Electric current (I): is the rate of movement of charge through a conductor.

$$I = \frac{Q}{t}$$

Generally, if I is the current in amperes and t the time in seconds during which the current flows, then $I \times t$ represents the quantity of electrical charge in coulombs,
1 ampere = 1 coulomb per second.

potential difference (p.d.) or voltage (V): For a continuous current to flow between two points in a circuit a **potential difference (p.d.)** or **voltage**, ***V***, is required between them; a complete conducting path is necessary to and from the source of electrical energy. The unit of p.d. is the **volt**, **v**.

Resistance (R): The flow of electric current is subject to friction. This friction, or opposition, is called **resistance *R*** and is the property of a conductor that limits current. The unit of resistance is the **ohm (Ω)**; 1 ohm is defined as the resistance which will have a current of 1 ampere flowing through it when 1 volt is connected across it.

Conductance (G): is the reciprocal of resistance and is measured in mho (   ) or siemens (S). Thus

$$G = \frac{1}{R}$$

Power (P): The unit of power is the watt (W) where one watt is one joule per second. Power is defined as the rate of doing work or transferring energy. Thus,

$$P = \frac{W}{t}$$

where W is the work done or energy transferred, in joules, and t is the time, in seconds. Thus,

energy, in joules, $W = Pt$

==

Multiples and submultiples of units:

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount. The eight most common multiples and submultiples, with their meaning, are listed below:

Prefix	Name	Meaning
T	tera	multiply by 1 000 000 000 000 (i.e.$\times 10^{12}$)
G	gega	multiply by 1 000 000 000 (i.e.$\times 10^{9}$)
M	mega	multiply by 1 000 000 (i.e.$\times 10^{6}$)
k	kilo	multiply by 1000 (i.e. $\times 10^{3}$)
m	milli	divide by 1000 (i.e. $\times 10^{-3}$)
µ	micro	divide by 1 000 000 (i.e. $\times 10^{-6}$)
N	nano	divide by 1 000 000 000 (i.e. $\times 10^{-9}$)
P	pico	divide by 1 000 000 000 000 (i.e. $\times 10^{-12}$)

Sources

Ideal voltage and current sources can be further described as either independent sources or dependent sources.

Independent source: in which the value of the voltage or current supplied is specified by the value of the independent source alone.

- **Independent voltage source:** An ideal voltage source is independent of the current through it. If a copper wire were connected across its ends the current through it would be infinite. The symbol for an ideal voltage source is shown in Fig.1.

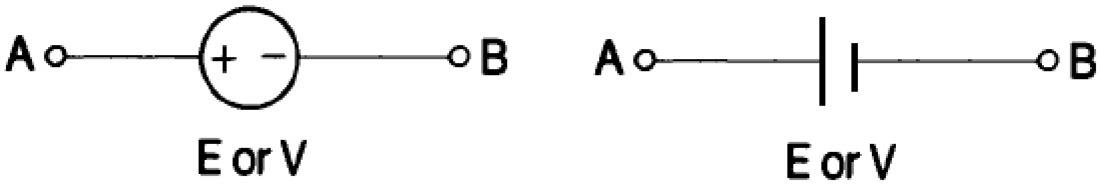

Fig.1

- **Independent current source:** An ideal current source is independent of the voltage across it and if its two ends are not connected to an external circuit the potential difference across it would be infinite. The symbol for a current generator is shown in Fig. 2..

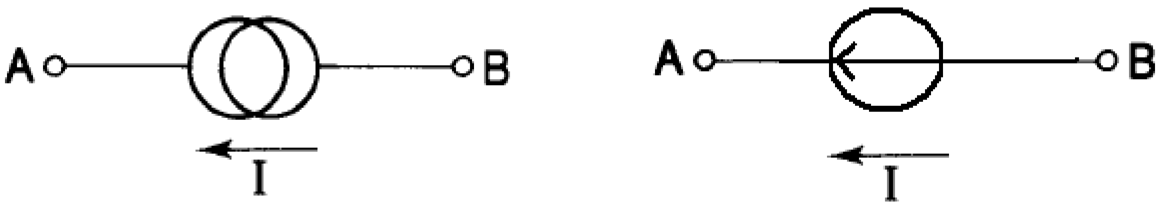

Fig.2

Dependent source (or controlled source): in which the value of the voltage or current cannot specify unless we know the value of the voltage or current on which it depends in the circuit.

- **Dependent voltage source:** The symbol for a dependent voltage source is shown in Fig. 3.
- **Dependent current source:** The symbol for a dependent current source is shown in Fig. 4.

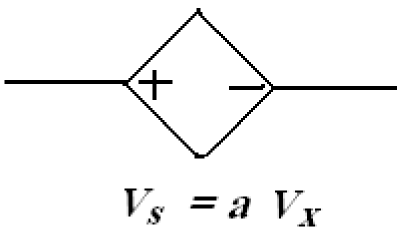

Fig. 3

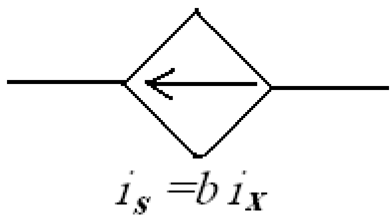

Fig. 4

1- Ohms law

1.1 Ohms law:

Ohm's law states that the current (*I*) flowing in a circuit is directly proportional to the applied voltage *(V)* and inversely proportional to the resistance *R*, provided the temperature remains constant. Thus,

$$I = \frac{V}{R} \quad \text{or} \quad V = IR \quad \text{or} \quad R = \frac{V}{I} \qquad \text{...(1.1)}$$

Example1.1: The current flowing through a resistor is 0.8A when a p.d. of 20V is applied. Determine the value of the resistance.

Solution:

$$\text{resistance } R = \frac{V}{I} = \frac{20}{0.8} = \frac{200}{8} = \mathbf{25\,\Omega}$$

Example1.2: Determine the p.d. which must be applied to a 2 kΩ resistor in order that a current of 10mA may flow.

Solution:

$$\text{Resistance } R = 2\,k\Omega = 2 \times 10^3 = 2000\,\Omega$$

$$\text{Current I} = 10\,\text{mA} = 10 \times 10^{-3}\,\text{A}$$

$$V = IR = (0.01)(2000) = \mathbf{20\ V}$$

Example1.3: A coil has a current of 50mA flowing through it when the applied voltage is 12V. What is the resistance of the coil?

Solution:

$$\text{Resistance, } R = \frac{V}{I} = \frac{12}{50 \times 10^{-3}}$$

$$= \frac{12 \times 10^3}{50} = \frac{12\,000}{50} = \mathbf{240\,\Omega}$$

Example1.4: A 100V battery is connected across a resistor and causes a current of 5mA to flow. Determine the resistance of the resistor. If the voltage is now reduced to 25V, what will be the new value of the current flowing?

Solution:

$$\text{Resistance } R = \frac{V}{I} = \frac{100}{5 \times 10^{-3}} = \frac{100 \times 10^3}{5}$$

$$= 20 \times 10^3 = \mathbf{20\,k\Omega}$$

Current when voltage is reduced to 25 V,

$$I = \frac{V}{R} = \frac{25}{20 \times 10^3} = \frac{25}{20} \times 10^{-3} = \mathbf{1.25\,mA}$$

1.2 Electrical power:

Power *P* in an electrical circuit is given by the product of potential difference *V* and current *I*. The unit of power is the watt,W. Hence:

$$P = V \times I \text{ watts} \quad \text{...(1.2)}$$

From Ohm's law, $V = IR$. Substituting for V in equation (1.2) gives:

$P = (IR) \times I$

i.e. $$P = \frac{V^2}{R} \text{ watts}$$

There are thus three possible formulae which may be used for calculating power.

Example 1.5: A 100W electric light bulb is connected to a 250V supply. Determine (a) the current flowing in the bulb, and (b) the resistance of the bulb.

Solution:

Power $P = V \times I$, from which, current $I = \dfrac{P}{V}$

(a) Current $I = \dfrac{100}{250} = \dfrac{10}{25} = \dfrac{2}{5} = \mathbf{0.4\,A}$

(b) Resistance $R = \dfrac{V}{I} = \dfrac{250}{0.4} = \dfrac{2500}{4} = \mathbf{625\,\Omega}$

Example1.6: Calculate the power dissipated when a current of 4mA flows through a resistance of 5 kΩ.

Solution:

$$\text{Power } P = I^2R = (4 \times 10^{-3})^2(5 \times 10^3)$$
$$= 16 \times 10^{-6} \times 5 \times 10^3$$
$$= 80 \times 10^{-3}$$
$$= \mathbf{0.08\,W} \text{ or } \mathbf{80\,mW}$$

Alternatively, since $I = 4 \times 10^{-3}$ and $R = 5 \times 10^3$ then from Ohm's law, voltage

$$V = IR = 4 \times 10^{-3} \times 5 \times 10^3 = 20\,V$$

Hence,

$$\text{power } P = V \times I = 20 \times 4 \times 10^{-3}$$
$$= \mathbf{80\,mW}$$

Example 1.7: A current of 5A flows in the winding of an electric motor, the resistance of the winding being 100Ω Determine (a) the p.d. across the winding, and (b) the power dissipated by the coil.

Solution:

(a) Potential difference across winding,

$$V = IR = 5 \times 100 = \mathbf{500\,V}$$

(b) Power dissipated by coil,

$$P = I^2R = 5^2 \times 100$$
$$= \mathbf{2500\,W} \text{ or } \mathbf{2.5\,kW}$$

(Alternatively, $P = V \times I = 500 \times 5$
$= \mathbf{2500\,W}$ or $\mathbf{2.5\,kW}$)

1.3 Electrical energy

Electrical energy = power × time ……………………………………..(1.3)

If the power is measured in watts and the time in seconds then the unit of energy is watt-seconds or joules. If the power is measured in kilowatts and the time in hours then the unit of energy is kilowatt-hours, often called the 'unit of electricity'. The 'electricity meter' in the home records the number of kilowatt-hours used and is thus an energy meter.

Example 1.8: A 12V battery is connected across a load having a resistance of 40Ω Determine the current flowing in the load, the power consumed and the energy dissipated in 2 minutes.

Solution:

$$\text{Current } I = \frac{V}{R} = \frac{12}{40} = \mathbf{0.3\,A}$$

$$\text{Power consumed, } P = VI = (12)(0.3) = \mathbf{3.6\,W.}$$

$$\begin{aligned}\text{Energy dissipated} &= \text{power} \times \text{time}\\ &= (3.6\,\text{W})(2 \times 60\,\text{s})\\ &= \mathbf{432\,J}\ (\text{since } 1\,\text{J} = 1\,\text{Ws})\end{aligned}$$

Example 1.9: Electrical equipment in an office takes a current of 13A from a 240V supply. Estimate the cost per week of electricity if the equipment is used for 30 hours each week and 1 kWh of energy costs 12.5dinar.

Solution:

$$\begin{aligned}\text{Power} = VI \text{ watts} &= 240 \times 13\\ &= 3120\,\text{W} = 3.12\,\text{kW}\end{aligned}$$

$$\begin{aligned}\text{Energy used per week} &= \text{power} \times \text{time}\\ &= (3.12\,\text{kW}) \times (30\,\text{h})\\ &= 93.6\,\text{kWh}\end{aligned}$$

Cost at 12.5 dinar per kWh=93.6×12.5=1170 dinar.

1.4 Resistance and resistivity:

The resistance of an electrical conductor depends on four factors, these being:

(a) the length of the conductor (*l*),

(b) the cross-sectional area of the conductor (*a*),

(c) the type of material and

(d) the temperature of the material.

Resistance, *R*, is directly proportional to length of a conductor, i.e. *R*α*l*. Thus, for example, if the length of a piece of wire is doubled, then the resistance is doubled. Resistance, *R*, is inversely proportional to cross-sectional area of a conductor, i.e. *R*α1/*a*. Thus, for example, if the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since *R*α*l* and *R*α1/*a* then *R*α*l*/*a*. By inserting a constant of proportionality into this relationship the type of material used may be taken into account. The constant of proportionality is known as the resistivity of the material and is given the symbol ρ (Greek rho). Thus,

$$\text{resistance} \quad R = \frac{\rho l}{a} \text{ ohms} \qquad \text{……………………………………………………(1.4)}$$

P is measured in ohm metres (Ω.m).

Example 1.10: Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100mm2. Take the resistivity of aluminium to be 0.03×10^{-6} Ω.m.

Solution:

Length $l = 2\text{ km} = 2000\text{ m}$,
area $a = 100\text{ mm}^2 = 100 \times 10^{-6}\text{ m}^2$
and resistivity $\rho = 0.03 \times 10^{-6}\ \Omega\text{ m}$.

$$\text{Resistance } R = \frac{\rho l}{a}$$

$$= \frac{(0.03 \times 10^{-6}\ \Omega\text{ m})(2000\text{ m})}{(100 \times 10^{-6}\text{ m}^2)}$$

$$= \frac{0.03 \times 2000}{100}\ \Omega = \mathbf{0.6\ \Omega}$$

Example 1.11: Calculate the cross-sectional area, in mm2, of a piece of copper wire, 40m in length and having a resistance of 0.25Ω. Take the resistivity of copper as 0.02×10-6 Ω.m.

Solution:

Resistance $R = \rho l/a$ hence cross-sectional area

$$a = \frac{\rho l}{R} = \frac{(0.02 \times 10^{-6}\ \Omega\text{ m})(40\text{ m})}{0.25\ \Omega}$$

$$= 3.2 \times 10^{-6}\text{ m}^2$$

$$= (3.2 \times 10^{-6}) \times 10^{6}\text{ mm}^2 = \mathbf{3.2\text{ mm}^2}$$

Example 1.12: The resistance of 1.5 km of wire of cross-sectional area 0.17mm^2 is 150Ω. Determine the resistivity of the wire.

Solution:

Resistance, $R = \rho l/a$ hence

$$\text{resistivity } \rho = \frac{Ra}{l}$$

$$= \frac{(150\ \Omega)(0.17 \times 10^{-6}\text{ m}^2)}{(1500\text{ m})}$$

$$= \mathbf{0.017 \times 10^{-6}\ \Omega\text{ m}}$$

or $\mathbf{0.017\ \mu\Omega\text{ m}}$

2- Series and parallel networks

2.1 Series circuit:

Figure 2.1 shows three resistors *R*1, *R*2 and *R*3 connected end to end, i.e. in series, with a battery source of *V* volts. Since the circuit is closed a current *I* will flow and the p.d. across each resistor may be determined from the voltmeter readings *V*1, *V*2 and *V*3.

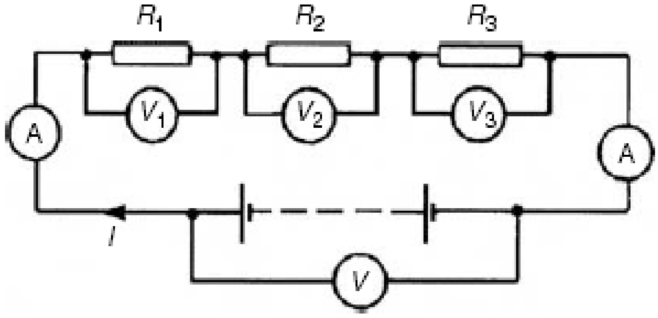

Figure 2.1

In a series circuit

(a) the current *I* is the same in all parts of the circuit and hence the same reading is found on each of the ammeters shown, and

(b) the sum of the voltages *V*1, *V*2 and *V*3 is equal to the total applied voltage, *V*, i.e.

$$V = V1 + V2 + V3 \quad \ldots\ldots(2.1)$$

From Ohm's law: *V*1 =*IR*1, *V*2 =*IR*2, *V*3 =*IR*3 and *V* =*IR* where *R* is the total circuit resistance. Since *V* = *V*1+*V*2+*V*3 then *IR* = *IR*1+*IR*2+*IR*3. Dividing throughout by *I* gives

$$R = R1 + R2 + R3 \quad \ldots\ldots(2.2)$$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistance's.

