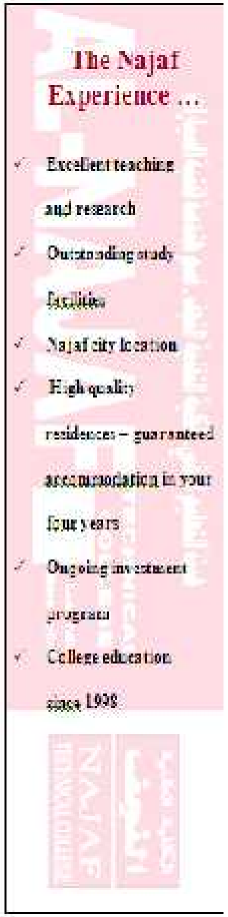


HEAT TRANSFER

انتقال الحرارة

اعداد الدكتور

علي شاكر باقر الجابري

HEAT TRANSFER انتقال الحرارة

CHAPTER TWO HEAT CONDUCTION EQUATION

1-1 Thermodynamics and Heat Transfer
1-2 Engineering Heat Transfer
1-3 Heat and Other Forms of Energy
Specific Heats of Gases, Liquids, and Solids
1-4 Heat Transfer Mechanisms
1-5 Conduction
1-6 Convection
1-6 Radiation
Problem-Solving Technique

2–1 GENERAL HEAT CONDUCTION EQUATION

Rectangular Coordinates

Consider a small rectangular element of length Δx, width Δy, and height Δz, as shown in Figure 2–21. Assume the density of the body is ρ and the specific heat is C. An *energy balance* on this element during a small time interval Δt can be expressed as

$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction at}\\ x, y, \text{and } z\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y, \text{and } z+\Delta z\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{array}\right)$$

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad \text{… 2–1}$$

Noting that the volume of the element is $V_{\text{element}} = \Delta x \Delta y \Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C \Delta x \Delta y \Delta z (T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g} V_{\text{element}} = \dot{g} \Delta x \Delta y \Delta z$$

Substituting into Eq. 2–1, we get

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g} \Delta x \Delta y \Delta z = \rho C \Delta x \Delta y \Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x \Delta y \Delta z$ gives

$$-\frac{1}{\Delta y \Delta z} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x \Delta z} \frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x \Delta y} \frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t} \qquad \text{… 2–2}$$

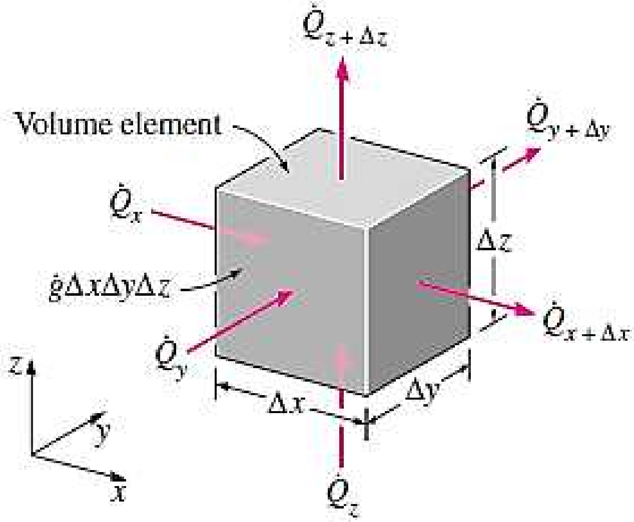


FIGURE 2–21
Three-dimensional heat conduction through a rectangular volume element.

Noting that the heat transfer areas of the element for heat conduction in the x, y, and z directions are $A_x = \Delta y \Delta z$, $A_y = \Delta x \Delta z$, and $A_z = \Delta x \Delta y$, respectively, and taking the limit as Δx, Δy, Δz and $\Delta t \rightarrow 0$ yields

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t} \quad \text{… 2–3}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0} \frac{1}{\Delta y \Delta z} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z} \frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z} \frac{\partial}{\partial x}\left(-k \Delta y \Delta z \frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right)$$

$$\lim_{\Delta y \to 0} \frac{1}{\Delta x \Delta z} \frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z} \frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z} \frac{\partial}{\partial y}\left(-k \Delta x \Delta z \frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right)$$

$$\lim_{\Delta z \to 0} \frac{1}{\Delta x \Delta y} \frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{\Delta x \Delta y} \frac{\partial Q_z}{\partial z} = \frac{1}{\Delta x \Delta y} \frac{\partial}{\partial z}\left(-k \Delta x \Delta y \frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)$$

Equation 2–3 is the general heat conduction equation in rectangular coordinates. In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{… 2–4}$$

where the property $\alpha = k/\rho C$ is again the *thermal diffusivity* of the material. Equation 2–4 is known as the **Fourier-Biot equation,** and it reduces to these forms under specified conditions:

(1) *Steady-state:* (called the **Poisson equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:* (called the **diffusion equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* (called the **Laplace equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Cylindrical Coordinates

The general heat conduction equation in cylindrical coordinates can be obtained from an energy balance on a volume element in cylindrical coordinates, shown in Figure 2–23, by following the steps just outlined. It can also be obtained directly from Eq. 2–3 by coordinate transformation using the following relations between the coordinates of a point in rectangular and cylindrical coordinate systems:

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad \text{and} \qquad z = z$$

After lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

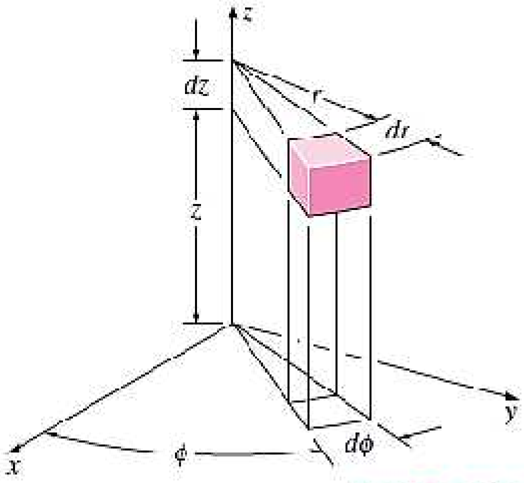


FIGURE 2–23
A differential volume element in cylindrical coordinates.

Spherical Coordinates

The general heat conduction equations in spherical coordinates can be obtained from an energy balance on a volume element in spherical coordinates, shown in Figure 2–24, by following the steps outlined above. It can also be obtained directly from Eq. 2–3 by coordinate transformation using the following relations between the coordinates of a point in rectangular and spherical coordinate systems:

$$x = r\cos\phi\sin\theta, \qquad y = r\sin\phi\sin\theta, \qquad \text{and} \qquad z = \cos\theta$$

Again after lengthy manipulations, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

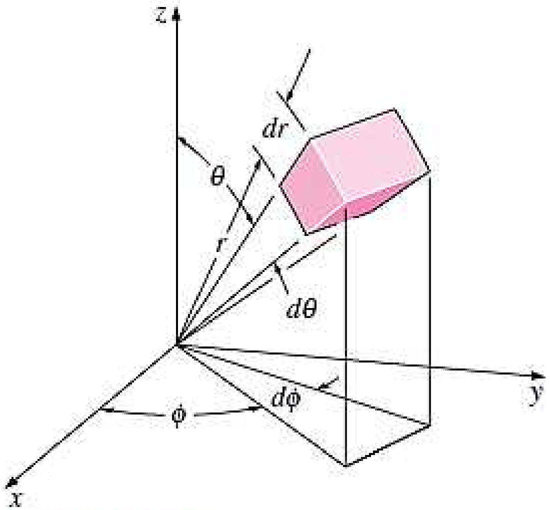


FIGURE 2–24
A differential volume element in spherical coordinates.

2–2 ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

Heat Conduction Equation in a Large Plane Wall

Consider a thin element of thickness Δx in a large plane wall, as shown in Figure 2–13. Assume the density of the wall is ρ, the specific heat is C, and the area of the wall normal to the direction of heat transfer is A. An *energy balance* on this thin element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \quad \text{... 2–5}$$

But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C A \Delta x (T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g} V_{\text{element}} = \dot{g} A \Delta x$$

Substituting into Equation 2–5, we get

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{g} A \Delta x = \rho C A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

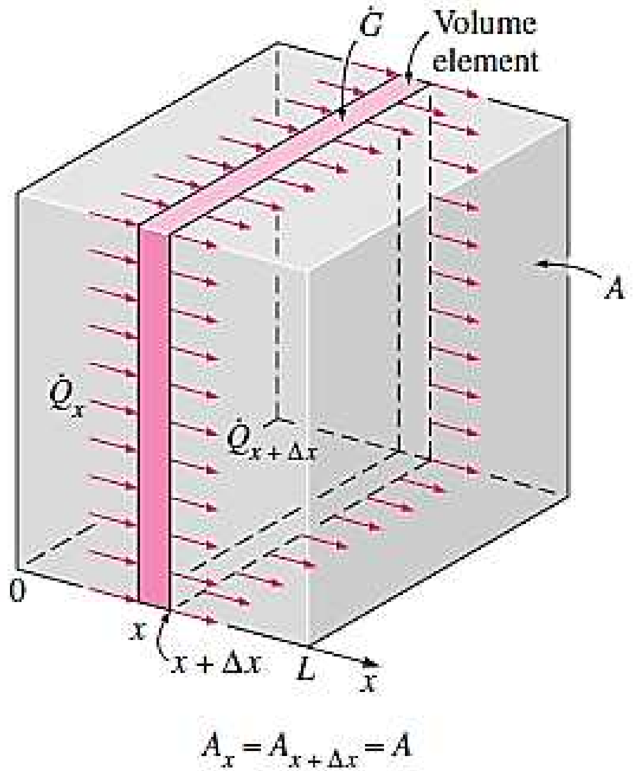


FIGURE 2–13
One-dimensional heat conduction through a volume element in a large plane wall.

