Question1

Sketch each of the following continuous-time signals. For each case, specify if the signal is causal/non-causal, periodic/non-periodic, odd/even. If the signal is periodic specify its period.

1. x(t) = 2sin(2πt)
2. x(t)= $\begin{cases} 3e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$
3. x(t)= $\begin{cases} 1 - e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$
4. x(t) = $\frac{1}{|t|}$

Answer

1. Non-causal, because it takes non-zero values for −∞ < t < ∞. Periodic with period 1. Odd because x(−t) = −x(t).

The fundamental period is T_0 = 2π/ω = 2π/2π = 1.

2. **HOME WORK.**
3. Causal, because it takes non-zero values for 0 ≤ t < ∞. Non-periodic. Neither odd nor even.
4. **HOME WORK.**

Question2

Sketch the signal

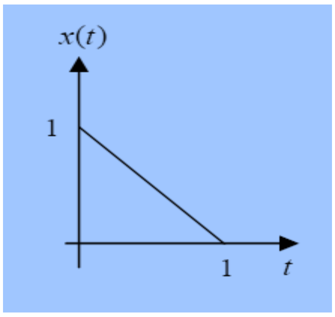
x(t)= $\begin{cases} 1 - t, & 0 \leq t \leq 1 \\ 0, & otherwise \end{cases}$

Now for each of the following describe briefly in words how each of the signals can be derived from the original signal x(t) .

1. x(t + 3)
2. x(t −1)
3. x(3t)
4. x(t / 3)
5. x(−t)
6. x(−t + 2)

Answer

1. Left shift by 3.
2. Right shift by 1.
3. Linearly compress by factor of 3.
4. Linearly stress (expand) by factor of 3.
5. Time reverse.
6. Time reverse and shift right by 2.



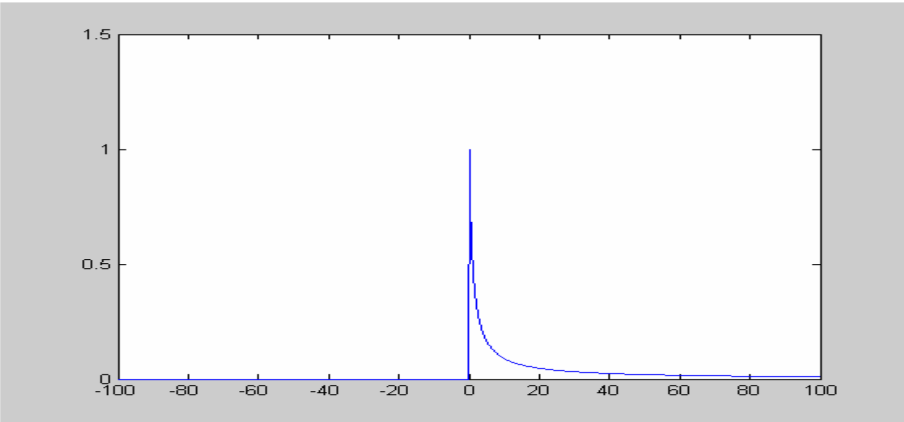
Question3

Sketch each of the following discrete-time signals. For each case, specify if the signal is causal/non-causal, periodic/non-periodic, odd/even. If the signal is periodic specify its period.

1. x[n] = $\cos(n\pi)$
2. x(t)= $\begin{cases} \frac{1}{n+1}, & n \geq 0 \\ 0, & n < 0 \end{cases}$
3. x(t)= $\begin{cases} 0.5^{-n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$

Answer

1. Non-causal, because it takes non-zero values for $-\infty < n < \infty$. Periodic with period 2. Even because $x[-n] = x[n]$. We all know how it looks like.
2. Causal, because it takes non-zero values for $0 \leq n < \infty$. Non-periodic. Neither odd nor even.
3. **HOME WORK.**



Question4

Consider a discrete-time signal x[n], fed as input into a system. The system produces the discrete-time output y[n] such that

$$y[n]= \begin{cases} x[n], & n \ \boldsymbol{even} \\ 0, & n \ \boldsymbol{odd} \end{cases}$$

(i) Is the system described above memoryless? Explain.
(ii) Is the system described above causal? Explain.
(iii) Are memoryless systems in general causal? Explain.
(iv) Are causal systems in general memoryless? Explain.

Answer

(i) It is memoryless since the output at time instant n depends on the input only at time instant n and not past or future time instants.

(ii) It is causal since the output at time instant n depends on the input only at time instant n and not future time instants.

(iii) Obviously yes.

(iv) No. If the output at time instant n depends on the input at time instant n and past time instants the system is causal but not memoryless.

Question5

State with a brief explanation if the following systems are linear/non-linear, causal/non-causal, time-invariant/ time-varying.

1. $y[n] = x[n] - x[n-1]$
2. $y[n] = ax[n] + b$
3. $y[n] = n^2x[n+2]$

Answer

1. Linear, causal, time invariant.

Proof

We see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$, then the input signal $a_1x_1[n]+a_2x_2[n]$ produces the output $y_3[n] = (a_1x_1[n]+a_2x_2[n]) - (a_1x_1[n-1]+a_2x_2[n-1]) = (a_1x_1[n] - a_1x_1[n]) + (a_2x_2[n] - a_2x_2[n-1]) = a_1y_1[n] + a_2y_2[n]$

Therefore, **the system is linear**.

If the input signal x[n] produces an output signal y[n] then the input signal $x[n-n_0]$ produces the output $y_1[n] = x[n-n_0] - x[n-1-n_0]$. We see that $y[n-n_0] = y_1[n]$.

Therefore, **the system is time invariant**.

2. **HOME WORK.**
3. Linear, non-causal, time varying.

We see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$, then the input signal $a_1x_1[n]+a_2y_2[n]$ produces the output $y3[n] = n^2(a_1x_1[n+2]+a_2y_2[n+2]) = (a_1\, n^2x_1[n+2]+a_2\, n^2y_2[n+2] = a_1y_1[n] + a_2y_2[n]$

Therefore, **the system is linear**.

If the input signal x[n] produces an output signal y[n] then the input signal $x[n-n_0]$ produces the output $y_1[n] = n^2x[n-n_0]$.

We see that $y[n-n_0] = (n-n_0)^2x[n-n_0] \neq y_1[n]$.

Therefore, **the system is time varying.**