BOOLEAN ALGEBRA AND LOGIC SIMPLIFICATION

BOOLEAN OPERATIONS AND EXPRESSIONS

Variable, complement, and literal are terms used in Boolean algebra. A variable is a symbol used to represent a logical quantity. Any single variable can have a ***1*** *or a* ***0*** *value. The complement is the inverse of a variable and is indicated by a bar over variable (overbar). For example, the complement of the variable* ***A*** *is* ***A.*** *If* ***A = 1,*** *then* ***A = 0.*** *If* ***A = 0****, then* ***A = 1****. The complement of the variable A is read as "not* ***A****" or "****A*** *bar." Sometimes a prime symbol rather than an overbar is used to denote the complement of a variable; for example, B' indicates the complement of B. A literal is a variable or the complement of a variable. In Boolean algebra, a*

Boolean Addition

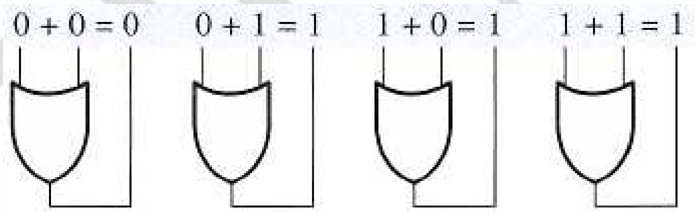
sum term is a sum of literals. In logic circuits, a sum term is produced by an OR operation with no AND operations involved. Some examples of sum terms are

$A + B$

$A + \overline{B}$,

$A + B + \overline{C}$, and

$\overline{A} + B + C + \overline{D}$.



EX//Determine the values of A, B, C, and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

For the solution must the variables equal (0) therefore

A=0, B=1 so that $\overline{B}$=0, C=0 and D=1 so that $\overline{D}$=0

$A+\overline{B}+C+\overline{D}= 0+\overline{1}+0+\overline{1} = 0+ 0+0+0= 0$

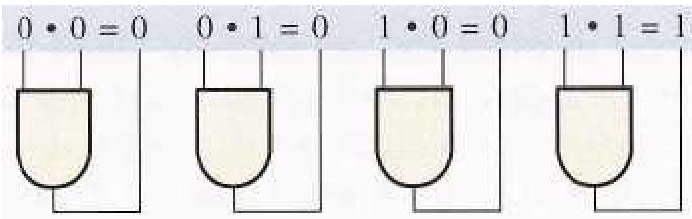
Boolean Multiplication

In Boolean algebra, a product term is the product of literals. In logic circuits, a product term is produced by an AND operation with no OR operations involved. Some examples of product terms are

AB, $A\overline{B}$, ABC, and $\overline{A}BC\overline{D}$.

A product term is equal to **1** only if each of the literals in the term is **1**.

A product term is equal to **0** when one or more of the literals are **0**.



EX// Determine the values of A, B, C, and D that make the product term $A\overline{B}C\overline{D}$ equal to 1?

For the solution to be 1, therefore A=1,B=0 so that $\overline{B}$=1,C=1,and D=0 so that $\overline{D}$=1.

$A\overline{B}C\overline{D}=1.\overline{0}.1.\overline{0}=1.1.1.1=1$

LAWS AND RULES OF BOOLEAN ALGEBRA

Laws of Boolean Algebra

The basic laws of Boolean algebra-the commutative laws for addition and multiplication, the associative laws for addition and multiplication, and the distributive law-are the same as in ordinary algebra.

Commutative Laws

The commutative law of addition for two variables is written as

A+B = B+A

This law states that the order in which the variables are ORed makes no difference. Remember, in Boolean algebra as applied to logic circuits, addition and the OR operation are the same.

(The symbol ≡ means "equivalent to.").

Fig.(3) Application of commutative law of addition.

