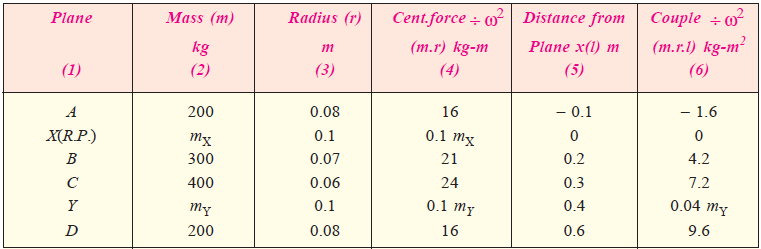
Draw force polygon as shown in Fig .(*b)*.

**Example: 2**

A shaft carries four masses *A*, *B*, *C* and *D* of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are *A* to *B* 45°, *B* to C 70° and *C* to *D* 120°. The balancing masses are to be placed in planes *X* and *Y*. The distance between the planes *A* and *X* is 100 mm, between *X* and *Y* is 400 mm and between *Y* and *D* is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

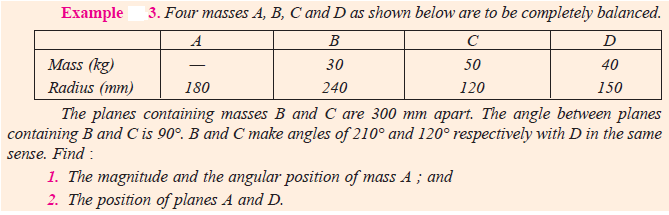
Solution





**Home work**

A shaft has three eccentrics, each 75 *mm* diameter and 25 *mm* thick, machined in one piece with the shaft. The central planes of the eccentric are 60 *mm* apart. The distance of the centres from the axis of rotation are 12 *mm*, 18 *mm* and 12 *mm* and their angular positions are 120° apart. The density of metal is 7000 *kg/m*3. Find the amount of out-of-balance force and couple at 600 r.p.m. If the shaft is balanced by adding two masses at a radius 75 *mm* and at distances of 100 *mm* from the central plane of the middle eccentric, find the amount of the masses and their angular positions.



***Solution:***

