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HEAT TRANSFER

انتقال الحرارة

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HEAT TRANSFER

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CHAPTER SIX HEAT EXCHANGERS

FUNDAMENTALS OF CONVECTION

PHYSICAL MECHANISM OF CONVECTION

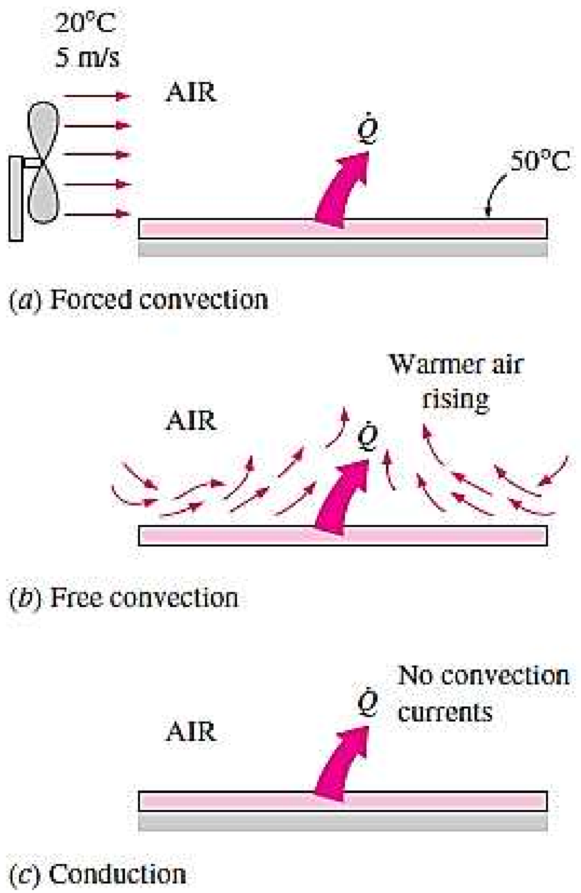


FIGURE 6–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

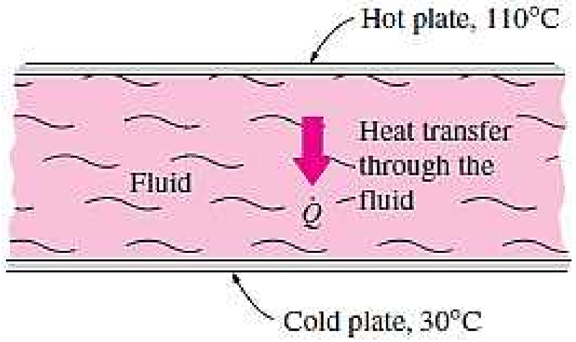


FIGURE 6–2
Heat transfer through a fluid sandwiched between two parallel plates.

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (\text{W/m}^2)$$

or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (\text{W})$$

where

h = convection heat transfer coefficient, W/m$^2 \cdot$ °C
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$\text{Nu} = \frac{hL_c}{k}$$

where k is the thermal conductivity of the fluid and L_c is the *characteristic length*. The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century, and it is viewed as the *dimensionless convection heat transfer coefficient.*

$$q_{\text{conv}} = h\Delta T$$

and

$$q_{\text{cond}} = k\frac{\Delta T}{L}$$

Taking their ratio gives $$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

Ernst Kraft Wilhelm Nusselt (1882–1957). This photograph, provided by his student, G. Lück, shows Nusselt at the Kesselberg waterfall in 1912. He was an avid mountain climber.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number,** defined as

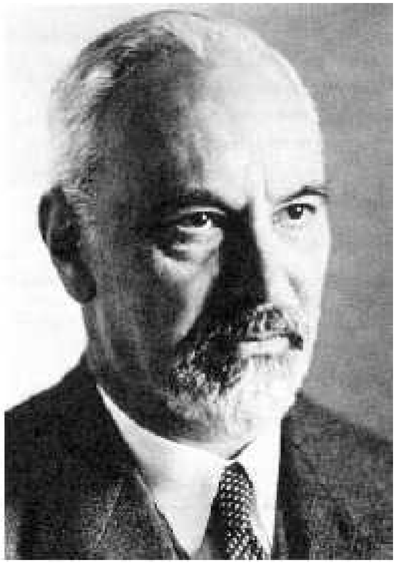
$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \qquad \textbf{(6-12)}$$

TABLE 6–2

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metals ($\Pr \ll 1$) and very slowly in oils ($\Pr \gg 1$) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.



Prandtl was educated at the Technical University in Munich and finished his doctorate there in 1900. He was given a chair in a new fluid mechanics institute at Göttingen University in 1904—the same year that he presented his historic paper explaining the boundary layer. His work at Göttingen, during the period up to Hitler’s regime, set the course of modern fluid mechanics and aerodynamics and laid the foundations for the analysis of heat convection.

Reynolds Number

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, free-stream velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number,** which is a *dimensionless* quantity, and is expressed for external flow as (Fig. 6–16)

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu}$$

where $\mathcal{V}$ is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge. Note that kinematic viscosity has the unit m^2/s, which is identical to the unit of thermal diffusivity, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

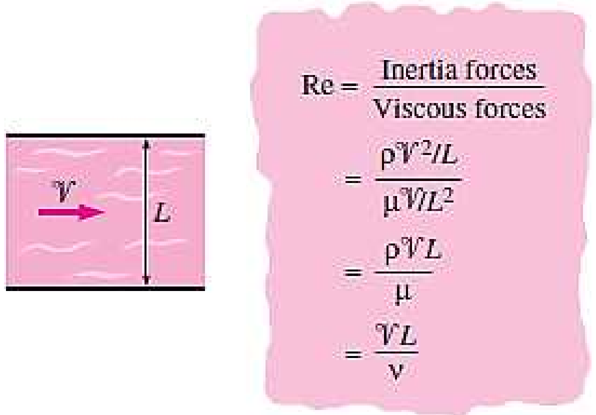
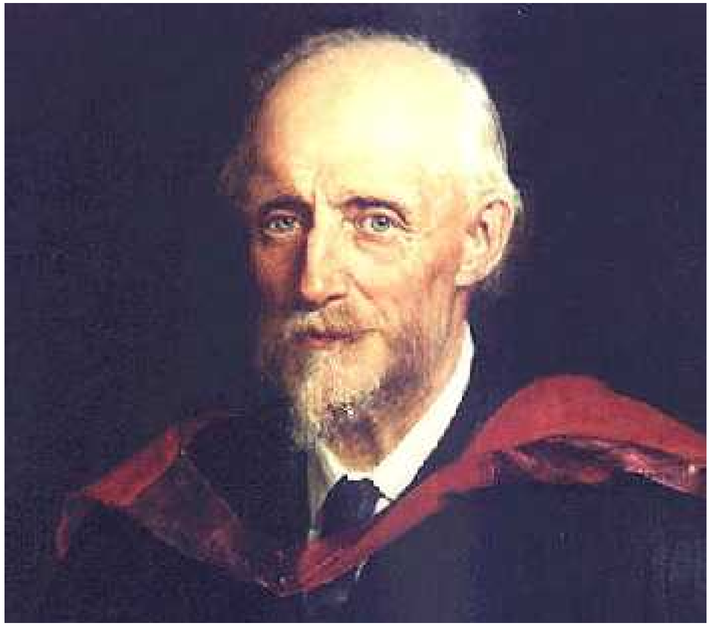


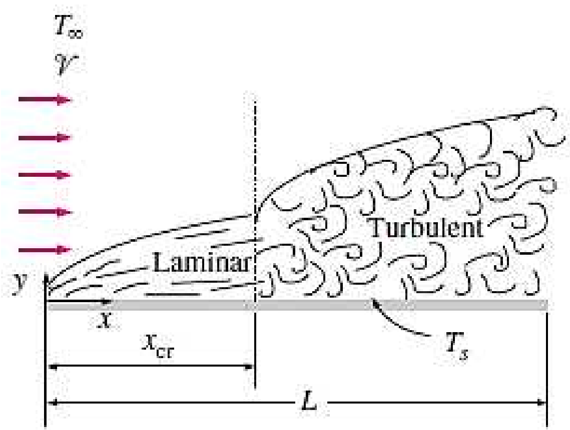
FIGURE 6–16
The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.



Osborne Reynolds (1842 to 1912) Reynolds was born in Ireland but he taught at the University of Manchester. He was a significant contributor to the subject of fluid mechanics in the late 19th C. His original laminar-to turbulent flow transition experiment, pictured below, was still being used as a student experiment at the University of Manchester in the 1970s.

