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# Thermodynamics

## الديناميكا الحرارية

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# Thermodynamics الديناميكا الحرارية

## CHAPTER TWO Properties of Pure Substances

2.1 The  $P$ - $v$ - $T$  Surface

2.2 The Liquid-Vapor Region

2.3 The Steam Tables

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Quiz No. 1

Quiz No. 2

Thermodynamic properties of a pure substance and the phase boundaries for solid, liquid, and vapor states are discussed. Phase equilibrium for vaporization (boiling liquid to vapor), with the opposite direction being condensation (vapor to liquid); sublimation (solid to vapor) or the opposite solidification (vapor to solid); and melting (solid to liquid) or the opposite solidifying (liquid to solid) should be recognized. The three-dimensional  $P$ - $v$ - $T$  surface and the two-dimensional representations in the  $(P, T)$ ,  $(T, v)$  and  $(P, v)$  diagrams, and the vaporization, sublimation, and fusion lines are related to the printed tables in Appendix B. Properties from printed and computer tables covering a number of substances are introduced, including two-phase mixtures, for which we use the mass fraction of vapor (quality). The ideal-gas law approximates the limiting behavior for low density. An extension of the ideal-gas law is shown with the compressibility factor  $Z$ , and other more complicated equations of state are mentioned.

You should have learned a number of skills and acquired abilities from studying this chapter that will allow you to

- Know phases and the nomenclature used for states and interphases.
- Identify a phase given a state  $(T, P)$ .
- Locate states relative to the critical point and know Tables A.2 (F.1) and 3.2.
- Recognize phase diagrams and interphase locations.
- Locate states in the Appendix B tables with any entry:  $(T, P)$ ,  $(T, v)$  or  $(P, v)$
- Recognize how the tables show parts of the  $(T, P)$ ,  $(T, v)$  or  $(P, v)$  diagrams.
- Find properties in the two-phase regions; use quality  $x$ .
- Locate states using any combination of  $(T, P, v, x)$  including linear interpolation.
- Know when you have a liquid or solid and the properties in Tables A.3, A.4 (F.2, F.3).
- Know when a vapor is an ideal gas (or how to find out).
- Know the ideal-gas law and Table A.5 (F.4).
- Know the compressibility factor  $Z$  and the compressibility chart Fig. D.1.
- Know the existence of more general equations of state.
- Know how to get properties from the computer program.

## Properties of Pure Substances

In this chapter the relationships between pressure, specific volume, and temperature will be presented for a pure substance. A pure substance is homogeneous, but may exist in more than one phase, with each phase having the same chemical composition. Water is a pure substance; the various combinations of its three phases (vapor, liquid, ice) have the same chemical composition. Air in the gas phase is a pure substance, but liquid air has a different chemical composition. Air is not a pure substance if it exists in more than one phase.

## 2.1 The $P$ - $v$ - $T$ Surface

It is well known that a substance can exist in three different phases: solid, liquid, and gas. Assume that a solid is contained in a piston-cylinder arrangement such that

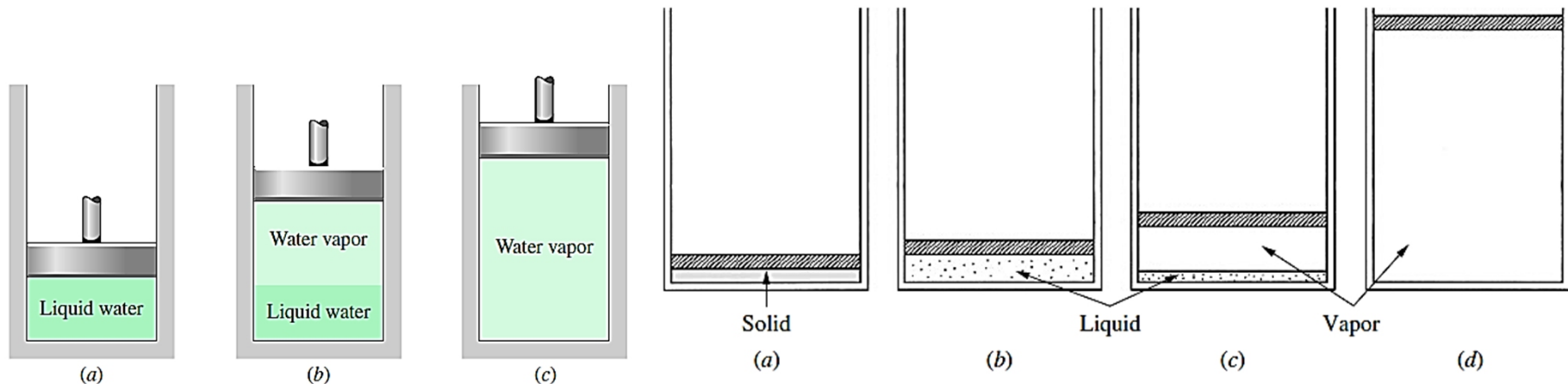


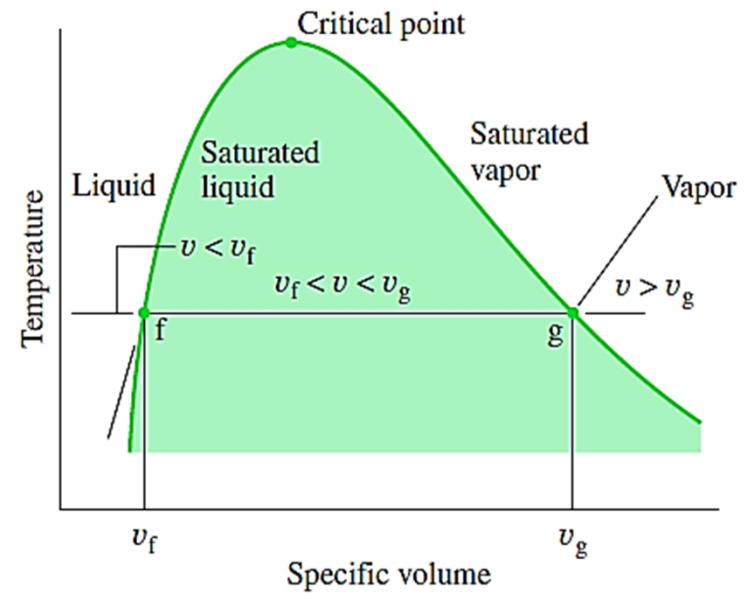
Figure 2.1 The solid, liquid, and vapor phases of a substance.

