Numerical Investigation of Free Convection Heat Transfer with non-Newtonian Fluid in Different Enclosures Geometries.

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ABSTRACT

In this investigation, study of a steady state tow-dimension free convection of non-Newtonian flow inside an elliptical enclosure with laminar flow has been studied numerically. Two kinds of boundary conditions were considered , first when the inner holes were under constant heat flux and second when inner holes heated to constant temperature while enclosure wall was insulated in all cases. Wide ranges of Rayleigh numbers(1.34E03, 1.34E04 and 1.34E05), power law index (0.1, 0.5,1 and 5) with Prandtl number(1.37, 6.37 and 15.37) are implemented.. The results presented in terms of isotherms to present the temperature field. Besides that, the relationships between Nusselt number and other essential parameters such as, Rayleigh numbers, Prandtl number and power law index are exhibited. The results proved that Nusselt number strongly affect by all parameters mentioned previously. The effect of alteration of power law index for non-Newtonian fluid on the temperature distribution was elucidated. Also, increasing of Prandtl number lead to increasing of Heat transfer rate for a given rest parameters, consequently.

Keywords— non-Newtonian, Free convection, Power law index, Constant heat flux.

INTRODUCTION

One of the most important of heat transfer modes is Free (or Natural) convection. This phenomenon, particularly depend on buoyancy force resulting from the gradients of density caused by the temperatures difference. Free convection can happen in enclosures, solar cavity receivers, oil containers, ventilation etc. [1]. The non-Newtonian fluids, commonly divided into two groups, first is called "Time independent fluid", which include Pseudo Plastic fluids, Plastic fluids and Dilatant fluids. While the second group "Time independent fluid" that contain Thixotropic fluids and Rheopectic fluids. However, several investigations studied the effect of using Non-Newtonian fluid instead of Newtonian one on rate of heat transfer. Irvine Glenn Reilly (1964) [2], was investigated experimentally free convection heat transfer of horizontal and vertical plates under constant wall temperature. And as compared with theoretical results, experimental results were expressed in terms of dimensionless quantities to satisfy the theoretical expression. KumariI et al. [3] were studied natural convection of laminar non-Newtonian with a power – law model flow over a vertical wavy(sinusoidal) surface numerically. The non-dimensional form governing equations besides isothermal boundary conditions were considered. It is found that the varies a periodic variation of local Nusselt number along the wavy surface and wavelength of local Nusselt number is varies proportionally with half of sinusoidal surface. The developing of thermally laminar flow of a Bingham plastic under a constant heat flux on a circular tube has been investigated analytically by Min and Yoo.[4]. Various parameters and non-dimensional values such as yield stress, Peclet number and Brinkman number were employed to describe the (Nu) and fully developed temperature. Particularly , it's shown that yield stress and viscous dissipation has considerable effect on the coefficient of heat transfer. Rita Choudhury and Alok Das [5] were investigated analytically steady state, natural convection in two-dimensional vertical channel with Waiters fluid (model B). one of the walls of channel was wavy wiles other was flat plate. Different parameters were showed in this study, such as rate of heat transfer, average Nusselt number and skin-friction. Ching-Yang Cheng [6]. Investigated the boundary layer flow of non-Newtonian with power-law fluids of free convection on a vertical cone with downward-pointing in porous media. Both constant wall temperature and constant heat flux boundary condition were investigated in this study. The similarity analysis was implemented. Its presented that increase in power-law viscosity will decrease of both thermal boundary conditions. In addition to, increasing of exponent of similarity will increase thickness of boundary layer and that will lead to decrease of heat flux of cone surface. A free convection of non-Newtonian fluid with a power law model over axisymmetric body is analyzed by A. A. Mohammadein [7]. An external magnetic field with uniform intensity was applied normally on the body surface. It's found that increase in magnetic field proportionally increase with Nusselt number. Sidhartha Bhowmick et al[8]. (2014), were studied the laminar of pseudo-plastic fluids flow with a modified power law heat transfer in a horizontal circular under constant heat flux numerically. The heat transfer mode was Mixed convection and a two-dimensional boundary-layer flow were considered. A double sweep technique based on finite difference method. The results showed the variation of decreasing of temperature in the boundary-layer less rapidly at the all flow region. Habibi et al. [9]. studied free convection heat transfer in square horizontal annulus duct. The flow descriptions were two dimensional, stead state non-Newtonian with power law model under constant temperature. A finite volume method was used to solve set of governing equations. The results showed that at power law index increase, the average Nusselt number associated decrease. Also, the variation of (Pr) has no effect on characteristics of heat transfer except for several cases. The free convection of laminar flow boundary layer with a power-law model as a non-Newtonian fluid over an isothermal plate were investigated numerically by Abhijit Guha and Kaustav Pradhan[10]. The plate was oriented horizontally. The most important parameters, that the temperature ,

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pressure and velocity depended, were generalized Prandtl number and index of power-law. The results showed that the heat transfer characteristics of dilute fluids, at the same generalized Prandtl number, is better than both pseudo-plastic fluids and Newtonian fluids. Sharaban Thohura et al.[11] studied numerically a free convection of non-Newtonian fluid with modified power law viscosity model along a vertical thin cylinder under constant wall temperature. The Navire-Stoke equations were transformed to non-dimensional form and solved by implicit FDM "Finite difference method". The results exposed that effectual increasing in shear-rate compared to Newtonian fluid beyond threshold value.

MATHEMATICAL FORMULATION

Both of schematic of physical model and the system of coordinates were showed in Fig.(1). It's a three different enclosure geometries (circular, square and elliptic) with a central big hole besides to four smaller equal holes in sides. The descriptions of flow were steady state, incompressible and laminar natural convection non-Newtonian fluid with constant fluid properties except for the density appears in the buoyancy term (Boussinesq approximation). However the viscous dissipation contribution in energy equation is considered negligible.