EXTERNAL FORCED CONVECTION

PARALLEL FLOW OVER FLAT PLATES



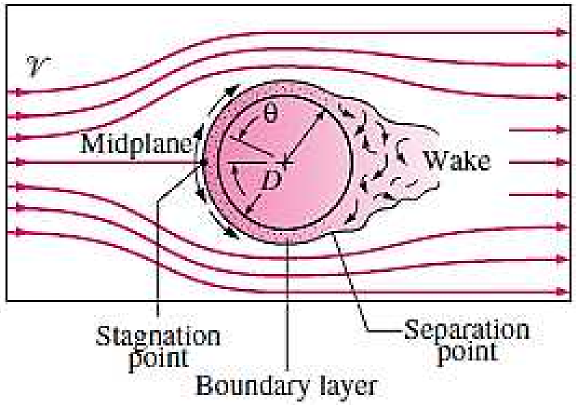
$$\mathrm{Re}_L = \mathcal{V}L/\nu$$

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Laminar: $\mathrm{Nu} = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$ $\qquad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $\mathrm{Nu} = \dfrac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$ $\qquad 0.6 \le \mathrm{Pr} \le 60$, $\quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

FLOW ACROSS CYLINDERS AND SPHERES

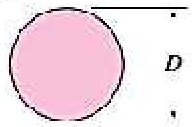
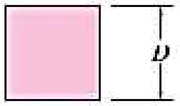
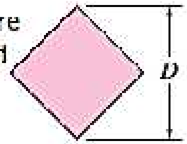
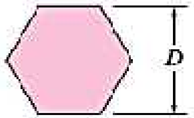
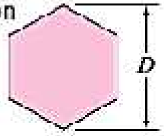
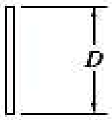
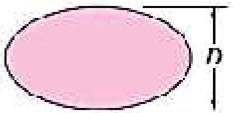


The average Nusselt number for flow across cylinders can be expressed compactly as

$$\mathrm{Nu}_{cyl} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

where $n = \frac{1}{3}$ and the experimentally determined constants C and m are given in Table 7–1 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and the Nusselt numbers for different geometries is as indicated on the figure. All fluid prop-

TABLE 7-1

Cross section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

INTERNAL FORCED CONVECTION

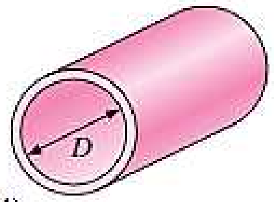
Laminar and Turbulent Flow In Tubes

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu}$$

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter**

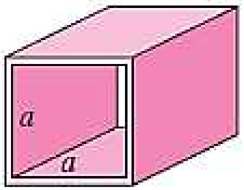
D_h

Circular tube:



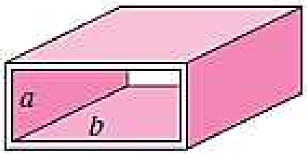
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



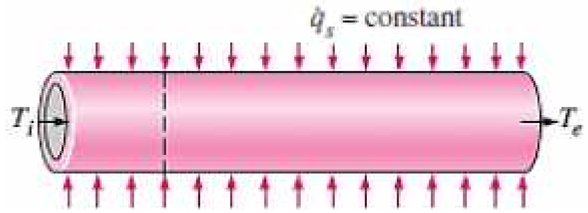
$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

$\mathrm{Re} < 2300$	laminar flow
$2300 \le \mathrm{Re} \le 10{,}000$	transitional flow
$\mathrm{Re} > 10{,}000$	turbulent flow

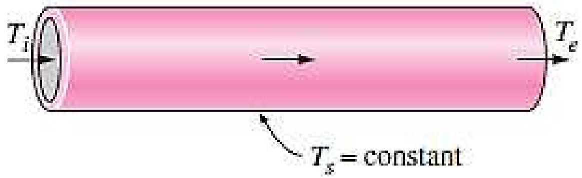
LAMINAR FLOW IN TUBES

1 Constant Surface Heat Flux ($\dot{q}_s$ = constant)

$$\mathrm{Nu} = \frac{hD}{k} = 4.36$$



2 Constant Surface Temperature (T_s = constant)



$$\mathrm{Nu} = \frac{hD}{k} = 3.66$$

TURBULENT FLOW IN TUBES

Smooth tubes: $f = (0.790 \ln \mathrm{Re} - 1.64)^{-2}$ $\quad 10^4 < \mathrm{Re} < 10^6$

The Nusselt number in turbulent flow is related to the friction factor through the *Chilton–Colburn analogy* expressed as

$$\mathrm{Nu} = 0.125 f \mathrm{Re}\mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix}$$

which is known as the *Colburn equation*. The accuracy of this equation can be improved by modifying it as

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube. This equation is known as the *Dittus–Boelter equation* [Dittus and Boelter (1930), Ref. 6] and it is preferred to the Colburn equation.

HEAT EXCHANGERS

Heat exchangers are devices that facilitate the *exchange of heat* between *two fluids* that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants.

Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix. In a car radiator, for example, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates outside attached to the tubes.

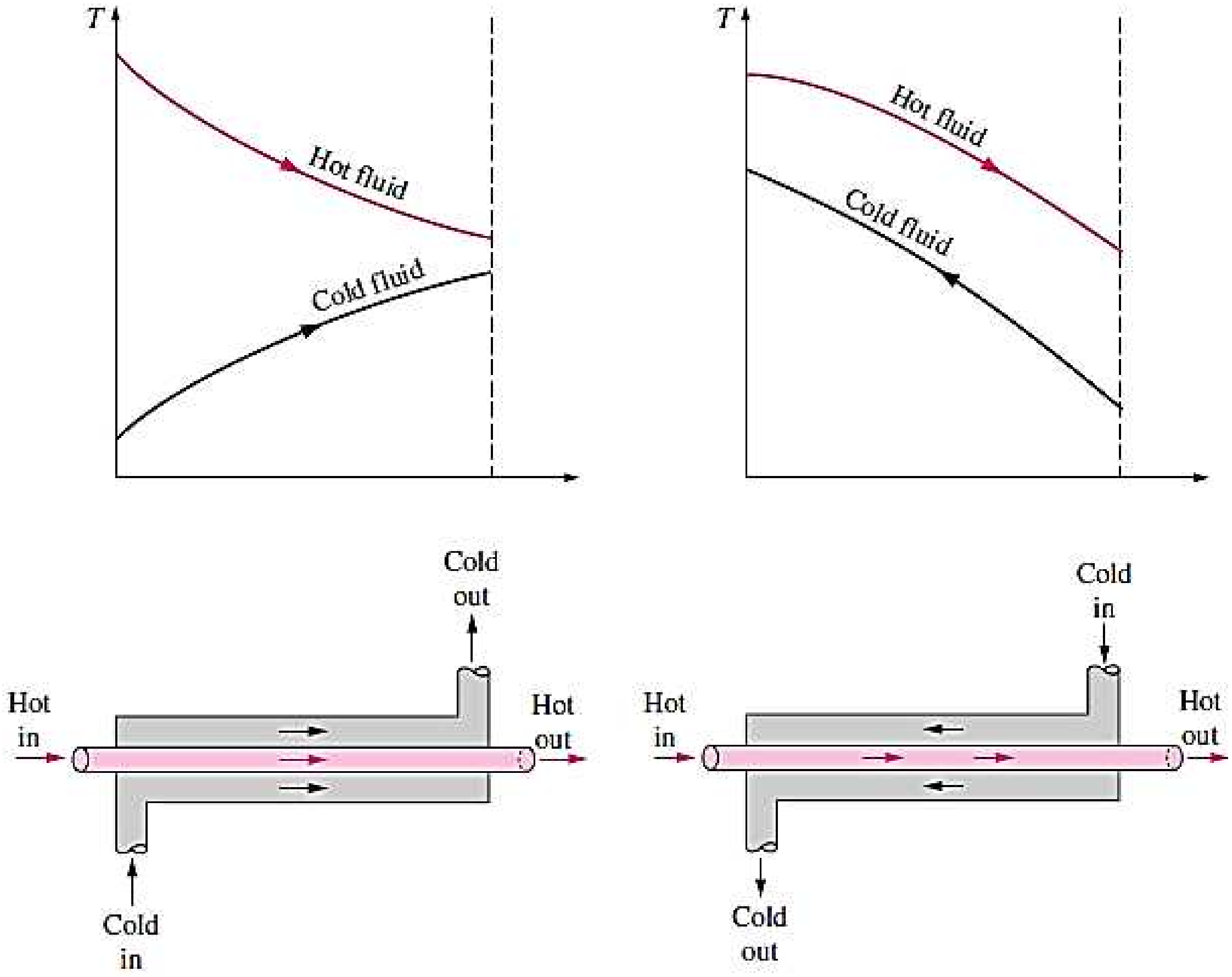
Heat transfer in a heat exchanger usually involves *convection* in each fluid and *conduction* through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* that accounts for the contribution of all these effects on heat transfer.

The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger. In the analysis of heat exchangers, it is usually convenient to work with the *logarithmic mean temperature difference LMTD,* which is an equivalent mean temperature difference between the two fluids for the entire heat exchanger.

Heat exchangers are manufactured in a variety of types, and thus we start this chapter with the *classification* of heat exchangers. We then discuss the determination of the overall heat transfer coefficient in heat exchangers, and the LMTD for some configurations. We then introduce the *correction factor F* to account for the deviation of the mean temperature difference from the LMTD in complex configurations. Next we discuss the effectiveness–NTU method, which enables us to analyze heat exchangers when the outlet temperatures of the fluids are not known. Finally, we discuss the selection of heat exchangers.