the pressure is maintained at a constant value; heat is added to the cylinder, causing the substance to experience all three phases, as in Fig. 2.1. We will record the temperature  $T$  and specific volume  $v$  during the experiment. Start with the solid at some low temperature, as in Fig. 2.2a; then add heat until it is all liquid ( $v$  does not increase very much). After all the solid is melted, the temperature of the liquid again rises until vapor just begins to form; this state is called the *saturated liquid* state.

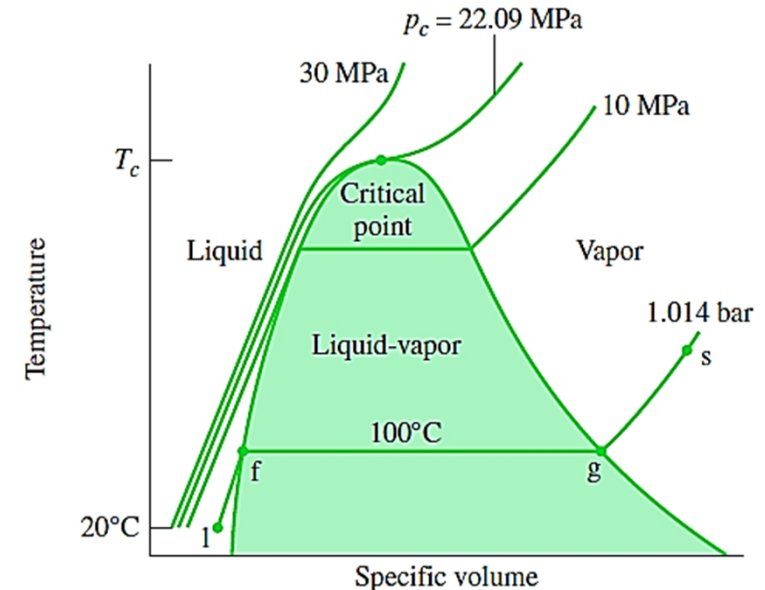
During the phase change from liquid to vapor,<sup>1</sup> called *vaporization*, the temperature remains constant as heat is added. Finally, all the liquid is vaporized and the state of *saturated vapor* exists, after which the temperature again rises with heat addition.

Note, the specific volumes of the solid and liquid are much less than the specific volume of vapor at relatively low pressures.

If the experiment is repeated a number of times using different pressures, a  $T-v$  diagram results, shown in Figure below. At pressures that exceed the pressure of the *critical point*, the liquid simply changes to a vapor without a constant-temperature vaporization process.



Projecting the liquid, two-phase liquid–vapor, and vapor regions of the  $p-v-T$  surface onto the temperature–specific volume plane results in a  $T-v$  diagram as in Figure below. Since consistent patterns are revealed in the  $p-v-T$  behavior of all pure substances, Figure below showing a  $T-v$  diagram for water can be regarded as representative.



The experiment could also be run by holding the temperature fixed and decreasing the pressure, as in Fig. 2.3a (the solid is not displayed). The solid would change to a liquid, and the liquid to a vapor, as in the experiment that led to Fig. 2.2. The  $T$ - $v$  diagram, with only the liquid and vapor phases shown, is displayed in Fig. 2.3b.

The process of melting, vaporization, and *sublimation* (the transformation of a solid directly to a vapor) are shown in Fig. 2.3c. Distortions are made in all three diagrams so that the various regions are displayed. The *triple point* is the point where all three phases exist in equilibrium together

Primary practical interest is in situations involving the liquid, liquid-vapor, and vapor regions. A saturated vapor lies on the *saturated vapor line* and a saturated liquid on the *saturated liquid line*. The region to the right of the saturated vapor line is the *superheated region*; the region to the left of the saturated liquid line is the *compressed liquid region* (also called the *subcooled liquid region*). A *supercritical state* is encountered when the pressure and temperature are greater than the critical values.