GOVERNING EQUATIONS

According to mathematical model and assumptions mentioned above, the governing equations (continuity, momentum and energy)in their primitive form are[12] and [13] are modeled as equations (1), (2) and (3), respectively

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\rho\left(u_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \nabla \tau_i + F \qquad (2)$$

$$(\rho C_p)\left(u_i \frac{\partial T}{\partial x_i}\right) = k\nabla^2 T \qquad (3)$$

However, The full description of applying the above three equations for Cartesian and cylindrical coordinates is stated in Table (1).

Coordinates system	Cartesian coordinates (x,y)	Cylindrical coordinates (r,θ)
Velocity	$V = ui_x + vi_y$	$\vec{V} = u\, i_r + v\, i_\theta$
Body Force	$F_x = 0$ $F_y = g\,\beta(T - T_{ref})$	$F_r = g\,Cos\theta\,\beta(T - T_o)$ $F_\theta = g\,Sin\theta\,\beta(T - T_o)$
1st order operator	$\nabla = \frac{\partial}{\partial x} i_x + \frac{\partial}{\partial y} i_y$	$\nabla = \frac{\partial}{\partial r} i_r + \frac{1}{r}\frac{\partial}{\partial \theta} i_\theta$
2nd order operator	$\nabla = \frac{\partial}{\partial x} i_x + \frac{\partial}{\partial y} i_y$	$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$

According to [14], the power law index of non-Newtonian fluid can be expressed as:

$$\tau = m\left(\frac{\partial u}{\partial y}\right)^n \qquad (4)$$

Where m and n are the consistency and powerful of the power law index respectively .

Thus, , the shear stresses for Cartesian coordinates are:

$$\tau_{xx} = 2m\left(\frac{\partial u}{\partial x}\right)^n \qquad (5)$$

$$\tau_{yy} = 2m\left(\frac{\partial v}{\partial y}\right)^n \qquad (6)$$

$$\tau_{xy} = \tau_{yx} = m\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^n \qquad (7)$$

And for the Cylindrical coordinates (r,θ)

$$\tau_{rr} = 2m\left(\frac{\partial v_r}{\partial r}\right)^n \qquad (8)$$

$$\tau_{\theta\theta} = \frac{2m}{r}\left(\frac{\partial v_\theta}{\partial \theta}\right)^n \qquad (9)$$

$$\tau_{r\theta} = \tau_{\theta r} = m\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r}\right)^n \qquad (10)$$

In addition to, the local Nusselt number as [15] remark is

1. For square enclosure

$$Nu = \frac{qD_h}{k\Delta T} = \frac{qL}{k\Delta T} \qquad (11)$$

2. For circular enclosure

$$Nu \frac{qd}{k\Delta T} \qquad (12)$$

3. For elliptical enclosure

$$Nu = \frac{qD_h}{k\Delta T} \qquad (13)$$

And

$$Ra = \frac{g\,\beta (D_h)^3 \Delta T}{\alpha\nu} \quad \text{is Rayleigh number} \qquad (14)$$

$$\Pr = \frac{\nu}{\alpha} \quad \text{is Prandtl number} \qquad (15)$$

BOUNDARY CONDITIONS

$u= v=0$ (no - slip condition)

$q'' = k\frac{\partial T}{\partial n}$ for supplied heat flux

$\frac{\partial T}{\partial n} = 0$ for insulated wall

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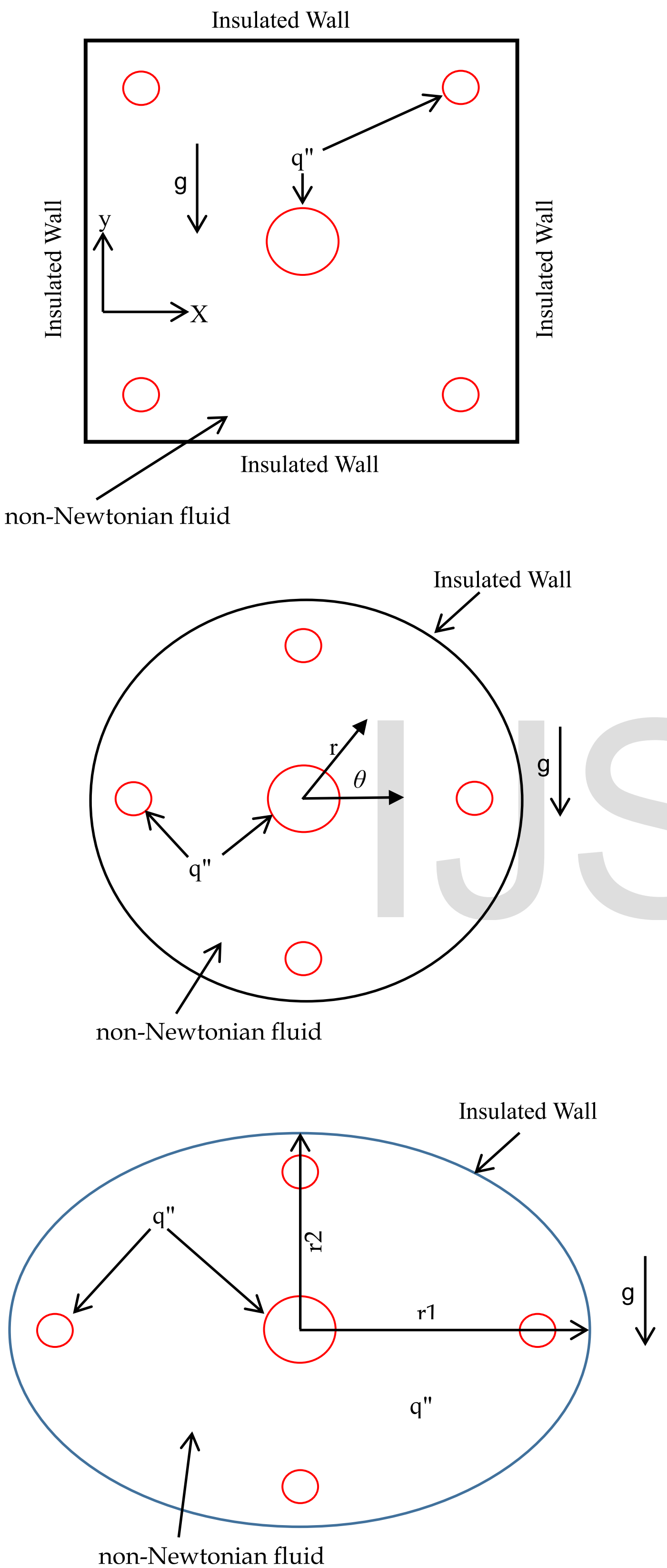