The commutative law of multiplication for two variables is

A.B = B.A

This law states that the order in which the variables are ANDed makes no difference. Fig.(4), il1ustrates this law as applied to the AND gate.

Fig.(4) Application of commutative law of multiplication

Associative Laws :

The associative law of addition is written as follows for three variables:

A + (B + C) = (A + B) + C

This law states that when ORing more than two variables, the result is the same regardless of the grouping of the variables. Fig.(4), illustrates this law as applied to 2-input OR gates.

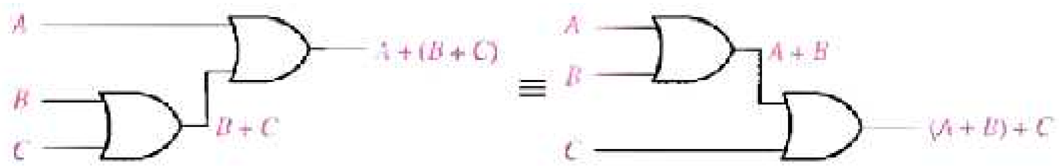


Fig.(5) Application of associative law of addition

The associative law of multiplication is written as follows for three variables:

A(BC) = (AB)C

This law states that it makes no difference in what order the variables are grouped when ANDing more than two variables. Fig.(6) illustrates this law as applied to 2 -input AND gates.

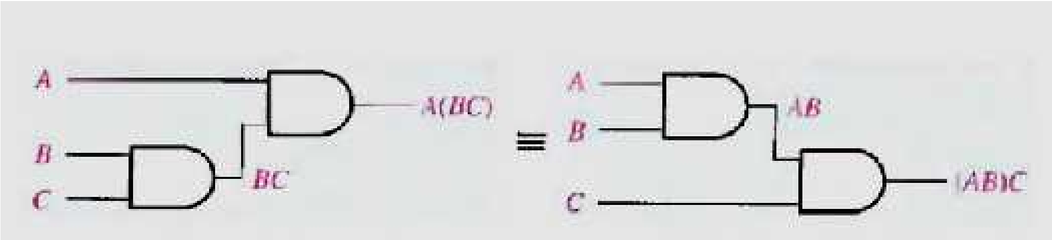


Fig.(6) Application of associative law of multiplication

Distributive Law:

The distributive law is written for three variables as follows:

A(B + C) = AB + AC

This law states that ORing two or more variables and then ANDing the result with a single variable is equivalent to ANDing the single variable with each of the two or more variables and then ORing the products. The distributive law also expresses the process of factoring in which the common variable A is factored out of the product terms, for example,

AB + AC = A(B + C).

Fig.(7) illustrates the distributive law in terms of gate

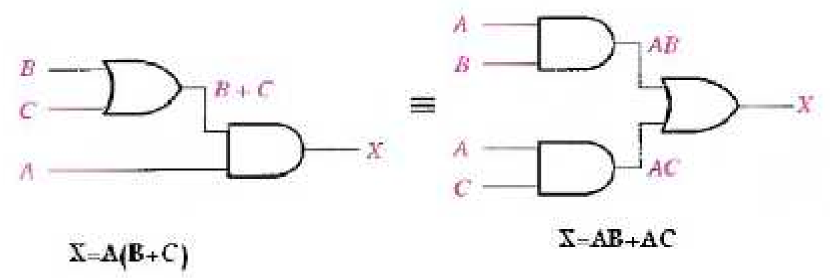


Fig.(7) Application of distributive law

Rules of Boolean Algebra

Table 1 *lists 12 basic rules that are useful in manipulating and simplifying Boolean expressions. Rules 1 through 9 will be viewed in terms of their application to logic gates. Rules 10 through 12 will be derived in terms of the simpler rules and the laws previously discussed.*