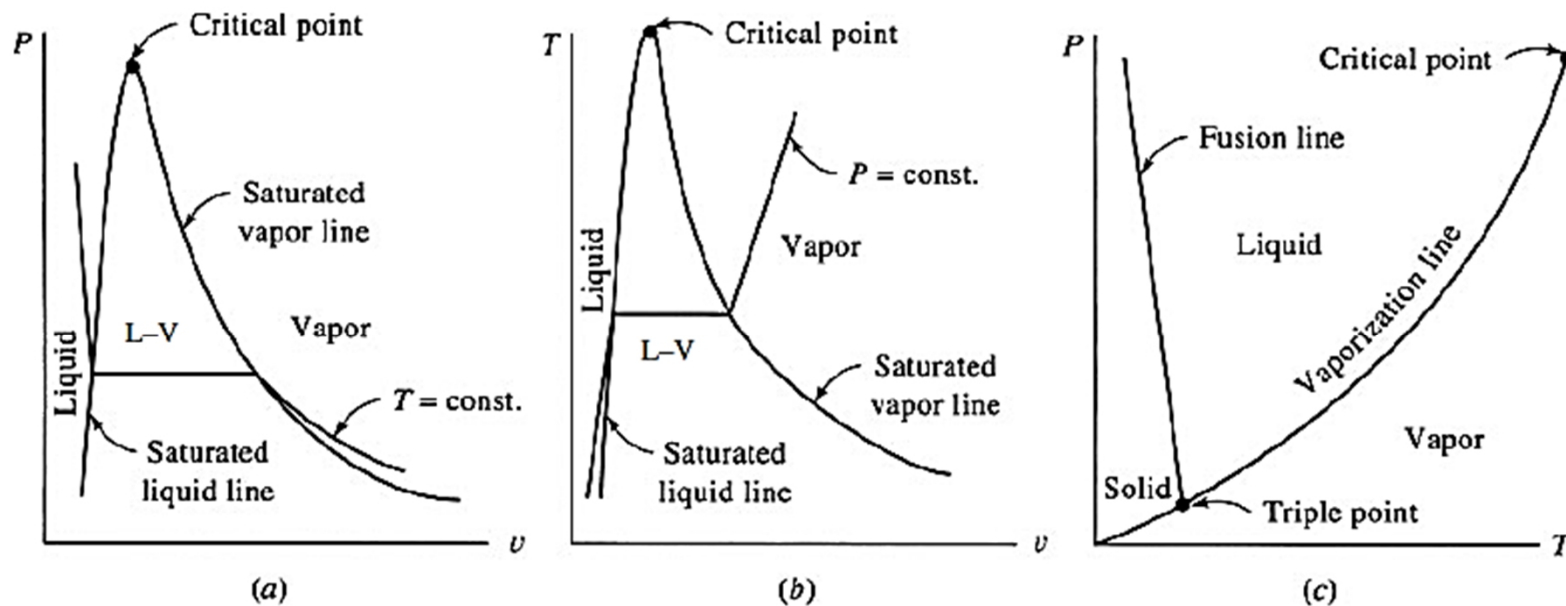


Figure 2.3 The (a)  $P$ - $v$ , (b)  $T$ - $v$ , and (c)  $P$ - $T$  diagrams.

## 2.2 The Liquid-Vapor Region

At any state  $(T, v)$  between saturated points  $f$  (state 1) and  $g$  (state 2), shown in Fig. 2.4, liquid and vapor exist in equilibrium. Let  $v_f$  and  $v_g$  represent, respectively, the specific volumes of the saturated liquid and the saturated vapor. Let  $m$  be the total mass of a system,  $m_f$  the amount of mass in the liquid phase, and  $m_g$  the amount of mass in the vapor phase. Then, for a state of the system represented by any  $(T, v)$ , such as state 3, the total volume of the mixture is the sum of the volume occupied by the liquid and that occupied by the vapor, or

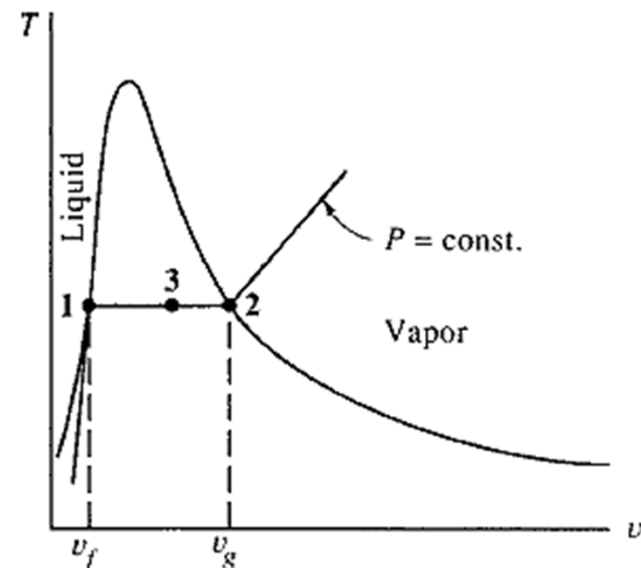
$$V = V_f + V_g \quad \text{or} \quad mv = m_f v_f + m_g v_g$$

The ratio of the mass of saturated vapor to the total mass is called the *quality* of the mixture, designated by the symbol  $x$ ; it is

$$x = \frac{m_g}{m}$$

We often refer to the region under the saturation lines as the *quality region*, or the *mixture region*, or the *wet region*; it is the only region where quality  $x$  has a meaning.

$$v = v_f + x(v_g - v_f)$$





Because the difference in saturated vapor and saturated liquid values frequently appears in calculations, we often let the subscript "fg" denote this difference; that is,

$$v_{fg} = v_g - v_f$$

Then, the specific volume can be written as

$$v = v_f + xv_{fg}$$

The percentage liquid by mass in a mixture is  $100(1 - x)$ , and the percentage vapor is  $100x$ .

## 2.3 The Steam Tables

Table C.1 Properties of Saturated H<sub>2</sub>O—Temperature Table

$T, ^\circ\text{C}$	$P, \text{MPa}$	$v, \text{m}^3/\text{kg}$		$u, \text{kJ/kg}$		$h, \text{kJ/kg}$			$s, \text{kJ/kg}\cdot\text{K}$		
		$v_f$	$v_g$	$u_f$	$u_g$	$h_f$	$h_{fg}$	$h_g$	$s_f$	$s_{fg}$	$s_g$
0.01	0.000611	0.001000	206.1	0.0	2375.3	0.0	2501.3	2501.3	0.0000	9.1571	9.1571
2	0.0007056	0.001000	179.9	8.4	2378.1	8.4	2496.6	2505.0	0.0305	9.0738	9.1043
5	0.0008721	0.001000	147.1	21.0	2382.2	21.0	2489.5	2510.5	0.0761	8.9505	9.0266
10	0.001228	0.001000	106.4	42.0	2389.2	42.0	2477.7	2519.7	0.1510	8.7506	8.9016
20	0.002338	0.001002	57.79	83.9	2402.9	83.9	2454.2	2538.1	0.2965	8.3715	8.6680
30	0.004246	0.001004	32.90	125.8	2416.6	125.8	2430.4	2556.2	0.4367	8.0174	8.4541
40	0.007383	0.001008	19.52	167.5	2430.1	167.5	2406.8	2574.3	0.5723	7.6855	8.2578
50	0.01235	0.001012	12.03	209.3	2443.5	209.3	2382.8	2592.1	0.7036	7.3735	8.0771
60	0.01994	0.001017	7.671	251.1	2456.6	251.1	2358.5	2609.6	0.8310	7.0794	7.9104
70	0.03119	0.001023	5.042	292.9	2469.5	293.0	2333.8	2626.8	0.9549	6.8012	7.7561
80	0.04739	0.001029	3.407	334.8	2482.2	334.9	2308.8	2643.7	1.0754	6.5376	7.6130