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{\partial \dot{Q}}{\partial x} = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$

Noting that the area A is constant for a plane wall, the one-dimensional transient heat conduction equation in a plane wall becomes

Variable conductivity:
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

Constant conductivity:
$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho C$ is the **thermal diffusivity** of the material and represents how fast heat propagates through a material. It reduces to the following forms under specified conditions

(1) *Steady-state:* ($\partial/\partial t = 0$)
$$\frac{d^2 T}{dx^2} + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:* ($\dot{g} = 0$)
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* ($\partial/\partial t = 0$ and $\dot{g} = 0$)
$$\frac{d^2 T}{dx^2} = 0$$

General, one dimensional:

$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\cancel{\frac{\dot{g}}{k}}}_{\text{No generation}}^{0} = \underbrace{\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}}_{\text{Steady-state}}^{0}$$

Steady, one-dimensional:

$$\frac{d^2 T}{dx^2} = 0$$

Heat Conduction Equation in a Long Cylinder

Now consider a thin cylindrical shell element of thickness Δr in a long cylinder, as shown in Figure 2–15. Assume the density of the cylinder is ρ, the specific heat is C, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r+\Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad \ldots\ 2\text{–}6$$

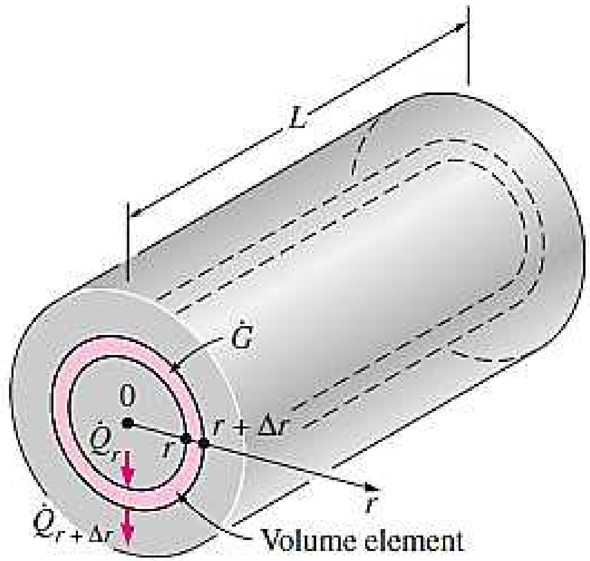


FIGURE 2–15
One-dimensional heat conduction through a volume element in a long cylinder.

The change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta r(T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}A\Delta r$$

Substituting into Eq. 2–6, we get

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{g}A\Delta r = \rho CA\Delta r\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 2\pi rL$. You may be tempted to express the area at the *middle* of the element using the *average* radius as $A = 2\pi(r + \Delta r/2)L$. But there is nothing we can gain from this complication since later in the analysis we will take the limit as $\Delta r \to 0$ and thus the term $\Delta r/2$ will drop out. Now dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{g} = \rho C\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields
$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 2\pi rL$, the one-dimensional transient heat conduction equation in a cylinder becomes

Variable conductivity: $$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

Constant conductivity: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where again the property $\alpha = k/\rho C$ is the thermal diffusivity of the material. Equation 2–26 reduces to the following forms under specified conditions

(1) *Steady-state:* $(\partial/\partial t = 0)$ $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$ $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{g} = 0)$ $$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

(*a*) The form that is ready to integrate

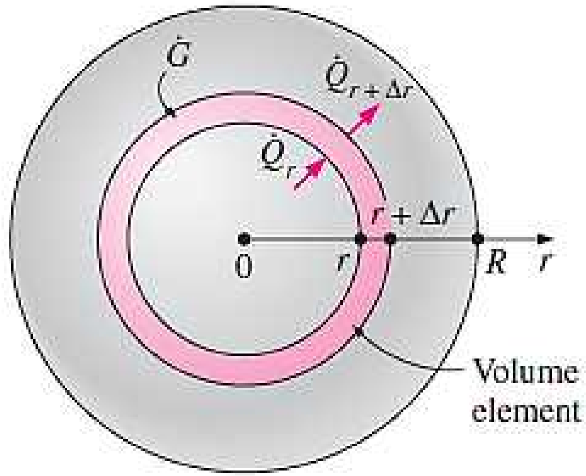
$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

(*b*) The equivalent alternative form

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

Heat Conduction Equation in a Sphere

Now consider a sphere with density ρ, specific heat C, and outer radius R. The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$, where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case also, and thus it varies with location. By considering a thin spherical shell element of thickness Δr and repeating the approach described above for the cylinder by using $A = 4\pi r^2$ instead of $A = 2\pi rL$, the one-dimensional transient heat conduction equation for a sphere is determined to be



Variable conductivity:
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

Constant conductivity:
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where again the property $\alpha = k/\rho C$ is the thermal diffusivity of the material It reduces to the following forms under specified conditions:

(1) *Steady-state:* $(\partial/\partial t = 0)$
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:* $(\dot{g} = 0)$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{g} = 0)$
$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \quad \text{or} \quad r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0$$

Combined One-Dimensional Heat Conduction Equation

An examination of the one-dimensional transient heat conduction equations for the plane wall, cylinder, and sphere reveals that all three equations can be expressed in a compact form as

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n k \frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

where $n = 0$ for a plane wall, $n = 1$ for a cylinder, and $n = 2$ for a sphere. In the case of a plane wall, it is customary to replace the variable r by x. This equation can be simplified for steady-state or no heat generation cases as described before.

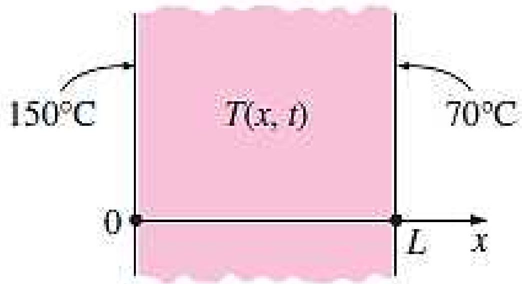
2-3 Boundary and Initial Conditions

1- Specified Temperature Boundary Condition

The *temperature* of an exposed surface can usually be measured directly and easily. Therefore, one of the easiest ways to specify the thermal conditions on a surface is to specify the temperature. For one-dimensional heat transfer through a plane wall of thickness L, for example, the specified temperature boundary conditions can be expressed as

$$T(0, t) = T_1$$
$$T(L, t) = T_2$$

where T_1 and T_2 are the specified temperatures at surfaces at $x = 0$ and $x = L$, respectively. The specified temperatures can be constant, which is the case for steady heat conduction, or may vary with time.