Fig.(1). The Enclosures Geometries and the Coordinates axis

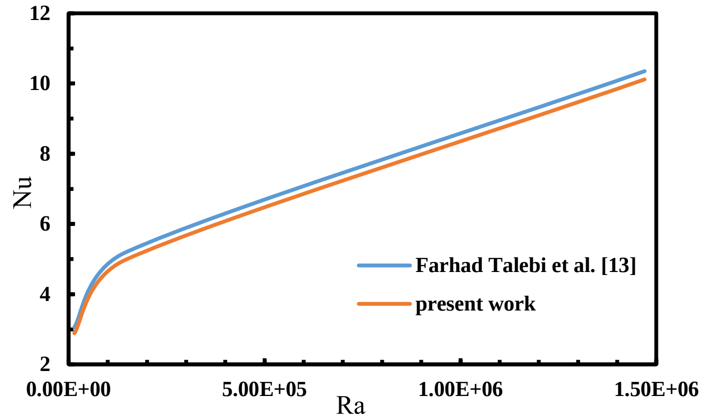

Fig.(2). Comparison of average Nusselt number between present work and Talebi et al. [13]

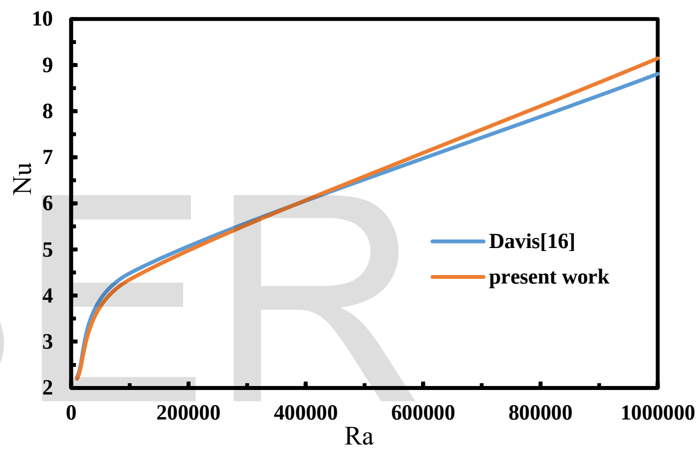

Fig.(3). Comparison of average Nusselt number between present work and G. De Vahl Davis. [16]

NUMERICAL SOLUTION

The aim of the computational solution is to study the parametric influences especially Ra, Pr and (*n*) on the heat transfer rate of the problem considered in the present work. The concurrent partial differential equations along with the boundary conditions associated were demonstrated and solved. Numerical solutions are introduced by adopting the software program (COMSOL 4.1) which is based on Finite Element Method for two dimensional natural convection in different shape enclosures as a result of supplying constant heat flux. The domain meshing and the differential discretization of the governing equations result in systems of algebraic equations that were solved by a PARDISO and GEOMATRIC MULTIGRID which are a semi-direct solver particularly effective or manipulating unsymmetrical crowded matrices by a LU-based decomposition technique.

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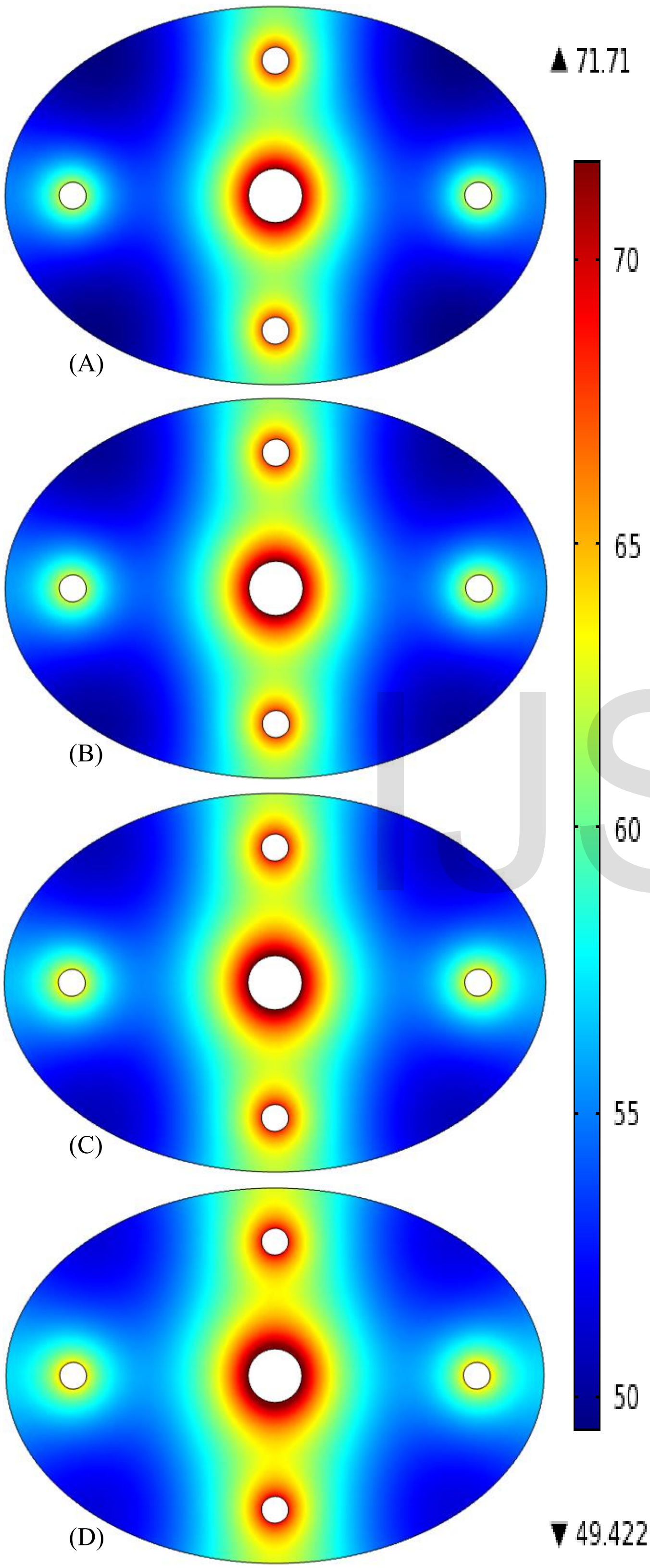