Table 1 Basic rules of Boolean algebra

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

1. ***A+0=A***

Fig8

X=A+0=A

2. ***A+1=1***

Fig9

X=A+1=1

3. ***A.0=0***

Fig10

X=A.0=0

4. ***A.1=A***

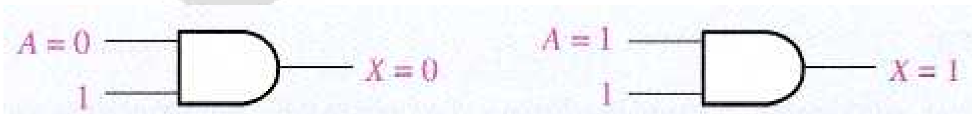


Fig11

X=A.1=A

5. ***A+A=A***

Fig12

X=A+A=A

6. $A+\overline{A}=1$

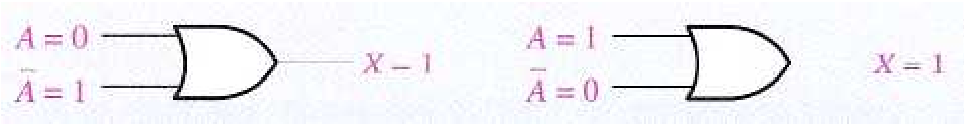


Fig13

$$X=A+\overline{A}=1$$

7. $A.\,A=A$

Fig14

$$X=A.\,A=A$$

8. $A.\overline{A}=0$

Fig15

9. $\overline{\overline{A}}=A$

Fig16

$$X=\overline{\overline{A}}=A$$

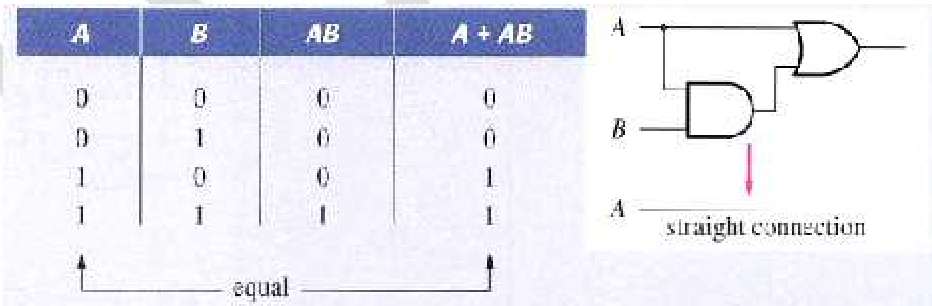
10. $A+AB=A$

$A+AB=A(1+B)$ *Factoring (distributive law)*

$=A\,.\,1$ *Rule 2:* $(1+B)=1$

$=A$ *Rule 4:* $A\,.\,1=A$

The proof is shown in Table 2, which shows the truth table and the resulting logic circuit simplification



A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Fig17

11. $A + \overline{A}B = A + B$

This rule can be proved as follows:

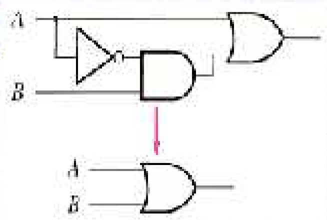
$A + \overline{A}B = (A + AB) + \overline{A}B$	*Rule 10:* $A = A + AB$
$= (AA + AB) + \overline{A}B$	*Rule 7:* $A = AA$
$= AA + AB + A\overline{A} + \overline{A}B$	*Rule 8: adding* $A\overline{A} = 0$
$= (A + \overline{A})(A + B)$	*Factoring*
$= 1.\ (A + B)$	*Rule 6:* $A + \overline{A} = 1$
$= A + B$	*Rule 4: drop the 1*

The proof is shown in Table 3, which shows the truth table and the resulting logic circuit simplification.