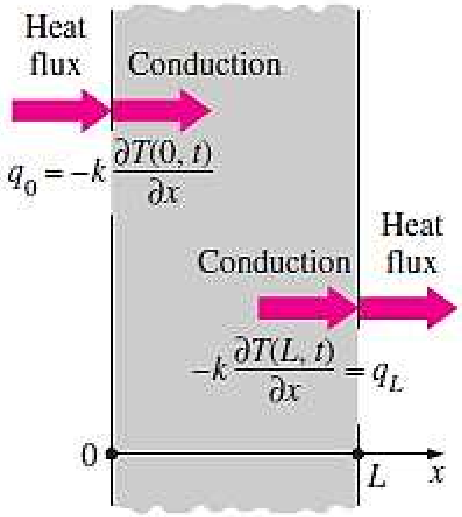
$$T(0, t) = 150°\text{C}$$
$$T(L, t) = 70°\text{C}$$

2- Specified Heat Flux Boundary Condition

When there is sufficient information about energy interactions at a surface, it may be possible to determine the rate of heat transfer and thus the *heat flux* $\dot{q}$ (heat transfer rate per unit surface area, W/m^2) on that surface, and this information can be used as one of the boundary conditions. The heat flux in the positive x-direction anywhere in the medium, including the boundaries, can be expressed by *Fourier's law* of heat conduction as

$$q = -k\frac{\partial T}{\partial x} = \begin{pmatrix}\text{Heat flux in the} \\ \text{positive } x\text{-direction}\end{pmatrix} \qquad (\text{W/m}^2)$$

Then the boundary condition at a boundary is obtained by setting the specified heat flux equal to $-k(\partial T/\partial x)$ at that boundary. The sign of the specified heat flux is determined by inspection: *positive* if the heat flux is in the positive direction of the coordinate axis, and *negative* if it is in the opposite direction. Note that it is extremely important to have the *correct sign* for the specified heat flux since the wrong sign will invert the direction of heat transfer and cause the heat gain to be interpreted as heat loss



For a plate of thickness L subjected to heat flux of 50 W/m^2 into the medium from both sides, for example, the specified heat flux boundary conditions can be expressed as

$$-k\frac{\partial T(0,t)}{\partial x} = 50 \quad \text{and} \quad -k\frac{\partial T(L,t)}{\partial x} = -50 \tag{2-48}$$

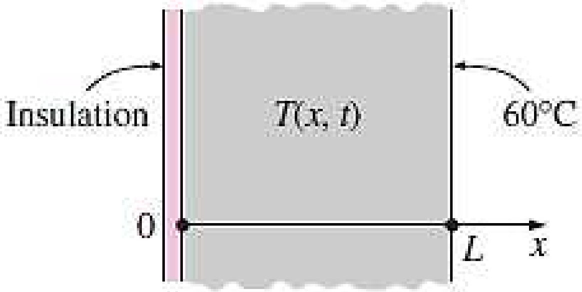
Note that the heat flux at the surface at $x = L$ is in the *negative* x-direction, and thus it is -50 W/m^2.

Special Case: Insulated Boundary

Some surfaces are commonly insulated in practice in order to minimize heat loss (or heat gain) through them. Insulation reduces heat transfer but does not totally eliminate it unless its thickness is infinity. However, heat transfer through a properly insulated surface can be taken to be zero since adequate insulation reduces heat transfer through a surface to negligible levels. Therefore, a well-insulated surface can be modeled as a surface with a specified heat flux of zero. Then the boundary condition on a perfectly insulated surface (at $x = 0$, for example) can be expressed as (Fig. 2–30)

$$k\frac{\partial T(0,t)}{\partial x} = 0 \quad \text{or} \quad \frac{\partial T(0,t)}{\partial x} = 0$$

That is, *on an insulated surface, the first derivative of temperature with respect to the space variable (the temperature gradient) in the direction normal to the insulated surface is zero.* This also means that the temperature function must be perpendicular to an insulated surface since the slope of temperature at the surface must be zero.

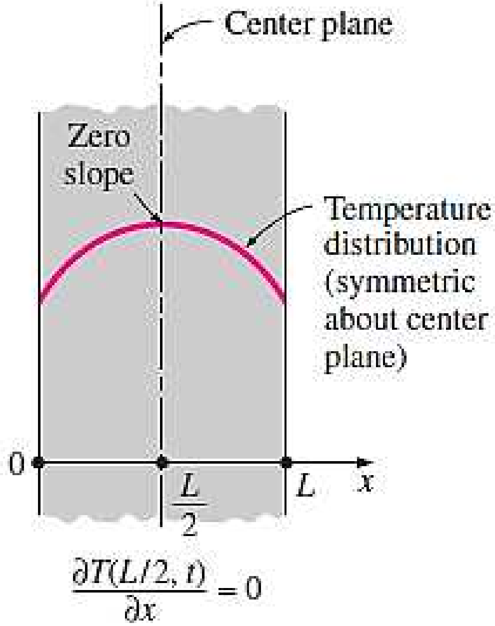


$$\frac{\partial T(0,t)}{\partial x} = 0$$
$$T(L,t) = 60°C$$

Another Special Case: Thermal Symmetry

Some heat transfer problems possess *thermal symmetry* as a result of the symmetry in imposed thermal conditions. For example, the two surfaces of a large hot plate of thickness L suspended vertically in air will be subjected to the same thermal conditions, and thus the temperature distribution in one half of the plate will be the same as that in the other half. That is, the heat transfer problem in this plate will possess thermal symmetry about the center plane at $x = L/2$. Also, the direction of heat flow at any point in the plate will be toward the surface closer to the point, and there will be no heat flow across the center plane. Therefore, the center plane can be viewed as an insulated surface, and the thermal condition at this plane of symmetry can be expressed as

$$\frac{\partial T(L/2, t)}{\partial x} = 0$$



which resembles the *insulation* or *zero heat flux* boundary condition. This result can also be deduced from a plot of temperature distribution with a maximum, and thus zero slope, at the center plane.

In the case of cylindrical (or spherical) bodies having thermal symmetry about the center line (or midpoint), the thermal symmetry boundary condition requires that the first derivative of temperature with respect to r (the radial variable) be zero at the centerline (or the midpoint).

3- Convection Boundary Condition

Convection is probably the most common boundary condition encountered in practice since most heat transfer surfaces are exposed to an environment at a specified temperature. The convection boundary condition is based on a *surface energy balance* expressed as

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface in a}\\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Heat convection}\\ \text{at the surface in}\\ \text{the same direction}\end{pmatrix}$$

For one-dimensional heat transfer in the x-direction in a plate of thickness L, the convection boundary conditions on both surfaces can be expressed as

$$-k\frac{\partial T(0,t)}{\partial x} = h_1[T_{\infty 1} - T(0,t)]$$
$$-k\frac{\partial T(L,t)}{\partial x} = h_2[T(L,t) - T_{\infty 2}] \qquad \text{… 2-7}$$

where h_1 and h_2 are the convection heat transfer coefficients and $T_{\infty 1}$ and $T_{\infty 2}$ are the temperatures of the surrounding mediums on the two sides of the plate, as shown in Figure 2–33.

In writing Eq. 2–7 for convection boundary conditions, we have selected the direction of heat transfer to be the positive x-direction at both surfaces. But those expressions are equally applicable when heat transfer is in the opposite direction at one or both surfaces since reversing the direction of heat transfer at a surface simply reverses the signs of *both* conduction and convection terms at that surface. This is equivalent to multiplying an equation by 1, which has no effect on the equality (Fig. 2–34).

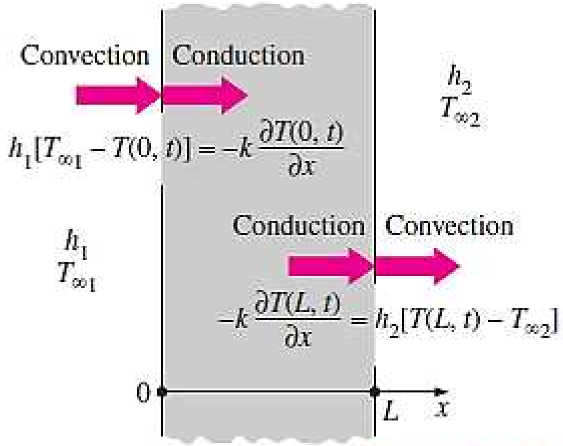


FIGURE 2–33

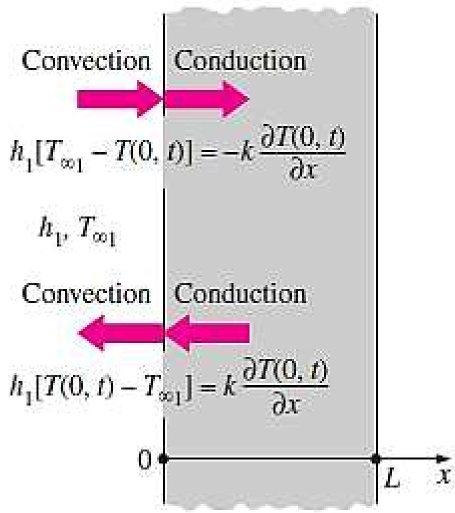
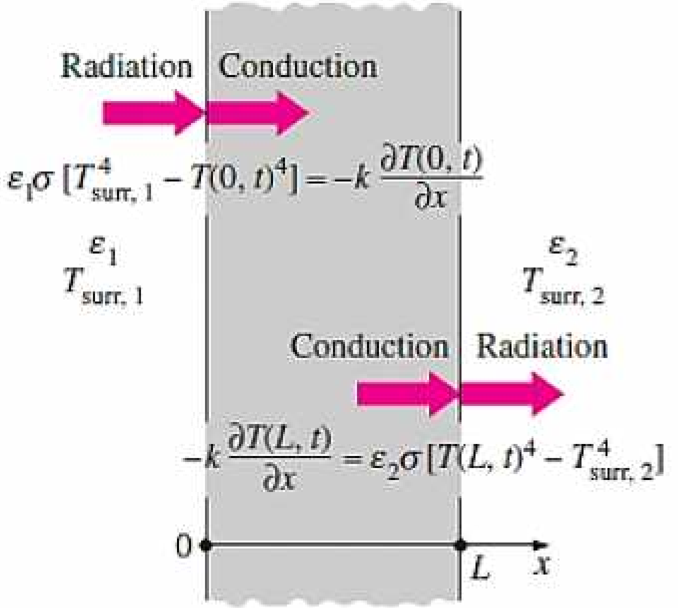