Table C.2 Properties of Saturated H<sub>2</sub>O—Pressure Table

P, MPa	T, °C	v, m <sup>3</sup> /kg		u, kJ/kg		h, kJ/kg			s, kJ/kg·K		
		v <sub>f</sub>	v <sub>g</sub>	u <sub>f</sub>	u <sub>g</sub>	h <sub>f</sub>	h <sub>fg</sub>	h <sub>g</sub>	s <sub>f</sub>	s <sub>fg</sub>	s <sub>g</sub>
0.0006	0.01	0.001000	206.1	0.0	2375.3	0.0	2501.3	2501.3	0.0000	9.1571	9.1571
0.0008	3.8	0.001000	159.7	15.8	2380.5	15.8	2492.5	2508.3	0.0575	9.0007	9.0582
0.001	7.0	0.001000	129.2	29.3	2385.0	29.3	2484.9	2514.2	0.1059	8.8706	8.9765
0.0012	9.7	0.001000	108.7	40.6	2388.7	40.6	2478.5	2519.1	0.1460	8.7639	8.9099
0.0014	12.0	0.001001	93.92	50.3	2391.9	50.3	2473.1	2523.4	0.1802	8.6736	8.8538
0.0016	14.0	0.001001	82.76	58.9	2394.7	58.9	2468.2	2527.1	0.2101	8.5952	8.8053
0.002	17.5	0.001001	67.00	73.5	2399.5	73.5	2460.0	2533.5	0.2606	8.4639	8.7245
0.003	24.1	0.001003	45.67	101.0	2408.5	101.0	2444.5	2545.5	0.3544	8.2240	8.5784
0.004	29.0	0.001004	34.80	121.4	2415.2	121.4	2433.0	2554.4	0.4225	8.0529	8.4754
0.006	36.2	0.001006	23.74	151.5	2424.9	151.5	2415.9	2567.4	0.5208	7.8104	8.3312
0.008	41.5	0.001008	18.10	173.9	2432.1	173.9	2403.1	2577.0	0.5924	7.6371	8.2295
0.01	45.8	0.001010	14.67	191.8	2437.9	191.8	2392.8	2584.6	0.6491	7.5019	8.1510
0.012	49.4	0.001012	12.36	206.9	2442.7	206.9	2384.1	2591.0	0.6961	7.3910	8.0871
0.014	52.6	0.001013	10.69	220.0	2446.9	220.0	2376.6	2596.6	0.7365	7.2968	8.0333
0.016	55.3	0.001015	9.433	231.5	2450.5	231.5	2369.9	2601.4	0.7719	7.2149	7.9868
0.018	57.8	0.001016	8.445	241.9	2453.8	241.9	2363.9	2605.8	0.8034	7.1425	7.9459
0.02	60.1	0.001017	7.649	251.4	2456.7	251.4	2358.3	2609.7	0.8319	7.0774	7.9093
0.03	69.1	0.001022	5.229	289.2	2468.4	289.2	2336.1	2625.3	0.9439	6.8256	7.7695
0.04	75.9	0.001026	3.993	317.5	2477.0	317.6	2319.1	2636.7	1.0260	6.6449	7.6709
0.06	85.9	0.001033	2.732	359.8	2489.6	359.8	2293.7	2653.5	1.1455	6.3873	7.5328
0.08	93.5	0.001039	2.087	391.6	2498.8	391.6	2274.1	2665.7	1.2331	6.2023	7.4354
0.1	99.6	0.001043	1.694	417.3	2506.1	417.4	2258.1	2675.5	1.3029	6.0573	7.3602
0.12	104.8	0.001047	1.428	439.2	2512.1	439.3	2244.2	2683.5	1.3611	5.9378	7.2980
0.14	109.3	0.001051	1.237	458.2	2517.3	458.4	2232.0	2690.4	1.4112	5.8360	7.2472
0.16	113.3	0.001054	1.091	475.2	2521.8	475.3	2221.2	2696.5	1.4553	5.7472	7.2025
0.18	116.9	0.001058	0.9775	490.5	2525.9	490.7	2211.1	2701.8	1.4948	5.6683	7.1631
0.2	120.2	0.001061	0.8857	504.5	2529.5	504.7	2201.9	2706.6	1.5305	5.5975	7.1280
0.3	133.5	0.001073	0.6058	561.1	2543.6	561.5	2163.8	2725.3	1.6722	5.3205	6.9927
0.4	143.6	0.001084	0.4625	604.3	2553.6	604.7	2133.8	2738.5	1.7770	5.1197	6.8967

Table C.3 Superheated Steam

<i>T</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>
	<i>P</i> = 0.010 MPa (45.81°C)				<i>P</i> = 0.050 MPa (81.33°C)				<i>P</i> = 0.10 MPa (99.63°C)			
Sat.	14.67	2438	2585	8.150	3.240	2484	2646	7.594	1.694	2506	2676	7.359
50	14.87	2444	2593	8.175	—	—	—	—	—	—	—	—
100	17.20	2516	2688	8.448	3.418	2512	2682	7.695	1.696	2507	2676	7.361
150	19.51	2588	2783	8.688	3.889	2586	2780	7.940	1.936	2583	2776	7.613
200	21.82	2661	2880	8.904	4.356	2660	2878	8.158	2.172	2658	2875	7.834
250	24.14	2736	2977	9.100	4.820	2735	2976	8.356	2.406	2734	2974	8.033
300	26.44	2812	3076	9.281	5.284	2811	3076	8.537	2.639	2810	3074	8.216
400	31.06	2969	3280	9.608	6.209	2969	3279	8.864	3.103	2968	3278	8.544
500	35.68	3132	3489	9.898	7.134	3132	3489	9.155	3.565	3132	3488	8.834
600	40.30	3302	3705	10.16	8.057	3302	3705	9.418	4.028	3302	3705	9.098