7–1 TYPES OF HEAT EXCHANGERS

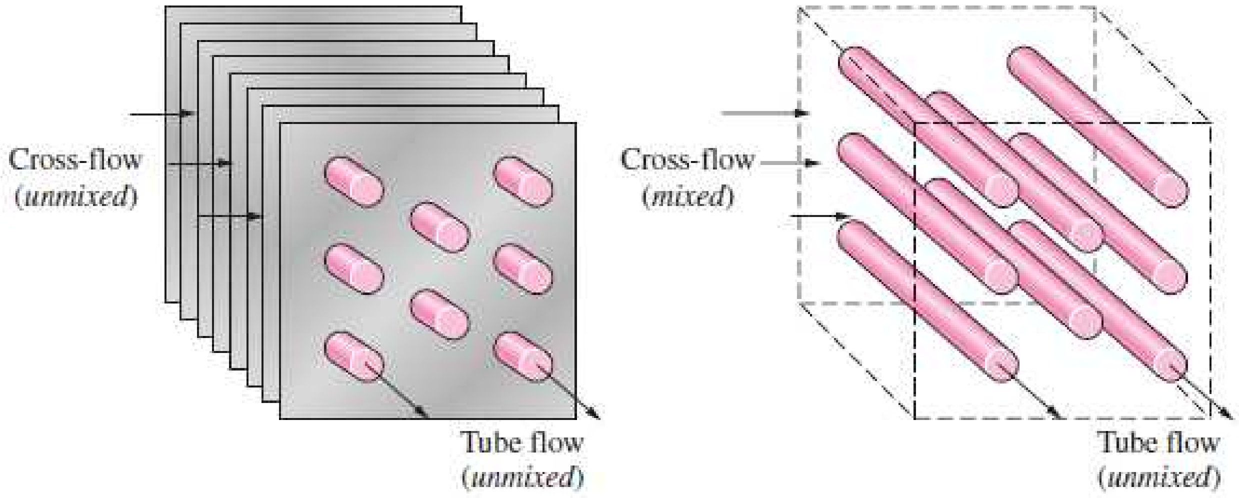
Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.



(*a*) Parallel flow

(*b*) Counter flow

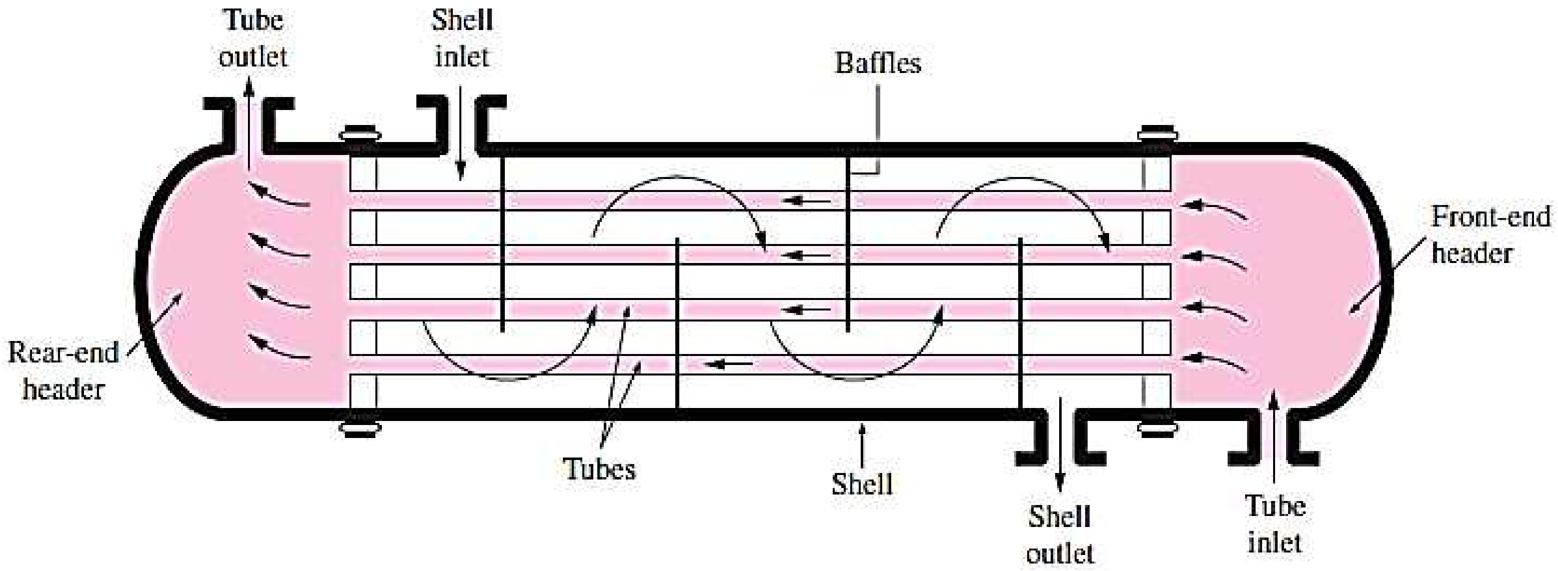
Different flow configurations in cross-flow heat exchangers.



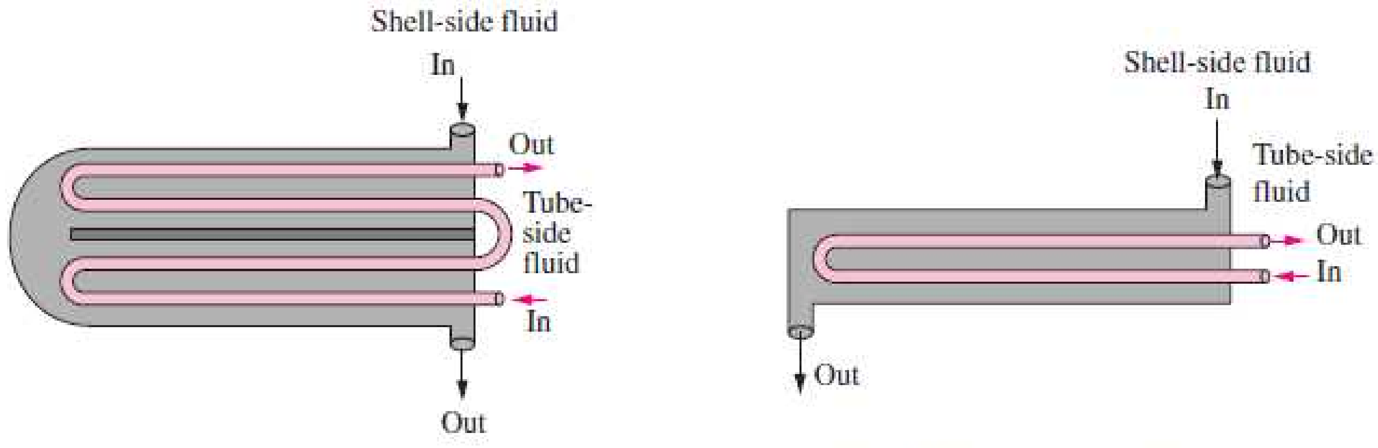
(*a*) Both fluids unmixed

(*b*) One fluid mixed, one fluid unmixed

The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).



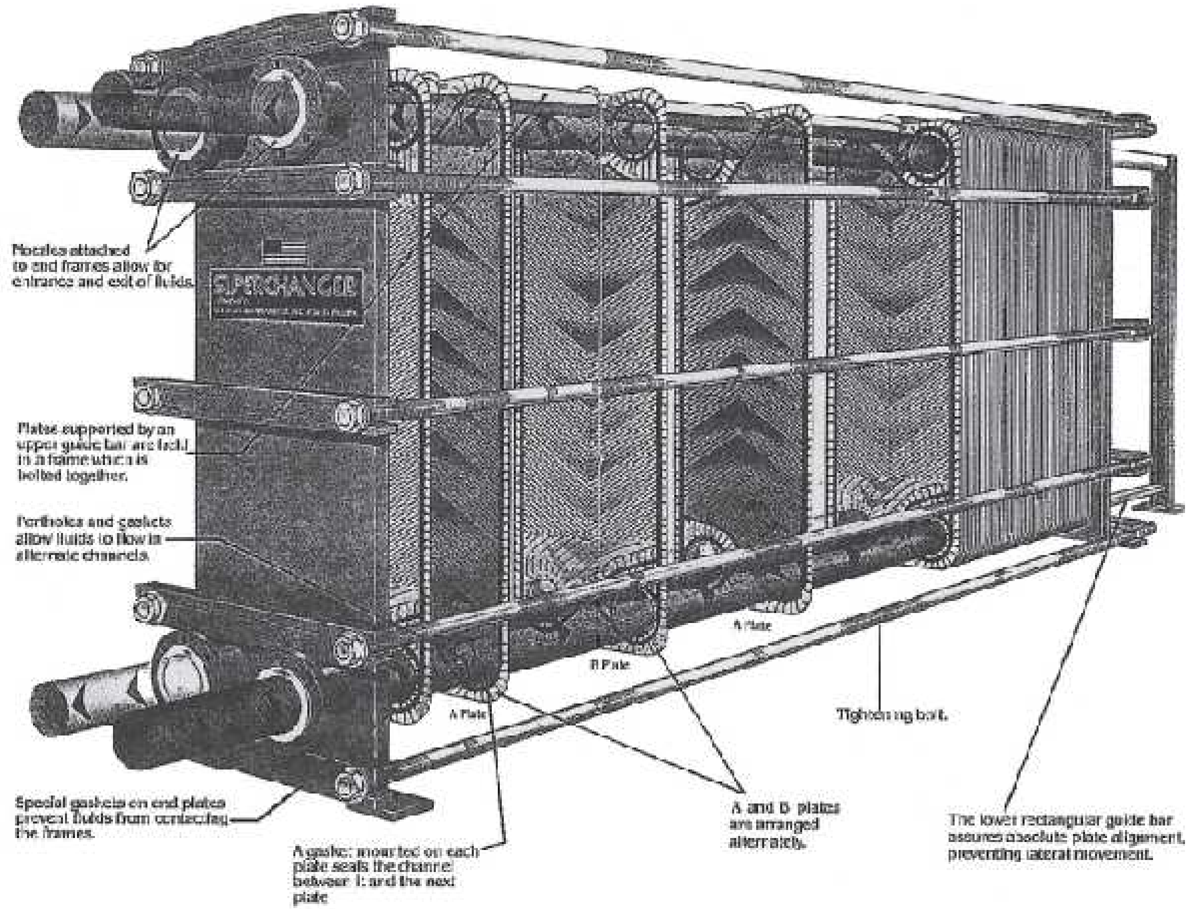
Multipass flow arrangements in shell-and-tube heat exchangers.