FIGURE 2–34

4- Radiation Boundary Condition

In some cases, such as those encountered in space and cryogenic applications, a heat transfer surface is surrounded by an evacuated space and thus there is no convection heat transfer between a surface and the surrounding medium. In such cases, *radiation* becomes the only mechanism of heat transfer between the surface under consideration and the surroundings. Using an energy balance, the radiation boundary condition on a surface can be expressed as

$$\begin{pmatrix} \text{Heat conduction} \\ \text{at the surface in a} \\ \text{selected direction} \end{pmatrix} = \begin{pmatrix} \text{Radiation exchange} \\ \text{at the surface in} \\ \text{the same direction} \end{pmatrix}$$



For one-dimensional heat transfer in the x-direction in a plate of thickness L, the radiation boundary conditions on both surfaces can be expressed as

$$-k\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma[T^4_{surr,1} - T(0,t)^4]$$

$$-k\frac{\partial T(L,t)}{\partial x} = \varepsilon_2\sigma[T(L,t)^4 - T^4_{surr,2}]$$

where ε_1 and ε_2 are the emissivities of the boundary surfaces, $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$ is the Stefan–Boltzmann constant, and $T_{surr,1}$ and $T_{surr,2}$ are the average temperatures of the surfaces surrounding the two sides of the plate, respectively. Note that the temperatures in radiation calculations must be expressed in K or R (not in °C or °F).

5- Interface Boundary Conditions

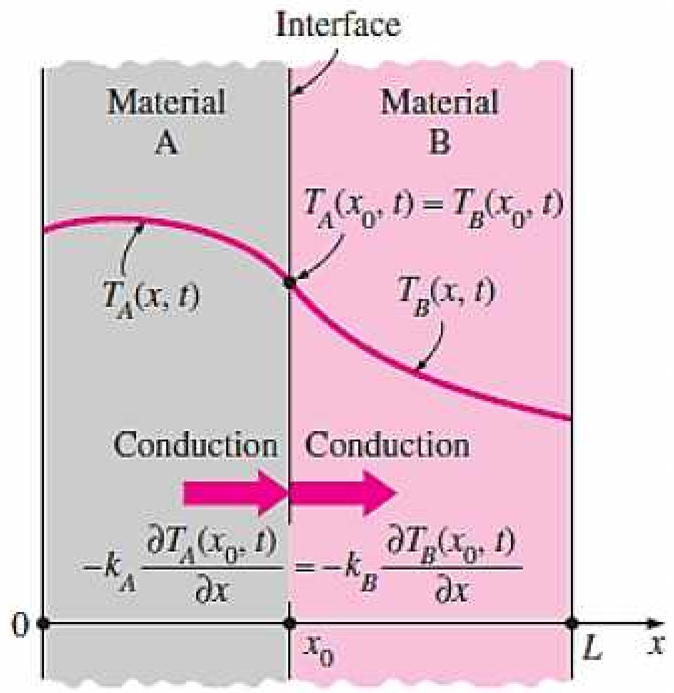
Some bodies are made up of layers of different materials, and the solution of a heat transfer problem in such a medium requires the solution of the heat transfer problem in each layer. This, in turn, requires the specification of the boundary conditions at each *interface*.

The boundary conditions at an interface are based on the requirements that (1) two bodies in contact must have the *same temperature* at the area of contact and (2) an interface (which is a surface) cannot store any energy, and thus the *heat flux* on the two sides of an interface *must be the same*. The boundary conditions at the interface of two bodies A and B in perfect contact at $x = x_0$ can be expressed as

$$T_A(x_0, t) = T_B(x_0, t)$$

$$-k_A \frac{\partial T_A(x_0, t)}{\partial x} = -k_B \frac{\partial T_B(x_0, t)}{\partial x}$$

where k_A and k_B are the thermal conductivities of the layers A and B, respectively. The case of imperfect contact results in thermal contact resistance, which is considered in the next chapter.



EXAMPLE 2–1 Heat Conduction in a Plane Wall

Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m · °C, and surface area $A = 15$ m^2. The two sides of the wall are maintained at constant temperatures of $T_1 = 120$°C and $T_2 = 50$°C, respectively, as shown in Figure 2–41. Determine (*a*) the variation of temperature within the wall and the value of temperature at $x = 0.1$ m and (*b*) the rate of heat conduction through the wall under steady conditions.

SOLUTION

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x*-direction, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 120°\text{C}$$

$$T(L) = T_2 = 50°\text{C}$$

Integrating the differential equation once with respect to *x* yields

$$\frac{dT}{dx} = C_1$$

Integrating one more time, we obtain

$$T(x) = C_1 x + C_2$$

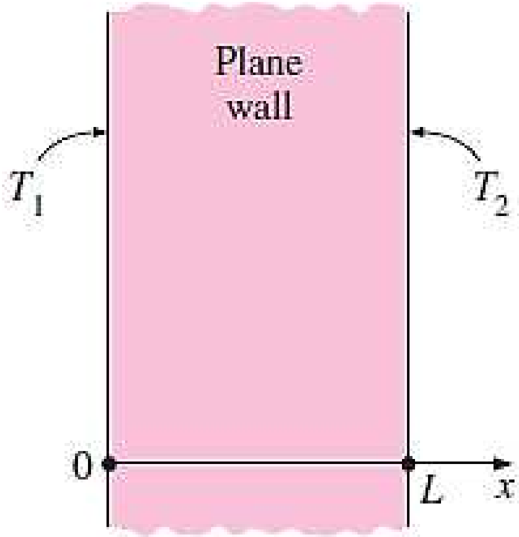


FIGURE 2–41

The first boundary condition can be interpreted as *in the general solution, replace all the x's by zero and T (x) by T*1

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

The second boundary condition can be interpreted as *in the general solution, replace all the x's by L and T (x) by T*2. That is,

$$T(L) = C_1 L + C_2 \quad \rightarrow \quad T_2 = C_1 L + T_1 \quad \rightarrow \quad C_1 = \frac{T_2 - T_1}{L}$$

Substituting the *C*1 and *C*2 expressions into the general solution, we obtain $T(x) = \dfrac{T_2 - T_1}{L} x + T_1$

Substituting the given information, the value of the temperature at x =0.1 m is determined to be

$$T(0.1\text{ m}) = \frac{(50 - 120)^\circ\text{C}}{0.2\text{ m}}(0.1\text{ m}) + 120^\circ\text{C} = 85^\circ\text{C}$$

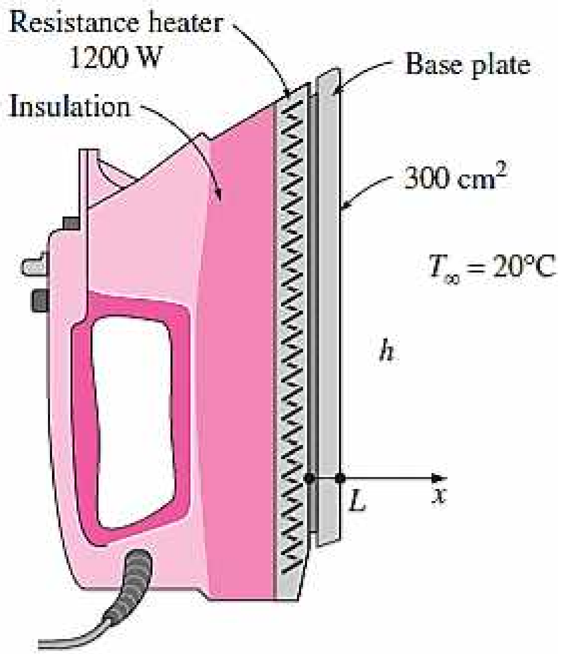
(*b*) The rate of heat conduction anywhere in the wall is determined from Fourier's law to be

$$\dot{Q}_{\text{wall}} = -kA\frac{dT}{dx} = -kAC_1 = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L}$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2\text{ W/m}\cdot{}^\circ\text{C})(15\text{ m}^2)\frac{(120 - 50)^\circ\text{C}}{0.2\text{ m}} = 6300\text{ W}$$

EXAMPLE 2–2 Heat Conduction in the Base Plate of an Iron

Consider the base plate of a 1200-W household iron that has a thickness of L = 0.5 cm, base area of A = 300 cm², and thermal conductivity of k = 15 W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside, and the outer surface loses heat to the surroundings at T_∞ = 20°C by convection,
Taking the convection heat transfer coefficient to be h = 80 W/m² · °C and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces.