Example 2.1: For the circuit shown in Fig. 2.2, determine (a) the battery voltage *V*, (b) the total resistance of the circuit, and (c) the values of resistors *R*1, *R*2 and *R*3, given that the p.d.'s across *R*1, *R*2 and *R*3 are 5V, 2V and 6V respectively.

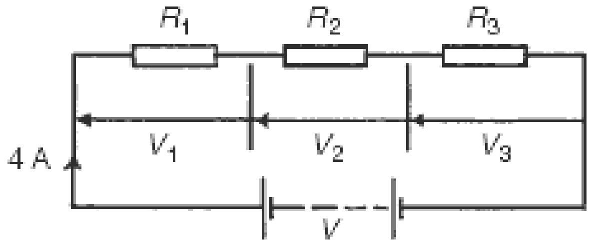

Figure 2.2

Solution:

(a) Battery voltage $V = V_1 + V_2 + V_3$

$$= 5 + 2 + 6 = \mathbf{13\ V}$$

(b) Total circuit resistance $R = \dfrac{V}{I} = \dfrac{13}{4} = \mathbf{3.25\ \Omega}$

(c) Resistance $R_1 = \dfrac{V_1}{I} = \dfrac{5}{4} = \mathbf{1.25\ \Omega}$

$$\text{Resistance } R_2 = \frac{V_2}{I} = \frac{2}{4} = \mathbf{0.5\ \Omega}$$

$$\text{Resistance } R_3 = \frac{V_3}{I} = \frac{6}{4} = \mathbf{1.5\ \Omega}$$

$$\text{(Check: } R_1 + R_2 + R_3 = 1.25 + 0.5 + 1.5 = 3.25\ \Omega = R\text{)}$$

Example 2.2: For the circuit shown in Fig. 2.3, determine the p.d. across resistor *R*3. If the total resistance of the circuit is 100Ω, determine the current flowing through resistor *R*1. Find also the value of resistor *R*2.

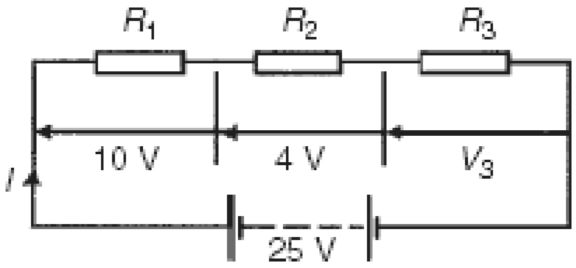

Figure 2.3

Solution:

P.d. across R_3, $V_3 = 25 - 10 - 4 = \mathbf{11\ V}$

$$\text{Current } I = \frac{V}{R} = \frac{25}{100} = \mathbf{0.25\ A},$$

which is the current flowing in each resistor

$$\text{Resistance } R_2 = \frac{V_2}{I} = \frac{4}{0.25} = \mathbf{16\ \Omega}$$

Example 2.3: A 12V battery is connected in a circuit having three series-connected resistors having resistance's of 4Ω, 9Ω and 11Ω. Determine the current flowing through, and the p.d. across the 9Ω resistor. Find also the power dissipated in the 11Ω resistor.

Solution:

The circuit diagram is shown in Fig. 2.4

$$\text{Total resistance } R = 4 + 9 + 11 = 24\ \Omega$$

$$\text{Current } I = \frac{V}{R} = \frac{12}{24} = \mathbf{0.5\ A},$$

which is the current in the 9 Ω resistor. P.d. across the 9 Ω resistor,

$$V_1 = I \times 9 = 0.5 \times 9 = \mathbf{4.5\ V}$$

Power dissipated in the 11Ω resistor,

$$P = I^2R = (0.5)^2(11) = (0.25)(11) = \mathbf{2.75\ W}$$

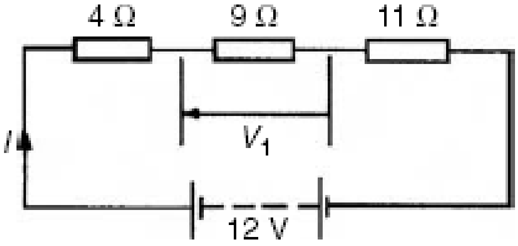

Figure 2.4

2.2 Voltage divider rule

The voltage distribution for the circuit shown in Fig. 2.5(a) is given by:

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V \text{ and } V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V \quad \text{.......................................(2.3)}$$

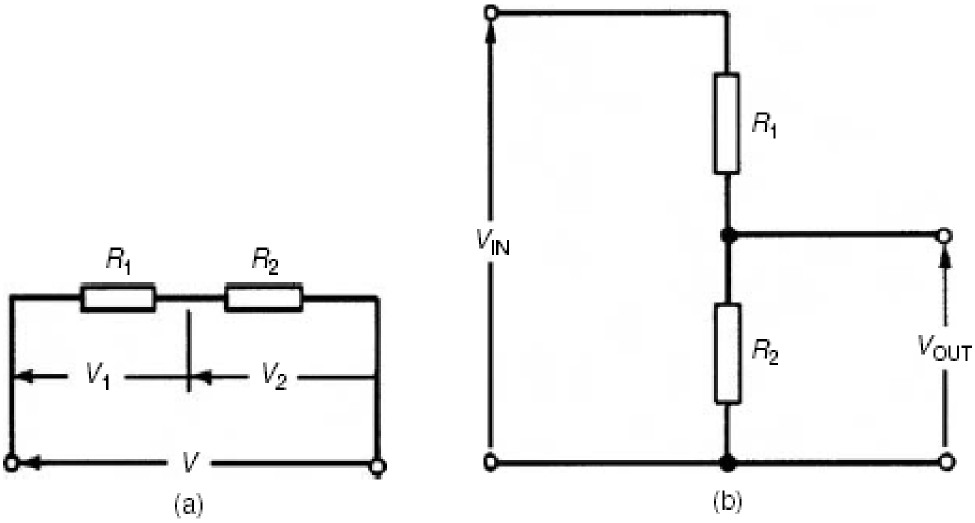

Figure 2.5

The circuit shown in Fig. 2.5(b) is often referred to as a **potential divider** circuit. Such a circuit can consist of a number of similar elements in series connected across a voltage source, voltages being taken from connections between the elements. Frequently the divider consists of two resistors as shown in Fig. 2.5(b), where

$$V_{OUT} = \left(\frac{R_2}{R_1 + R_2}\right)V_{IN} \quad \text{..(2.4)}$$

Example 2.4: Determine the value of voltage *V* shown in Fig. 2.6

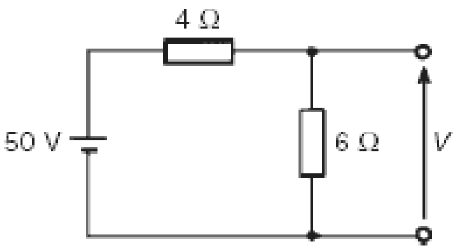

Figure 2.6

Solution:

Figure 2.6 may be redrawn as shown in Fig. 2.7, and

$$\text{voltage } V = \left(\frac{6}{6+4}\right)(50) = \mathbf{30\ V}$$

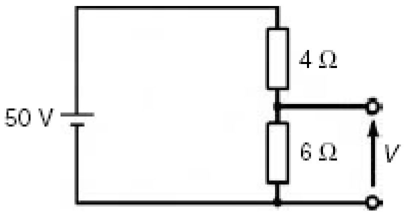

Figure 2.7

Example 2.4: Two resistors are connected in series across a 24V supply and a current of 3A flows in the circuit. If one of the resistors has a resistance of 2Ω determine (a) the value of the other resistor, and (b) the p.d. across the 2Ω resistor. If the circuit is connected for 50 hours, how much energy is used?

Solution:

The circuit diagram is shown in Fig. 2.8

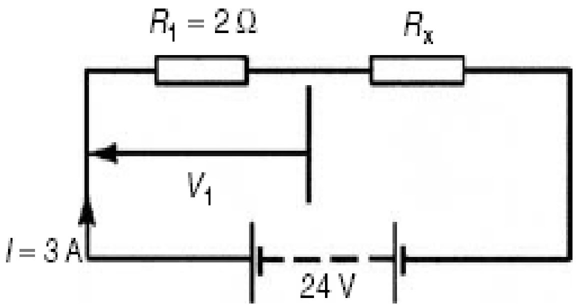

Figure 2.8

(a) Total circuit resistance

$$R = \frac{V}{I} = \frac{24}{3} = 8\ \Omega$$

Value of unknown resistance,

$$R_x = 8 - 2 = \mathbf{6\ \Omega}$$

(b) P.d. across 2 Ω resistor,

$$V_1 = IR_1 = 3 \times 2 = \mathbf{6\ V}$$

Alternatively, from above,

$$V_1 = \left(\frac{R_1}{R_1 + R_x}\right) \text{V}$$

$$= \left(\frac{2}{2+6}\right)(24) = 6\ \text{V}$$

$$\text{Energy used} = \text{power} \times \text{time}$$

$$= (V \times I) \times t$$

$$= (24 \times 3\ \text{W})(50\ \text{h})$$

$$= 3600\ \text{Wh} = \mathbf{3.6\ kWh}$$

Exercise 1: problems on series circuits

1. The p.d's measured across three resistors connected in series are 5V, 7V and 10V, and the supply current is 2A. Determine (a) the supply voltage, (b) the total circuit resistance and (c) the values of the three resistors. [(a) 22 V (b) 11 Ω (c) 2.5 Ω, 3.5 Ω, 5 Ω]

2. For the circuit shown in Fig. 2.9, determine the value of *V*1. If the total circuit resistance is 36Ω determine the supply current and the value of resistors *R*1, *R*2 and *R*3. [10V, 0.5A, 20 Ω, 10 Ω, 6 Ω]

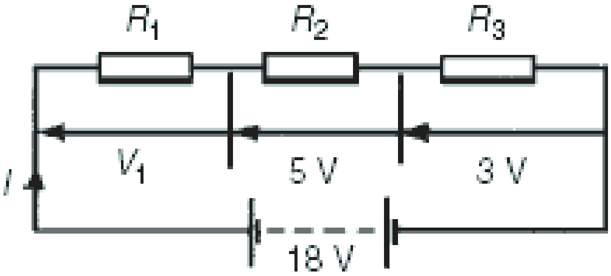

Figure 2.9

3. When the switch in the circuit in Fig. 2.10 is closed the reading on voltmeter 1 is 30V and that on voltmeter 2 is 10V. Determine the reading on the ammeter and the value of resistor *R*x. [4 A, 2.5 Ω]

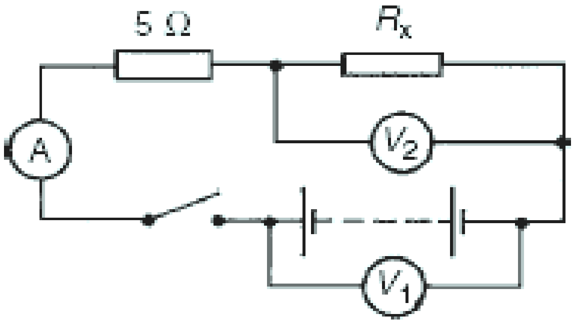

Figure 2.10

4. Calculate the value of voltage *V* in Fig. 2.11. [45 V]

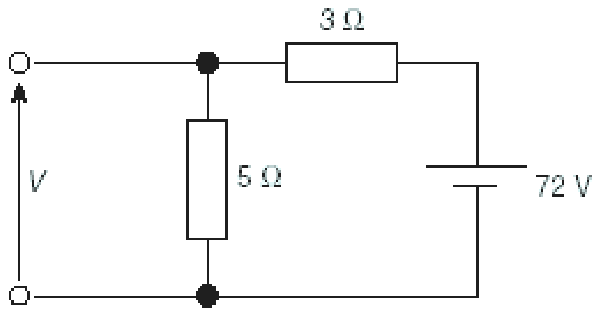

Figure 2.11

5. Two resistors are connected in series across an 18V supply and a current of 5A flows. If one of the resistors has a value of 2.4Ω determine (a) the value of the other resistor and (b) the p.d. across the 2.4Ω resistor. [(a) 1.2 Ω (b) 12 V]

6. An arc lamp takes 9.6A at 55V. It is operated from a 120V supply. Find the value of the stabilising resistor to be connected in series. [6.77 Ω]

7. An oven takes 15A at 240V. It is required to reduce the current to 12A. Find (a) the resistor which must be connected in series, and (b) the voltage across the resistor.

[(a) 4 Ω (b) 48 V]

2.3 Parallel circuit:

Figure 2.12 shows three resistors, *R*1, *R*2 and *R*3 connected across each other, i.e. in parallel, across a battery source of *V* volts.

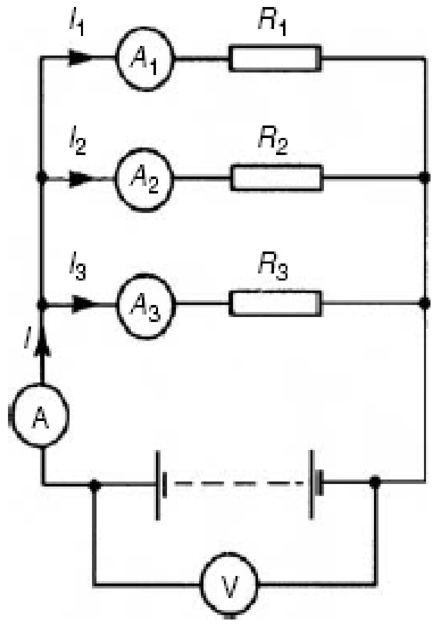

Figure 2.12

In a parallel circuit:

(a) the source p.d., *V* volts, is the same across each of the resistors, and

(b) the sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, *I*, i.e.

$$I = I1 + I2 + I3 \quad \text{..(2.5)}$$

From Ohm's law:

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3} \quad \text{and} \quad I = \frac{V}{R}$$

where *R* is the total circuit resistance.

Since $I = I1 + I2 + I3$ then

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Dividing throughout by *V* gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{..(2.6)}$$

This equation must be used when finding the total resistance *R* of a parallel circuit. For the special case of **two resistors in parallel**

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\text{Hence} \quad R = \frac{R_1 R_2}{R_1 + R_2} \quad \left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right) \quad \text{..(2.7)}$$

Example 2.5: For the circuit shown in Fig. 2.13, determine (a) the reading on the ammeter, and (b) the value of resistor *R*2.