(*b*) Two-shell passes and four-tube passes

(*a*) One-shell pass and two-tube passes

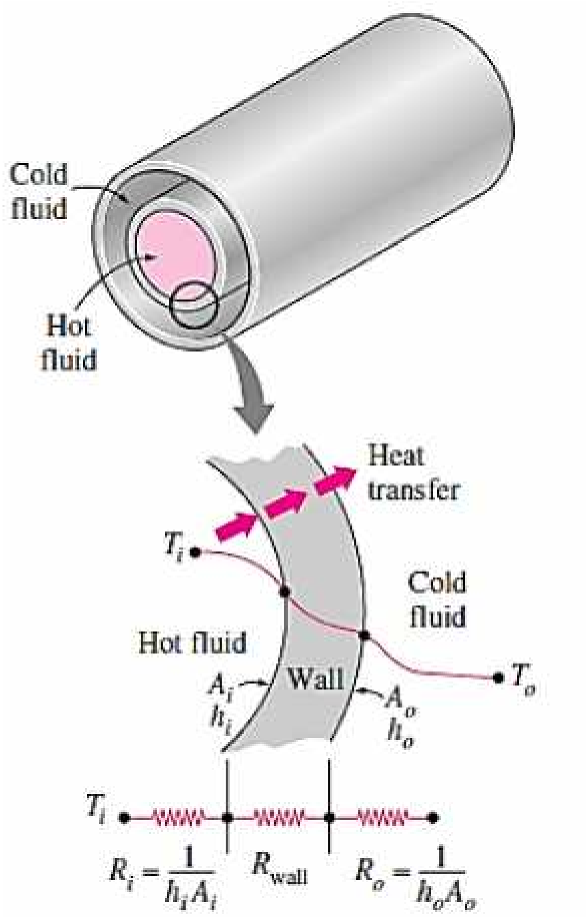
A plate-and-frame liquid-to-liquid heat exchanger (courtesy of Trante Corp.).



7–2 THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection,* through the wall by *conduction,* and from the wall to the cold fluid again by *convection.* Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure below. Here the subscripts *i* and *o* represent the inner and outer surfaces of the



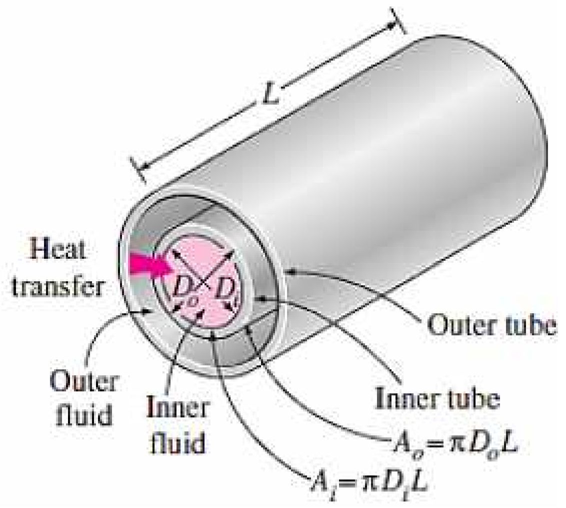
inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the *thermal resistance* of the tube wall in this case is

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi k L}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

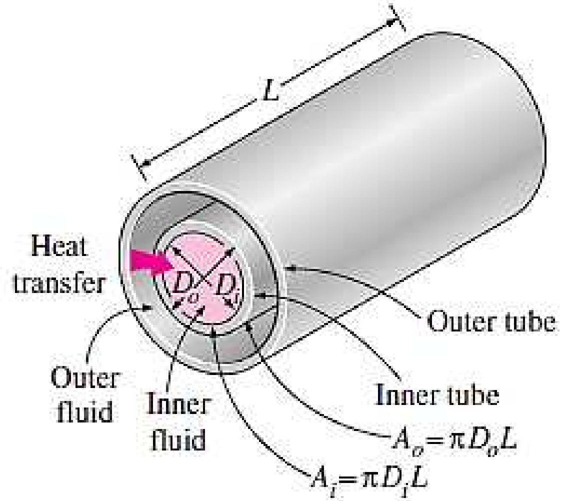
In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance *R*, and to express the rate of heat transfer between the two fluids as



$$\dot{Q} = \frac{\Delta T}{R} = UA\,\Delta T = U_i A_i\,\Delta T = U_o A_o\,\Delta T$$

where U is the **overall heat transfer coefficient,** whose unit is W/m$^2 \cdot$ °C, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, Above equation reduces to

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o}$$



Perhaps you are wondering why we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

Note that $U_iA_i = U_oA_o$, but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{\text{wall}} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A_s$).

Then above Eq. for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, h_i and h_o, are determined using the convection relations discussed in earlier chapters.

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m² · °C*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850‡
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

*Multiply the listed values by 0.176 to convert them to Btu/h · ft² · °F.

†Based on air-side surface area.

‡Based on water- or steam-side surface area.

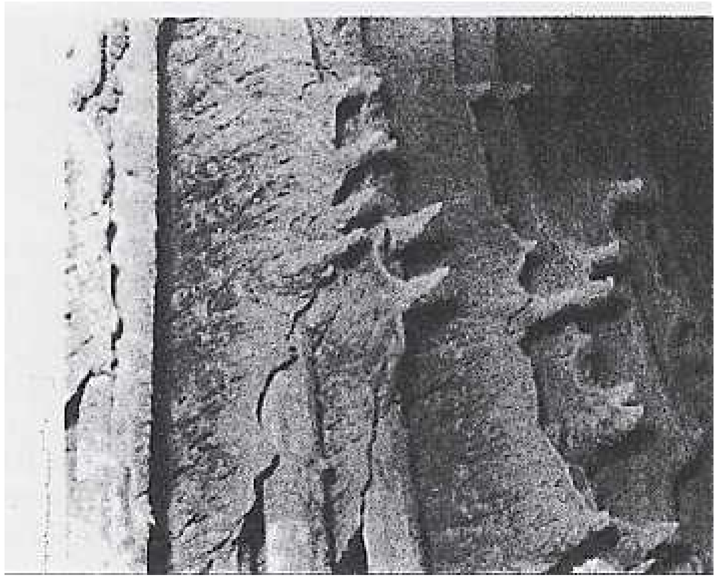
Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** *Rf* , which is a measure of the *thermal resistance* introduced by fouling. The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling.* In this case.

The overall heat transfer coefficient relation given above is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned shell-and-tube heat exchanger, it can be expressed as

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.



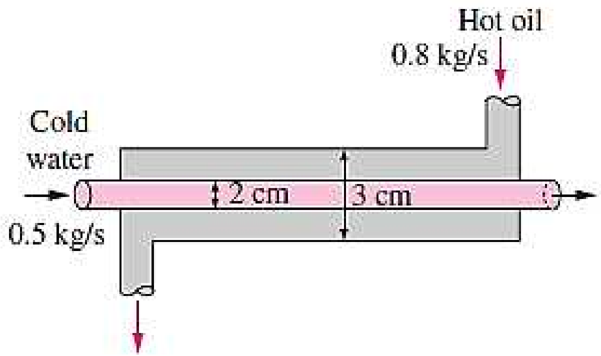
Representative fouling factors (thermal resistance due to fouling for a unit surface area)

(*Source:* Tubular Exchange Manufacturers Association.)

Fluid	R_f, $m^2 \cdot {}^\circ C/W$
Distilled water, sea water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

EXAMPLE 7–1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.