SOLUTION

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1200\text{ W}}{0.03\text{ m}^2} = 40{,}000\text{ W/m}^2$$

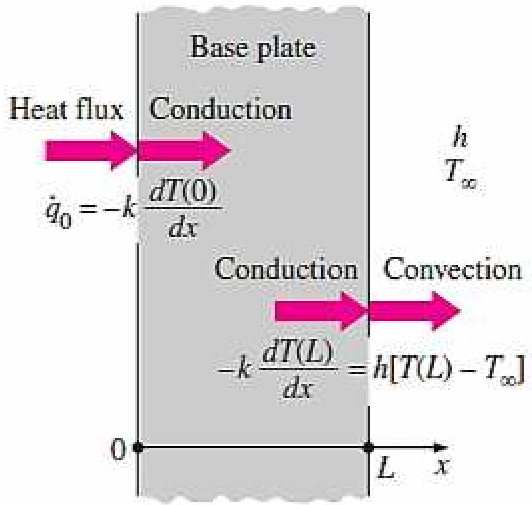
the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with the boundary co

$$-k\frac{dT(0)}{dx} = \dot{q}_0 = 40{,}000\text{ W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$



The general solution of the differential equation is again obtained by two successive integrations to be

$\frac{dT}{dx} = C_1$ and $T(x) = C_1x + C_2$ ...a

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition,

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \rightarrow -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2$, the application of the second boundary condition gives

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \quad \rightarrow \quad -kC_1 = h[(C_1L + C_2) - T_\infty]$$

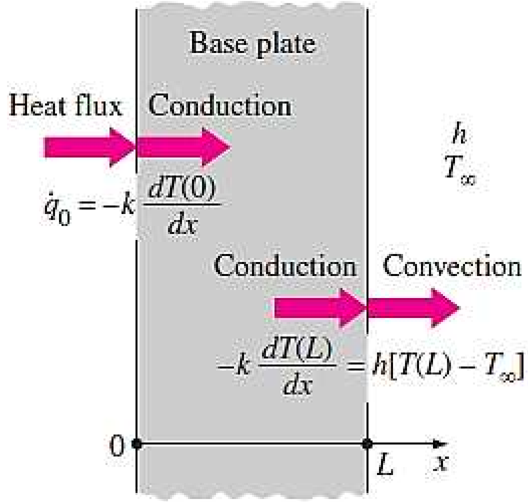
Substituting $C_1 = -\dot{q}_0/k$ and solving for C_2, we obtain

$$C_2 = T_\infty + \frac{\dot{q}_0}{h} + \frac{\dot{q}_0}{k}L$$

Now substituting C_1 and C_2 into the general solution (*a*) gives

$$T(x) = T_\infty + \dot{q}_0\left(\frac{L-x}{k} + \frac{1}{h}\right) \quad \text{...b}$$

which is the solution for the variation of the temperature in the plate. The temperatures at the inner and outer surfaces of the plate are determined by substituting $x = 0$ and $x = L$, respectively, into the relation (*b*):



$$T(0) = T_\infty + \dot{q}_0\left(\frac{L}{k} + \frac{1}{h}\right)$$

$$= 20°\text{C} + (40{,}000\ \text{W/m}^2)\left(\frac{0.005\ \text{m}}{15\ \text{W/m}\cdot°\text{C}} + \frac{1}{80\ \text{W/m}^2\cdot°\text{C}}\right) = 533°\text{C}$$

and

$$T(L) = T_\infty + \dot{q}_0\left(0 + \frac{1}{h}\right) = 20°\text{C} + \frac{40{,}000\ \text{W/m}^2}{80\ \text{W/m}^2\cdot°\text{C}} = 520°\text{C}$$

EXAMPLE 2–3 Heat Conduction in a Solar Heated Wall

Consider a large plane wall of thickness $L = 0.06$ m and thermal conductivity $k = 1.2$ W/m · °C in space. The wall is covered with white porcelain tiles that have an emissivity of $\varepsilon = 0.85$ and a solar absorptivity of $\alpha = 0.26$, as shown in Figure 2–48. The inner surface of the wall is maintained at $T_1 = 300$ K at all times, while the outer surface is exposed to solar radiation that is incident at a rate of $\dot{q}_{solar} = 800$ W/m². The outer surface is also losing heat by radiation to deep space at 0 K. Determine the temperature of the outer surface of the wall and the rate of heat transfer through the wall when steady operating conditions are reached. What would your response be if no solar radiation was incident on the surface?

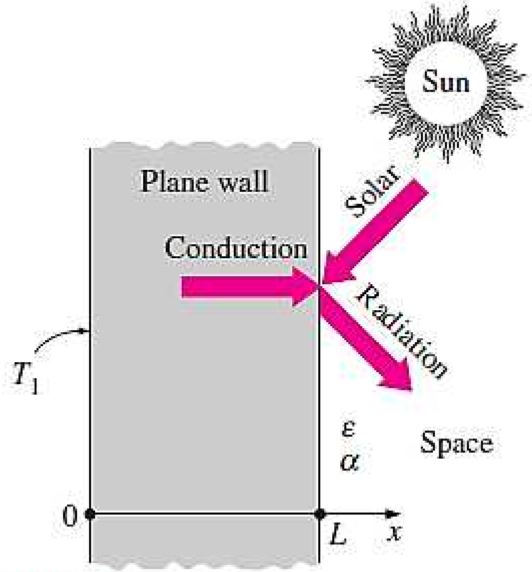


FIGURE 2–48

SOLUTION

Analysis Taking the direction normal to the surface of the wall as the x-direction with its origin on the inner surface, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 300 \text{ K}$$

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma[T(L)^4 - T_{space}^4] - \alpha\dot{q}_{solar}$$

where $T_{space} = 0$. The general solution of the differential equation is again obtained by two successive integrations to be

$$T(x) = C_1x + C_2 \quad (a)$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition yields

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2 = C_1L + T_1$, the application of the second boundary conditions gives

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha\dot{q}_{solar} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma(C_1L + T_1)^4 - \alpha\dot{q}_{solar}$$

The application of the second boundary condition in this case gives

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha q_{\text{solar}} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma T_L^4 - \alpha q_{\text{solar}}$$

Solving for C_1 gives

$$C_1 = \frac{\alpha q_{\text{solar}} - \varepsilon\sigma T_L^4}{k} \quad (b)$$

Now substituting C_1 and C_2 into the general solution (a), we obtain

$$T(x) = \frac{\alpha q_{\text{solar}} - \varepsilon\sigma T_L^4}{k}x + T_1 \quad (c)$$

At $x = L$ it becomes

$$T_L = \frac{\alpha q_{\text{solar}} - \varepsilon\sigma T_L^4}{k}L + T_1$$

Substituting the given values, we get

$$T_L = \frac{0.26 \times (800\ \text{W/m}^2) - 0.85 \times (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\, T_L^4}{1.2\ \text{W/m} \cdot \text{K}}(0.06\ \text{m}) + 300\ \text{K}$$

$$T_L = 310.4 - 0.240975\left(\frac{T_L}{100}\right)^4$$

$$T_L = 292.7\ \text{K}$$

Knowing the outer surface temperature and knowing that it must remain constant under steady conditions, the temperature distribution in the wall can be determined by substituting the T_L value above into Eq. (c):

$$T(x) = \frac{0.26 \times (800\ \text{W/m}^2) - 0.85 \times (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(292.7\ \text{K})^4}{1.2\ \text{W/m} \cdot \text{K}}x + 300\ \text{K}$$

$$T(x) = (-121.5\ \text{K/m})x + 300\ \text{K}$$

the steady rate of heat conduction through the wall can be determined from

$$\dot{q} = k\frac{T_0 - T_L}{L} = (1.2\ \text{W/m} \cdot \text{K})\frac{(300 - 292.7)\ \text{K}}{0.06\ \text{m}} = 146\ \text{W/m}^2$$

EXAMPLE 2–4 Heat Loss through a Steam Pipe

Consider a steam pipe of length $L = 20$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 8$ cm, and thermal conductivity $k = 20$ W/m · °C, as shown in Figure 2–50. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 150°C$ and $T_2 = 60°C$, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

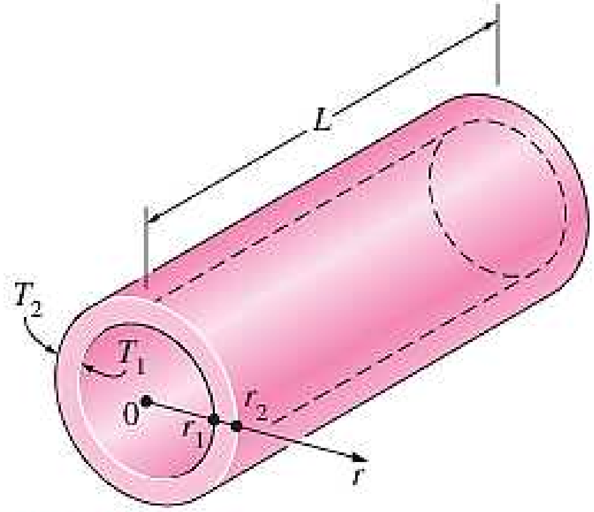


FIGURE 2–50

SOLUTION

The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 150°C$$

$$T(r_2) = T_2 = 60°C$$

Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Again integrating with respect to r gives

$$T(r) = C_1 \ln r + C_2 \quad (a)$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (a) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \quad \text{and} \quad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)}\ln r_1$$

