Question1

Determine the discrete-time Fourier transform of $x(n) = (0.5)^n\, u(n)$.

Answer

- **HOME WORK.**

Question2

A sequence s(n) defined by $s(n) = \begin{cases} 1\,, & n = 0 \\ \frac{1}{2}\,, & n = \pm 1 \\ 0\,, & |n| > 1 \end{cases}$ is the input to a digital filter characterized by $w(n) + \alpha w(n-1) = s(n) + \beta s(n-1)$ where w(n) is the output response of the filter. Find the Fourier transform $W(e^{jw})$ of the output.

Answer

$$w(n) + \alpha w(n-1) = s(n) + \beta s(n-1)$$

$$W(e^{j\omega}) + \alpha W(e^{j\omega})e^{-j\omega} = S(e^{j\omega}) + \beta S(e^{j\omega})e^{-j\omega}$$

$$W(e^{j\omega})[1 + \alpha e^{-j\omega}] = S(e^{j\omega})[1 + \beta e^{-j\omega}]$$

$$W(e^{j\omega}) = S(e^{j\omega})\frac{[1 + \beta e^{-j\omega}]}{[1 + \alpha e^{-j\omega}]}$$

$$S(e^{j\omega}) = \sum_{n=-\infty}^{\infty} s(n).e^{-j\omega n} = \frac{1}{2}e^{+j\omega} + 1 + \frac{1}{2}e^{-j\omega} = 1 + \cos w = 2cos^2 w$$

$$\boldsymbol{W(e^{j\omega}) = 2cos^2 w\frac{[1 + \beta e^{-j\omega}]}{[1 + \alpha e^{-j\omega}]}}$$

Question3

Determine the IDFT of X(k) = {3, (2+j), 1, (2-j)}.

Answer

- **HOME WORK.**

Question4

If you have the following sequence $x(n) = \begin{cases} 2^n\,, & n = 0, 2, 4, \ldots \\ 3^n\,, & n = 1, 3, 5, \ldots \\ 0\,, & n < 0 \end{cases}$

Find the DFT of the sequence.

Answer

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n).e^{-j\omega n} = \sum_{n=-\infty}^{-1} 0.e^{-j\omega n} + \sum_{n=0}^{\infty} x(n).e^{-j\omega n}$$

$$= 0 + \sum_{n=0,2,4,\dots}^{\infty} 2^n.e^{-j\omega n} + \sum_{n=1,3,5,\dots}^{\infty} 3^n.e^{-j\omega n} = \sum_{n=0,2,4,\dots}^{\infty} (2.e^{-j\omega})^n + \sum_{n=1,3,5,\dots}^{\infty} (3.e^{-j\omega})^n$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (2.e^{-j\omega})^{2n} + \sum_{n=0}^{\infty} (3.e^{-j\omega})^{2n+1}$$

$$= \sum_{n=0}^{\infty} (4.e^{-j2\omega})^n + \sum_{n=0}^{\infty} (3.e^{-j\omega})^{2n}.(3.e^{-j\omega})^1 = \sum_{n=0}^{\infty} (4.e^{-j2\omega})^n + 3e^{-j\omega}\sum_{n=0}^{\infty} (9.e^{-j2\omega})^n$$

$$\boldsymbol{X(e^{j\omega}) = \frac{1}{1-4e^{-j2\omega}} + \frac{3e^{-j\omega}}{1-9e^{-j2\omega}}}$$

NOTES

$$1 - \sum_{n=0,2,4,\dots}^{\infty} a^n = \sum_{n=0}^{\infty} a^{2n}$$

$$2 - \sum_{n=1,3,5,\dots}^{\infty} a^n = \sum_{n=0}^{\infty} a^{2n+1}$$

$$3 - \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Property	Aperiodic Signal	Fourier Transform
	$x[n]$ $y[n]$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ periodic with period 2π
Linearity	$Ax[n]+By[n]$	$AX(e^{j\omega})+BY(e^{j\omega})$
Time shifting	$x[n-n_0]$	$X(e^{j\omega})e^{-j\omega n_0}$
Frequency shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time reversal	$x[-n]$	$X(e^{-j\omega})$
Time expansion	$x_{(k)}[n]=\begin{cases} x[n/k], & \text{if } n \text{ is a multiple of } k \\ 0, & \text{if } n \text{ is not a multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Differentiation in time	$x[n]-x[n-1]$	$(1-e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j\omega}}X(e^{j\omega})$ $+\pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega-2\pi k)$
Differentiation in frequency	$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
Conjugate symmetry for real signals	$x[n]$ real	$\begin{cases} X(e^{j\omega})=X^*(e^{-j\omega}) \\ \mathrm{Re}\{X(e^{j\omega})\}=\mathrm{Re}\{X(e^{-j\omega})\} \\ \mathrm{Im}\{X(e^{j\omega})\}=-\mathrm{Im}\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert=\lvert X(e^{-j\omega})\rvert \\ \angle X(e^{j\omega})=-\angle X(e^{-j\omega}) \end{cases}$
Real and even signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Real and odd signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-odd decomposition of real signals	$\begin{cases} x_e[n] & \mathrm{Even}\{x[n]\}\ [x[n]\ \text{real}] \\ x_o[n] & \mathrm{Odd}\{x[n]\}\ [x[n]\ \text{real}] \end{cases}$	$\mathrm{Re}\{X(e^{j\omega})\}$ $j\,\mathrm{Im}\{X(e^{j\omega})\}$
Parseval's relation for aperiodic signals $\sum_{n=-\infty}^{+\infty}\lvert x[n]\rvert^2=\frac{1}{2\pi}\int_{2\pi}\lvert X(e^{j\omega})\rvert^2 d\omega$		