Table C.4 Compressed Liquid

<i>T</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>u</i>	<i>h</i>	<i>s</i>
	<b><i>P</i> = 5 MPa (264.0°C)</b>				<b><i>P</i> = 10 MPa (311.1°C)</b>				<b><i>P</i> = 15 MPa (342.4°C)</b>			
0	0.000998	0.04	5.04	0.0001	0.000995	0.09	10.04	0.0002	0.000993	0.15	15.05	0.0004
20	0.001000	83.65	88.65	0.296	0.000997	83.36	93.33	0.2945	0.000995	83.06	97.99	0.2934
40	0.001006	167.0	172.0	0.570	0.001003	166.4	176.4	0.5686	0.001001	165.8	180.78	0.5666
60	0.001015	250.2	255.3	0.828	0.001013	249.4	259.5	0.8258	0.001010	248.5	263.67	0.8232
80	0.001027	333.7	338.8	1.072	0.001024	332.6	342.8	1.0688	0.001022	331.5	346.81	1.0656
100	0.001041	417.5	422.7	1.303	0.001038	416.1	426.5	1.2992	0.001036	414.7	430.28	1.2955
120	0.001058	501.8	507.1	1.523	0.001055	500.1	510.6	1.5189	0.001052	498.4	514.19	1.5145
140	0.001077	586.8	592.2	1.734	0.001074	584.7	595.4	1.7292	0.001071	582.7	598.72	1.7242
160	0.001099	672.6	678.1	1.938	0.001095	670.1	681.1	1.9317	0.001092	667.7	684.09	1.9260
180	0.001124	759.6	765.2	2.134	0.001120	756.6	767.8	2.1275	0.001116	753.8	770.50	2.1210
200	0.001153	848.1	853.9	2.326	0.001148	844.5	856.0	2.3178	0.001143	841.0	858.2	2.3104
	<b><i>P</i> = 20 MPa (365.8°C)</b>				<b><i>P</i> = 30 MPa</b>				<b><i>P</i> = 50 MPa</b>			
0	0.000990	0.19	20.01	0.0004	0.000986	0.25	29.82	0.0001	0.000977	0.20	49.03	0.0014
20	0.000993	82.77	102.6	0.2923	0.000989	82.17	111.8	0.2899	0.000980	81.00	130.02	0.2848
40	0.000999	165.2	185.2	0.5646	0.000995	164.0	193.9	0.5607	0.000987	161.9	211.21	0.5527
60	0.001008	247.7	267.8	0.8206	0.001004	246.1	276.2	0.8154	0.000996	243.0	292.79	0.8052
80	0.001020	330.4	350.8	1.0624	0.001016	328.3	358.8	1.0561	0.001007	324.3	374.70	1.0440
100	0.001034	413.4	434.1	1.2917	0.001029	410.8	441.7	1.2844	0.001020	405.9	456.89	1.2703
120	0.001050	496.8	517.8	1.5102	0.001044	493.6	524.9	1.5018	0.001035	487.6	539.39	1.4857
140	0.001068	580.7	602.0	1.7183	0.001062	576.9	608.8	1.7088	0.001052	569.8	622.35	1.6815

### EXAMPLE 2.1

Determine the volume change when 10 kg of saturated water is completely vaporized at a pressure of (a) 1 kPa, (b) 260 kPa, and (c) 10 000 kPa.

#### Solution

Table C.2 provides the necessary values. The quantity being sought is  $\Delta V = mv_{fg}$  where  $v_{fg} = v_g - v_f$ . Note that  $P$  is given in MPa.

(a) 1 kPa = 0.001 MPa. Thus,  $v_{fg} = 129.2 - 0.001 = 129.2 \text{ m}^3/\text{kg}$ .

$$\therefore \Delta V = 1292 \text{ m}^3$$

(b) At 0.26 MPa we must interpolate<sup>2</sup> if we use the tables. The tabulated values are used at 0.2 MPa and 0.3 MPa:

$$v_g = \frac{0.26 - 0.2}{0.3 - 0.2} (0.6058 - 0.8857) + 0.8857 = 0.718 \text{ m}^3/\text{kg}$$

The value for  $v_f$  is, to four decimal places, 0.0011 m<sup>3</sup>/kg at 0.2 MPa and at 0.3 MPa; hence, no need to interpolate for  $v_f$ . We then have

$$v_{fg} = 0.718 - 0.0011 = 0.717 \text{ m}^3/\text{kg}. \quad \therefore \Delta V = 7.17 \text{ m}^3$$

(c) At 10 MPa,  $v_{fg} = 0.01803 - 0.00145 = 0.01658 \text{ m}^3/\text{kg}$  so that

$$\Delta V = 0.1658 \text{ m}^3$$

### EXAMPLE 2.2

Four kilograms of water are placed in an enclosed volume of  $1 \text{ m}^3$ . Heat is added until the temperature is  $150^\circ\text{C}$ . Find the (a) pressure, (b) mass of the vapor, and (c) volume of the vapor.

#### Solution

Table C.1 is used. The volume of 4 kg of saturated vapor at  $150^\circ\text{C}$  is  $0.3928 \times 4 = 1.5712 \text{ m}^3$ . Since the given volume is less than this, we assume the state to be in the quality region.

- (a) In the quality region, the pressure is given as  $P = 475.8 \text{ kPa}$ , the value next to the temperature.
- (b) To find the mass of the vapor we must determine the quality. It is found from Eq. (2.3), using  $v = 1/4 = 0.25 \text{ m}^3/\text{kg}$ :

$$\begin{aligned}v &= v_f + x(v_g - v_f) \\0.25 &= 0.00109 + x(0.3928 - 0.00109) \quad \therefore x = 0.6354\end{aligned}$$

Using Eq. (2.2), the vapor mass is

$$m_g = mx = 4 \times 0.6354 = 2.542 \text{ kg}$$

- (c) Finally, the volume of the vapor is found from

$$V_g = v_g m_g = 0.3928 \times 2.542 = 0.998 \text{ m}^3$$

Note: In mixtures where the quality is not close to zero, the vapor phase occupies most of the volume. In this example, with a quality of 63.5 percent, it occupies 99.8 percent of the volume.