Substituting them into Eq. (*a*) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\right)(T_2 - T_1) + T_1$$

The rate of heat loss from the steam is simply the total rate of heat conduction through the pipe, and is determined from Fourier's law to be

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

The numerical value of the rate of heat conduction through the pipe is determined by substituting the given values

$$\dot{Q} = 2\pi(20\ \text{W/m}\cdot{}^\circ\text{C})(20\ \text{m})\frac{(150 - 60)^\circ\text{C}}{\ln(0.08/0.06)} = \mathbf{786\ kW}$$

EXAMPLE 2–5 Heat Conduction through a Spherical Shell

Consider a spherical container of inner radius $r_1 = 8$ cm, outer radius $r_2 =$ 10 cm, and thermal conductivity $k = 45$ W/m · °C, as shown in Figure 2–52 The inner and outer surfaces of the container are maintained at constant temperatures of $T_1 = 200°C$ and $T_2 = 80°C$, respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.

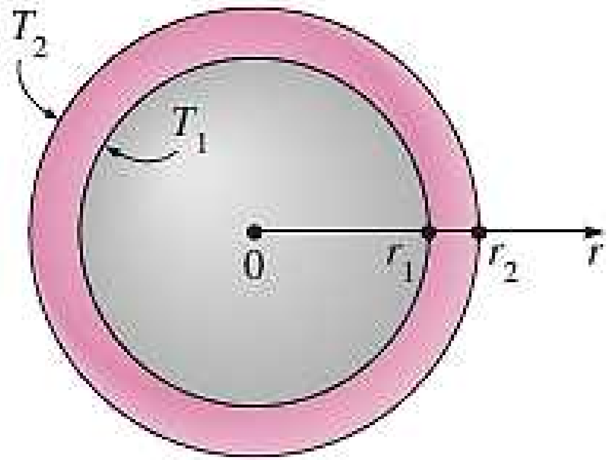


FIGURE 2–52

SOLUTION

The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad \text{with boundary conditions}$$

$$T(r_1) = T_1 = 200°C$$

$$T(r_2) = T_2 = 80°C$$

Integrating the differential equation once with respect to r yields

$$r^2\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r^2 to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

Again integrating with respect to r gives

$$T(r) = -\frac{C_1}{r} + C_2 \qquad (a)$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in the relation above by the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad -\frac{C_1}{r_1} + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad -\frac{C_1}{r_2} + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2) \qquad \text{and} \qquad C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting into Eq. (*a*), the variation of temperature within the spherical shell is determined to be

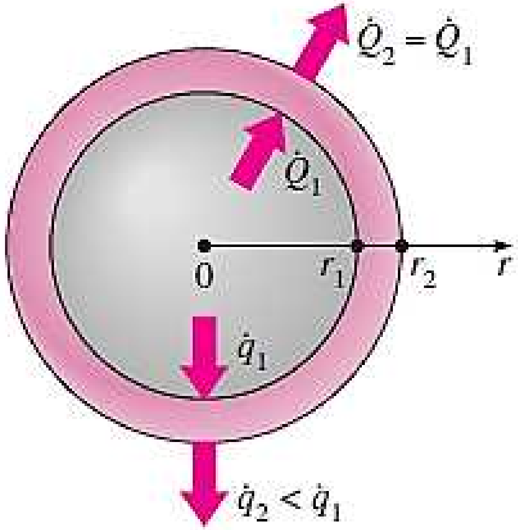
$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

The rate of heat loss from the container is simply the total rate of heat conduction through the container wall and is determined from Fourier's law

$$\dot{Q}_{\text{sphere}} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = 4\pi k r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1}$$

The numerical value of the rate of heat conduction through the wall is deter mined by substituting the given values to be

$$\dot{Q} = 4\pi(45\ \text{W/m}\cdot{}^\circ\text{C})(0.08\ \text{m})(0.10\ \text{m})\frac{(200 - 80)^\circ\text{C}}{(0.10 - 0.08)\ \text{m}} = 27{,}140\ \text{W}$$



$$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.14\ \text{kW}}{4\pi(0.08\ \text{m})^2} = 337.5\ \text{kW/m}^2$$

$$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.14\ \text{kW}}{4\pi(0.10\ \text{m})^2} = 216.0\ \text{kW/m}^2$$

2-4 HEAT GENERATION IN A SOLID

EXAMPLE 2–6 Heat Conduction in a Two-Layer Medium

Consider a long resistance wire of radius $r_1 = 0.2$ cm and thermal conductivity $k_{wire} = 15$ W/m · °C in which heat is generated uniformly as a result of resistance heating at a constant rate of $\dot{g} = 50$ W/cm³ (Fig. 2–61). The wire is embedded in a 0.5-cm-thick layer of ceramic whose thermal conductivity is $k_{ceramic} = 1.2$ W/m · °C. If the outer surface temperature of the ceramic layer is measured to be $T_s = 45$°C, determine the temperatures at the center of the resistance wire and the interface of the wire and the ceramic layer under steady conditions.

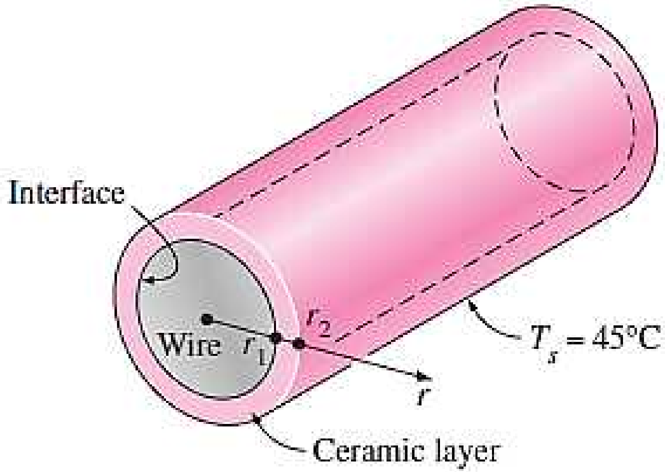


FIGURE 2–61

SOLUTION

Analysis Letting T_I denote the unknown interface temperature, the heat transfer problem in the wire can be formulated as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{wire}}{dr}\right) + \frac{\dot{g}}{k} = 0$$

$$T_{wire}(r_1) = T_I$$

$$\frac{dT_{wire}(0)}{dr} = 0$$

$$T_{wire}(r) = T_I + \frac{\dot{g}}{4k_{wire}}(r_1^2 - r^2) \qquad (a)$$

Noting that the ceramic layer does not involve any heat generation and its outer surface temperature is specified, the heat conduction problem in that layer can be expressed as

$$\frac{d}{dr}\left(r\frac{dT_{ceramic}}{dr}\right) = 0$$

$$T_{ceramic}(r_1) = T_I$$

$$T_{ceramic}(r_2) = T_s = 45°C$$

$$T_{ceramic}(r) = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_s - T_I) + T_I \qquad (b)$$

We have already utilized the first interface condition by setting the wire and ceramic layer temperatures equal to T_I at the interface $r = r_1$. The interface temperature T_I is determined from the second interface condition that the heat flux in the wire and the ceramic layer at $r = r_1$ must be the same:

$$-k_{\text{wire}} \frac{dT_{\text{wire}}(r_1)}{dr} = -k_{\text{ceramic}} \frac{dT_{\text{ceramic}}(r_1)}{dr} \quad \rightarrow \quad \frac{\dot{g}r_1}{2} = -k_{\text{ceramic}} \frac{T_s - T_I}{\ln(r_2/r_1)} \left(\frac{1}{r_1}\right)$$

Solving for T_I and substituting the given values, the interface temperature is determined to be

$$T_I = \frac{\dot{g}r_1^2}{2k_{\text{ceramic}}} \ln\frac{r_2}{r_1} + T_s$$

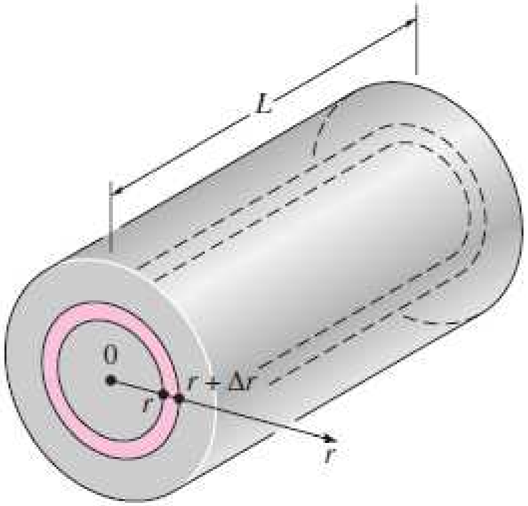
$$= \frac{(50 \times 10^6 \text{ W/m}^3)(0.002 \text{ m})^2}{2(1.2 \text{ W/m} \cdot {}^\circ\text{C})} \ln\frac{0.007 \text{ m}}{0.002 \text{ m}} + 45^\circ \text{ C} = \mathbf{149.4^\circ C}$$