SOLUTION

Properties The properties of water at 45°C are

$$\rho = 990 \text{ kg/m}^3 \qquad \Pr = 3.91$$
$$k = 0.637 \text{ W/m}\cdot{}^\circ\text{C} \qquad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2/\text{s}$$

The properties of oil at 80°C are

$$\rho = 852 \text{ kg/m}^3 \qquad \Pr = 490$$
$$k = 0.138 \text{ W/m}\cdot{}^\circ\text{C} \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The schematic of the heat exchanger is given in Figure . The overall heat transfer coefficient *U* can be determined from Eq. Below:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$ and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2/\text{s}} = 53{,}490$$

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$\mathrm{Nu} = \frac{hD_h}{k} = 0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

Then,

$$h = \frac{k}{D_h}\mathrm{Nu} = \frac{0.637\ \mathrm{W/m\cdot{}^\circ C}}{0.02\ \mathrm{m}}(240.6) = 7663\ \mathrm{W/m^2\cdot{}^\circ C}$$

Now we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852\ \mathrm{kg/m^3} \qquad \nu = 37.5\times10^{-6}\ \mathrm{m^2/s}$$
$$k = 0.138\ \mathrm{W/m\cdot{}^\circ C} \qquad \mathrm{Pr} = 490$$

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01\ \mathrm{m}$$

The mean velocity and the Reynolds number in this case are

$$\mathcal{V}_m = \frac{\dot m}{\rho A_c} = \frac{\dot m}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8\ \mathrm{kg/s}}{(852\ \mathrm{kg/m^3})[\frac{1}{4}\pi(0.03^2 - 0.02^2)]\ \mathrm{m^2}} = 2.39\ \mathrm{m/s}$$

and

$$\mathrm{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2.39\ \mathrm{m/s})(0.01\ \mathrm{m})}{37.5\times10^{-6}\ \mathrm{m^2/s}} = 637$$

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 13–3 by interpolation to be

$$\mathrm{Nu} = 5.45$$

$$h_o = \frac{k}{D_h}\mathrm{Nu} = \frac{0.138\ \mathrm{W/m\cdot{}^\circ C}}{0.01\ \mathrm{m}}(5.45) = 75.2\ \mathrm{W/m^2\cdot{}^\circ C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663\ \mathrm{W/m^2\cdot{}^\circ C}} + \frac{1}{75.2\ \mathrm{W/m^2\cdot{}^\circ C}}} = \mathbf{74.5\ W/m^2\cdot{}^\circ C}$$

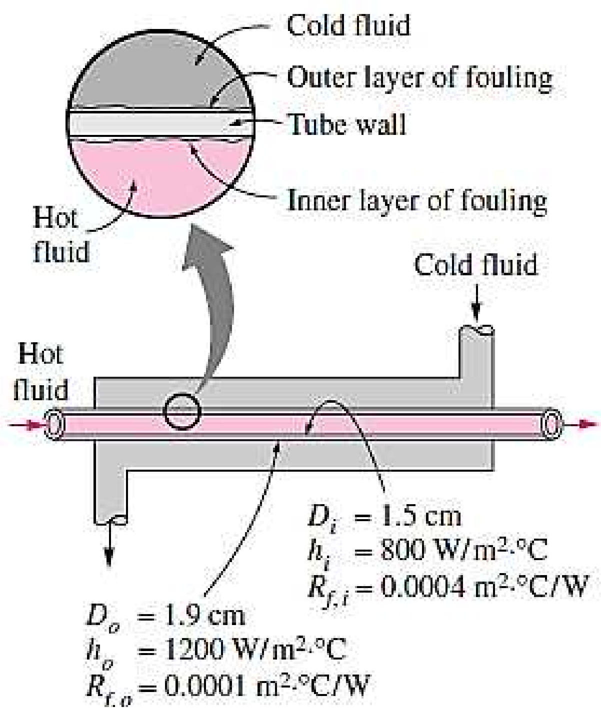
TABLE 13–3

Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal (Kays and Perkins, Ref. 8.)

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

EXAMPLE 7–2 Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m · °C) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m² · °C on the inner surface of the tube and h_o = 1200 W/m² · °C on the outer surface. For a fouling factor of $R_{f,i}$ = 0.0004 m² · °C/W on the tube side and $R_{f,o}$ = 0.0001 m² · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.



SOLUTION

The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$\begin{aligned} R &= \frac{1}{(800 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0471 \text{ m}^2} \\ &\quad + \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m})} \\ &\quad + \frac{0.0001 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0597 \text{ m}^2)} \\ &= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396){}^\circ\text{C/W} \\ &= \mathbf{0.0532{}^\circ C/W} \end{aligned}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficient based on the inner and outer surfaces of the tube are determined again from Eq. 13-8 to be

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0471\ \text{m}^2)} = \mathbf{399\ W/m^2 \cdot {}^\circ C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0597\ \text{m}^2)} = \mathbf{315\ W/m^2 \cdot {}^\circ C}$$

Discussion Note that the two overall heat transfer coefficients differ significantly (by 27 percent) in this case because of the considerable difference between the heat transfer surface areas on the inner and the outer sides of the tube. For tubes of negligible thickness, the difference between the two overall heat transfer coefficients would be negligible.

7–3 ANALYSIS OF HEAT EXCHANGERS

THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation $\dot{Q} = UA_s\,\Delta T_m$.

A- parallel-flow double-pipe heat exchanger

energy balance on each fluid in a differential section of the heat exchanger can be expressed as

$$\delta\dot{Q} = -\dot{m}_h C_{ph}\, dT_h \qquad \text{and} \qquad \delta\dot{Q} = \dot{m}_c C_{pc}\, dT_c$$

Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h C_{ph}} \qquad \text{and} \qquad dT_c = \frac{\delta\dot{Q}}{\dot{m}_c C_{pc}}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

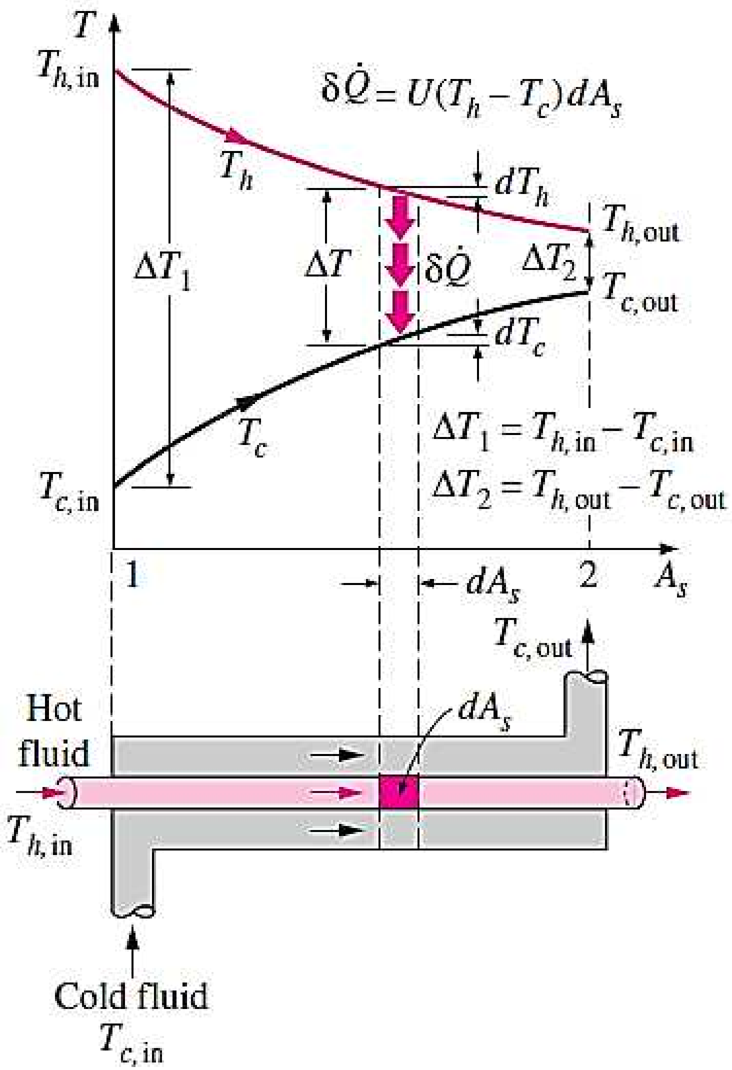
The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

$$\delta\dot{Q} = U(T_h - T_c)\, dA_s$$

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\int_1^2 \frac{d(T_h - T_c)}{(T_h - T_c)} = -U\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)\int_1^2 dA$$



$$\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

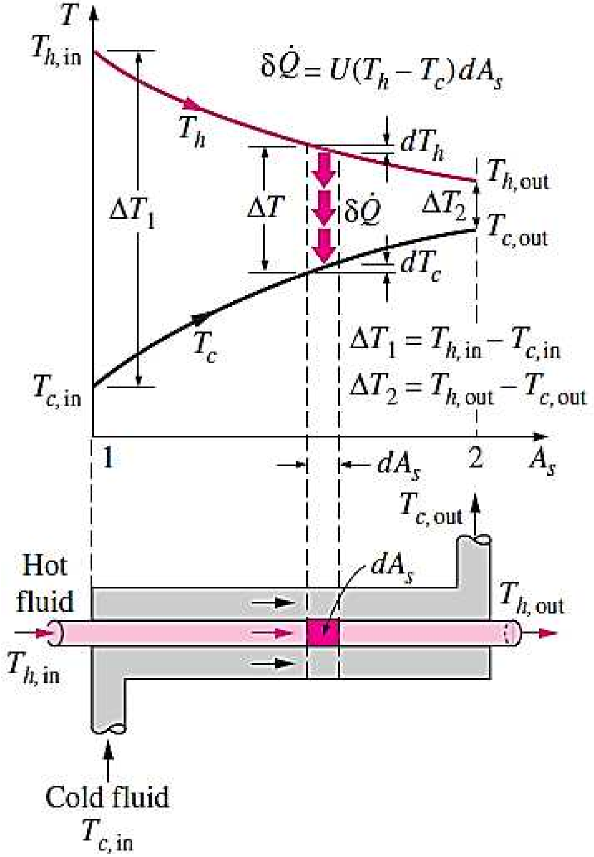
$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right) = -UA\left(\frac{(T_{h1} - T_{h2})}{\dot{Q}} + \frac{(T_{c2} - T_{c1})}{\dot{Q}}\right) = UA\left(\frac{(T_{h2} - T_{h1}) - (T_{c2} - T_{c1})}{\dot{Q}}\right)$$