Fig.(4). Variation of Temp. disterbuion for Ra=1.34E03 and Pr=6.37 where(A) *n*=0.1(B) *n*=0.5 (C) *n*=1 (D) *n*=5

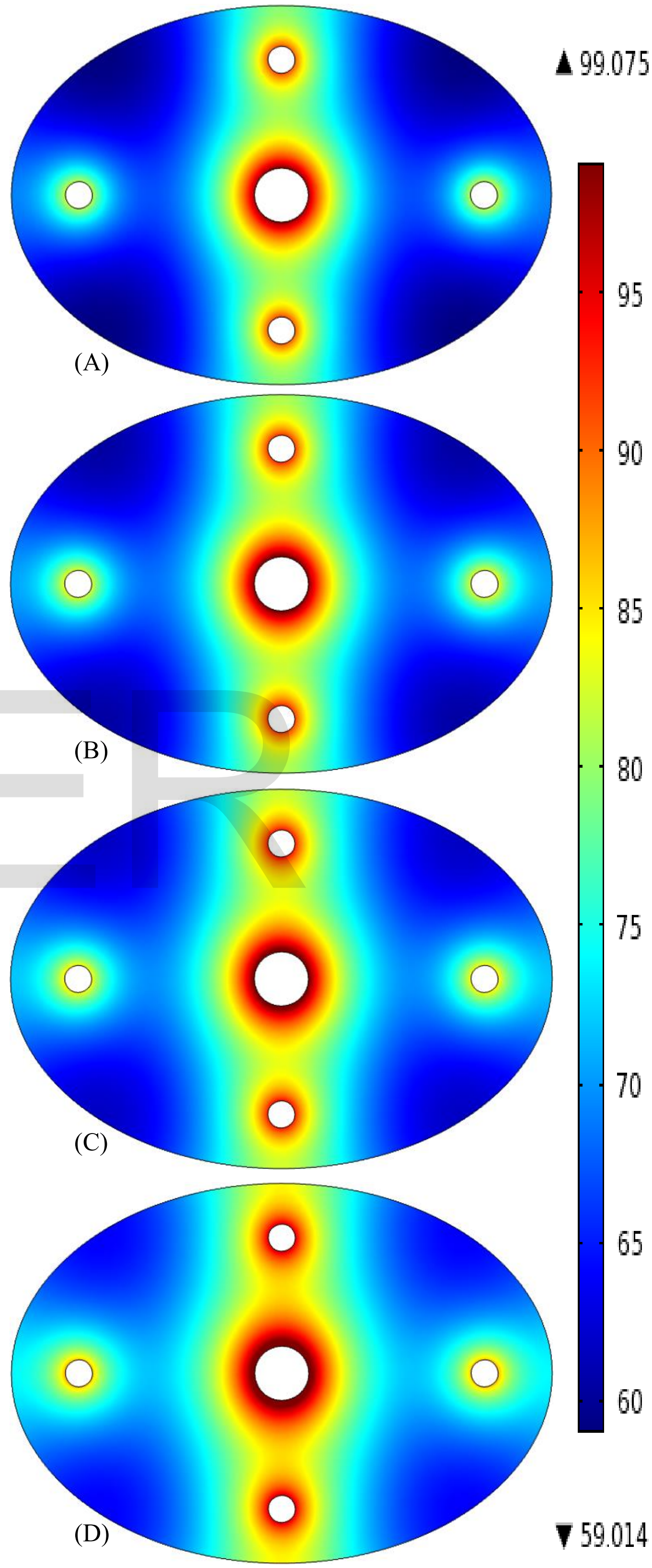

Fig.(5). Variation of Temp. disterbuion for Ra=1.34E05 and Pr=6.37 where(A) *n*=0.1(B) *n*=0.5 (C) *n*=1 (D) *n*=5

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VALIDATION

To show the validation of applied package software with the Average Nusselt number versus Rayleigh number by comparing the results with those of free convection heat transfer in enclosures with no holes for Newtonian fluid predicted byFarhad Talebi et al. [13] at figure (2). The comparisons have been in different values of Rayleigh number and Prandtl number, a good agreement is shown for the shape of square enclosure. Another validation of the computational model is performed by comparing the predicted evolution of the Nusselt number with the numerical results of G. De Vahl Davis (1983) [16] for square enclosure as it presented in Fig. (3). The results showed a good agreement.

RESULTS AND DISCUSSION

The temperature distribution for different values of system parameters are illustrated in Figs. (4) to (9). In this case, the energy is transported from hot walls of holes to others parts of containers.