Knowing the interface temperature, the temperature at the centerline ($r = 0$) is obtained by substituting the known quantities into Eq. (a),

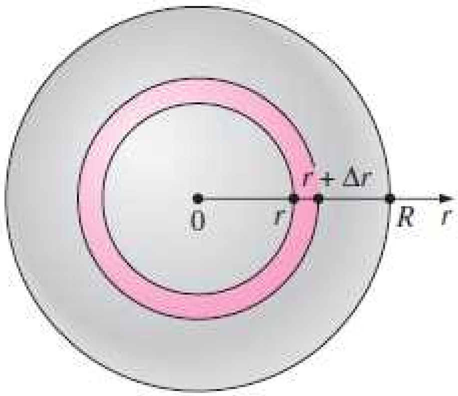
$$T_{\text{wire}}(0) = T_I + \frac{\dot{g}r_1^2}{4k_{\text{wire}}} = 149.4^\circ\text{C} + \frac{(50 \times 10^6 \text{ W/m}^3)(0.002 \text{ m})^2}{4 \times (15 \text{ W/m} \cdot {}^\circ\text{C})} = \mathbf{152.7^\circ C}$$

Problems

★ Starting with an energy balance on a cylindrical shell volume element, derive the steady one-dimensional heat conduction equation for a long cylinder with constant thermal conductivity in which heat is generated at a rate of $\dot{g}$.



★ Starting with an energy balance on a spherical shell volume element, derive the one-dimensional transient heat conduction equation for a sphere with constant thermal conductivity and no heat generation.



★ Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right) + \dot{g} = 0$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

★ Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

★ Consider a medium in which the heat conduction equation is given in its simplest form as

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

★ Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

★ Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = 0$$

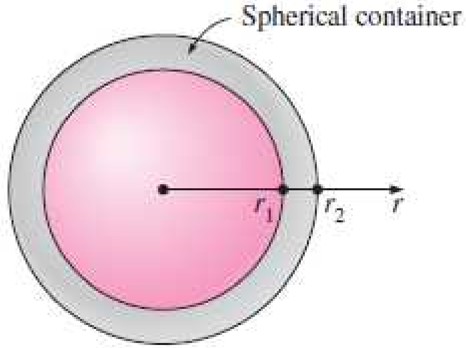
(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

★ Consider a medium in which the heat conduction equation is given in its simplest form as

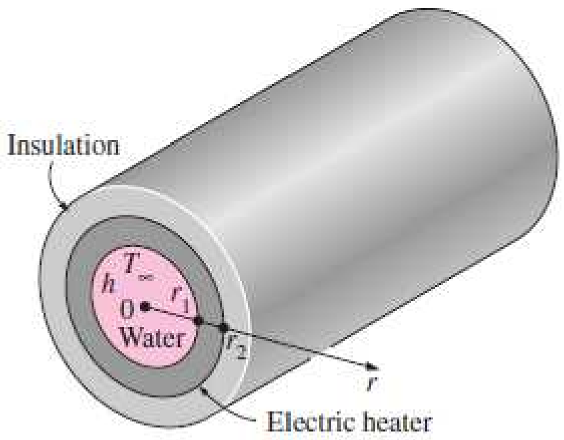
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial t}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

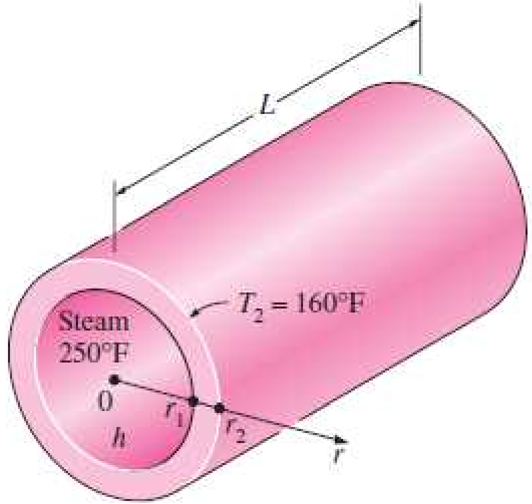
★ Consider a spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k. Express the boundary condition on the inner surface of the container for steady one-dimensional conduction for the following cases: (*a*) specified temperature of 50°C, (*b*) specified heat flux of 30 W/m^2 toward the center, (*c*) convection to a medium at T_∞ with a heat transfer coefficient of h.



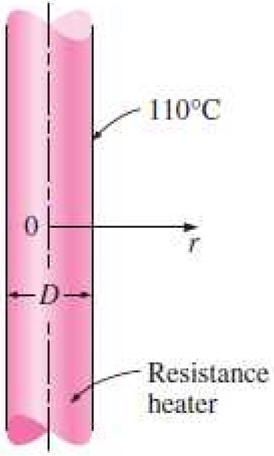
Water flows through a pipe at an average temperature of T_∞ = 50°C. The inner and outer radii of the pipe are r_1 = 6 cm and r_2 = 6.5 cm, respectively. The outer surface of the pipe is wrapped with a thin electric heater that consumes 300 W per m length of the pipe. The exposed surface of the heater is heavily insulated so that the entire heat generated in the heater is transferred to the pipe. Heat is transferred from the inner surface of the pipe to the water by convection with a heat transfer coefficient of h = 55 W/m^2 · °C. Assuming constant thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of the heat conduction in the pipe during steady operation. Do not solve.



Consider a steam pipe of length L = 15 ft, inner radius r_1 = 2 in., outer radius r_2 = 2.4 in., and thermal conductivity k = 7.2 Btu/h · ft · °F. Steam is flowing through the pipe at an average temperature of 250°F, and the average convection heat transfer coefficient on the inner surface is given to be h = 1.25 Btu/h · ft^2 · °F . If the average temperature on the outer

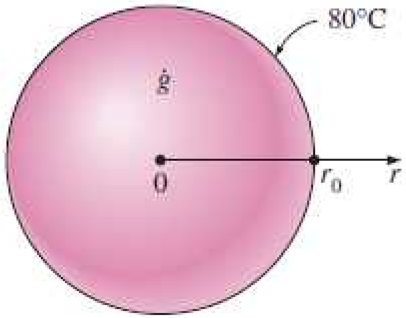


★ A 2-kW resistance heater wire with thermal conductivity of $k = 20$ W/m · °C, a diameter of $D = 5$ mm, and a length of $L = 0.7$ m is used to boil water. If the outer surface temperature of the resistance wire is $T_s = 110$°C, determine the temperature at the center of the wire.

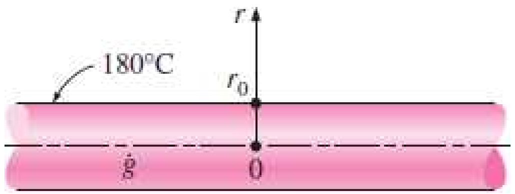


★ In a nuclear reactor, 1-cm-diameter cylindrical uranium rods cooled by water from outside serve as the fuel. Heat is generated uniformly in the rods ($k = 29.5$ W/m · °C) at a rate of 7×10^7 W/m^3. If the outer surface temperature of rods is 175°C, determine the temperature at their center.

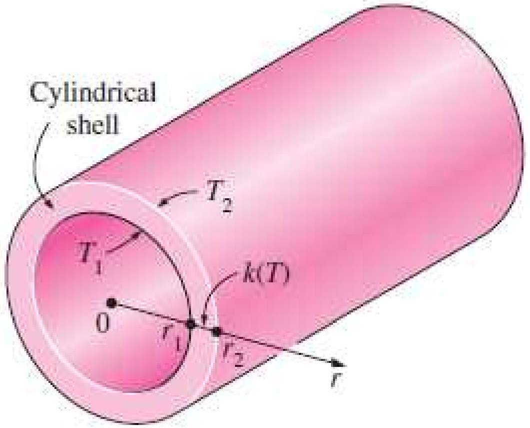
★ Consider a homogeneous spherical piece of radioactive material of radius $r_0 = 0.04$ m that is generating heat at a constant rate of $\dot{g} = 4 \times 10^7$ W/m^3. The heat generated is dissipated to the environment steadily. The outer surface of the sphere is maintained at a uniform temperature of 80°C and the thermal conductivity of the sphere is $k = 15$ W/m · °C. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the sphere, (*b*) obtain a relation for the variation of temperature in the sphere by solving the differential equation, and (*c*) determine the temperature at the center of the sphere.