### EXAMPLE 2.3

Two kilograms of water are heated at a pressure of 220 kPa to produce a mixture with quality  $x = 0.8$ . Determine the final volume occupied by the mixture.

#### Solution

Use Table C.2. To determine the appropriate values at 220 kPa, linearly interpolate between 0.2 and 0.25 MPa. This provides, at 220 kPa,

$$v_g = \frac{220 - 200}{300 - 200} (0.6058 - 0.8857) + 0.8857 = 0.830 \text{ m}^3/\text{kg}$$
$$\therefore v_f = 0.0011 \text{ m}^3/\text{kg}$$

Note, no interpolation is necessary for  $v_f$  since, for both pressures,  $v_f$  is the same to four decimal places. Using Eq. (2.3), we now find

$$v = v_f + x(v_g - v_f) = 0.0011 + 0.8 \times (0.830 - 0.0011) = 0.662 \text{ m}^3/\text{kg}$$

The total volume occupied by 2 kg is

$$V = mv = 2 \text{ kg} \times 0.662 \text{ m}^3/\text{kg} = 1.32 \text{ m}^3$$



**EXAMPLE 2.4**

Two kilograms of water are contained in a constant-pressure cylinder held at 2.2 MPa. Heat is added until the temperature reaches 800°C. Determine the final volume of the container.

**Solution**

Use Table C.3. Since 2.2 MPa lies between 2 MPa and 2.5 MPa, the specific volume is interpolated to be

$$v = 0.2467 + 0.4(0.1972 - 0.2467) = 0.227 \text{ m}^3/\text{kg}$$

The final volume is then

$$V = mv = 2 \times 0.227 = 0.454 \text{ m}^3$$

The linear interpolation above results in a less accurate number than the numbers in the table. So, the final number has fewer significant digits.

## 2.4 Equations of State

When the vapor of a substance has relatively low density, the pressure, specific volume, and temperature are related by an *equation of state*,

$$Pv = RT$$

where, for a particular gas, the *gas constant* is  $R$ . A gas for which this equation is valid is called an *ideal gas* or sometimes a *perfect gas*. Note that when using the above equation of state the pressure and temperature must be expressed as absolute quantities.

The gas constant  $R$  is related to a *universal gas constant*  $\bar{R}$ , which has the same value for all gases, by the relationship

$$R = \frac{\bar{R}}{M}$$

where  $M$  is the *molar mass*, values of which are tabulated in Tables B.2 and B.3. The *mole* is that quantity of a substance (i.e., that number of atoms or molecules) having a mass which, measured in grams, is numerically equal to the atomic or molecular weight of the substance. In the SI system it is convenient to use instead the kilomole (kmol), which amounts to  $x$  kilograms of a substance of molecular weight  $x$ . For instance, 1 kmol of carbon (with a molecular weight of 12) is a mass of 12 kg; 1 kmol of oxygen is 32 kg. Stated otherwise,  $M = 32$  kg/kmol for  $O_2$ . The value  $R = 8.314$  kJ/kmol·K. For air  $M$  is 28.97 kg/kmol, so that for air  $R$  is 0.287 kJ/ kg·K, or 287 J/kg·K, a value used extensively in calculations involving air.

Other forms of the ideal-gas equation are

$$PV = mRT \quad P = \rho RT \quad PV = n\bar{R}T$$

where  $n$  is the number of moles.

The compressibility factor  $Z$  helps us in determining whether or not the ideal-gas equation can be used. It is defined as

$$Z = \frac{Pv}{RT}$$

and is displayed in Fig. 2.5 for nitrogen. This figure is acceptable for air also, since air is composed mainly of nitrogen. If  $Z = 1$ , or very close to 1, the ideal-gas equation can be used. If  $Z$  is not close to 1, then above Eq. may be used.

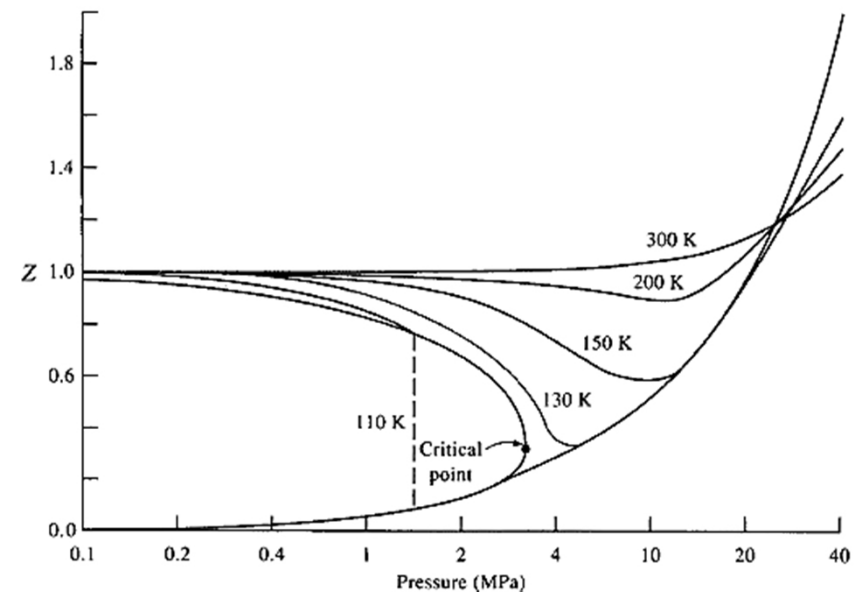


Figure 2.5 The compressibility factor.