Or $$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right) = UA\left(\frac{(T_{h2} - T_{h1}) - (T_{c2} - T_{c1})}{\dot{Q}}\right) = \frac{UA}{\dot{Q}}\left[(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})\right]$$

Then the heat transfer equation becomes

$$\dot{Q} = AU \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}} = AU(\Delta T)_m$$

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}}$$



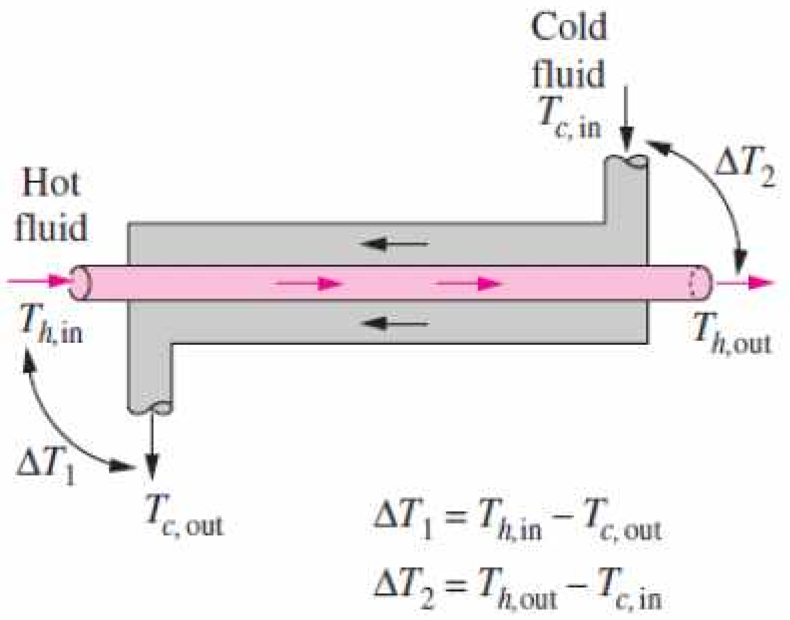
$$\boxed{(\Delta T)_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}}}$$ **log mean temperature difference,**

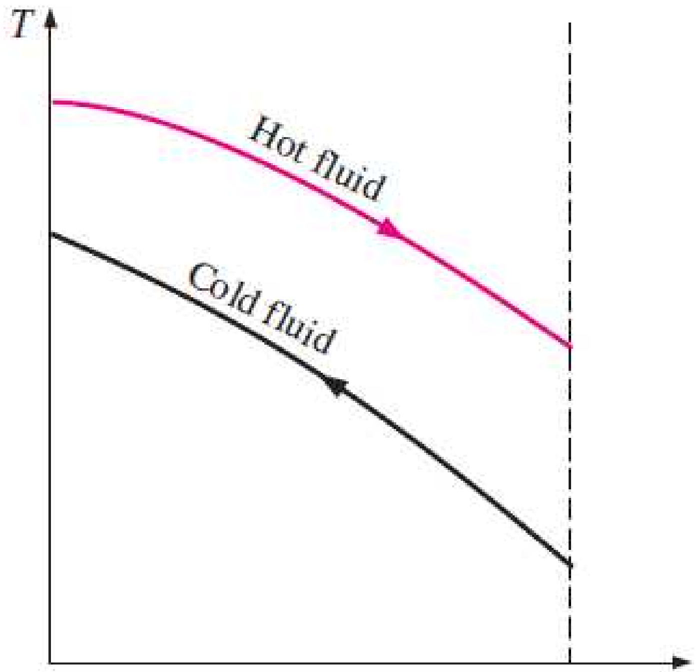
Counter Flow Heat Exchanger

$$(\Delta T)_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln \dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}}$$

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}}$$

Counter-Flow Heat Exchangers





Example

Determine the area required in parallel flow heat exchanger to cool oil from 60°C to 30°C using water available at 20°C. The outlet temperature of the water is 26°C. The rate of flow of oil is 10kg/s. The specific heat of the oil is 2200J/kg.K. The overall heat transfer coefficient U is 300W/m².°C. Compare the area required for a counter flow exchanger.

Solution

$$\dot{Q} = UA_s\, \Delta T_{\text{lm}}$$

$$(\Delta T)_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}}$$

$$\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = 10 \times 2200(60 - 30) = 660000\text{W}$$

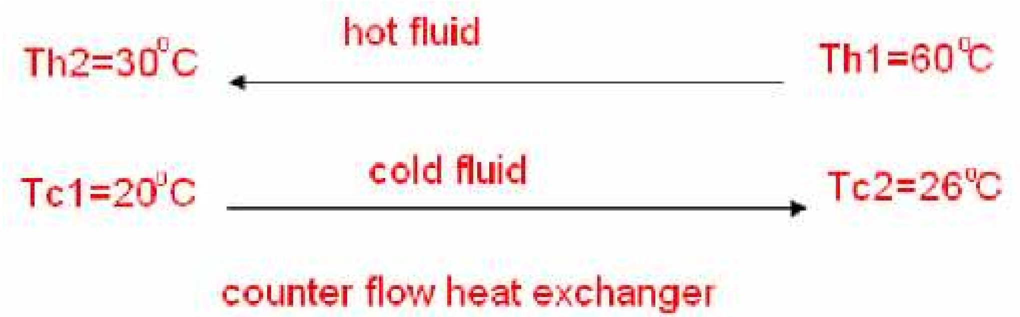
$$(\Delta T)_m = 15.635°\,C$$

$$A = \frac{\dot{Q}}{U(\Delta T)_m} = \frac{660000}{300(15.635)} = 140.71 m^2$$

For counter flow

$$(\Delta T)_m = 19.611°\,C$$

$$A = \frac{\dot{Q}}{U(\Delta T)_m} = \frac{660000}{300(19.611)} = 112.18 m^2$$



The flow rates of hot and cold water streams running through a parallel flow heat exchanger are 0.25kg/s and 0.6kg/s respectively. The inlet temperatures on the hot and colds sides are 80°C and 30°C respectively. The exit temperature of hot water is 55°C. If the heat transfer coefficients on both sides are 700W/m²°C, calculate the area of the heat exchanger.

Solution

the specific hat of water to be constant and is $c^c = c^h = 4180 J/kg.^oC$

$$\dot{Q} = AU(\Delta T)_m \quad \text{Where}$$

$$\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = (0.25kg/s)(4180J/kg.^oC)(80-55)^oC = 26125\text{W}$$

And

$$\dot{Q} = \dot{m}_c c_c (T_{c2} - T_{c1}) \rightarrow 26125W = (0.6kg/s)(4180J/kg.^oC)(T_{c2} - 30)^oC$$

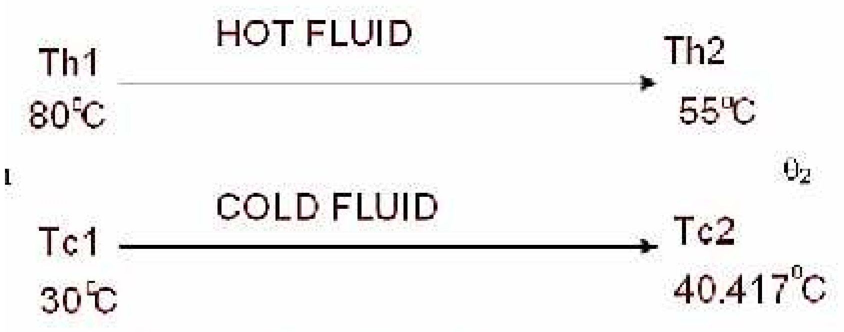
$$T_{c2} = 40.417^\circ C$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{700} + \frac{1}{700}} = 350W/m^{2o}C$$

The log-mean-temperature-difference

$$(\Delta T)_m = 28.744^\circ C$$

$$A = \frac{\dot{Q}}{U(\Delta T)_m} = \frac{26125W}{(350W/m^{2o}C)(28.744^\circ C)} = 2.6\text{m}^2$$



Parallel Flow Heat Exchanger

Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor

The log mean temperature difference T_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for *cross-flow* and *multi pass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.
In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as

$$\Delta T_{lm} = F\, \Delta T_{lm,\, CF}$$

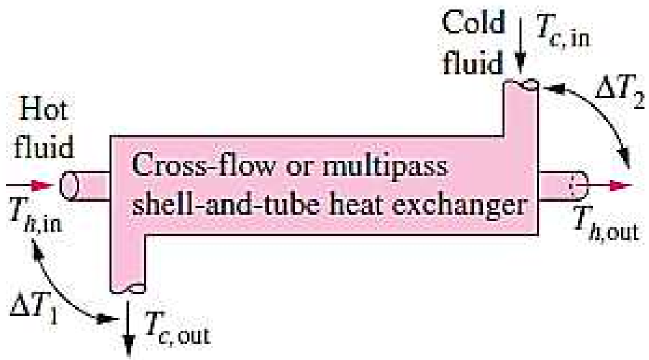
where F is the **correction factor,** which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{lm,\, CF}$ is the log mean temperature difference for the case of a *counter-flow* heat exchanger with the same inlet and outlet temperatures