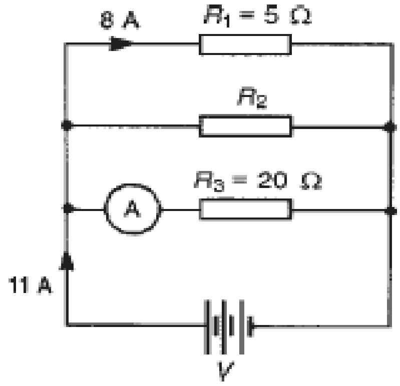

Figure 2.13

Solution:

P.d. across R_1 is the same as the supply voltage V

Hence supply voltage, $V = 8 \times 5 = 40\,\text{V}$

(a) Reading on ammeter,

$$I = \frac{V}{R_3} = \frac{40}{20} = \mathbf{2\,A}$$

(b) Current flowing through $R_2 = 11 - 8 - 2 = 1\,\text{A}$.

Hence

$$R_2 = \frac{V}{I_2} = \frac{40}{1} = \mathbf{40\,\Omega}$$

Example 2.6: For the circuit shown in Fig. 2.14, find (a) the value of the supply voltage *V* and (b) the value of current *I*.

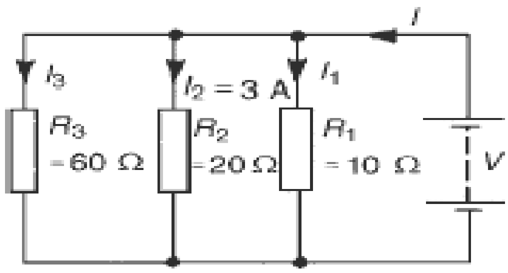

Fig. 2.14

Solution:

(a) P.d. across 20 Ω resistor $= I_2R_2 = 3 \times 20 = 60\,\text{V}$, hence supply voltage $\mathbf{V = 60\,V}$ since the circuit is connected in parallel

(b) Current $I_1 = \dfrac{V}{R_1} = \dfrac{60}{10} = 6\,\text{A}$, $\quad I_2 = 3\,\text{A}$

and $\quad I_3 = \dfrac{V}{R_3} = \dfrac{60}{60} = 1\,\text{A}$

Current $I = I_1 + I_2 + I_3$ hence

$\mathbf{I = 6 + 3 + 1 = 10\,A}$.

Alternatively,

$$\frac{1}{R} = \frac{1}{60} + \frac{1}{20} + \frac{1}{10} = \frac{1+3+6}{60} = \frac{10}{60}$$

Hence total resistance

$$R = \frac{60}{10} = 6\,\Omega, \text{ and current}$$

$$I = \frac{V}{R} = \frac{60}{6} = 10\,\text{A}$$

Example 2.6: Given four 1Ω resistors, state how they must be connected to give an overall resistance of (a) 1/4 Ω (b) 1 Ω (c) 4/3 Ω (d) 5/2 Ω , all four resistors being connected in each case.

Solution:

(a) All four in parallel (see Fig. 2.15), since

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{4}{1} \text{ i.e. } R = \frac{1}{4}\,\Omega$$

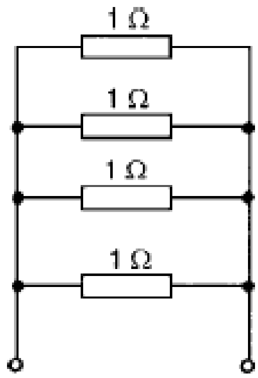

Fig. 2.15

(b) Two in series, in parallel with another two in series (see Fig. 2.16), since 1Ω and 1Ω in series gives 2 Ω, and 2 Ω in parallel with 2 Ω gives

$$\frac{2 \times 2}{2+2} = \frac{4}{4} = 1\,\Omega$$

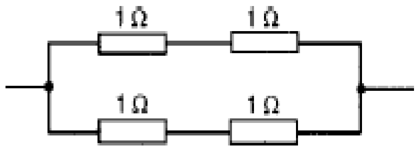

Fig. 2.16

(c) Three in parallel, in series with one (see Fig. 2.17), since for the three in parallel,

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1}$$

i.e. $R = \frac{1}{3}\,\Omega$ and $\frac{1}{3}\,\Omega$ in series with $1\,\Omega$ gives $1\frac{1}{3}\,\Omega$

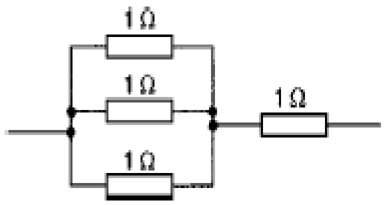

Fig. 2.17

(d) Two in parallel, in series with two in series (see Fig. 2.18), since for the two in parallel

$$R = \frac{1 \times 1}{1+1} = \frac{1}{2}\,\Omega$$

and $\frac{1}{2}\,\Omega$, $1\,\Omega$ and $1\,\Omega$ in series gives $2\frac{1}{2}\,\Omega$

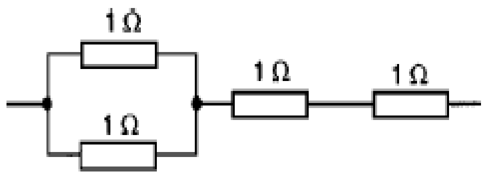

Fig. 2.18

Example 2.7: Find the equivalent resistance for the circuit shown in Fig. 2.19

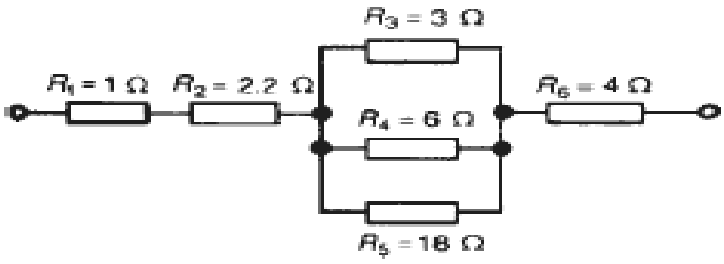

Fig. 2.19

Solution:

$R3$, $R4$ and $R5$ are connected in parallel and their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{18} = \frac{6+3+1}{18} = \frac{10}{18}$$

hence $R=(18/10)=1.8\Omega$. The circuit is now equivalent to four resistors in series and the equivalent circuit resistance=1+2.2+1.8+4=9Ω

2.2 Current divider rule

For the circuit shown in Fig. 2.20, the total circuit resistance, R_T is given by

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

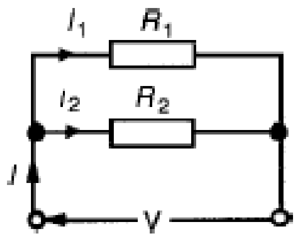

Fig. 2.20

and $$V = IR_T = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Current $$I_1 = \frac{V}{R_1} = \frac{I}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$= \left(\frac{\mathbf{R_2}}{\mathbf{R_1 + R_2}}\right)(\mathbf{I})$$

Similarly,

current $$I_2 = \frac{V}{R_2} = \frac{I}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$= \left(\frac{\mathbf{R_1}}{\mathbf{R_1 + R_2}}\right)(\mathbf{I})$$

Summarising, with reference to Fig. 2.20

$$I_1 = \left(\frac{\mathbf{R_2}}{\mathbf{R_1 + R_2}}\right)(\mathbf{I})$$

and $$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)(\mathbf{I})$$

....(2.8)

Example 2.8: For the series-parallel arrangement shown in Fig. 2.21, find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor.

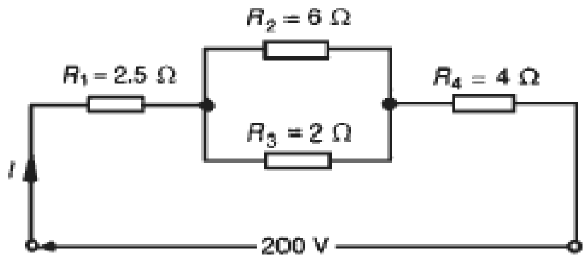

Fig. 2.21

Solution:

(a) The equivalent resistance R_x of R_2 and R_3 in parallel is:

$$R_x = \frac{6 \times 2}{6 + 2} = 1.5\ \Omega$$

The equivalent resistance R_T of R_1, R_x and R_4 in series is:

$$R_T = 2.5 + 1.5 + 4 = 8\ \Omega$$

Supply current

$$I = \frac{V}{R_T} = \frac{200}{8} = \mathbf{25\ A}$$

(b) The current flowing through R_1 and R_4 is **25 A**. The current flowing through R_2

$$= \left(\frac{R_3}{R_2 + R_3}\right) I = \left(\frac{2}{6+2}\right) 25$$
$$= \mathbf{6.25\ A}$$

The current flowing through R_3

$$= \left(\frac{R_2}{R_2 + R_3}\right) I$$
$$= \left(\frac{6}{6+2}\right) 25 = \mathbf{18.75\ A}$$

(c) The equivalent circuit of Fig. 5.21 is shown in Fig. 5.22

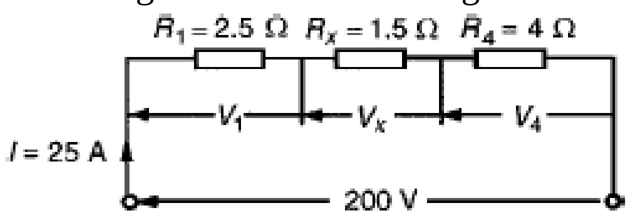

Fig. 2.22

p.d. across R_1, i.e.

$$V_1 = IR_1 = (25)(2.5) = \mathbf{62.5\ V}$$

p.d. across R_x, i.e.

$$V_x = IR_x = (25)(1.5) = \mathbf{37.5\ V}$$

p.d. across R_4, i.e.

$$V_4 = IR_4 = (25)(4) = \mathbf{100\ V}$$

Hence the p.d. across R_2

$$= \text{p.d. across } R_3 = \mathbf{37.5\ V}$$

Example 2.9: For the circuit shown in Fig. 2.23 calculate (a) the value of resistor R_x such that the total power dissipated in the circuit is 2.5kW, (b) the current flowing in each of the four resistors.

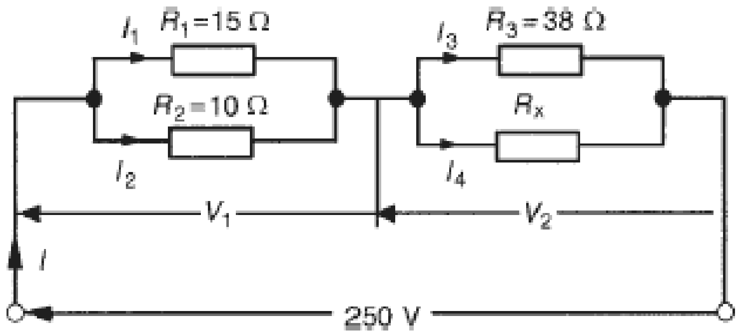

Fig. 2.23

Solution:

(a) Power dissipated $P = VI$ watts, hence

$$2500 = (250)(I)$$

$$\text{i.e. } I = \frac{2500}{250} = 10\,\mathbf{A}$$

From Ohm's law,

$$R_T = \frac{V}{I} = \frac{250}{10} = 25\,\Omega,$$

where R_T is the equivalent circuit resistance. The equivalent resistance of R_1 and R_2 in parallel is

$$\frac{15 \times 10}{15 + 10} = \frac{150}{25} = 6\,\Omega$$

The equivalent resistance of resistors R_3 and R_x in parallel is equal to $25\,\Omega - 6\,\Omega$, i.e. $19\,\Omega$. There are three methods whereby R_x can be determined.

Method 1

The voltage $V_1 = IR$, where R is 6 Ω, from above, i.e. $V_1 = (10)(6) = 60$ V. Hence

$$V_2 = 250\text{ V} - 60\text{ V} = 190\text{ V}$$
$$= \text{p.d. across } R_3$$
$$= \text{p.d. across } R_x$$
$$I_3 = \frac{V_2}{R_3} = \frac{190}{38} = 5\text{ A}.$$

Thus $I_4 = 5$ A also, since $I = 10$ A. Thus

$$\mathbf{R_x} = \frac{V_2}{I_4} = \frac{190}{5} = \mathbf{38\ \Omega}$$

Method 2

Since the equivalent resistance of R_3 and R_x in parallel is 19 Ω,

$$19 = \frac{38R_x}{38 + R_x} \quad \left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$$

Hence

$$19(38 + R_x) = 38R_x$$
$$722 + 19R_x = 38R_x$$
$$722 = 38R_x - 19R_x = 19R_x$$
$$= 19R_x$$

Thus $$\mathbf{R_x} = \frac{722}{19} = \mathbf{38\ \Omega}$$

Method 3

When two resistors having the same value are connected in parallel the equivalent resistance is always half the value of one of the resistors. Thus, in this case, since $R_T = 19\ \Omega$ and $R_3 = 38\ \Omega$, then $R_x = 38\ \Omega$ could have been deduced on sight.

(b) Current $$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$
$$= \left(\frac{10}{15 + 10}\right)(10)$$
$$= \left(\frac{2}{5}\right)(10) = \mathbf{4\ A}$$

Current $$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I = \left(\frac{15}{15 + 10}\right)(10)$$
$$= \left(\frac{3}{5}\right)(10) = \mathbf{6\ A}$$

From part (a), method 1, $\mathbf{I_3 = I_4 = 5\ A}$

Example 2.10: For the arrangement shown in Fig. 2.24, find the current I_x.

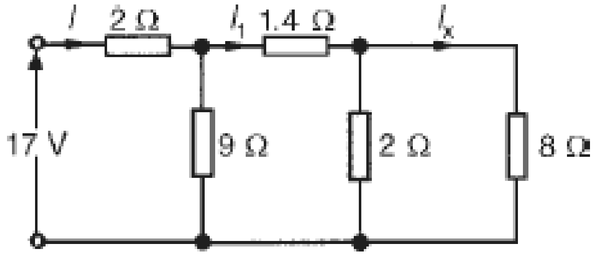

Fig. 2.24

Solution:

The circuit is gradually reduced in stages as shown in Fig. 2.24(a)–(d).

- From Fig. 2.25(d),

$$I = \frac{17}{4.25} = 4\,\text{A}$$

- From Fig. 2.25(b),

$$I_1 = \left(\frac{9}{9+3}\right)(I) = \left(\frac{9}{12}\right)(4) = 3\,\text{A}$$

- From Fig. 2.24

$$I_x = \left(\frac{2}{2+8}\right)(I_1) = \left(\frac{2}{10}\right)(3) = \mathbf{0.6\,A}$$

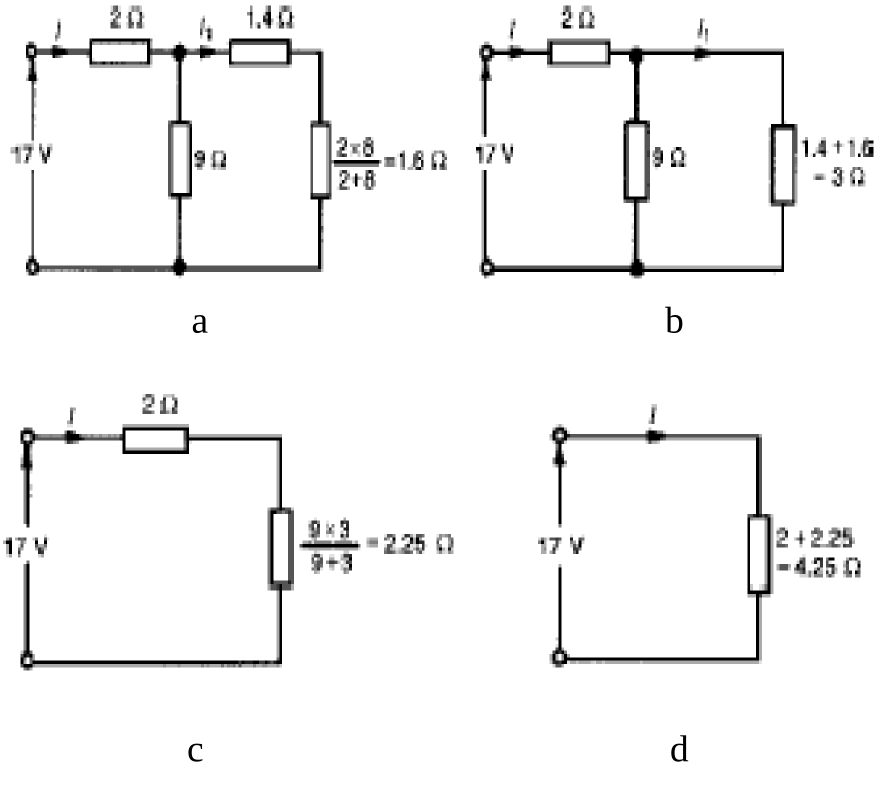

Fig. 2.25

Exercise 2: problems on series and parallel circuits

1. For the circuit shown in Fig. 2.26 determine (a) the reading on the ammeter, and (b) the value of resistor *R*. [2.5 A, 2.5 Ω]

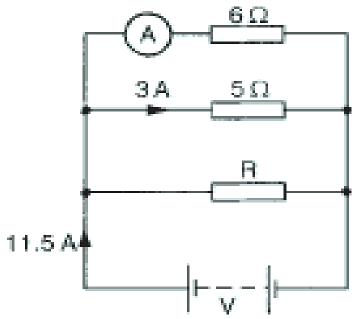

Fig. 2.26

2. Find the equivalent resistance between terminals C and D of the circuit shown in Fig. 2.27. [27.5 Ω]

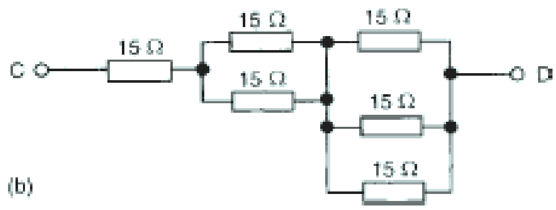

Fig. 2.27

3. Determine the currents and voltages indicated in the circuit shown in Fig. 2.28.
[$I_1 = 5$ A, $I_2 = 2.5$ A, $I_3 = 1\frac{2}{3}$ A, $I_4 = \frac{5}{6}$ A
$I_5 = 3$ A, $I_6 = 2$ A, $V_1 = 20$ V, $V_2 = 5$ V,
$V_3 = 6$ V]

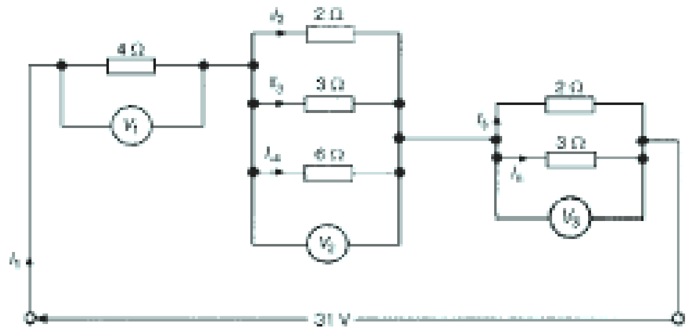

Fig. 2.28

4. Find the current *I* in Fig. 2.29. [1.8 A]

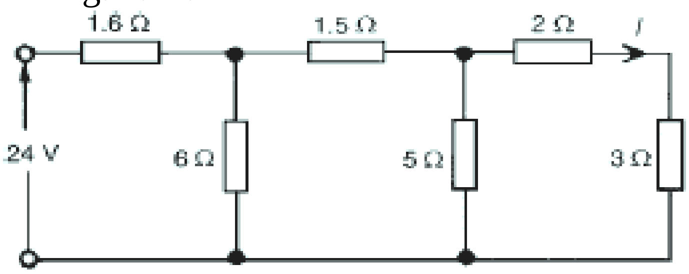

Fig. 2.29

3- Delta-Star Transformation

Three resistors connected as shown in Fig. 3.1-a are said to be connected in delta (or mesh), while three resistors connected as shown in Fig. 3.1-b are said to be star connected. It is often necessary to convert from a delta connection to an equivalent star.

The star circuit will be equivalent to the delta connection if the resistance measured between any two terminals in the star is identical to the resistance measured between the same two terminals in the delta. For this to be the case, the total resistance between terminals 1 and 2 (R_{12}) in the delta circuit must equal the total resistance (R1 + R2) between the same two terminals in the star circuit. Similarly for the resistances between the other two pairs of terminals. We will deal with these in turn.

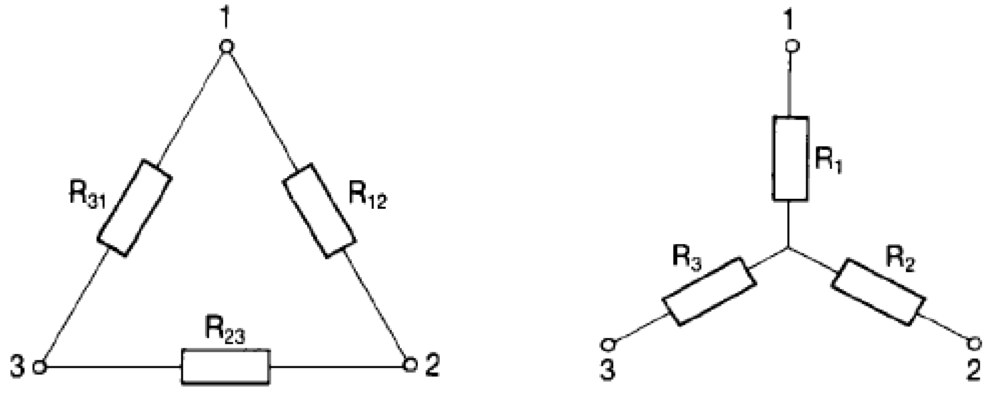

Fig. 3.1

3.1 Delta to star transformation:

$$R_1 = R_{31}R_{12}/(R_{12} + R_{23} + R_{31}) \quad \ldots(3.1)$$

$$R_2 = R_{12}R_{23}/(R_{12} + R_{23} + R_{31}) \quad \ldots(3.2)$$

$$R_3 = R_{23}R_{31}/(R_{12} + R_{23} + R_{31}) \quad \ldots(3.3)$$

An easy way to remember the rules for changing from delta to star is to draw the star set inside the delta set as shown in Fig. 3.2. Any star equivalent resistor is then given as 'the product of the two delta resistors on either side divided by the sum of all three delta resistors'.

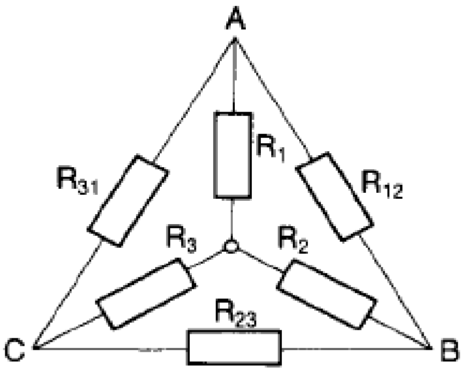

Fig. 3.2

3.2 star-delta transformation

We can use the same diagrams of Fig 3.1and obtain the reverse transformation.

$$R_{12} = R_1 + R_2 + R_1R_2/R_3 \quad \text{…………………………………………………...…(3.4)}$$

$$R_{23} = R_2 + R_3 + R_2R_3/R_1 \quad \text{…………………………………………………...…(3.5)}$$

$$R_{31} = R_3 + R_1 + R_3R_1/R_2 \quad \text{…………………………………………………...…(3.6)}$$

Example 3.1: Determine the total resistance (i.e. the resistance between terminals A and B) and current I_L in the circuit of Fig. 3.3.

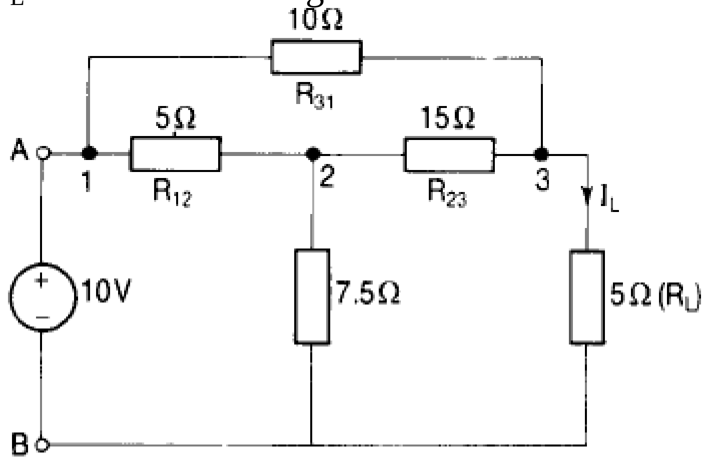

Fig. 3.3

Solution:

The resistors R12 , R23 and R31 are connected in delta. This is shown more clearly in Fig. 3.4 in which the star equivalent resistors are shown as Ra, Rb and Rc. Using the delta-star transformation, we have that

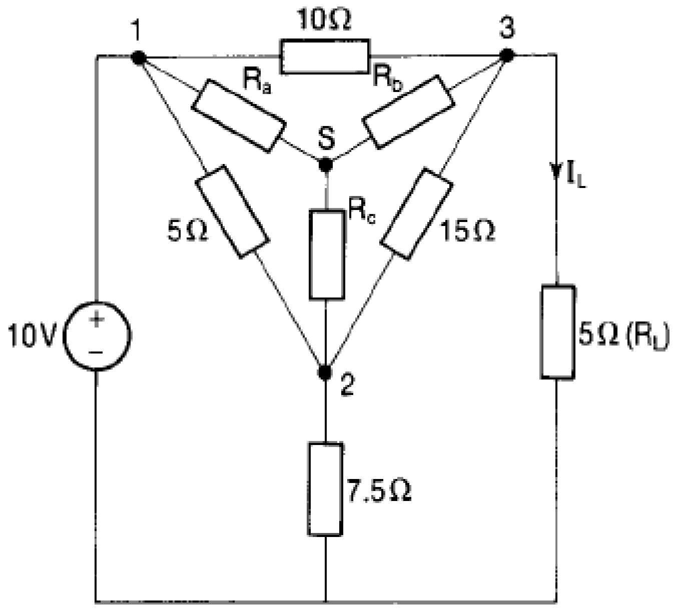

Fig. 3.4

Ra= 10x5/30 = 1.67Ω

Rb= 10x15/30 = 5Ω

Rc= 5x15/30 =2.5Ω

The circuit now simplifies to that of Fig. 3.6 via Fig. 3.5.

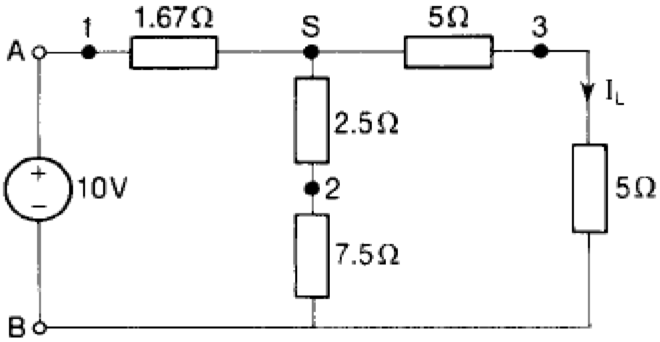

Fig. 3.5

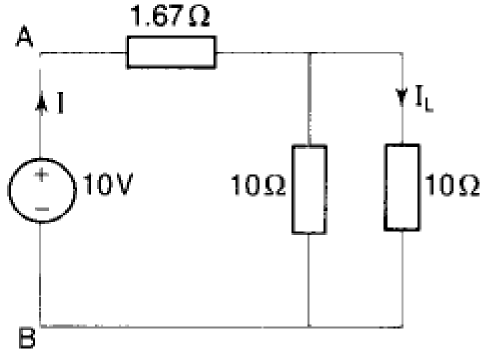

Fig. 3.6

The equivalent resistance between A and B is then given by

$$R_{AB} = R_a + 10 \times 10/(10 + 10) = 1.67 + 5 = 6.67\ \Omega$$

The current I_L in Fig. 3.3 is the same as that in Fig. 3.6 and is obtained by current divider rule. Since the two parallel connected resistors are equal in value, the total current I will divide equally between them. Now

$I = V/R_{AB} = (10/6.67) = 1.5$ A

so that $I_L = 0.75$ A

4- D.C.circuit theory

The laws which determine the currents and voltage drops in d.c. networks are:
(a) Ohm's law (see section 1.1),
(b) the laws for resistors in series and in parallel (see sections 2.1 and 2.3), and
(c) Kirchhoff's laws (see Section 4.1 following).
(d) Maxwell loop current method or mesh analysis (see Section 4.2 following).
(e) Nodal analysis (see Section 4.3 following).

In addition, there are a number of circuit theorems which have been developed for solving problems in electrical networks. These include:
(i) Superposition theorem (see Section 5.1),
(ii) Thevenin's theorem (see Section 5.2),
(iii) Norton's theorem (see Section 5.3),
(iv) Maximum power transfer theorem (see Section 5.4)
(v) Milliman's theorem (see Section 5.5),
(vi) Substitution theorem (see Section 5.6), and
(v) Reciprocity theorem (see Section 5.7).

4.1 Kirchhoff's laws :

(a) **Current Law (KCL)**. At any node (junction) in an electric circuit the sum of currents flowing towards that node is equal to the sum of the currents flowing away from the node. This means that the algebraic sum of the currents meeting at a node is equal to zero, i.e.

$$\Sigma I = 0 \quad \ldots\ldots(4.1)$$

Applying the law to the node shown in Fig. 4.1, we see that

$$I_1 + I_2 = I_3 + I_4 + I_5$$
$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

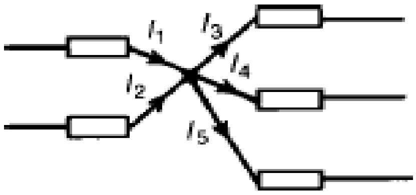

Figure 4.1

(b) Voltage Law (KVL). In any closed loop in a network, the algebraic sum of the voltages taken around the loop is equal to zero, i.e.

$$\Sigma V = 0 \quad \ldots\ldots(4.2)$$

In Figure 4.2, application of KVL around the loop (moving anticlockwise around the loop) gives the following equation:

$$-E_2 - IR_1 - IR_2 - IR_3 + E_1 = 0$$

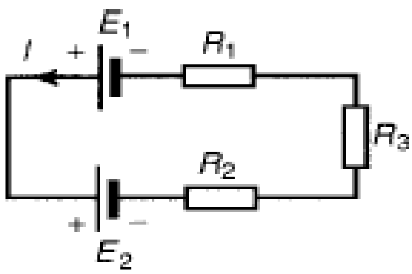

Figure 4.2

Example 4.1: (a) Find the unknown currents marked in Fig. 4.3(a). (b) Determine the value of e.m.f. E in Fig. 4.3(b).

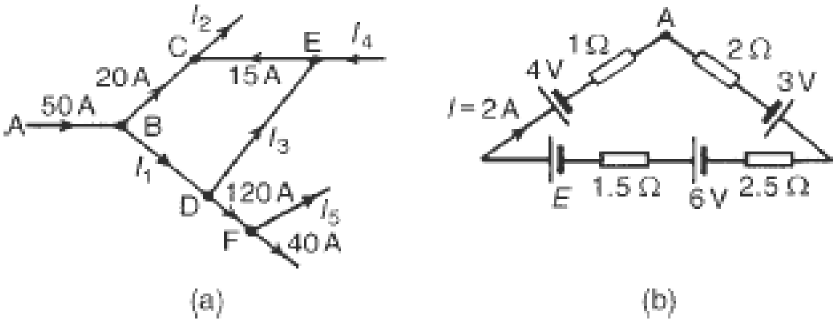

Figure 4.3

Solution:

(a) Applying Kirchhoff's current law:

For junction B: $50 = 20 + I_1$.

Hence $\mathbf{I_1 = 30\,A}$

For junction C: $20 + 15 = I_2$.

Hence $\mathbf{I_2 = 35\,A}$

For junction D: $I_1 = I_3 + 120$

i.e. $30 = I_3 + 120$.

Hence $\mathbf{I_3 = -90\,A}$

(i.e. in the opposite direction to that shown in Fig. 4.3(a))

For junction E: $I_4 + I_3 = 15$

i.e. $I_4 = 15 - (-90)$.

Hence $\mathbf{I_4 = 105\,A}$

For junction F: $120 = I_5 + 40$.

Hence $\mathbf{I_5 = 80\,A}$

(b) Applying Kirchhoff's voltage law and moving clockwise around the loop of Fig. 4.3(b) starting at point A:

$-2I + 3 - 2.5I + 6 - 1.5I + E - 4 - I = 0$

since I = 2A

Hence E = 9V

Example 4.2: Use Kirchhoff's laws to determine the currents flowing in each branch of the network shown in Fig. 4.4.

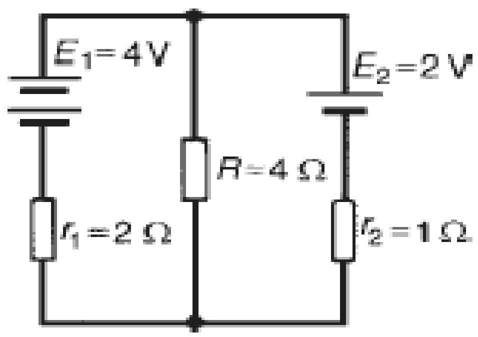

Figure 4.4

Solution:

For solution any problem by Kirchhoff's laws, the following procedure helping you:

1. Use Kirchhoff's current law and label current directions on the original circuit diagram. The directions chosen are arbitrary (for the current sources, current values and direction are given). This is shown in Fig. 4.5 where the three branch currents are expressed in terms of I_1 and I_2 only, since the current through R is $(I_1 + I_2)$

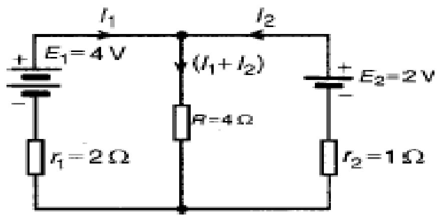

Figure 4.5

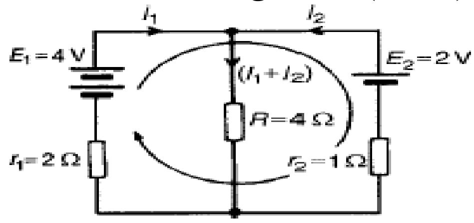

Figure 4.6

2. Apply Kirchhoff's voltage law to each loop (write equation for each loop). A useful convention is to take the voltage drop (going from positive to negative) as negative and the voltage rise (going from negative to positive) as positive.

-From loop 1 (left loop)of Fig. 4.5, and moving in a clockwise direction (the direction chosen does not matter), gives

$$-4(I_1+I_2) - 2I_1 + 4 = 0$$

i.e $\quad -6I_1 - 4I_2 = -4 \quad$ ....(1)

-From loop 2 (right loop) of Fig. 4.5, and moving in an clockwise direction (once again, the choice of direction does not matter), gives:

$$-2 + I_2 + 4(I_1+I_2) = 0$$

i.e $\quad 4I_1 + 5I_2 = 2 \quad$ ....(2)

3. Solve equations (1) and (2) for I_1 and I_2. (number of equations equals number of variables).

Multiplying eq. (1) by 2 and eq. (2) by 3 gives:

$-12I_1 - 8I_2 = -8 \quad$ ....(3)

$12I_1 + 15I_2 = 6 \quad$ ....(4)

eq. (1) + eq. (2) gives:

$7I_2 = -2 \quad$ ....(3)

hence $I_2 = -2/7 = \mathbf{-0.286\ A}$

(i.e. I_2 is flowing in the opposite direction to that shown in Fig. 4.5).

From (1) $\quad 6I_1 + 4(-0.286) = 4$

$$6I_1 = 4 + 1.144$$

Hence $\quad I_1 = \frac{5.144}{6} = \mathbf{0.857\ A}$

Current flowing through resistance R is

$$(I_1 + I_2) = 0.857 + (-0.286)$$

$$= \mathbf{0.571\ A}$$

Note that a third loop is possible, as shown in Fig. 4.6, giving a third equation which can be used as a check:

$$2 + I_2 - 2I_1 + 4 = 0$$

$$2I_1 - I_2 = 2$$

[Check: $2I_1 - I_2 = 2(0.857) - (-0.286) = 2$]

Example 4.3: For the bridge network shown in Fig. 4.7 determine the currents in each of the resistors.

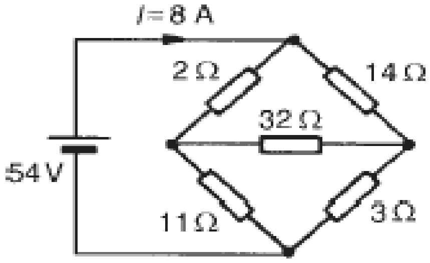

Figure 4.7

Solution:

Let the current in the 2Ω resistor be I_1, then by Kirchhoff's current law, the current in the 14 Ω resistor is $(I - I_1)$. Let the current in the 32 Ω resistor be I_2 as shown in Fig. 4.8. Then the current in the 11Ω resistor is $(I_1 - I_2)$ and that in the 3 Ω resistor is $(I - I_1 + I_2)$. Applying Kirchhoff's voltage law to loop (*abedfa*) gives:

$$-2I_1 - 11(I_1 - I_2) + 54 = 0$$

i.e. $\quad -13I_1 + 11I_2 = -54$ …………………(1)

Applying Kirchhoff's voltage law to loop (*bcea*) gives:

$$-14(I - I_1) + 32I_2 + 2I_1 = 0$$

However $\quad I = 8\text{A}$

Hence $\quad -14(8 - I_1) + 32I_2 + 2I_1 = 0$

i.e. $\quad 16I_1 + 32I_2 = 112$ …………………(2)

Equations (1) and (2) are simultaneous equations with two unknowns, I_1 and I_2. solving the above equations gives:

$$I_1 = 5A \quad and \quad I_2 = 1A$$

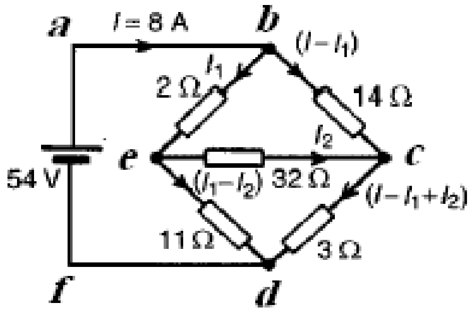

Figure 4.8

Hence, the current flowing in the 2 Ω resistor

$$= I_1 = 5\text{ A}$$

the current flowing in the 14 Ω resistor

$$= (I - I_1) = 8 - 5 = 3\text{ A}$$

the current flowing in the 32 Ω resistor

$$= I_2 = 1\text{ A}$$

the current flowing in the 11 Ω resistor

$$= (I_1 - I_2) = 5 - 1 = 4\text{ A}$$

and the current flowing in the 3 Ω resistor

$$= I - I_1 + I_2 = 8 - 5 + 1 = 4\text{ A}$$

Example4.4: Use Kirchhoff's laws to determine the v_o in the network shown in Figure 4.9.

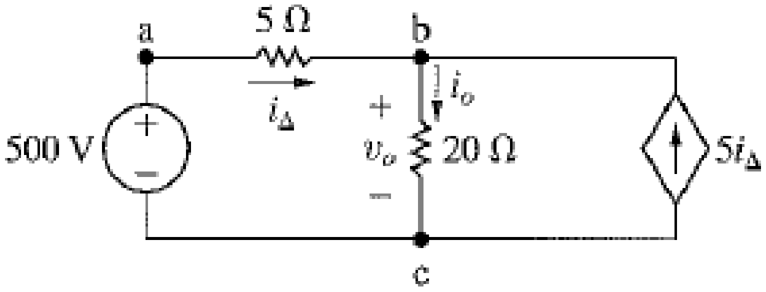

Figure 4.9

Solution:

A look at the circuit reveals that :

- Once we can know i_o , we can calculate v_o using Ohm's law.
- Once we can know i_Δ , we also know the current supplied by the dependent source $5i_\Delta$.
- The current in the 500V source is i_Δ .

We can apply KVL around loop *abca* gives:

$$-5i_\Delta - 20\, i_o + 500 = 0$$

$$-5i_\Delta - 20\, i_o = -500 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

We can apply KCL at node b gives:

$$5i_\Delta + i_\Delta = i_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving eqs. 1 and 2 for the currents, we get

$$i_\Delta = 4A$$

$$i_o = 24$$

Using Ohm's law for the 20Ω resistor, we can solve for the voltage v_o .

$$v_o = 20\, i_o = 480\ V$$

Exercise 3: problems on Kirchhoff's laws

1. Find currents I_3, I_4 and I_6 in Fig. 4.10. [$I_3 = 2$ A, $I_4 = -1$ A, $I_6 = 3$ A]

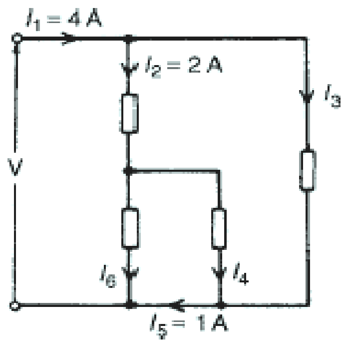

Figure 4.10

2. For the circuit shown in Fig. 11, find i_1, v, total power generated, and total power absorbed. Ans. (0.000025A, -2V, 6.150mW, 6.150mW)

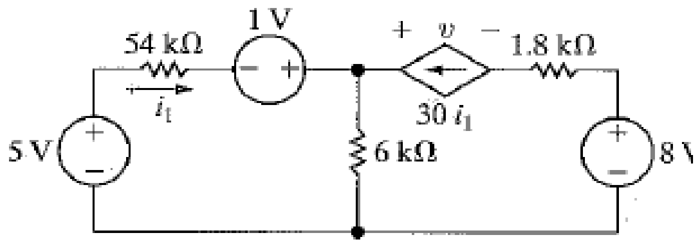

Figure 4.11

3. The current i_Φ shown in the circuit of Fig.(12) is 2A. Calculate: v_s , the power absorbed by the independent voltage source, the power delivered by the independent current source, the power delivered by the controlled current source, and the total power dissipated in the two resistor. Ans. (70V, 210W, 300W, 40W, 130W)

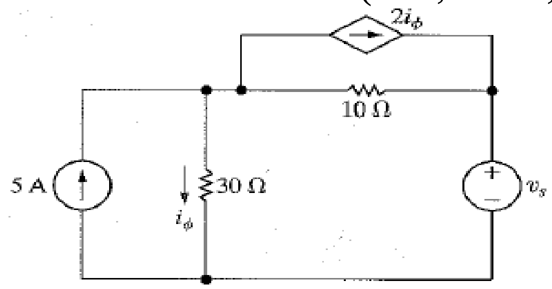

Figure 4.12

4. For the circuit shown in Fig. 4.13, use Kirchhoff's laws to find:

a) v_1, v_2, and i_1 .

b) How much power is delivered to the circuit by the 15A source?

c) Repeat (b) for the 5A source. Ans. (60V, 10V, 10A, 900W, -50W)

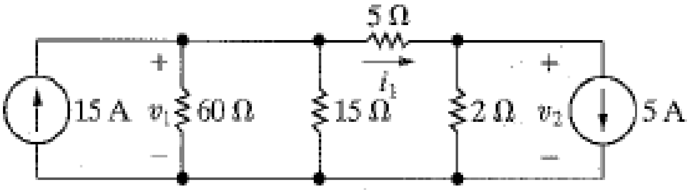

Figure 4.13

5. For the circuit shown in Fig. 4.14, use Kirchhoff's laws to find v. Ans. (15V)

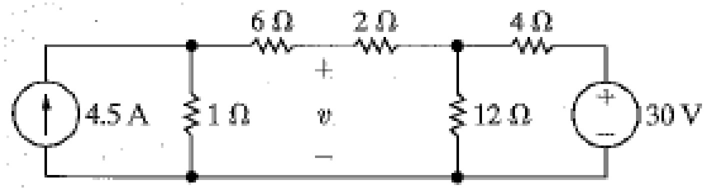

Figure 4.14

6. In the circuit shown in Fig. 4.15 a) Use Kirchhoff's laws to find the power associated with each source. b) State whether the source is delivering power to the circuit or extracting power from the circuit. Ans.(a- P_{50v} = -150W, P_{3i_1}= -144W, P_{5A}= -80W, b- all sources delivering power to the circuit)

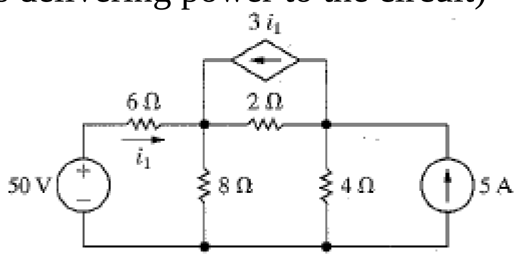

Figure 4.15

7. For the circuit shown in Fig. 4.16, use Kirchhoff's laws to find i_Φ . Ans. (2A)

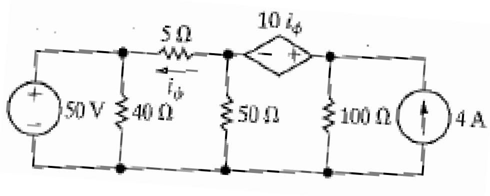

Figure 4.16

8. Use Kirchhoff's laws to find v_o .   Ans. (24V)

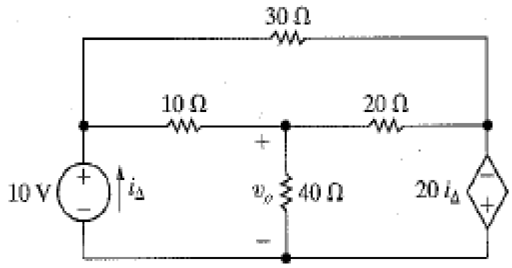

Figure 4.17

9. Use Kirchhoff's laws to find v .   Ans. (8V)

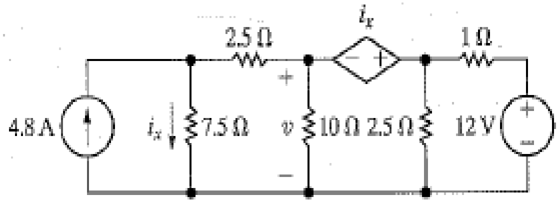

Figure 4.18

10. Use Kirchhoff's laws to find v_1 .   Ans. (48V)

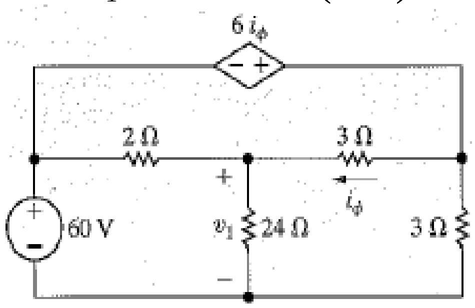

Figure 4.19

11. Use Kirchhoff's laws to find how much the power is being delivered to the dependent voltage source.   (Ans. -36W)

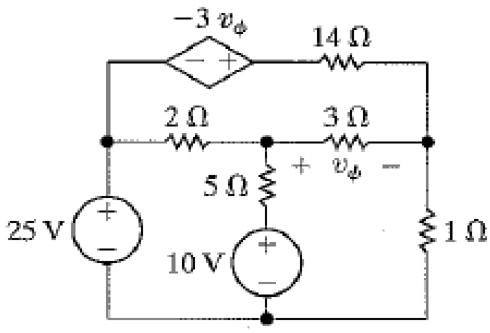

Figure 4.20

12. Use Kirchhoff's laws to find the v_o .   Ans. (16V)

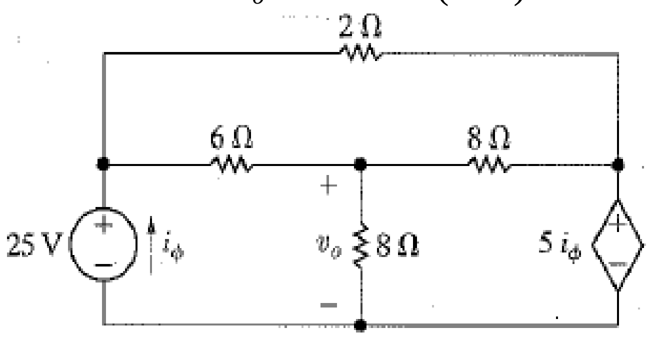

Figure 4.21

13. Find the power dissipated in the 300Ω resistor. Ans.(16.57W)

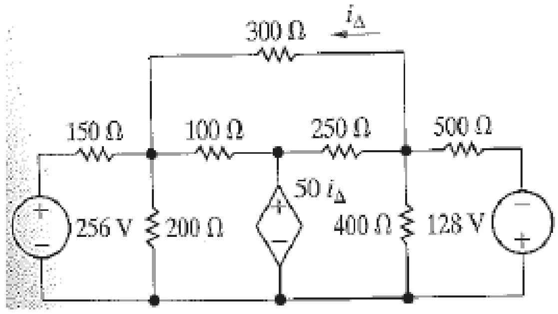

Figure 4.22

14. Use Kirchhoff's laws to find the v_o in the circuit shown. Ans. (173V)

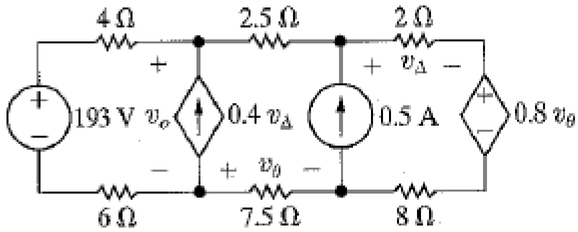

Figure 4.23

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4.2 Loop Current Method

In this method, one distinct current variable is assigned to each independent loop. The element currents are then calculated in terms of the loop currents. Using the element currents and values, element voltages are calculated. Thus, in this method, the number of simultaneous equations to be solved are equal to the number of independent loops. Examples below illustrate this technique.

Example 4.5: In the circuit in Figure 4.24, find the voltage across the 3Ω resistor.

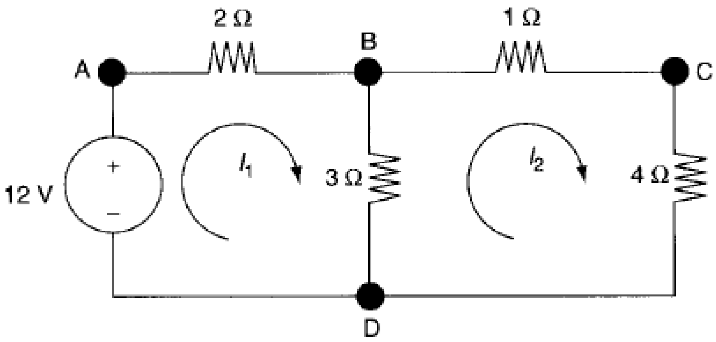


Figure 4.24

Solution:

First, note that there are two independent loops, which are the two meshes in the circuit, and that loop currents 1_1 and I_2 are assigned as shown in the diagram. Then calculate the element currents and the element voltages:

$$5\ I_1 - 3\ I_2 = 12$$
$$-3\ I_1 + 8\ I_2 = 0.$$

Solving the two equations, you get $I_1 = 96/31A$, and $I_2 = 36/31A$. The voltage across the 3Ω resistor is

$$3(I_1\text{-}I_2\ ) = 3(96/31 - 36/31) = 180/31\ A.$$

Special case 1:

When one of the elements in a loop is a current source, the voltage across it cannot be written using the *v-i* relation of the element. In this case, the voltage across the current source should be treated as an unknown variable to be determined. If a current source is present in only one loop and is not common to more than one loop, then the current of the loop in which the current source is present should be equal to the value of the current source and hence is known. To determine the remaining currents, there is no need to write the KVL equation for the current source loop. However, to determine the voltage of the current source, a KVL equation for the current source loop needs to be written. This case is presented in example 4.6.

Example 4.6: Analyze the circuit shown in Figure 4.25 to find the voltage across the current sources.

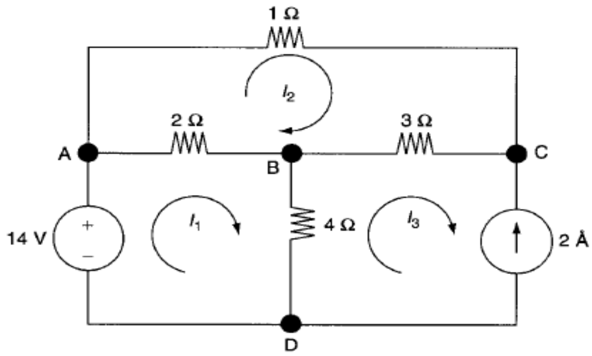


Figure 4.25

Solution:

The loop currents are assigned as shown. It is easily seen that I_3 = -2. Writing KVL equations for loops 1 and 2, you get:

Loop 1 $6I_1-2I_2-4*(-2)=14$ → $6I_1-2I_2=6$

Loop 2 $6I_2-2I_1-3*(-2)=0$ → $-2I_1+6I_2=-6$

Solving the two equations simultaneously, you get $I_1 = 3/4\ A$ and $I_2 = -3/4\ A$. To find the V_{CD} across the current source, write the KVL equation for the loop 3 as:

$$4(I_3 - I_1) + 3(I_3 - I_2) + V_{CD} = 0 =>$$

$$V_{CD} = 4\ I_1 + 3\ I_2 - 7\ I_3 = 14.75\ V.$$

Special case 2:

This case concerns a current source that is common to more than one loop. The solution to this case is illustrated in example 4.7.

Example 4.7: In the circuit shown in Figure 4.26, find the voltage across the current sources.

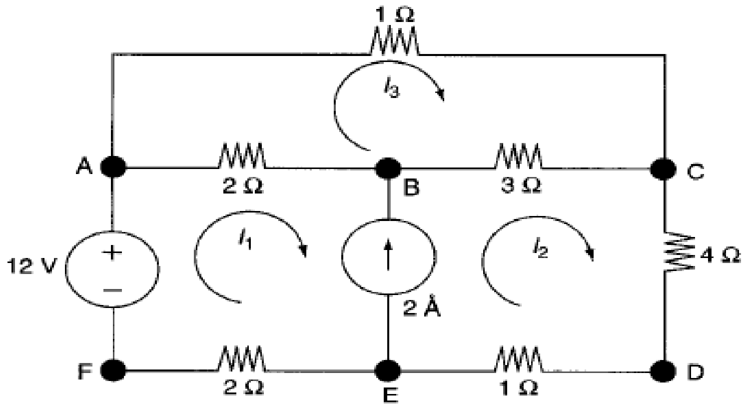


Figure 4.26

Solution:
the 2A current source is common to loops 1 and 2. One method of writing KVL equations is to treat V_{BE} as an unknown and write three KVL equations. In addition, you can write the current of the current source as $I_2 - I_1 = 2$, giving a fourth equation. Solving the four equations simultaneously, you determine the values of I_1, I_2, I_3 and V_{BE}. These equations are the following:

Loop 1 $4I_1 - 2I_3 = 12 - V_{BE}$ → $4I_1 - 2I_3 - V_{BE} = 12$ ………(1)
Loop 2 $8I_2 - 3I_3 = V_{BE}$ → $8I_2 - 3I_3 - V_{BE} = 0$ ………(2)
Loop 3 $6I_3 - 2I_1 - 3I_2 = 0$ → $-2I_1 - 3I_2 + 6I_3 = 0$ ………(3)
Current source relation: $-I_1 - I_2 = 2$ ………..(4)

Solving the above four equations results in $I_1 = 0.13$ A, $I_2 = 2.13$ A, $I_3 = 1.11$ A, and $V_{BE} = 13.70$ V.

Alternative method for special case 2 (Super **loop** method): This method eliminates the need to add the voltage variable as an unknown. When a current source is common to loops 1 and 2, then KVL is applied on a new loop called the super loop. The super loop is obtained from combining loops 1 and 2 (after deleting the common elements) as shown in Figure 4.27. For the circuit considered in example 4.7, the loop (*ABCDEFA*) is the super loop obtained by combining loops 1 and 2. The KVL is applied on this super loop instead of KVL being applied for loop 1 and loop 2 separately. The following is the KVL equation for super loop (*ABCDEFA*):

$2(I_1 - I_3) + 3(I_2 - I_3) + 4I_2 + I_2 + 2I_1 = 12$ →

$4I_1 + 8I_2 - 5I_3 = 12$ ………..(1)

The KVL equation around loop 3 is written as:

$-2I_1 - 3I_2 + 6I_3 = 0$ ………(2)

Current source relation:

$-I_1 - I_2 = 2$ ………(3)

Solving the above three equations simultaneously produces equations $I_1 = 0.13$ A, $I_2 = 2.13$ A, $I_3 = 1.11$ A.

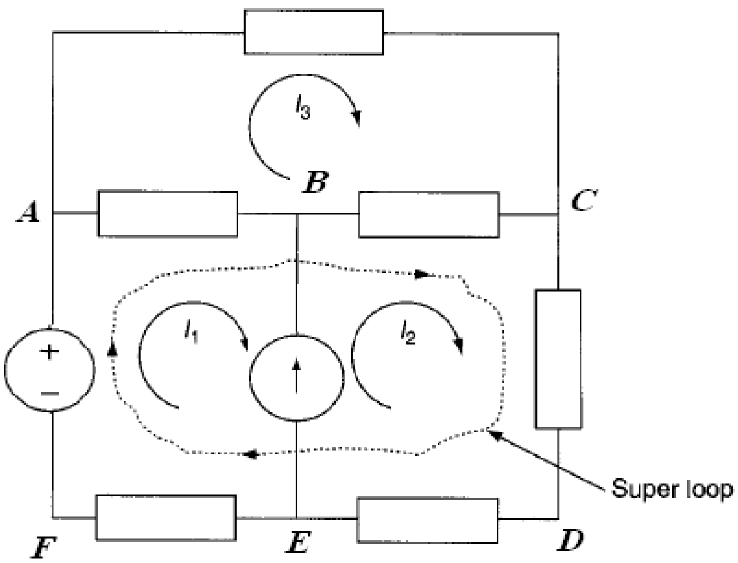


Figure 4.27

Example 4.8: Use the loop current method to determine the power dissipated in the 4Ω resistor in the circuit shown in Fig. 4.28.

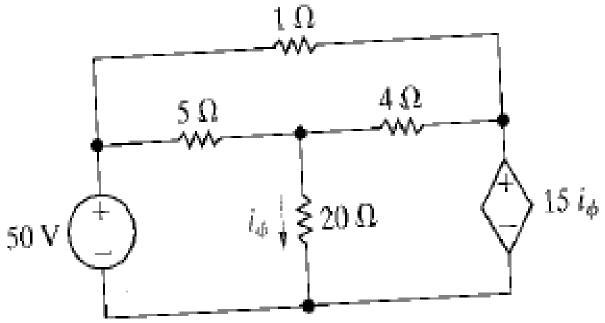
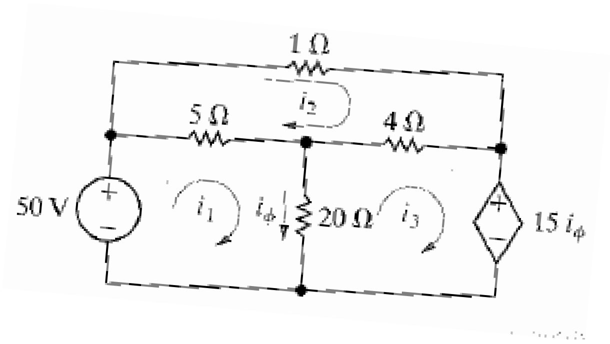


Figure 4.27

Solution:



$25i_1-5i_2-20i_3=50$ ………………..(1)

$10i_2-5i_1-4i_3=0$ ………………..(2)

$24i_3-20i_1-4i_3= -15i_\Phi$ ………………..(3)

$i_1-i_3=i_\Phi$ ………………...(4)

because we are calculating the power dissipated in the 4Ω resistor , we compute the loop currents i_2 and i_3 :

i_2=26A, i_3=28A

$P_{4\Omega}=(i_3-i_2)^2$x$(4)=4$x$4=16W$

Exercise 4: problems on loop current method

-Repeat all problems in exercise 3 using loop current method.

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4.3 Node Voltage Method (Nodal Analysis):

In this method, one node is chosen as the reference, node whose voltage is assumed as zero, and the voltages of other nodes are expressed with respect to the reference node. For example, in Figure 4.28, the voltage of node G is chosen as the reference node, and then the voltage of node A is $V_A = V_{AG}$ and that of node B is $V_B = V_{BG}$ and so on. Then, for every element between two nodes, the element voltages may be expressed as the difference between the two node voltages. For example, the voltage of element R_{AB} is $V_{AB} = V_A- V_B$. Similarly $V_{BC} =V_B- V_C$ and so on. Then the current through the element R_{AB} can be determined using the *v-i* characteristic of the element as $I_{AB} = V_{AB}/R_{AB}$. Once the currents of all elements are known in terms of node voltages, KCL is applied for each node except for the reference node, obtaining a total of (N-1) equations where N is the total number of nodes.

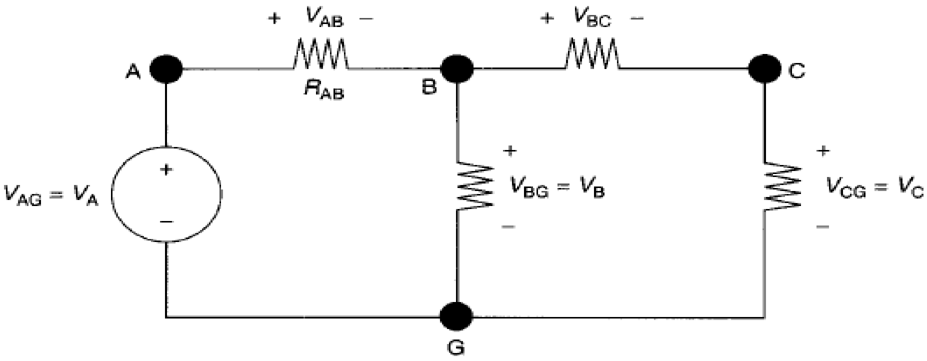


Figure 4.28

Special Case 1:

In branches where voltage sources are present, the *v-i* relation cannot be used to find the current. Instead, the current is left as an unknown. Because the voltage of the element is known, another equation can be used to solve the added unknown. When the element is a current source, the current through the element is known. There is no need to use the *v-i* relation. The calculation is illustrated in the following example.

Example 4.7: In Figure 4.29, solve for the voltages V_A, V_B, and V_C with respect to the reference node G.

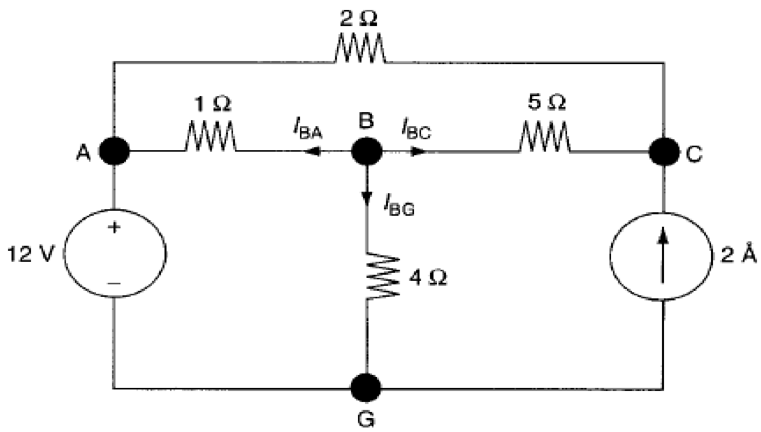


Figure 4.29

Solution:

At node A, $V_A = 12$. ……………………(1)

At node B, KCL yields:

$$I_{BA} + I_{BG} + I_{BC} = 0 =>$$

$$(V_B - V_A)/1 + V_B/4 + (V_B - V_C)/5 = 0 =>$$

$$-V_A + (1 + 1/4 + 1/5)V_B - V_C/5 = 0.$$ **……………(2)**

Similarly at node C, KCL yields:

$$V_A/2 - V_B/5 + (1/5 + 1/2)V_C = 2.$$ **…………………..(3)**

Solving the above three equations simultaneously results in $V_A = 12$ V, $V_B = 10.26$ V, and $V_C = 14.36$ V.

Super Node: When a voltage source is present between two non-reference nodes, a super node may be used to avoid introducing an unknown variable for the current through the voltage source. Instead of applying KCL to each of the two nodes of the voltage source element, KCL is applied to an imaginary node consisting of both the nodes together. This imaginary node is called a **super node**. In Figure 4.30, the super node is shown by a dotted closed shape. KCL on this super node is given by:

$I_{AB} + I_{BG} + I_{CG} + I_{CA} = 0 \quad \Rightarrow \quad (V_B - V_A)/1 + V_B/3 + V_C/4 + (V_C - V_A)/2 = 0$

In addition to this equation, the two voltage constraint equations, $V_A=10$ and V_B-V_C =5, are used to solve for V_B and V_C as $V_B = 9V$ and $V_C = 4V$.

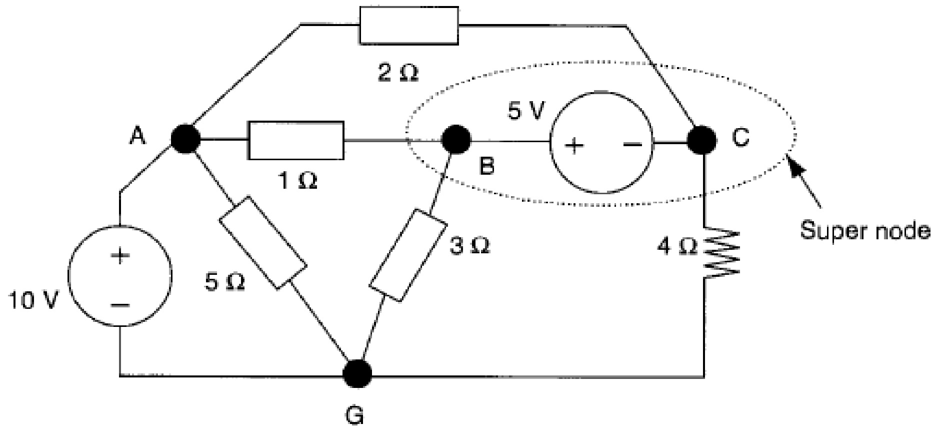


Figure 4.30

Example 4.8: Use the node voltage method to find the power dissipated in the 5Ω resistor In Figure 4.31.

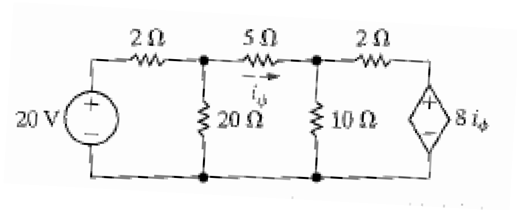


Figure 4.31

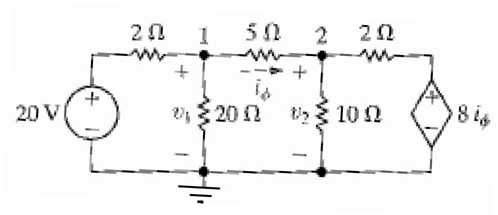
Solution:

Summing the currents away from node 1 generates the equation:

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \Rightarrow v_1(\frac{1}{2} + \frac{1}{20} + \frac{1}{5}) - \frac{v_2}{5} - \frac{20}{2} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Summing the currents away from node 2 yields:

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{v_2 - 8i_\Phi}{2} = 0 \Rightarrow v_2(\frac{1}{2} + \frac{1}{10} + \frac{1}{5}) - \frac{v_1}{5} - \frac{8i_\Phi}{2} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$



To eliminate i_Φ we must express this controlling current in term of the node voltages,

or: $\quad i_\Phi = \frac{v_1 - v_2}{5} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$

Substituting eq.3 into eqs. 1 and 2 simplifies the two node voltage equations to

$0.75v_1 - 0.2v_2 = 10$,

$-v_1 + 1.6v_2 = 0$

Solving for v1 and v2 gives:

$v_1 = 16V$ and $v_2 = 10V$

Then, $i_\Phi = \frac{16 - 10}{5} = 1.2A$, $\quad P_{5\Omega} = 1.44 * 5 = 7.2W$

Exercise 5: problems on nodal analysis

-Repeat all problems in exercise 3 using nodal method.

5- Superposition Theorem

Superposition theorem may be stated as follows: in any linear network containing more than one source of voltage or current, the current in any element (or voltage across it) of the network may be found by determining the current (voltage) in that element when each source acts alone and then adding the results algebraically. When removing, in turn, all the sources except one, any voltage source must be replaced by a short circuit and any current source must be replaced by an open circuit.
The superposition theorem is demonstrated in the following worked problems:
Example 5.1: Determine the current in each branch of the network by using the superposition theorem.

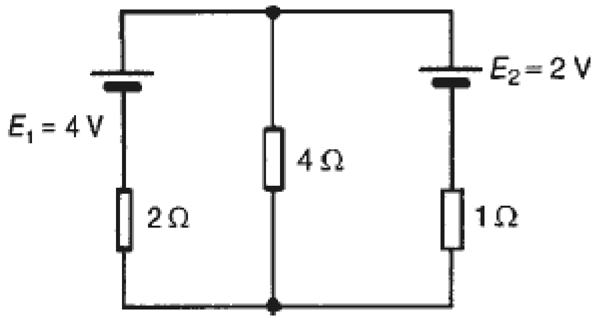


Figure 5.1

Solution:
1. Redraw the original circuit with source E_2 removed, being replaced by (s.c), as shown in Fig. 5.2 (a).

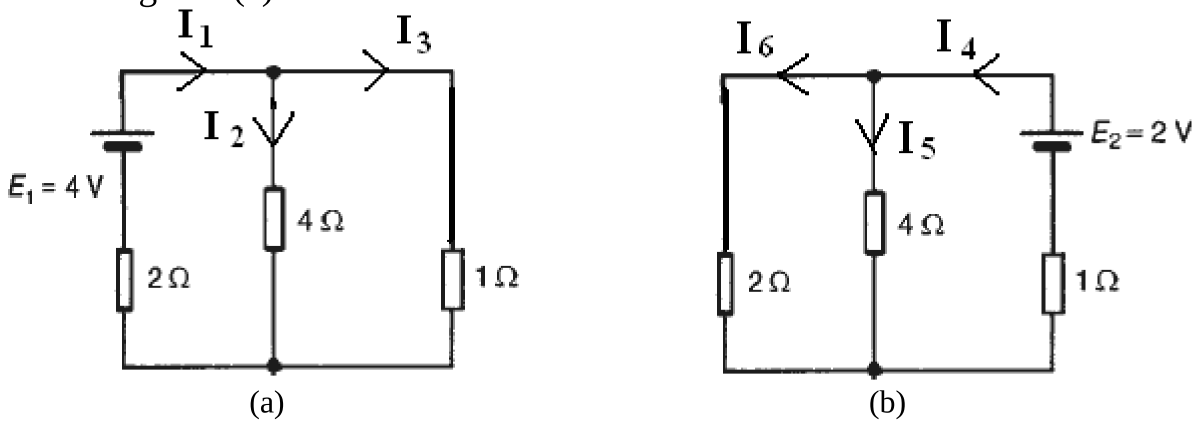


Figure 5.2

2. Label the currents in each branch and their directions as shown in Fig. 5.2(a) and determine their values. (Note that the choice of current directions depends on the battery polarity, which, by convention is taken as flowing from the positive battery terminal as shown).
The equivalent resistance (R_T)= [(4×1)/(4+1)] + 2 =2.8Ω

$\boldsymbol{I_1 = E_1/R_T}$ = 4/2.8 = 1.429A

by current division

$$I_2 = \left(\frac{1}{4+1}\right) I_1 = \frac{1}{5}(1.429) = 0.286\,\text{A}$$

$$\text{and} \quad I_3 = \left(\frac{4}{4+1}\right) I_1 = \frac{4}{5}(1.429) = 1.143\,\text{A}$$

3. Redraw the original circuit with source E_2 removed, being replaced by (s.c) only, as shown in Fig. 5.2(b).

4. Label the currents in each branch and their directions as shown in Fig. 5.2(b) and determine their values.
The equivalent resistance (R_T)= [(4×2)/(4+2)] + 1 =1.333Ω

$I_4 = E_2/R_T$ = 2/1.333 = 0.857A

by current division

$$I_5 = \left(\frac{2}{2+4}\right) I_4 = \frac{2}{6}(0.857) = 0.286\,\text{A}$$

$$I_6 = \left(\frac{4}{2+4}\right) I_4 = \frac{4}{6}(0.857) = 0.571\,\text{A}$$

5. Determine the algebraic sum of the currents flowing in each branch.
Resultant current flowing through (2Ω) = $I_1 - I_6$ = 1.429 – 0.571= 0.858A
Resultant current flowing through (1Ω) = $I_4 - I_3$ = 0.857 – 1.143 = – 0.286A
Resultant current flowing through (4Ω) = $I_2 - I_5$ = 0.286 – 0.286 = 0.572A

Example 5.2: Determine the current flowing in the 8Ω resistor in the circuit of Fig. 5.3.

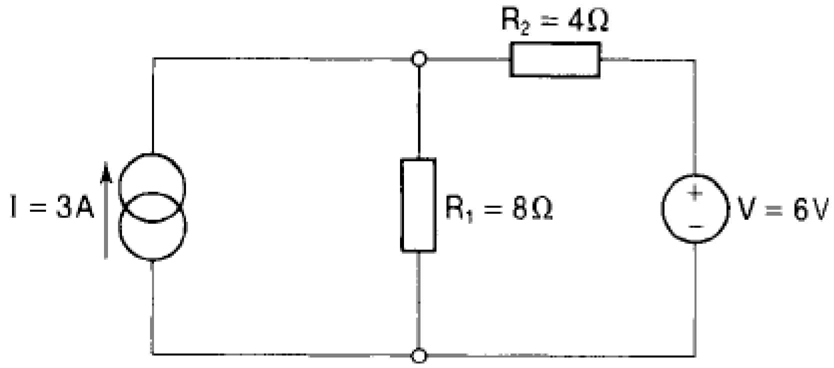


Figure 5.3

Solution:
First we replace the current source (I) by an open circuit, giving the circuit of Fig. 5.4 (a). From this circuit we see that the current

$I_a = V/(R1 + R2)$= 6 / 1 2 = 0.5 A.

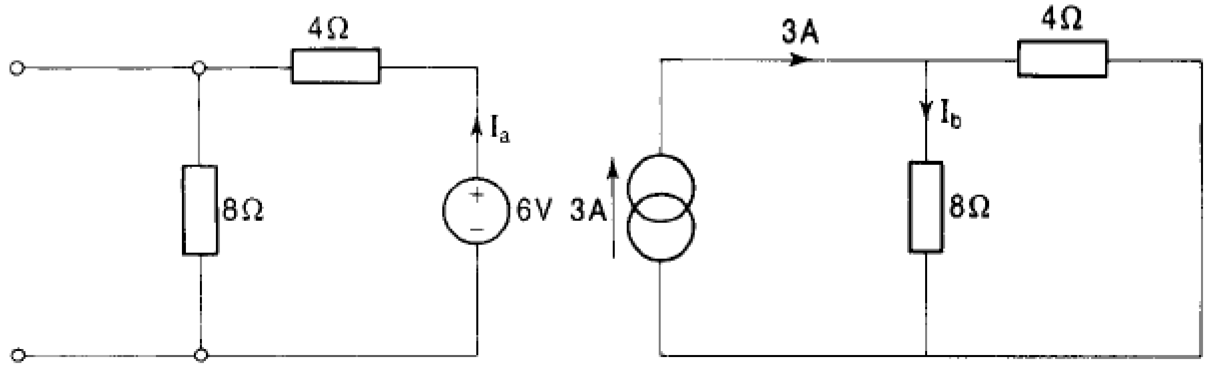


Figure 5.4

Next we reconnect the current source and replace the battery (V) by a short circuit to give the circuit of Fig. 5.4 (b). By current division we obtain:

$I_b = [4/(4 + 8)] \times 3 = 1\ A$

When both sources are acting together the current through 8Ω resistor.

