AL FURAT AL AWSAT TECHNICAL UNIVERSITY

NAJAF COLLEGE OF TECHNOLOGY

DEPARTMENT OF TECHNICAL COMMUNICATIONS ENGINEERING

# DIGITAL SIGNAL PROCESSING

## 3rd YEAR

Academic Responsible

HAYDER S. RASHID

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# Realization of Digital Filters

### 3. Cascade (Series) Realization

By expressing the numerator and the denominator polynomials of the transfer function H(z) as a products of polynomials of lower degree, a digital filter often realized as a cascade of low-order filter sections.

Consider the following transfer function

$$H(z) = H_1(z).H_2(z).H_3(z) \dots H_k(z)$$

Where $H_k(z)$ is chosen to be the first or the second-order transfer function (section), which is defined by

$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1}}{1 + a_{k1} Z^{-1}}$$ First-order transfer function.

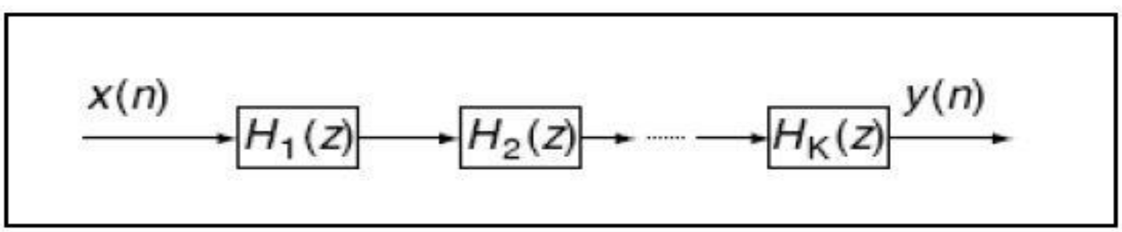
$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1} + b_{k2} Z^{-2}}{1 + a_{k1} Z^{-1} + a_{k2} Z^{-2}}$$ Second-order transfer function.

The transform Y(Z) of the output y(n) is given by the product of the system function H(Z) and the transform X(Z) of the input x(n) as:

$$Y(Z) = H(Z)\,X(Z)$$

$$\boldsymbol{Y(z)} = [H_1(z).H_2(z).H_3(z) \dots H_k(z)].X(z)$$

**The block diagram of the cascade, or series, realization is depicted in figure below.**




**Cascade realization**

**<u>Note:</u>** various different cascade realizations of H(z) can be obtained by different pole-zero polynomial pairings.

## 4. Parallel Realization

An IIR transfer function can be realized in a parallel form by making use of the partial fraction expansion of the transfer function. We can convert H(z) into the following form:

$$H(z) = H_1(z) + H_2(z) + H_3(z) + \cdots + H_k(z)$$

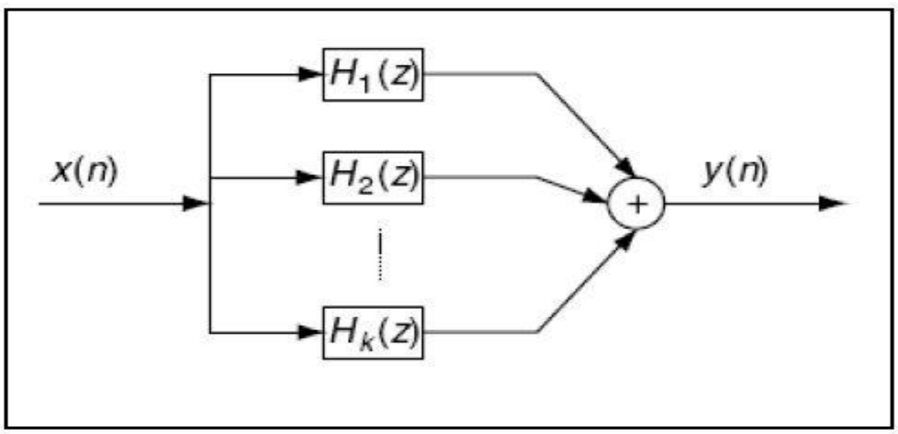
Where $H_k(z)$ is defined as the first or the second-order transfer function (section) given by

$$H_k(Z) = \frac{b_{k0}}{1 + a_{k1} Z^{-1}}$$

Or

$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1}}{1 + a_{k1} Z^{-1} + a_{k2} Z^{-2}}$$

The resulting parallel realization is illustrated in the block diagram in figure below.