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,\, in} - T_{c,\, out}$$

$$\Delta T_2 = T_{h,\, out} - T_{c,\, in}$$

The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, $F \leq 1$. The limiting value of $F = 1$ corresponds to the counter-flow heat exchanger. Thus, the correction factor F for a heat exchanger is *a measure of deviation of the ΔT_{lm} from the corresponding values for the counter-flow case.*



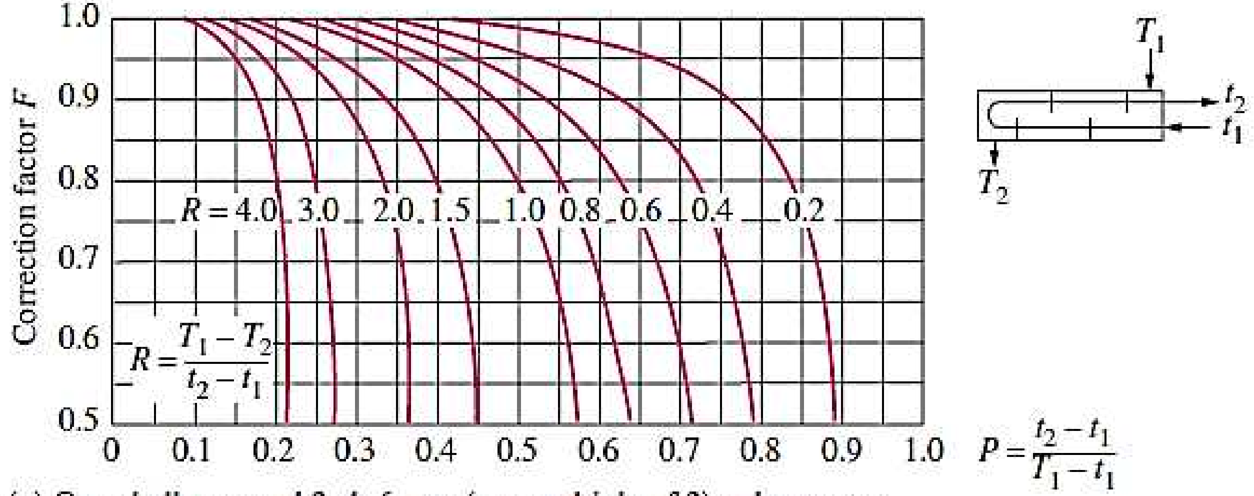
Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

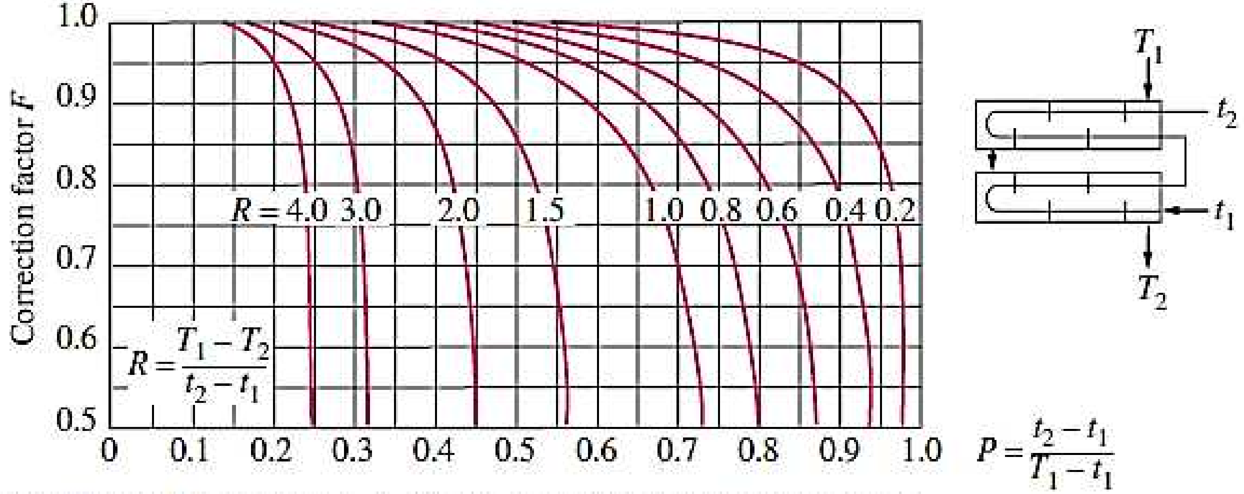
where $$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out}$$
$$\Delta T_2 = T_{h,out} - T_{c,in}$$

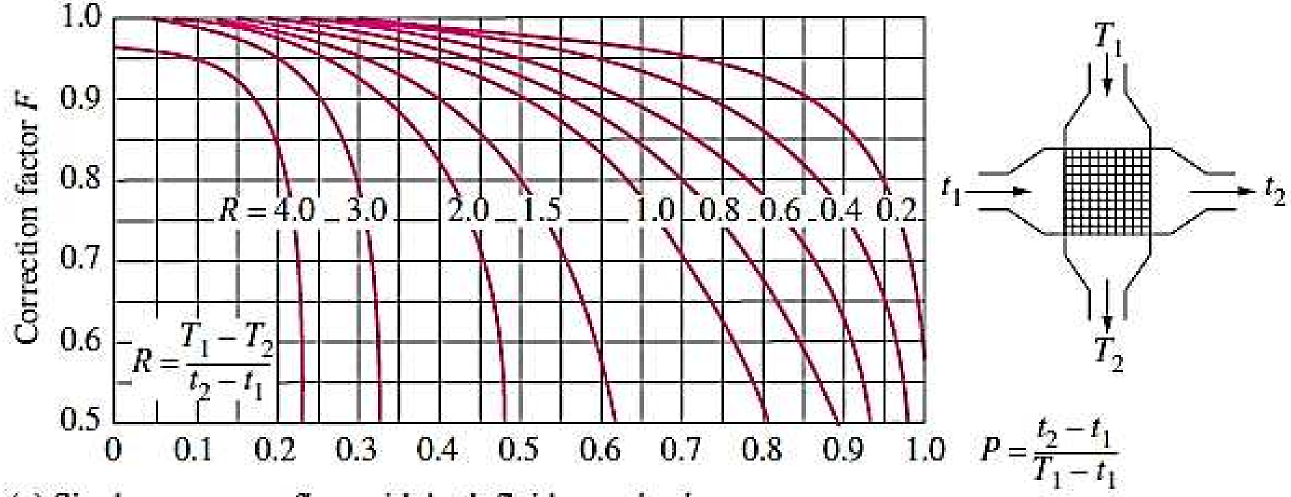
and $F = \ldots$ (Fig. 13–18)



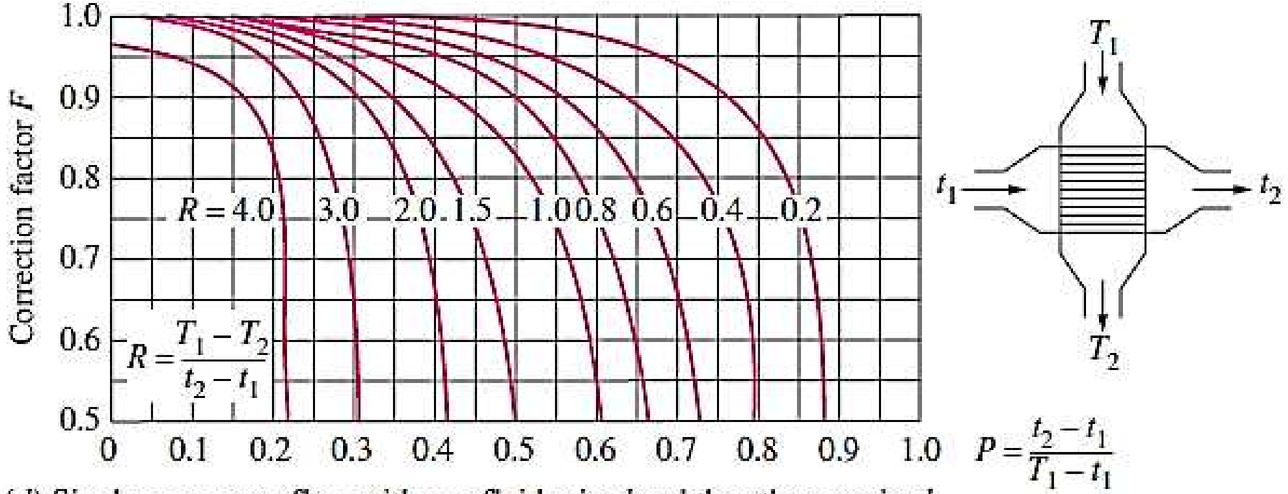
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

EXAMPLE Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 13–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m² · °C on the glycerin (shell) side and 160 W/m² · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m² · °C/W occurs on the outer surfaces of the tubes.