The variation of the power law index for 0,0.5,1 and 5 in square enclosure with two values of (Ra) are exhibited in Figures (4) and (5). Various sections in a ellipse enclosure were made to typify the effect of increasing of power law index on temperature distributions. It's clear from those figures that there some regions (near of holes) which have higher temperature corresponding with others. However, when Rayleigh number increases, the effect of buoyancy-driven convection increases with earlier terminating of the heat transfer process. On the other hand, increasing the power law index of the non-Newtonian fluid enhances heat transfer rate with monotonic reduction in the elapsing time.

Figures (6) and (7) show the temperature distribution for different values of system parameters in case of circular enclosure. These were shown that heavy concentration of hot region near the hole for all cases. Its observed that as the Ra and / or (n) increase, the temperature distribution shift towards the inside of enclosure. The shift is more pronounced as the Ra increase more and more. Furthermore, the temperature gradient near the hot hole increase rapidly as the Ra increase. Figures (8) and (9) demonstrate the effect of power law index (n) on heat transfer progress in two different values of (Ra) and Pr=6.37. The considerable augmentation of heat downturn near holes can be expanded due increasing in (n). also, the increasing in power law index is affirmative parameter to enhance heat transfer between hot holes and non-Newtonian fluid inside enclosure. From this figure, its clear that increase of Rayleigh number present an extra augmentation for heat transfer rate.

The average Nusselt number in different enclosures geometry are presented in figures (10). Its seen that for Ra and (Pr) is small, the rate of increase in Nu_a is relatively small too. Then the Nu_a is rabidly increasing as Ra or (Pr) increases expressing the existence and increasing of convective heat transfer. This behavior is in coincidence with that of different values of (Pr). The possible reason of this behavior is as follow, for low Ra , the flow consists of small single cell rotating slowly in the enclosure so it characterize a very weak convective flow, and available areas for direct contact between the insolating wall and turning hot flow increase slowly. For high Ra, however, as Ra increase, the streamlines move closer to the insolating walls and the direct contact areas increase, at the same time, the convection motion

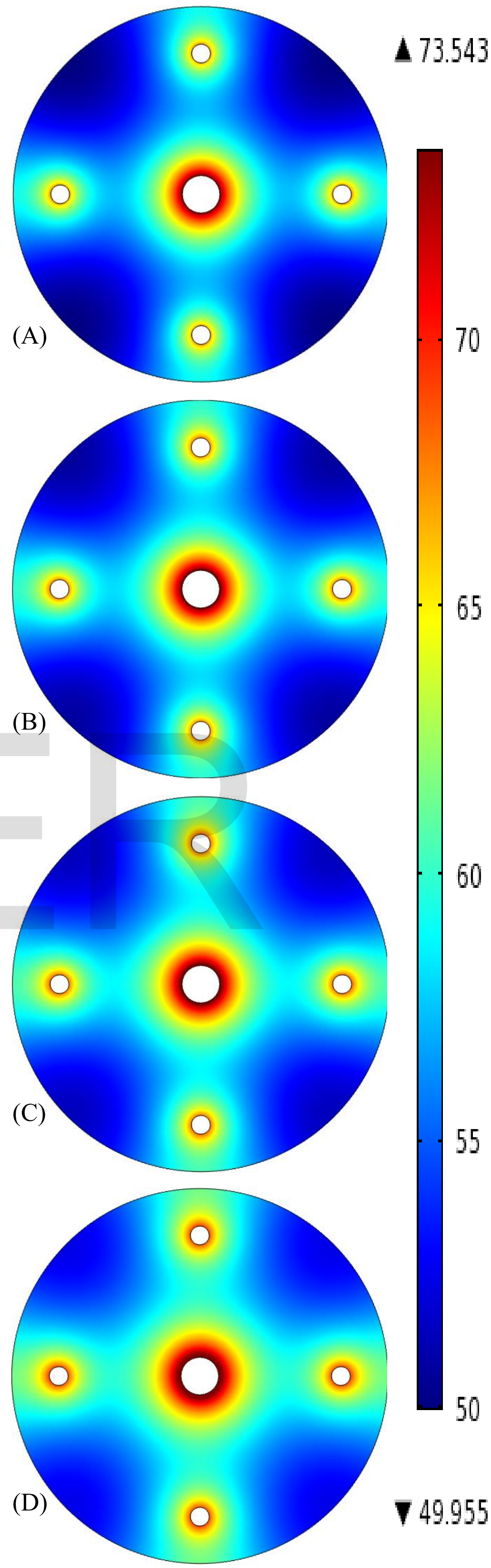

Fig.(6). Variation of Temp. disterbuion for Ra=1.34E03 and Pr=6.37 where(A) n=0.1(B) n=0.5 (C) n=1 (D) n=5

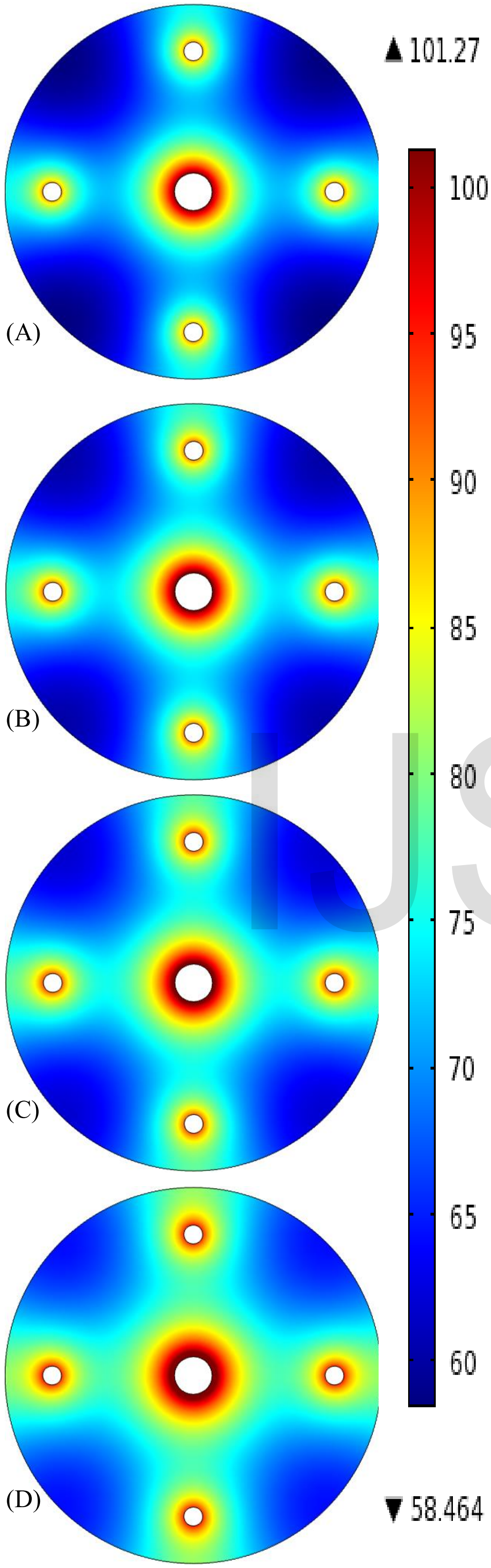

Fig.(7). Variation of Temp. disterbuion for Ra=1.34E05 and Pr=6.37 where(A) *n*=0.1(B) *n*=0.5 (C) *n*=1 (D) *n*=5

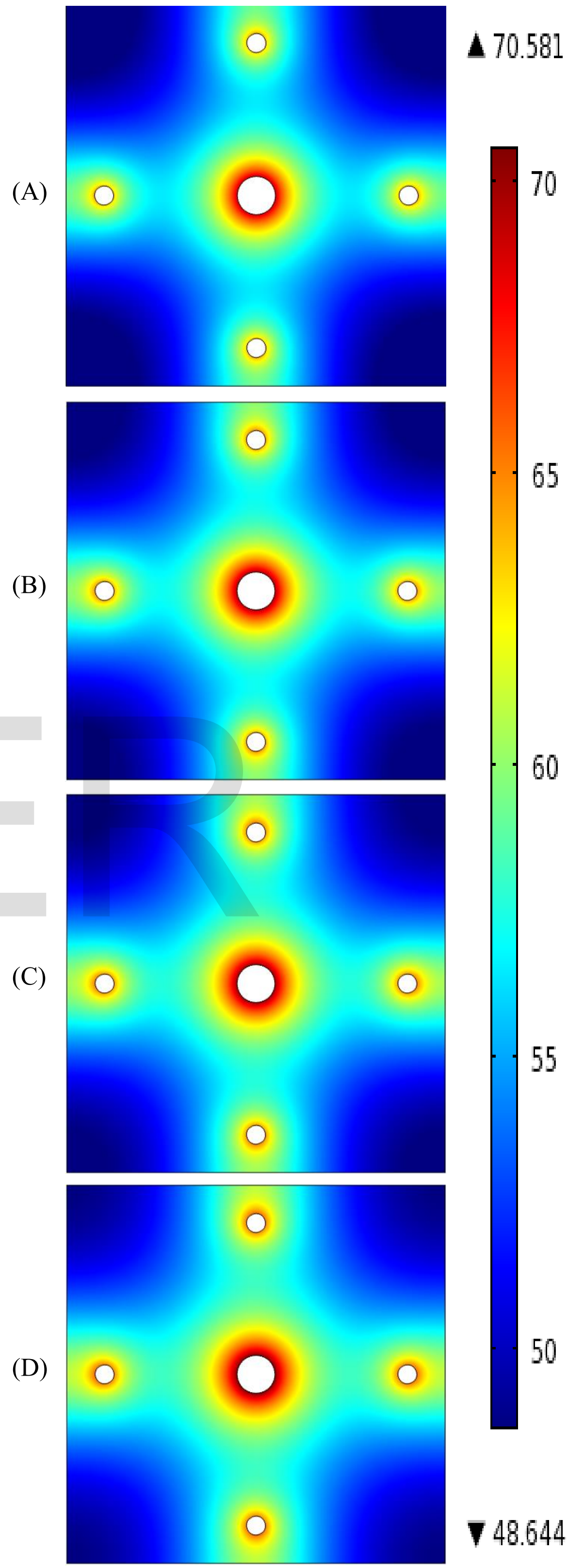

Fig.(8). Variation of Temp. disterbuion for Ra=1.34E03 and Pr=6.37 where(A) *n*=0.1(B) *n*=0.5 (C) *n*=1 (D) *n*=5

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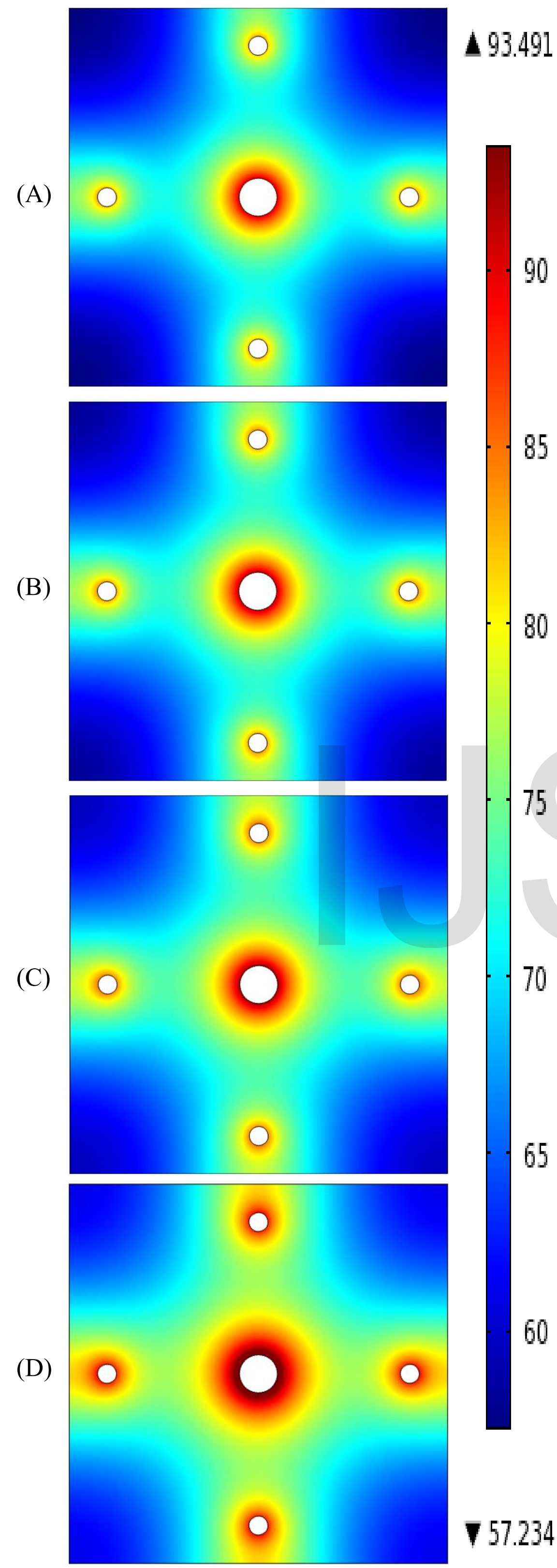

Fig.(9). Variation of Temp. disterbuion for Ra=1.34E05 and Pr=6.37 where(A) n=0.1(B) n=0.5 (C) n=1 (D) n=5

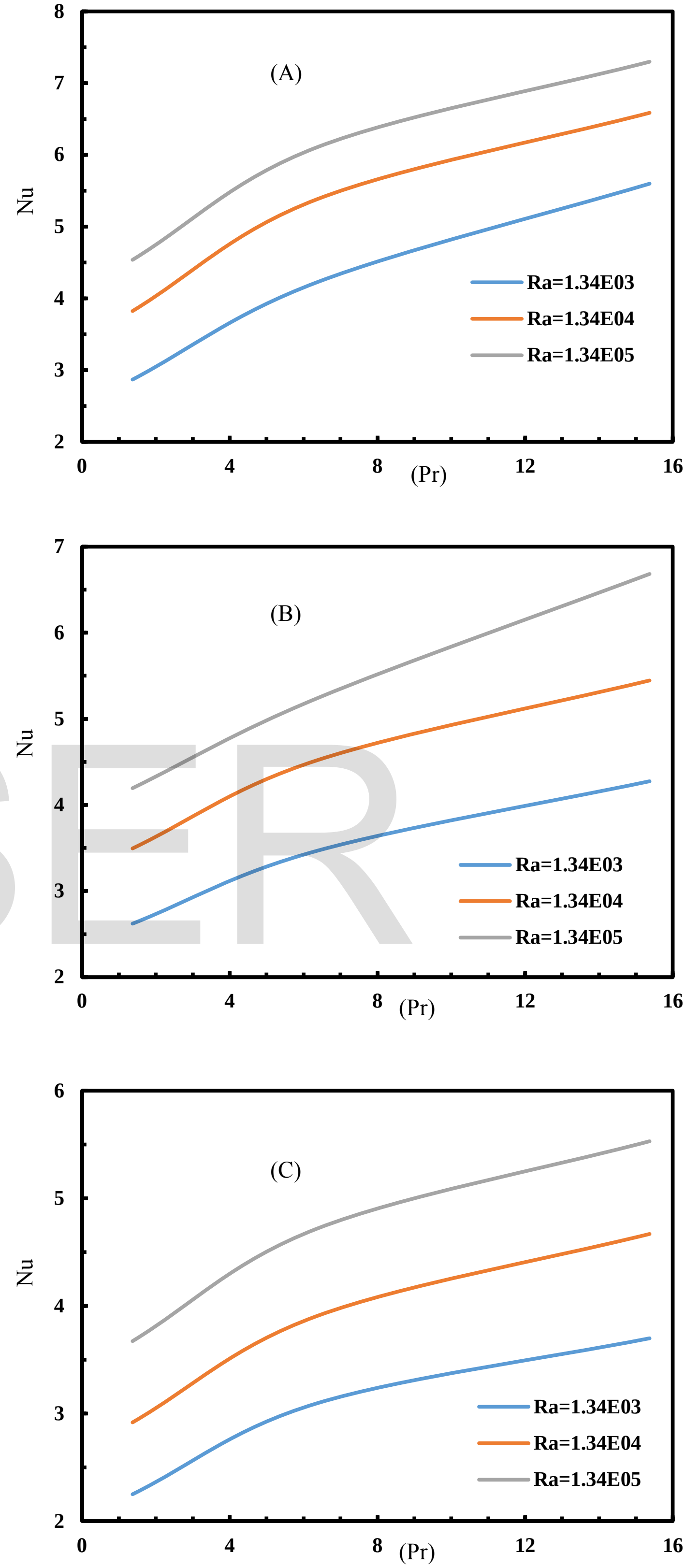

Fig.(10). Variation of (Nu_a) with (Pr) at (n)=0.5 where (A) Circular Enclosure (B) square Enclosure (C) Elliptic Enclosure

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increases. As already indicated by the temperature field, the Nusselt number for the circular enclosure is higher than that rest types.The variation of stream function with alteration of Rayleigh number compared for different power law index and enclosure geometries were illustrated In figures (11). However, at low Ra, the Pr and (*n*) have small effect on stream function. That perhaps due to dominance of conduction regime. At higher Ra, the stream function increase with increasing (*n*) and\or Pr . it is also seen that ψ_{max} increase and reach the maximum value at Ra =1.34E05, Pr=15.37 and (*n*)=1. In addition that, its observed that the peak value of stream function depends on (*n*) and Rayleigh number.

6.Conclusion

The numerical solutions for heat transfer Natural convection of power law index non- Newtonian fluid had been presented. Three types of enclosures were used square, circular and elliptic one included holes with different sizes and positions under constant heat flux. Its proved that Nusselt number is a strong function of Rayleigh number, Prandtl number and power law index. It's found that increasing in (Pr) will lead to increase local and average Nusselt number, consecutively heat transfer rate , at the given rest parameters. The increase of (*n*) will lead to decreasing in time elapsed for steady state conditions and augmentation of Nusselt number. As the Rayleigh number increase, the value of (*n*) at which maximum average Nusselt number take place shift low value of (*n*) for all values of (Pr), nevertheless for small (Ra), it hasn't direct effect on heat transfer rate because in this situation, the convection is very weak and the dominant mode of heat transfer nearly conduction.

Nomenclature
Symbols Variables

Symbol	Description	Units
C_p	Heat capacity	KJ/Kg. K
D	Diameter of enclosure	m
D_h	Hydraulic diameter	m
F	Body force	N/m3
g	Gravitational acceleration	m/s^2
H	Heat Transfer Coefficient	$W/m^2.K$
K	Thermal Conductivity	W/m.K
L	Length of Tube	m
m	Fluid consistancy indix for power-law model index	Kg/sec(*n*).m
n	Fluid powerful for power-law model index	dimensionless
Nu	Local Nusselt Number	dimensionless
Nu_a	Average Nusselt Number	dimensionless
P	pressure	Pa
Pr	Prandtl number	dimensionless
q"	Heat Flux	W/m^2
r	Radial Coordinate	M
Ra	Rayleigh number	dimensionless
T	Temperature	*K*
u	Axial Velocity	m/s
v	Radial Velocity	m/s
x	Horizontal Coordinate	m
y	Vertical Coordinate	m

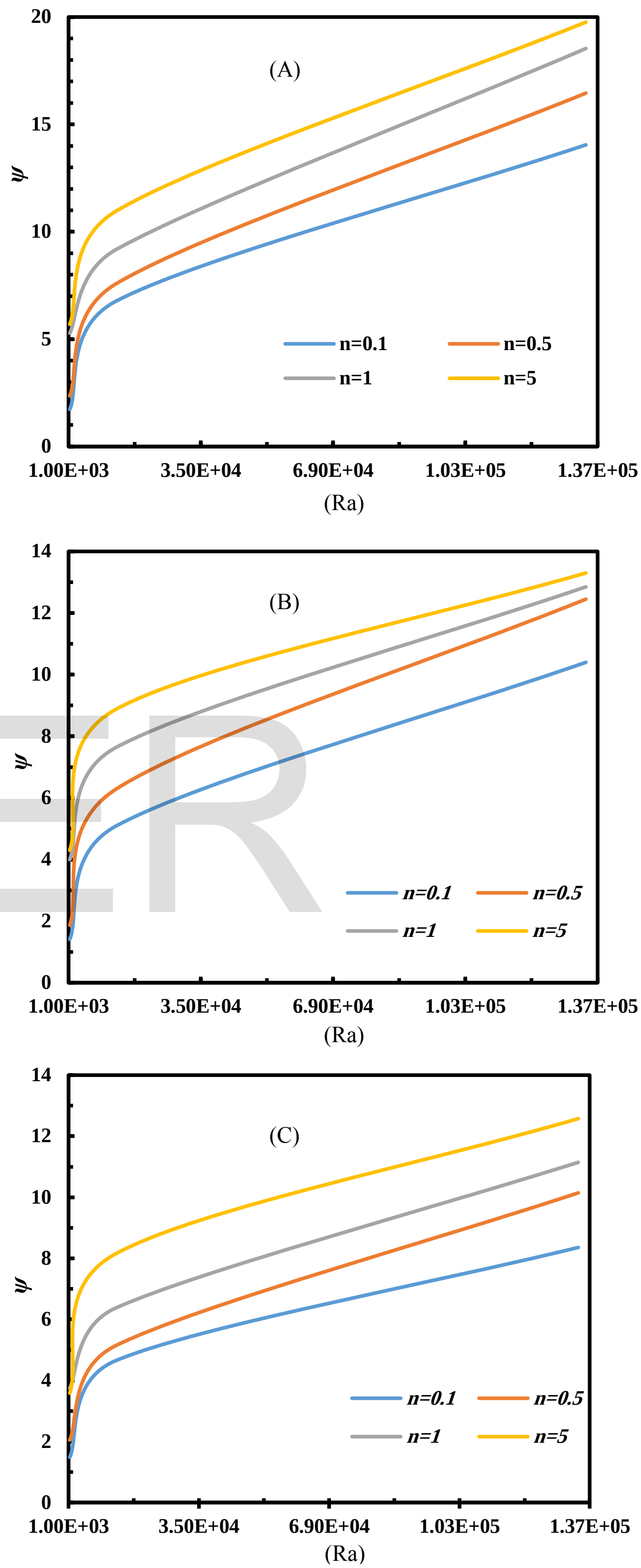

Fig.(11). Variation of (ψ) with (Ra) at (Pr)=6.37 where (A) Circular Enclosure (B) square Enclosure (C) Elliptic Enclosure

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Greek Symbols

Symbol	Description	Units
μ	Dynamic viscosity	m^2/s
ρ	Density	Kg/m^3
τ	Stresses	Pa
ψ	Stream function	m^2/s
θ	Angular coordinate	dimensionless
β	Thermal expansion coefficient	$1/K$
α	Thermal diffusivity	m^2/s
∇	Laplacian operator	dimensionless
ΔT	Temperature difference	K
ν	Kinematic viscosity of fluid	m^2/s

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