**Parallel realization**

## Example

Given a second-order transfer function

Perform the filter realizations and write the difference equations using the following realizations:

1. Cascade form realization.
2. Parallel form realization.

$$\boldsymbol{H(z) = \frac{0.5(1 - Z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}}$$

**Solution:**

To achieve the cascade (series) form realization, we factor H(z) into two first-order sections to yield

$$\boldsymbol{H(z) = \frac{0.5(1 - Z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}} = \frac{(0.5 - 0.5\,Z^{-1})\ (1 + Z^{-1})}{(1 + 0.4\,Z^{-1})\ (1 + 0.9\,Z^{-1})}}$$

$$\boldsymbol{H_1(z) = \frac{(0.5 - 0.5\,Z^{-1})}{(1 + 0.4\,Z^{-1})}}$$

$$\boldsymbol{H_2(z) = \frac{(1 + Z^{-1})}{(1 + 0.9\,Z^{-1})}}$$

Using $H_1(Z)$ and $H_2(Z)$ with the direct-form II realization we achieve the cascade form. The difference equations for the direct-form II realization have two cascaded sections, expressed as

**Section 1:**

$$H_1(z) = \frac{(0.5 - 0.5\,Z^{-1})}{(1 + 0.4\,Z^{-1})}$$

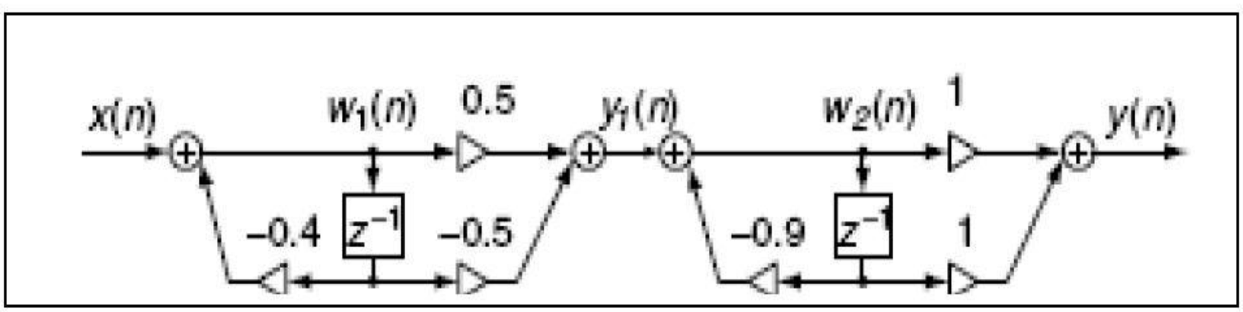
$$w_1(n) = x(n) - 0.4\,w_1(n-1)$$

$$y_1(n) = 0.5\,w_1(n) - 0.5\,w_1(n-1)$$

**Section 2:**

$$w_2(n) = y_1(n) - 0.9\,w_2(n-1)$$

$$y(n) = w_2(n) - w_2(n-1)$$




**Cascade realization**

The obtained $H_1(Z)$ and $H_2(Z)$ are not unique selections for realization. For example, there is another way of choosing them to yield the same H(Z).

In order to yield the parallel form of realization, we need to make use of the partial fraction expansion,

$$\frac{H(Z)}{Z} = \frac{0.5(Z^2 - 1)}{Z(Z + 0.4)(Z + 0.9)} = \frac{A}{Z} + \frac{B}{Z + 0.4} + \frac{C}{Z + 0.9}$$

$$H(Z) = -1.39 + \frac{2.1Z}{Z + 0.4} + \frac{-0.21Z}{Z + 0.9} = -1.39 + \frac{2.1Z}{1 + 0.4Z^{-1}} + \frac{-0.21}{1 + 0.9Z^{-1}}$$

$$H(z) = -1.39 + \frac{2.1}{1+0.4Z^{-1}} - \frac{0.21}{1+0.9Z^{-1}} \Longrightarrow H(z) = H_1(z) + H_2(z) + H_3(z)$$

Using the direct form II for each section, we can obtain the parallel realization. The difference equations for the direct-form II realization have three parallel sections, expressed as:

$y_1(n) = -1.39\ x(n)$

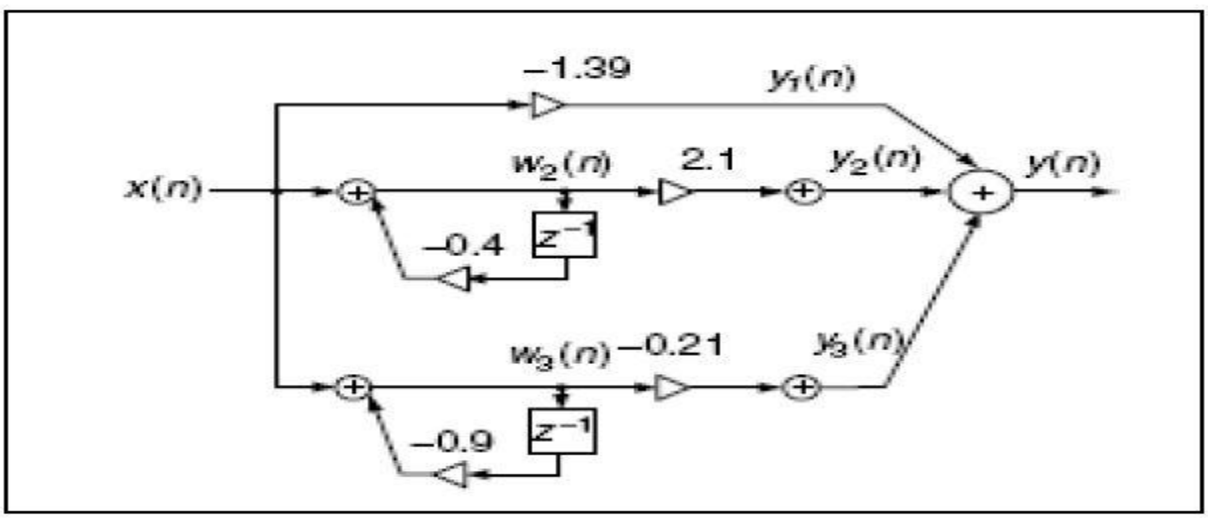
$w_2(n) = x(n) - 0.4\ w_2(n-1)$

$y_2(n) = 2.1\ w_2(n)$

$w_3(n) = x(n) - 0.9\ w_3(n-1)$

$y_3(n) = -0.21\ w_3(n)$

$y(n) = y_1(n) + y_2(n) + y_3(n)$




**Parallel realization**

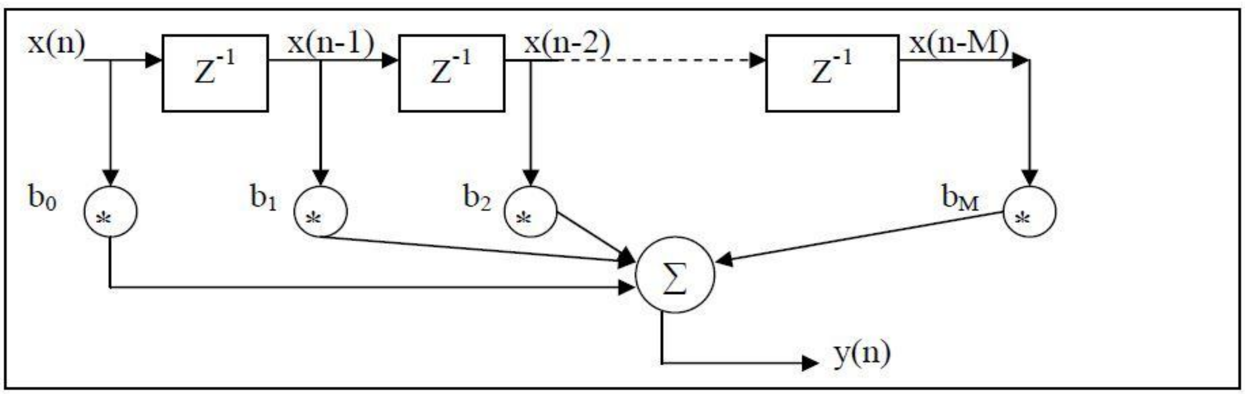
## Realization of FIR Filters

A causal FIR filter is characterized by:

$$\mathrm{H(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

The output is simply a weighted sum of present and past input values, as shown in figure below.




## Example

Realize a system whose transfer function is given by:

$$\boldsymbol{H(z)} = \frac{\mathbf{2Z^4 + Z^3 + 0.8Z^2 + 2Z + 8}}{\mathbf{Z^4}}$$

## Solution

Multiply the transfer function by $(\mathrm{Z}^{-4}/\mathrm{Z}^{-4})$

$$\boldsymbol{H(z) = 2 + Z^{-1} + 0.8Z^{-2} + 2Z^{-3} + 8Z^{-4}}$$

Then, the difference equation is

**y(n) = 2x(n) + x(n-1) + 0.8x(n-2) + 2x(n-3) + 8x(n-4)**