The compressibility factor can be determined for any gas by using a generalized compressibility chart presented in App. G. In the generalized chart the *reduced pressure*  $PR$  and *reduced temperature*  $TR$  must be used. They are calculated from

$$P_R = \frac{P}{P_{cr}} \quad T_R = \frac{T}{T_{cr}} \quad v_R = v \frac{P_{cr}}{RT_{cr}}$$

where  $P_{cr}$  and  $T_{cr}$  are the critical-point pressure and temperature, respectively, of Table B.3, and  $v_R$  is the pseudo reduced volume in App. G.

### EXAMPLE 2.5

An automobile tire with a volume of  $0.6 \text{ m}^3$  is inflated to a gage pressure of  $200 \text{ kPa}$ . Calculate the mass of air in the tire if the temperature is  $20^\circ\text{C}$  using the ideal-gas equation of state.

#### Solution

Air is assumed to be an ideal gas at the conditions of this example. In the ideal-gas equation,  $PV = mRT$ , we use absolute pressure and absolute temperature. Thus, using  $P_{\text{atm}} = 100 \text{ kPa}$  (to use a pressure of  $101 \text{ kPa}$  is unnecessary; the difference of 1 percent is not significant in most engineering problems):

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 200 + 100 = 300 \text{ kPa} \quad \text{and} \quad T = T_c + 273 = 20 + 273 = 293 \text{ K}$$

The mass is then calculated to be

$$m = \frac{PV}{RT} = \frac{(300\,000 \text{ N/m}^2)(0.6 \text{ m}^3)}{(287 \text{ N} \cdot \text{m/kg} \cdot \text{K})(293 \text{ K})} = 2.14 \text{ kg}$$

Be careful to be consistent with the units in the above equation.

## 2.5 Equations of State for a Nonideal Gas

Nonideal-gas behavior occurs when the pressure is relatively high ( $> 2$  MPa for many gases) or when the temperature is near the saturation temperature. There are no acceptable criteria that can be used to determine if the ideal-gas equation can be used or if nonideal-gas equations must be used. Usually a problem is stated in such a way that it is obvious that nonideal-gas effects must be included; otherwise a problem is solved assuming an ideal gas.

The *van der Waals equation of state* accounts for the volume occupied by the gas molecules and for the attractive forces between molecules. It is

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

where the constants  $a$  and  $b$  are related to the critical-point data of Table B.3 by

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} \quad b = \frac{RT_{cr}}{8P_{cr}}$$

These constants are also presented in Table B.7 to simplify calculations.

An improved equation is the Redlich-Kwong equation of state:

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)\sqrt{T}}$$

where the constants are also included in Table B.7 and are given by

$$a = 0.4275 \frac{R^2T_{cr}^{2.5}}{P_{cr}} \quad b = 0.0867 \frac{RT_{cr}}{P_{cr}}$$

### EXAMPLE 2.6

Calculate the pressure of steam at a temperature of 500°C and a density of 24 kg/m<sup>3</sup> using (a) the ideal-gas equation, (b) the van der Waals equation, (c) the Redlich-Kwong equation, (d) the compressibility factor, and (e) the steam table.

#### Solution

- (a) Using the ideal-gas equation,

$$P = \rho RT = 24 \times 0.462 \times 773 = 8570 \text{ kPa}$$

where the gas constant for steam is found in Table B.2.

- (b) Using values for  $a$  and  $b$  from Table B.7, the van der Waals equation provides

$$P = \frac{RT}{v-b} - \frac{a}{v^2} = \frac{0.462 \times 773}{\frac{1}{24} - 0.00169} - \frac{1.703}{\frac{1}{24^2}} = 7950 \text{ kPa}$$

- (c) Using values for  $a$  and  $b$  from Table B.7, the Redlich-Kwong equation gives

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)\sqrt{T}} = \frac{0.462 \times 773}{\frac{1}{24} - 0.00117} - \frac{43.9}{\frac{1}{24}(\frac{1}{24} + 0.00117)\sqrt{773}} = 7930 \text{ kPa}$$

- (d) The compressibility factor is found from the generalized compressibility chart of App. G. To use the chart we must know the reduced temperature and pressure (using Table B.3):

$$T_R = \frac{T}{T_{cr}} = \frac{773}{647.4} = 1.19 \qquad P_R = \frac{P}{P_{cr}} = \frac{8570}{22\,100} = 0.39$$

where we have used the ideal-gas pressure from part (a). Using the compressibility chart (it is fairly insensitive to the precise values of  $T_R$  and  $P_R$  so estimates of these values are quite acceptable) and Eq. (2.9), we find

$$P = \frac{ZRT}{v} = \frac{0.93 \times 0.462 \times 773}{1/24} = 7970 \text{ kPa}$$

- (e) The steam table provides the best value for the pressure. Using  $T = 500^\circ\text{C}$  and  $v = 1/24 = 0.0417 \text{ m}^3/\text{kg}$ , we find  $P = 8000 \text{ kPa}$ .

Note that the ideal-gas law has an error of 7.1 percent, and the error of each of the other three equations is less than 1 percent.

