**AL FURAT AL AWSAT TECHNICAL UNIVERSITY**

**NAJAF COLLEGE OF TECHNOLOGY**

**DEPARTMENT OF TECHNICAL COMMUNICATIONS ENGINEERING**

# DIGITAL SIGNAL PROCESSING

## 3rd YEAR

**Academic Responsible**

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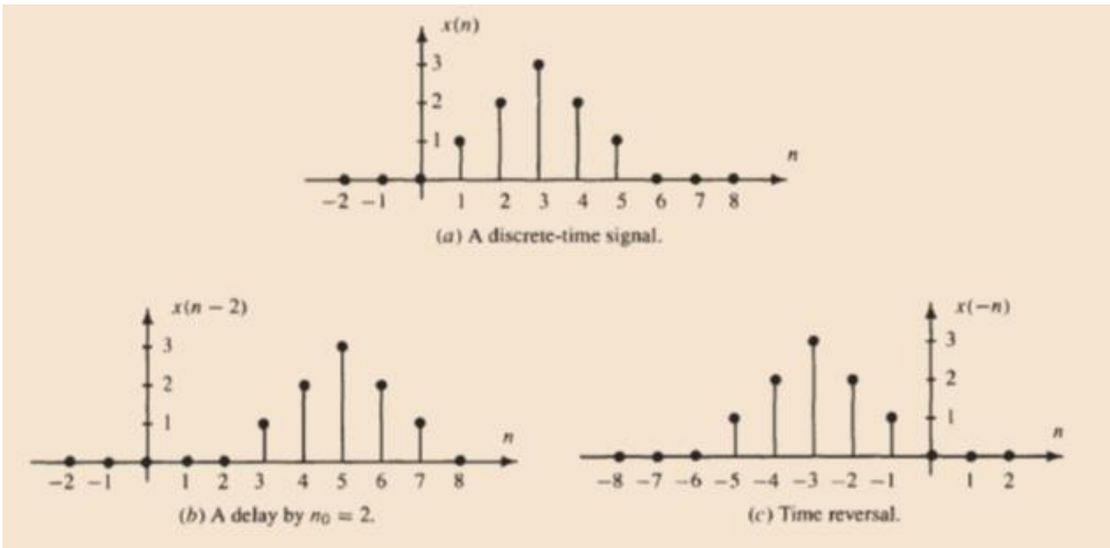
# Representation of an arbitrary sequence

Any arbitrary sequence x[n] can be represented in terms of delayed and scaled impulse sequence δ[n] as shown in the figure




(a) A discrete-time signal.

(b) A delay by $n_0 = 2$.

(c) Time reversal.

## Example

Represent the sequence x[n] = {4, 2, -1, 1, 3, 2, 1, 5} as sum of shifted unit impulse.
$\uparrow$

**Solution** Given x[n] = {4, 2, -1, 1, 3, 2, 1, 5}; n = -3 -2 -1 0 1 2 3 4
$\uparrow$

x[n] = x[-3]δ[n+3] + x[-2] δ[n+2] + x[-1] δ[n+1] +x[0] δ[n] + x[1] δ[n-1] + x[2] δ[n-2] + x[3] δ[n-3] + x[4] δ[n-4] = 4 δ[n+3] +2 δ[n+2] - δ[n-1] + δ[n] +3 δ[n-1] + 2 δ[n-2] + δ[n-3] +5 δ[n-4]

# Discrete time LTI systems: The convolution sum

## The convolution sum representation of LTI systems

Consider a discrete LTI system with input

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

**According to the definition of linearity**, the response (output) of the system to the signal x[n] = $\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$, (which is a linear combination of shifted unit impulses δ[n-k]) is equal to the sum of the scaled responses of the system to each of the shifted unit impulses. If we denote the response of the system to the shifted unit impulse *δ[n- k]* with $h_k[n]$, then the response to the input x[n] will be y[n] = $\sum_{k=-\infty}^{+\infty} x[k]h_k[n]$. Note that *$h_0[n]$* will be the response (output) of the system to the unshifted impulse δ[n]. Moreover, **according to the definition of time-invariance** the condition *$h_k[n] = h_0[n-k]$* holds. For simplicity we drop the subscript on *$h_0[n]$* and define the so called **impulse response** (or unit impulse response) *$h[n] = h_0[n]$*, that is the response of the system when the input is the unit impulse δ[n]. We then conclude that for an LTI system we have