Table 3

A	B	$\overline{A}B$	$A + \overline{A}B$	A + B
0	0	0	0	0
0	1	1	[illegible]	1
1	0	0	[illegible]	1
1	1	0	1	1

equal



12. $(A + B)(A + C) = A + BC$

This rule can be proved as follows:

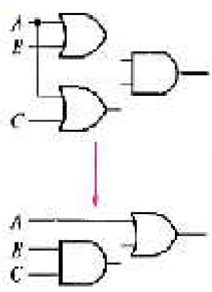
It $(A + B)(A + C) = AA + AC + AB + BC$	*Distributive law*
$= A + AC + AB + BC$	*Rule 7:* $AA = A$
$= A(1 + C) + AB + BC$	*Factoring (distributive law)*
$= A.\ 1 + AB + BC$	*Rule 2:* $1 + C = 1$
$= A(1 + B) + BC$	*Factoring (distributive law)*
$= A.\ 1 + BC$	*Rule 2:* $1 + B = 1$
$= A + BC$	*Rule 4:* $A \,.\, 1 = A$

The proof is shown in Table 4, which shows the truth table and the resulting logic circuit simplification.

Table 4

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	[illegible]	0	0	0	0
0	1	1	[illegible]	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



DEMORGAN'S THEOREMS

DeMorgan, a mathematician who knew Boole, proposed two theorems that are an important part of Boolean algebra. In practical terms. DeMorgan's theorems provide mathematical verification of the equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates.

One of DeMorgan's theorems is stated as follows:

The complement of a product of variables is equal to the sum of the complements of the variables,

Stated another way,

The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.

The formula for expressing this theorem for two variables is

$\overline{XY} = \overline{X} + \overline{Y}$

DeMorgan's second theorem is stated as follows:

The complement of a sum of variables is equal to the product of the complements of the variables.

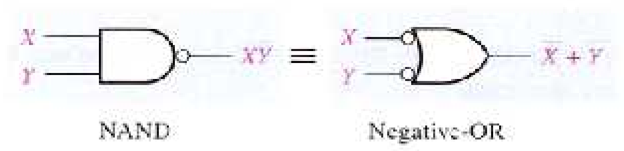
Stated another way,

The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables,

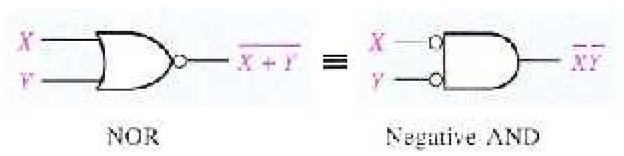
The formula for expressing this theorem for two variables is

$\overline{X + Y} = \overline{X}\,\overline{Y}$

Fig.(18) shows the gate equivalencies and truth tables for the two equations above.



Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Fig18

Gate equivalencies and the corresponding truth tables thatillustrate DeMorgan's theorems.

As stated, DeMorgan's theorems also apply to expressions in which there are more than two variables. The following examples illustrate the application of DeMorgan's theorems to 3-variable and 4-variable expressions.

EX//Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ *and* $\overline{X + Y + Z}$.

$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

$\overline{X + y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$\overline{\overline{A+B\overline{C}}+D(\overline{E+\overline{F}})}$

Step 1. *Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable.*

Let

$\overline{A+B\overline{C}}=X$ *and* $D(\overline{E+\overline{F}})=Y$

Step 2. *Since* $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$\overline{\overline{A+B\overline{C}}+D(\overline{E+\overline{F}})} = (\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})})$

Step 3. *Use rule 9 ($A = \overline{\overline{A}}$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).*

$(\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$

Step 4. *Applying DeMorgan's theorem to the second term***,**

$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$

Step 5. *Use rule 9 ($A = \overline{\overline{A}}$) to cancel the double bars over the* $E + \overline{F}$ *part of the term.*

$(A + B\overline{C})(D + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$

***EX//** Apply DeMorgan's theorems to each of the following expressions:*

(a) $\overline{(A + B + C)D}$ (b) $\overline{ABC + DEF}$ (c) $\overline{A\overline{B} + \overline{C}D + EF}$

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$