## Quiz No. 1

- The phase change from a vapor to a liquid is referred to as
  - vaporization
  - condensation
  - sublimation
  - melting
- The volume occupied by 4 kg of 200°C steam at a quality of 80 percent is nearest
  - 0.004 m<sup>3</sup>
  - 0.104 m<sup>3</sup>
  - 0.4 m<sup>3</sup>
  - 4.1 m<sup>3</sup>
- For a specific volume of 0.2 m<sup>3</sup>/kg, the quality of steam, if the absolute pressure is 630 kPa, is nearest
  - 0.44
  - 0.50
  - 0.59
  - 0.66
- Saturated liquid water occupies a volume of 1.2 m<sup>3</sup>. Heat is added until it is completely vaporized. If the pressure is held constant at 600 kPa, the final volume is nearest
  - 344 m<sup>3</sup>
  - 290 m<sup>3</sup>
  - 203 m<sup>3</sup>
  - 198 m<sup>3</sup>
- Saturated steam is heated in a rigid tank from 70 to 800°C.  $P_2$  is nearest
  - 100 kPa
  - 200 kPa
  - 300 kPa
  - 400 kPa
- Determine the final volume if 3 kg of water is heated at a constant pressure until the quality is 60 percent. The pressure is 270 kPa.
  - 1.07 m<sup>3</sup>
  - 1.24 m<sup>3</sup>
  - 1.39 m<sup>3</sup>
  - 2.93 m<sup>3</sup>
- A vertical circular cylinder holds a height of 1 cm of liquid water and 100 cm of vapor. If  $P = 200$  kPa, the quality is nearest
  - 0.01
  - 0.1
  - 0.4
  - 0.8
- A rigid vessel with a volume of 10 m<sup>3</sup> contains a water-vapor mixture at 400 kPa at 60 percent quality. The pressure is lowered to 300 kPa by cooling the vessel. Find  $m_g$  at state 2.
  - 16.5 kg
  - 19.5 kg
  - 23.8 kg
  - 29.2 kg
- The mass of air in a tire with a volume of 0.2 m<sup>3</sup> with a gage pressure of 280 kPa at 25°C is nearest
  - 7.8 kg
  - 0.889 kg
  - 0.732 kg
  - 0.655 kg
- Estimate the difference in density between the inside and outside of a house in the winter when  $P = 100$  kPa,  $T_{\text{inside}} = 20^\circ\text{C}$ , and  $T_{\text{outside}} = -20^\circ\text{C}$ . (Would you expect any tendency for air exchange due to this difference?)
  - 0.19 kg/m<sup>3</sup>
  - 0.17 kg/m<sup>3</sup>
  - 0.15 kg/m<sup>3</sup>
  - 0.09 kg/m<sup>3</sup>



11. Estimate the pressure of nitrogen at a temperature of 220 K and a specific volume of  $0.04 \text{ m}^3/\text{kg}$  using the van der Waals equation.
  - (A) 1630 kPa
  - (B) 1600 kPa
  - (C) 1580 kPa
  - (D) 1540 kPa
12. Ten kilograms of  $400^\circ\text{C}$  steam are contained in a 734-L tank. Calculate the error in the pressure if it is calculated using the ideal-gas equation (rather than the more accurate steam tables).
  - (A) 1%
  - (B) 4%
  - (C) 6%
  - (D) 18%
4. Calculate the specific volume of water at  $221^\circ\text{C}$  if the quality is 85 percent.
  - (A)  $0.072 \text{ m}^3/\text{kg}$
  - (B)  $0.074 \text{ m}^3/\text{kg}$
  - (C)  $0.76 \text{ m}^3/\text{kg}$
  - (D)  $0.78 \text{ m}^3/\text{kg}$
5. Five kilograms of steam occupy a volume of  $10 \text{ m}^3$ . If the temperature is  $86^\circ\text{C}$ , the quality and the pressure are nearest
  - (A) 0.76, 60 kPa
  - (B) 0.69, 68 kPa
  - (C) 0.71, 63 kPa
  - (D) 0.72, 61 kPa
6. Water at an initial quality of 50 percent is heated at constant volume from 100 kPa to  $200^\circ\text{C}$ . What is  $P_2$ ?
  - (A) 162 kPa
  - (B) 260 kPa
  - (C) 370 kPa
  - (D) 480 kPa

## Quiz No. 2

1. The point that connects the saturated-liquid line to the saturated-vapor line is called the
  - (A) triple point
  - (B) critical point
  - (C) superheated point
  - (D) compressed liquid point
2. Estimate the volume occupied by 10 kg of water at  $200^\circ\text{C}$  and 2 MPa.
  - (A)  $0.099 \text{ m}^3$
  - (B)  $0.016 \text{ m}^3$
  - (C) 11.6 L
  - (D) 11.8 L
3. The specific volume of water at  $200^\circ\text{C}$  and 80 percent quality is nearest
  - (A)  $0.06 \text{ m}^3/\text{kg}$
  - (B)  $0.08 \text{ m}^3/\text{kg}$
  - (C)  $0.1 \text{ m}^3/\text{kg}$
  - (D)  $0.12 \text{ m}^3/\text{kg}$
7. Find the quality of steam at  $120^\circ\text{C}$  if the vapor occupies 1200 L and the liquid occupies 2 L.
  - (A) 42%
  - (B) 28%
  - (C) 18%
  - (D) 4%
8. Two kilograms of steam are contained in a piston-cylinder arrangement. The 20-mm-diameter 48-kg piston is allowed to rise with no friction until the temperature reaches  $260^\circ\text{C}$ . The final volume is nearest
  - (A)  $0.13 \text{ m}^3$
  - (B)  $0.29 \text{ m}^3$
  - (C)  $0.34 \text{ m}^3$
  - (D)  $0.39 \text{ m}^3$

9. Estimate the temperature of 2 kg of air contained in a 40 L volume at 2 MPa.
- (A) 125 K
  - (B) 140 K
  - (C) 155 K
  - (D) 175 K
10. The gage pressure reading on an automobile tire is 240 kPa when the tire temperature is  $-30^{\circ}\text{C}$ . The automobile is driven to a warmer climate and the tire temperature increases to  $65^{\circ}\text{C}$ . Estimate the gage pressure in the tire making reasonable assumptions.
- (A) 480 kPa
  - (B) 370 kPa
  - (C) 320 kPa
  - (D) 280 kPa
11. Estimate the pressure of nitrogen at a temperature of 220 K and a specific volume of  $0.04 \text{ m}^3/\text{kg}$  using the compressibility factor.
- (A) 1630 kPa
  - (B) 1620 kPa
  - (C) 1580 kPa
  - (D) 1600 kPa
12. Ten kilograms of  $100^{\circ}\text{C}$  steam are contained in a 17000-L tank. Calculate the error in the pressure using the ideal-gas equation.
- (A) 1%
  - (B) 4%
  - (C) 6%
  - (D) 18%