$$\mathbf{y[n]} = \sum_{k=-\infty}^{+\infty} \boldsymbol{x[k]h[n-k]}$$

The above function y[n] is the **convolution sum** or **superposition sum** of the sequences x[n] and h[n]. This operation is represented symbolically as x[n]*h[n]. From the above result we see that an LTI system is completely characterized by its response to the unit impulse. Note also that

$$\mathbf{y[n]} = \sum_{k=-\infty}^{+\infty} \boldsymbol{x[k]h[n-k]} = \sum_{k=-\infty}^{+\infty} \boldsymbol{x[n-k]h[k]}$$

$$\boldsymbol{n = n_1 + n_2 - 1}$$

Where $n_1$ = number of samples of x(n), $n_2$ = number of samples of h(n), and n = total number of samples.

The basic idea behind convolution is to use the system's response to a simple input signal to calculate the response to more complex signals.

## Graphical method:

The convolution sum of two sequences can be found by using the following steps:

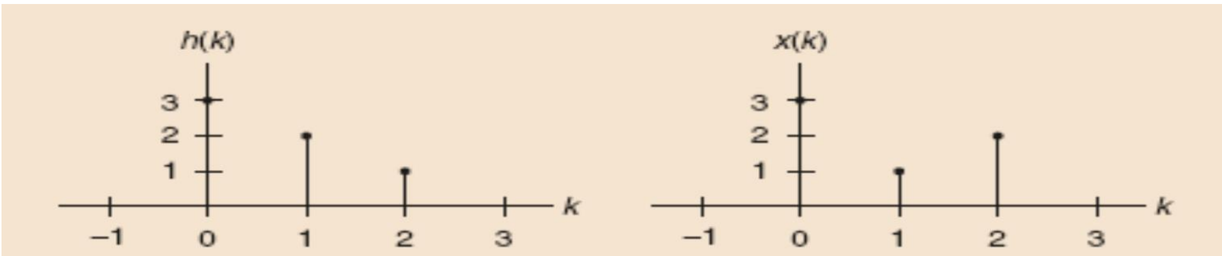
1. Choose an initial value of n, the starting time for evaluating the output sequence y[n]. If x[n] starts at $n = n_1$ and h[n] starts at $n = n_2$ then $n = n_1 + n_2$ is a good choice.
2. Express both sequences in terms of the index *k*.
3. Fold (flip) *h[k]* about *k = 0* to obtain *h[-k]* and shift by *n* to the right if *n* is positive and left if *n* is negative to obtain *h[n-k]*.
4. Multiply the two sequences *x[k]* and *h[n-k]* element by element and sum up the products to get y[n].
5. Increment the index *n*, shift the sequence *h[n-k]* to right by one sample and do **step4**.
6. Repeat **step5** until the sum of products is zero for all the remaining values of *n*.

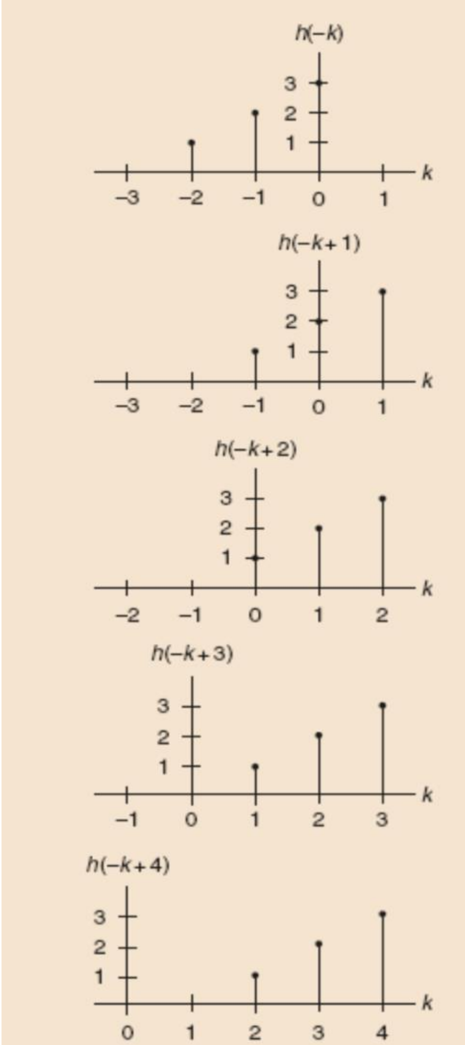
**Example**

Using graphical and direct method find y[n] if the input x[n] = {**3**, 1, 2} and h[n] = {**3**, 2, 1}.

The bold number shows where n=0.

**Solution:**



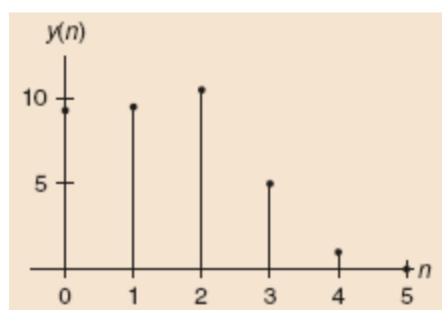

**3x3=9**

**2x3+3x1=9**

**1x3+2x1+3x2=11**

**0x3+1x1+2x2+3x0=5**

**0x3+0x1+1x2+2x0=2**



**Direct method**

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

x[n] = {**3**, 1, 2} , h[n] = {**3**, 2, 1}

Total no. of samples n = $n_1 + n_2$ -1 = 3 +3 -1 = 5 samples

$$n = 0,\ y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 3\times 3 + 1\times 0 + 2\times 0 = 9,$$
$$n = 1,\ y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 3\times 2 + 1\times 3 + 2\times 0 = 9,$$
$$n = 2,\ y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 3\times 1 + 1\times 2 + 2\times 3 = 11,$$
$$n = 3,\ y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) = 3\times 0 + 1\times 1 + 2\times 2 = 5.$$
$$n = 4,\ y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) = 3\times 0 + 1\times 0 + 2\times 1 = 2,$$
$$n \geq 5,\ y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) = 3\times 0 + 1\times 0 + 2\times 0 = 0.$$

**Table lookup method**

y(0) = 9

y(1) = 9

y(2) = 11

y(3) = 5

y(4) = 2

| | 3 | 2 | 1 |
|---|---|---|---|
| 3 | 9 | 6 | 3 |
| 1 | 3 | 2 | 1 |
| 2 | 6 | 4 | 2 |

## Continuous time LTI systems: The convolution integral

### The convolution integral representation of LTI systems

Consider a continuous LTI system with input x(t) = $\int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau$. Following analysis similar to that of discrete systems, it is straightforward to show that the output of the system is given as

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(t-\tau)h(\tau)d\tau$$

with h(t) the response of the system when the input is the continuous unit impulse δ(t).

The above is the convolution integral or superposition integral and the operation on the right hand side is known as the convolution of the signals x(t) and h(t). This operation is represented symbolically as in the discrete case with $x(t)*h(t)$.

## Basic Properties of DT Convolution

### 1- Commutativity

Convolution is a commutative operation, meaning signals can be convolved in any order.

$$x(n)*h(n) = h(n)*x(n)$$

This quite naturally is true of the convolution sums themselves, as well.

$$\sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

### 2- Associativity

Convolution is associative, meaning that convolution operations in series can be done in any order.

$$(x(n)*h(n))*g(n) = x(n)*(h(n)*g(n))$$

This is significant because it means systems in series can be reordered.

### 3- Distributivity

Convolution is distributive over addition.

$$x(n)*[h(n) + g(n)] = x(n)*h(n) + x(n)*g(n)$$

This is significant to all parallel connections because it means the following two arrangements are equivalent.