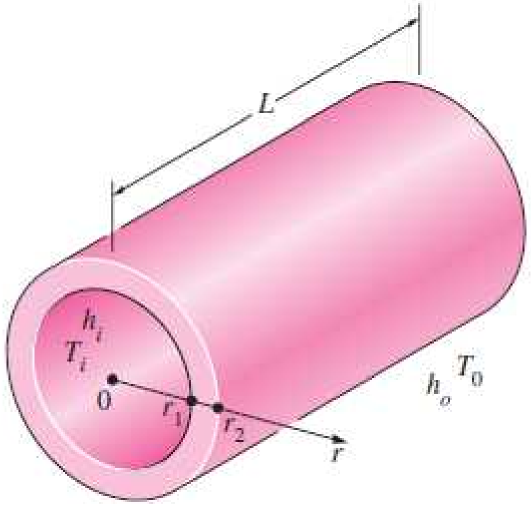
★ A long homogeneous resistance wire of radius $r_0 = 5$ mm is being used to heat the air in a room by the passage of electric current. Heat is generated in the wire uniformly at a rate of $\dot{g} = 5 \times 10^7$ W/m^3 as a result of resistance heating. If the temperature of the outer surface of the wire remains at 180°C, determine the temperature at $r = 2$ mm after steady operation conditions are reached. Take the thermal conductivity of the wire to be $k = 8$ W/m · °C. *Answer:* 212.8°C



★ Consider a cylindrical shell of length L, inner radius r_1, and outer radius r_2 whose thermal conductivity varies linearly in a specified temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 and β are two specified constants. The inner surface of the shell is maintained at a constant temperature of T_1, while the outer surface is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for (*a*) the heat transfer rate through the wall and (*b*) the temperature distribution $T(r)$ in the shell.

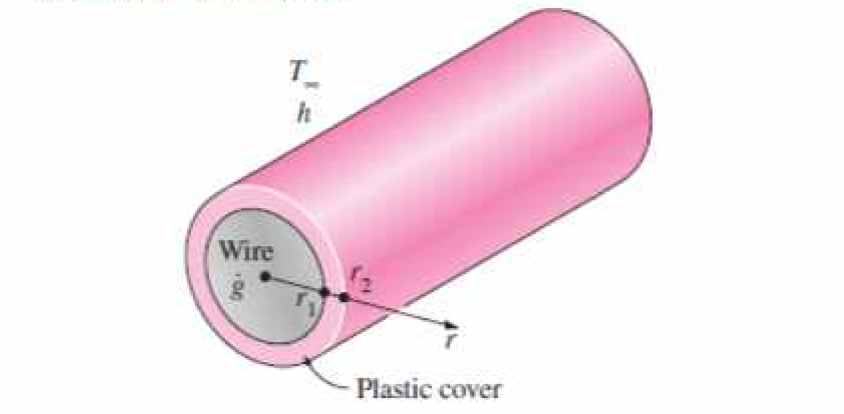


★ Consider a steam pipe of length L, inner radius r_1, outer radius r_2, and constant thermal conductivity k. Steam flows inside the pipe at an average temperature of T_i with a convection heat transfer coefficient of h_i. The outer surface of the pipe is exposed to convection to the surrounding air at a temperature of T_0 with a heat transfer coefficient of h_o. Assuming steady one-dimensional heat conduction through the pipe, (*a*) express the differential equation and the boundary conditions for heat conduction through the pipe material, (*b*) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (*c*) obtain a relation for the temperature of the outer surface of the pipe.

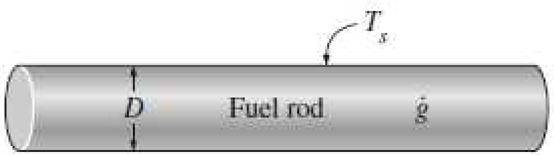


Consider a long resistance wire of radius $r_1 = 0.3$ cm and thermal conductivity $k_{\text{wire}} = 18$ W/m · °C in which heat is generated uniformly at a constant rate of $\dot{g} = 1.5$ W/cm^3 as a result of resistance heating. The wire is embedded in a 0.4-cm-thick layer of plastic whose thermal conductivity is $k_{\text{plastic}} = 1.8$ W/m · °C. The outer surface of the plastic cover loses heat by convection to the ambient air at $T_\infty = 25$°C with an average combined heat transfer coefficient of $h = 14$ W/m^2 · °C. Assuming one-dimensional heat transfer, determine the temperatures at the center of the resistance wire and the wire-plastic layer interface under steady conditions.

Answers: 97.1°C, 97.3°C



In a nuclear reactor, heat is generated in 1-cm-diameter cylindrical uranium fuel rods at a rate of 4×10^7 W/m^3. Determine the temperature difference between the center and the surface of the fuel rod. *Answer:* 9.0°C



Consider a 20-cm-thick large concrete plane wall ($k = 0.77$ W/m · °C) subjected to convection on both sides with $T_{\infty 1} = 27$°C and $h_1 = 5$ W/m^2 · °C on the inside, and $T_{\infty 2} = 8$°C and $h_2 = 12$ W/m^2 · °C on the outside. Assuming constant thermal conductivity with no heat generation and negligible radiation, (*a*) express the differential equations and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperatures at the inner and outer surfaces of the wall.

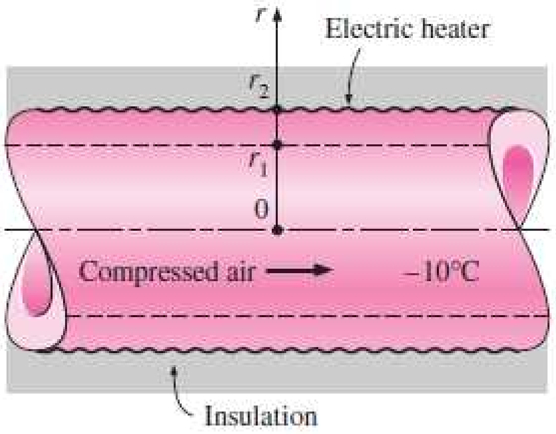
Consider a water pipe of length $L = 12$ m, inner radius $r_1 = 15$ cm, outer radius $r_2 = 20$ cm, and thermal conductivity $k = 20$ W/m · °C. Heat is generated in the pipe material uniformly by a 25-kW electric resistance heater. The inner and outer surfaces of the pipe are at $T_1 = 60$°C and $T_2 = 80$°C, respectively. Obtain a general relation for temperature distribution inside the pipe under steady conditions and determine the temperature at the center plane of the pipe.

Heat is generated uniformly at a rate of 2.6×10^6 W/m^3 in a spherical ball ($k = 45$ W/m · °C) of diameter 30 cm. The ball is exposed to iced-water at 0°C with a heat transfer coefficient of 1200 W/m^2 · °C. Determine the temperatures at the center and the surface of the ball.

A 6-m-long 2-kW electrical resistance wire is made of 0.2-cm-diameter stainless steel ($k = 15.1$ W/m · °C). The resistance wire operates in an environment at 30°C with a heat transfer coefficient of 140 W/m^2 · °C at the outer surface. Determine the surface temperature of the wire (*a*) by using the applicable relation and (*b*) by setting up the proper differential equation and solving it. *Answers:* (*a*) 409°C, (*b*) 409°C

★ When a long section of a compressed air line passes through the outdoors, it is observed that the moisture in the compressed air freezes in cold weather, disrupting and even completely blocking the air flow in the pipe. To avoid this problem, the outer surface of the pipe is wrapped with electric strip heaters and then insulated.

Consider a compressed air pipe of length $L = 6$ m, inner radius $r_1 = 3.7$ cm, outer radius $r_2 = 4.0$ cm, and thermal conductivity $k = 14$ W/m · °C equipped with a 300-W strip heater. Air is flowing through the pipe at an average temperature of −10°C, and the average convection heat transfer coefficient on the inner surface is $h = 30$ W/m² · °C. Assuming 15 percent of the heat generated in the strip heater is lost through the insulation, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (*b*) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (*c*) evaluate the inner and outer surface temperatures of the pipe. *Answers: (c) −3.91°C, −3.87°C*



★ Consider a large plane wall of thickness $L = 0.4$ m, thermal conductivity $k = 2.3$ W/m · °C, and surface area $A = 20$ m². The left side of the wall is maintained at a constant temperature of $T_1 = 80$°C while the right side loses heat by convection to the surrounding air at $T_\infty = 15$°C with a heat transfer coefficient of $h = 24$ W/m² · °C. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the rate of heat transfer through the wall. *Answer: (c) 6030 W*

★ A spherical metal ball of radius r_0 is heated in an oven to a temperature of T_i throughout and is then taken out of the oven and allowed to cool in ambient air at T_∞ by convection and radiation. The emissivity of the outer surface of the cylinder is ε, and the temperature of the surrounding surfaces is T_{surr}. The average convection heat transfer coefficient is estimated to be h. Assuming variable thermal conductivity and transient one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary