ShiftRows Transformation

Forward and Inverse Transformations

The **forward shift row transformation**, called ShiftRows, is depicted in Figure 8.1.

- The first row of **State** is not altered.
- For the second row, a 1-byte circular left shift is performed. For the third row, a 2-byte circular left shift is performed. For the fourth row, a 3-byte circular left shift is performed. The following is an example of ShiftRows:

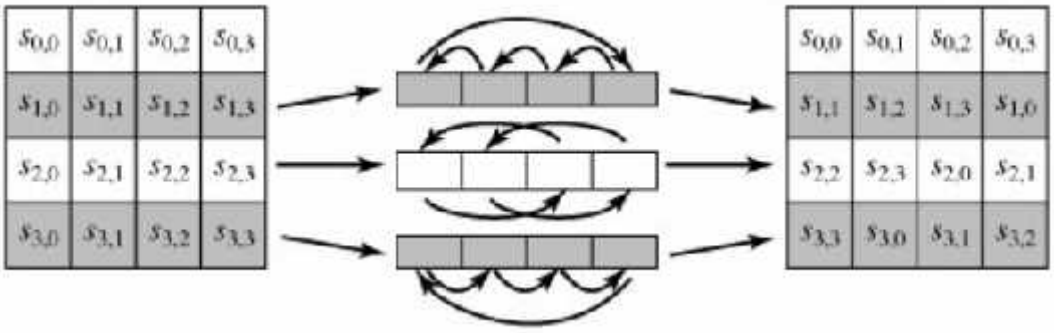


Figure (8-1)Shiftrow transformation

- The **inverse shift row transformation**, called InvShiftRows, performs the circular shifts in the opposite direction for each of the last three rows, with a one-byte circular right shift for the second row, and so on.

Example:

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

MixColumns Transformation

Forward and Inverse Transformations

The **forward mix column transformation**, called MixColumns, operates on each column individually.

- Each byte of a column is mapped into a new value that is a function of all four bytes in that column. The transformation can be defined by the following matrix multiplication on **State** (Figure 8-2):

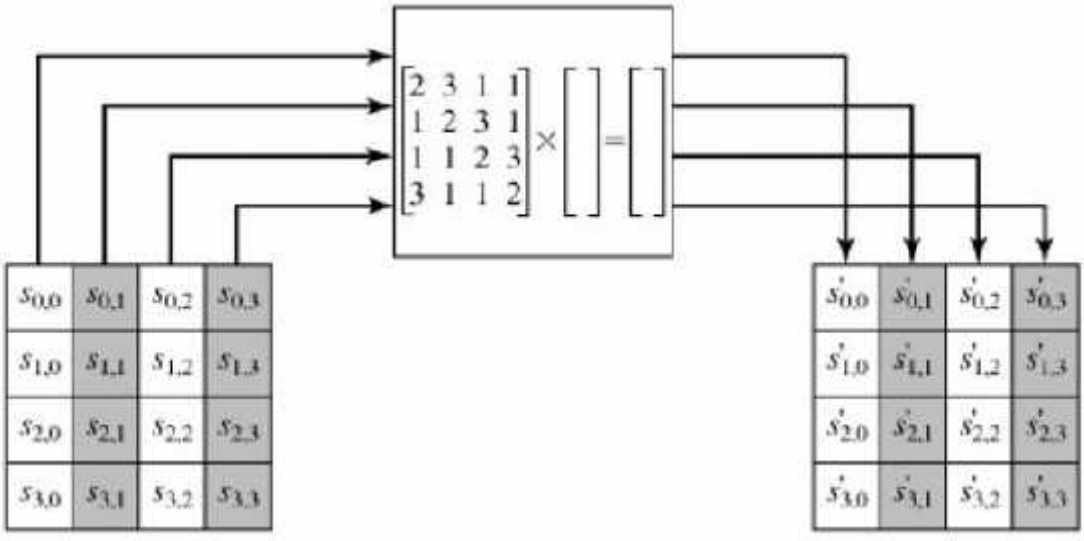


Figure (8-2) Mixcolumn transformation

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \quad \text{.....(8-1)}$$

- Each element in the product matrix is the sum of products of elements of one row and one column. In this case, the individual additions and multiplications [6] are performed in GF(2

transformation on a single column $j(0 \le j \le 3)$ of **State** can be expressed as:

$$s'_{0,j} = (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j}$$
$$s'_{1,j} = s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j}$$
$$s'_{2,j} = s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j})$$
$$s'_{3,j} = (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})$$

The AES document describes another way of characterizing the MixColumns transformation, which is in terms of polynomial arithmetic. In the standard, MixColumns is defined by considering each column of **State** to be a four-term polynomial with coefficients in GF(2^8).

Example:

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

Let us verify the first column of this example, in GF(2), **addition is the bitwise XOR operation** and that **multiplication** can be performed according to the rule established in Equation (8.2). It follows that multiplication by x (i.e., 00000010) can be implemented as a 1-bit left shift followed by a conditional bitwise XOR with (00011011), which represents ($x^4 + x^3 + x + 1$). To summarize,

$$x \times f(x) = \begin{cases} (b_6 b_5 b_4 b_3 b_2 b_1 b_0 0) & \text{if } b_7 = 0 \\ (b_6 b_5 b_4 b_3 b_2 b_1 b_0 0) \oplus (00011011) & \text{if } b_7 = 1 \end{cases} \quad \text{.....(8-2)}$$

In particular, multiplication of a value by *x* (i.e., by {02}) can be implemented as a 1bit left shift followed by a conditional bitwise XOR with (0001 1011) if the leftmost bit of the original 8value (prior to the shift) is 1. Thus, to verify the MixColumns transformation on the first column, we need to show that

$$\{02\} \cdot \{87\} \oplus (\{03\} \cdot \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\}$$
$$\{87\} \oplus (\{02\} \cdot \{6E\}) \oplus (\{03\} \cdot \{46\}) \oplus \{A6\} = \{37\}$$
$$\{87\} \oplus \{6E\} \oplus (\{02\} \cdot \{46\}) \oplus (\{03\} \cdot \{A6\}) = \{94\}$$
$$(\{03\} \cdot \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \cdot \{A6\}) = \{ED\}$$

For the first equation, we have

$\{02\} \cdot \{87\} = (0000\ 1110) \oplus (0001\ 1011) = (0001\ 0101)$; and

$\{03\} \cdot \{6E\} = \{6E\} \oplus (\{02\} \cdot \{6E\}) = (0110\ 1110) \oplus (1101\ 1100) = (1011\ 0010)$. Then

$$\begin{aligned} \{02\} \cdot \{87\} &= 0001\ 0101 \\ \{03\} \cdot \{6E\} &= 1011\ 0010 \\ \{46\} &= 0100\ 0110 \\ \{A6\} &= 1010\ 0110 \\ & 0100\ 0111 = \{47\} \end{aligned}$$

The other equations can be similarly verified.

- The **inverse mix column transformation**, called InvMixColumns, is defined by the following matrix multiplication:

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \quad \text{…..(8-3)}$$

It is not immediately clear that Equation (8-1) is the **inverse** of Equation (8-3). We need to show that:

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

All multiplication and addition operation are done in the GF(2^8).

AddRoundKey Transformation

Forward and Inverse Transformations

In the **forward add round key transformation**, called AddRoundKey,

- The 128 bits of **State** are bitwise XORed with the 128 bits of the round key. As shown in Figure 7.3b, the operation is viewed as a columnwise operation between the 4 bytes of a **State** column and one word of the round key; it can also be viewed as a byte-level operation.

Example of AddRoundKey:

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

⊕

AC	19	28	57
77	FA	D1	5C
66	DC	29	00
F3	21	41	6A

=

EB	59	8B	1B
40	2E	A1	C3
F2	38	13	42
1E	84	E7	D2

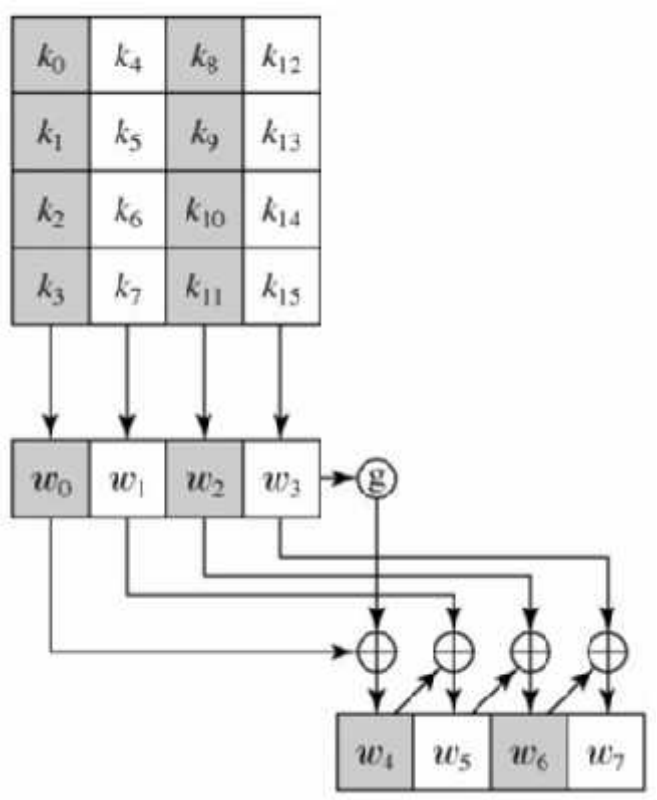
The first matrix is **State**, and the second matrix is the round key.

- The **inverse add round key transformation** is identical to the forward add round key transformation, because the XOR operation is its own inverse.

AES Key Expansion

Key Expansion Algorithm

The AES key expansion algorithm takes as input a 4-word (16-byte) key and produces a linear array of 44 words (176 bytes). This is sufficient to provide a 4-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher. Figure 8.3 illustrates the generation of the first eight words of the expanded key, using the symbol **g** to represent that complex function.



The function **g** consists of the following subfunctions:

1. **RotWord** performs a one-byte circular left shift on a word. This means that an input word [b0,b1, b2, b3] is transformed into [b1, b2, b3, b0].
2. **SubWord** performs a byte substitution on each byte of its input word, using the S-box (Table 5.4a).
3. The result of steps 1 and 2 is XORed with a round constant, Rcon[j].

The round constant is a word in which the three rightmost bytes are always 0. Thus the effect of an XOR of a word with Rcon is to only perform an XOR on the leftmost byte of the word. The round constant is different for each round and is defined as:

Rcon[j] = (RC[j], 0, 0, 0), with RC[1] = 1, $RC[j] = 2 \cdot RC[j - 1]$ and with multiplication defined over the field GF(2^8). The values of RC[j] in hexadecimal are:

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Example:

Suppose that the round key for round 8 is

EA D2 73 21 B5 8D BA D2 31 2B F5 60 7F 8D 29 2F

Then the first 4 bytes (first column) of the round key for round 9 are calculated as follows:

i (decimal)	temp	After RotWord	After SubWord	Rcon (9)	After XOR with Rcon	w[i-4]	w[i] = temp ⊕ w[i-4]
36	7F8D292F	8D292F7F	5DA515D2	1B000000	46A515D2	EAD27321	AC7766F3