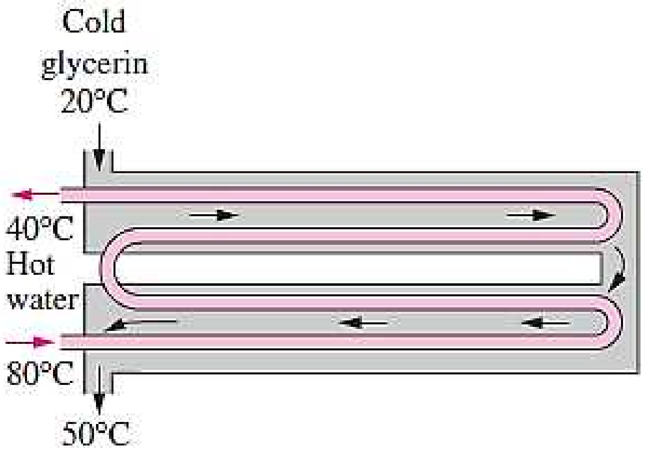


FIGURE 13–21

SOLUTION

$$A_s = \pi DL = \pi(0.02 \text{ m})(60 \text{ m}) = 3.77 \text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_s F \,\Delta T_{\text{lm, }CF}$$

where F is the correction factor and $\Delta T_{\text{lm, }CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

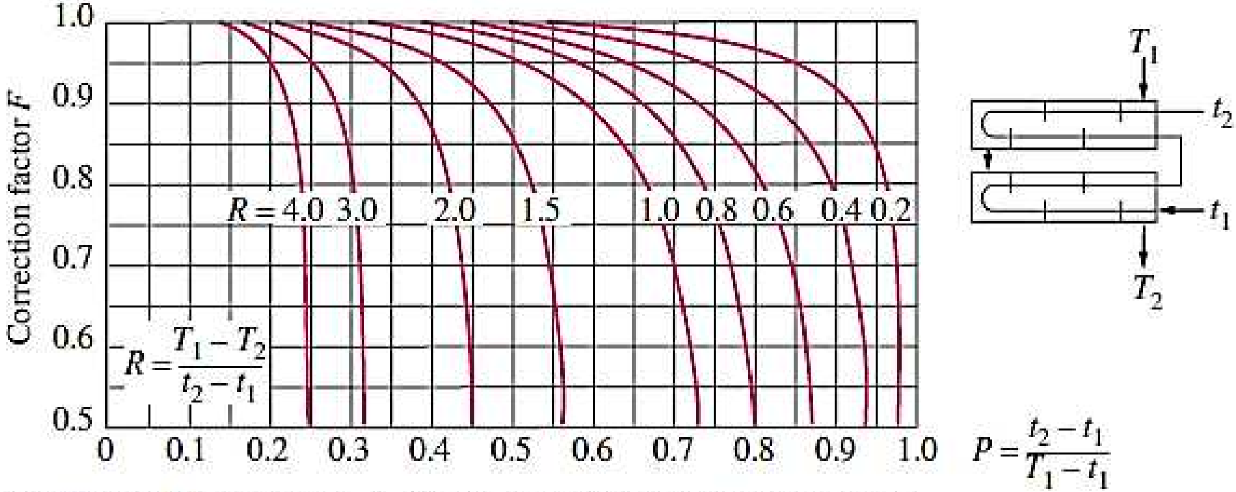
$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}} = (80 - 50)°\text{C} = 30°\text{C}$$

$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}} = (40 - 20)°\text{C} = 20°\text{C}$$

$$\Delta T_{\text{lm, }CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7°\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned}\right\} F = 0.91$$



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes (*a*) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2 \cdot {}^\circ\text{C}}} = 21.6\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F\, \Delta T_{\text{lm},\, CF} = (21.6\ \text{W/m}^2 \cdot {}^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = 1830\ \text{W}$$

(*b*) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is

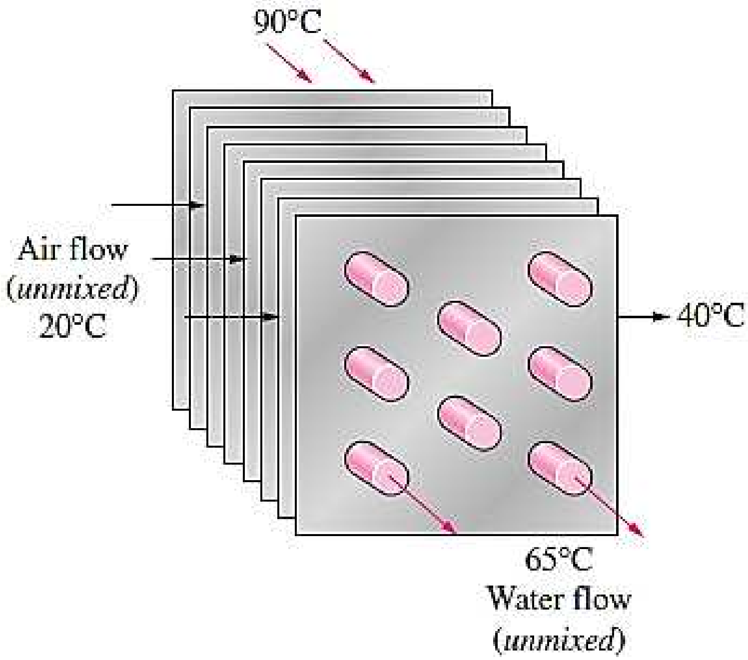
$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2 \cdot {}^\circ\text{C}} + 0.0006\ \text{m}^2 \cdot {}^\circ\text{C/ W}}$$
$$= 21.3\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UA_s F\, \Delta T_{\text{lm},\, CF} = (21.3\ \text{W/m}^2 \cdot {}^\circ\text{C})(3.77\ \text{m}^2)(0.91)(24.7^\circ\text{C}) = 1805\ \text{W}$$

EXAMPLE Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 13–22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.



SOLUTION

Properties The specific heat of water at the average temperature of (90 + 65)/2 = 77.5°C is 4.195 kJ/kg · °C.

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{water}} = (0.6\ \text{kg/s})(4.195\ \text{kJ/kg}\cdot{}^\circ\text{C})(90 - 65)^\circ\text{C} = 62.93\ \text{kW}$$

The tube-side heat transfer area is the total surface area of the tubes, and is determined from

$$A_i = n\pi D_i L = (40)\pi(0.005\ \text{m})(0.65\ \text{m}) = 0.408\ \text{m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfe coefficient can be determined from

$$\dot{Q} = U_i A_i F\,\Delta T_{\text{lm},\,CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F\,\Delta T_{\text{lm},\,CF}}$$

where F is the correction factor and $\Delta T_{\text{lm},\,CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

$$\Delta T_1 = T_{h,\,in} - T_{c,\,out} = (90 - 40)°C = 50°C$$
$$\Delta T_2 = T_{h,\,out} - T_{c,\,in} = (65 - 20)°C = 45°C$$
$$\Delta T_{lm,\,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.6°C$$

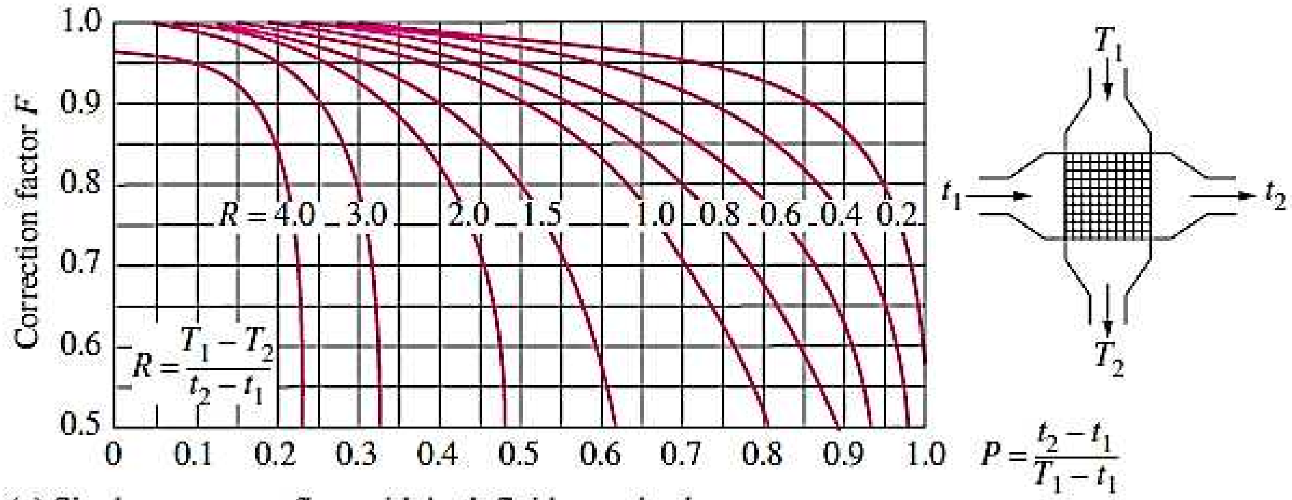
and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80 \end{aligned}\right\} F = 0.97$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F\,\Delta T_{lm,\,CF}} = \frac{62{,}930 \text{ W}}{(0.408 \text{ m}^2)(0.97)(47.6°C)} = 3341 \text{ W/m}^2\cdot°C$$

Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.



(c) Single-pass cross-flow with both fluids *unmixed*