$R_{8\Omega} = I_a + I_b = 0.5 + 1 = 1.5\ A$

Example 5.3:. For the circuit in Figure 5.5, determine the voltage across the 3Ω resistor.

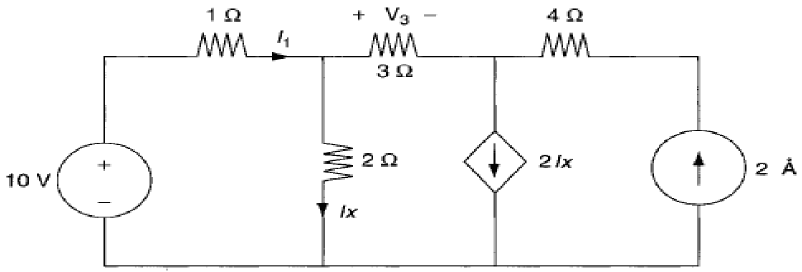


Figure 5.5

Solution:

First, the voltage source is activated and current source is deactivated (replaced by an open circuit) as shown in Figure 5.6 (a). Let V_{31} be the voltage across the 3Ω resistor in this circuit.

Apply (KVL) around closed loop (abcda)

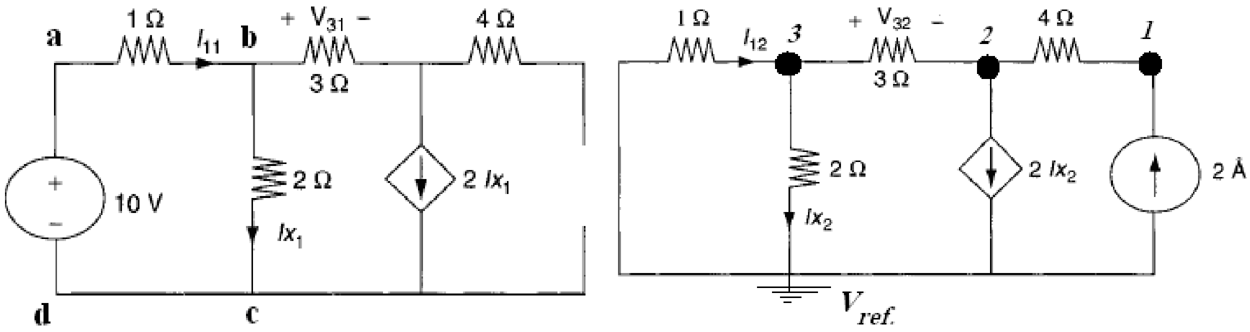
$- I_{11} – 2I_{x1}+10 = 0$ ……………..(1)

Apply (KCL) at node (b)

$I_{11} = I_{x1} + 2I_{x1}$ ⟶ $I_{11} =3 I_{x1}$ ……………..(2)

Substituting eq. (2) in eq.(1) ⟶ $-3I_{x1} – 2I_{x1}+10 = 0$

$I_{x1} =2A$ ⟶ $V_{31}=2 \times 2 \times 3= 12V$



a b

Figure 5.6

Next the current source is activated and voltage source is deactivated (replaced by a short circuit) as shown in Figure 5.6 (b). Let V_{32} be the voltage across the 3Ω resistor in this circuit. Using nodal method to find this voltage.

At node 1: $(1/4) V_1 – (1/4)V_2 = 2$ ⟶ $V_1 = 8 + V_2$ ………(1)

At node 2: $[(1/4) + (1/3)]V_2 – (1/4)V_1 – (1/3) V_3 = – 2i_{X2}$

⟶ $(7/12)V_2 – (1/4)V_1 – (1/3)V_3 = –2i_{x2}$ ………(2)

At node 3: $[(1/1) + (1/3) +(1/2)]V_3 – (1/3)V_2 = 0$

⟶ $(11/6)V_3 – (1/3)V_2 = 0$ ⟶ $V_3 = (2/11)V_2$ ………(3)

From Ohm's law : $i_{x2} = V_3/2 = (1/11)V_2$ ..…..…(4)

Substituting eqs. (1) , (3) and (4) in eq.(2)

$(7/12)V_2 – (1/4)(8 + V_2) – (1/3) (2/11)V_2 = – (2/11)V_2$ ⟶ $V_2= (22/5)$ V

$V_3 = (2/11) \times (22/5) = 0.8 V$

$V_{32}= V_3 – V_2 = 0.8 – (22/5) = – 3.6V$

Then you determine that the voltage across the 3Ω resistor in the given complete circuit is $V_{3Ω} = V_{31} + V_{32} = 12 – 3.6 = 8.4$ V

6- Thevenin's and Norton's Theorems

The following points involving d.c. circuit analysis need to be appreciated before proceeding with problems using Thevenin's and Norton's theorems:

(i) The open-circuit voltage, *E*, across terminals AB in Fig. 6.1 is equal to 10V, since no current flows through the 2Ω resistor and hence no voltage drop occurs.

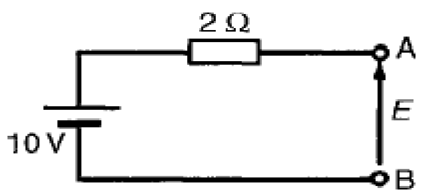


Figure 6.1

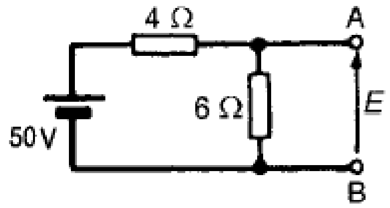


Figure 6.2

(ii) The open-circuit voltage, *E*, across terminals AB in Fig. 6.2 is the same as the voltage across the 6Ω resistor.

by voltage division in a series circuit,

$$E = \left(\frac{6}{6+4}\right)(50)$$

$$= 30V$$

(iii) The resistance 'looking-in' at terminals AB in Fig. 6.3 (a) is obtained by reducing the circuit in stages as shown in Figures 6.3(b) to (d). Hence the equivalent resistance across terminals AB is 7Ω.

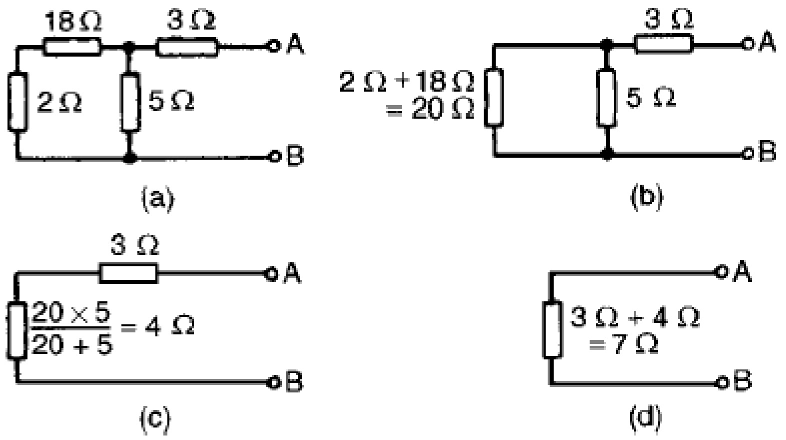


Figure 6.3

(iv) If the terminals AB is open circuit in the circuit shown in Fig. 6.4, the 3Ω resistor carries no current and the voltage across the 20 Ω resistor is 10V. Then,

$$E = \left(\frac{4}{4+6}\right) \times 10 = \mathbf{4\,V}$$

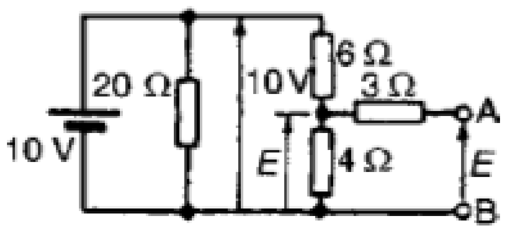
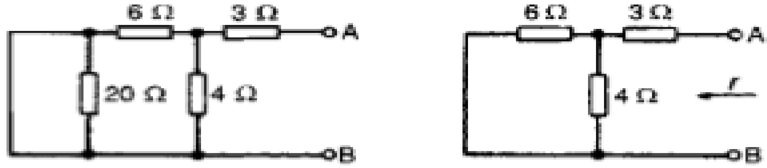


Figure 6.4

(v) If the 10V battery in Fig. 6.4 is removed and replaced by a short-circuit, as shown in Fig. 6.5 (a), then the 20Ω resistor may be removed. The reason for this is that a short circuit has zero resistance, and 20 Ω in parallel with zero ohms gives an equivalent resistance of (20×0)/(20+0) i.e. 0 Ω. The circuit is then as shown in Fig. 6.5 (b). The equivalent resistance across AB,

$$r = \frac{6 \times 4}{6+4} + 3 = 2.4 + 3 = \mathbf{5.4\ \Omega}$$



(a) (b)

Figure 6.5

(vi) To find the voltage across AB in Fig. 6.6: Since the 20V supply is across the 5 Ω and 15 Ω resistors in series then, by voltage division, the voltage drop across AC,

$$V_{AC} = \left(\frac{5}{5+15}\right)(20) = 5\,\mathrm{V}$$

Similarly,

$$V_{CB} = \left(\frac{12}{12+3}\right)(20) = 16\,\mathrm{V}$$

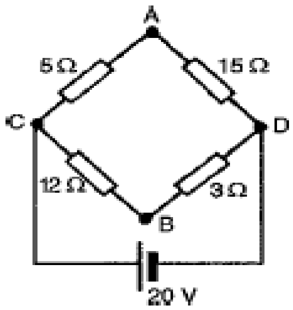


Figure 6.6

VC is at a potential of +20V.

$V_A = V_C - V_{AC} = +20 - 5 = 15\mathrm{V}$

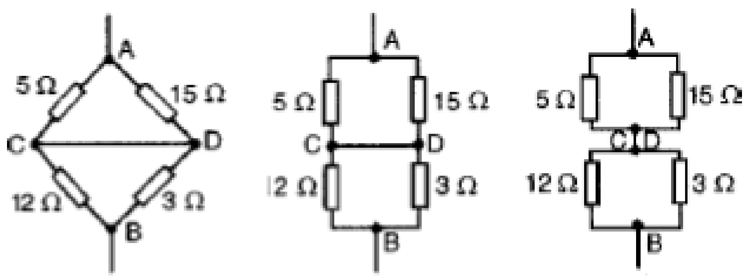
and $V_B = V_C - V_{BC} = +20 - 16 = 4\mathrm{V}$.

Hence the voltage between AB is VA -VB = 15-4=11V and current would flow from A to B since A has a higher potential than B.

(vii) In Fig. 6.7(a), to find the equivalent resistance across AB the circuit may be redrawn as in Figs. 6.7 (b) and (c). From Fig. 6.7 (c), the equivalent resistance across AB:

$$= \frac{5 \times 15}{5+15} + \frac{12 \times 3}{12+3}$$

$$= 3.75 + 2.4 = \mathbf{6.15\ \Omega}$$

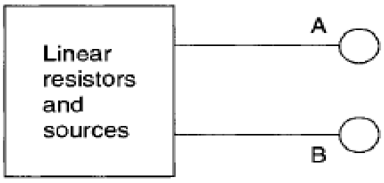


(a) (b) (c)

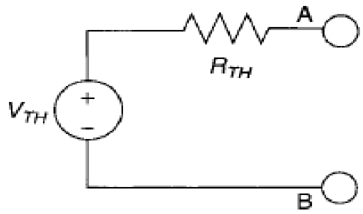
Figure 6.7

6.1 Thevenin's* theorem:

Thevenin's theorem states: any linear network containing an element connected to two terminals A and B may be represented by an equivalent circuit between those terminals consisting of an voltage source (V_{th}) in series with a resistor (R_{th}) as shown in Fig. 6.8.



a) Linear Network with Two Terminals



b) Equivalent Circuit Across the Terminals

Figure 6.8

(V_{th}) is the open circuit voltage between the terminals A and B with the element removed and (R_{th}) is the resistance between the terminals A and B with the element removed and with all sources removed (ideal voltage sources are replaced by a short circuit and ideal current sources are replaced by an open circuit).

The procedure adopted when using Thevenin's theorem is summarised below:

(i) remove the portion of the network across which the Thevenin equivalent circuit is to be found,

(ii) determine the open-circuit voltage, $V_{o.c}$, across the two terminals from where the portion has been removed. It is also called Thevenin voltage (V_{Th}),

(iii) Compute the resistance of the network as looked into from these two terminals after all sources have been removed (voltage sources are replaced by a short circuit and current sources are replaced by an open circuit). It is also called Thevenin resistance (R_{Th}),

(iv) Draw the Thevenin equivalent circuit and connect back the portion of the circuit previously removed to its terminals,

(v) Finally, calculate the values of the currents or voltages from the equivalent circuit.

Example 6.1: Use Thevenin's theorem to find the current flowing in the 10 Ω resistor for the circuit shown in Fig 6.9.

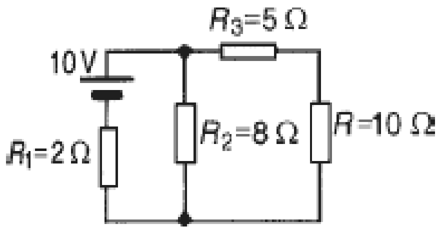


Figure 6.9

Solution:

(i) The 10 Ω resistance is removed from the circuit as shown in Fig. 6.10(a)

(ii) There is no current flowing in the 5 Ω resistor and current I_1 is given by

$$I_1 = \frac{10}{R_1 + R_2} = \frac{10}{2+8} = 1\,\mathrm{A}$$

Voltage across $R_2 = I_1R_2 = 1\times8 = 8V$. Hence voltage across AB, i.e. Thevenin voltage (V_{Th}) =8V.

iii) Removing the source of e.m.f. gives the circuit of Fig. 6.10 (b) Resistance (R_{Th}),

---

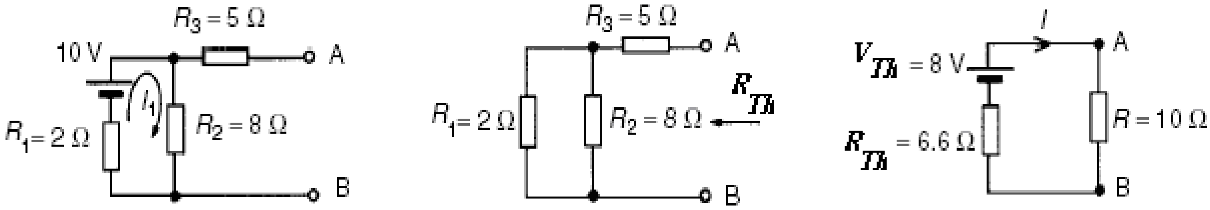
* Thevenin, a French engineer, developed work by Helmholtz and published this theorem in 1883.

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 5 + \frac{2\times 8}{2+8} = 5 + 1.6 = 6.6\Omega$$

(iv) The equivalent Thevenin's circuit is shown in Fig. 6.10 (c)

$$I = \frac{V_{Th}}{R_{Th} + R} = \frac{8}{10+6.6} = 0.482A$$

Hence the current flowing in the 10 Ω resistor of Fig. 6.9 is **0.482A**.



(a) (b) (c)

Figure 6.10

Example 6.2: Use Thevenin's theorem to determine the current *I* flowing in the 4Ω resistor shown in Fig. 6.11. Find also the power dissipated in the 4Ω resistor.

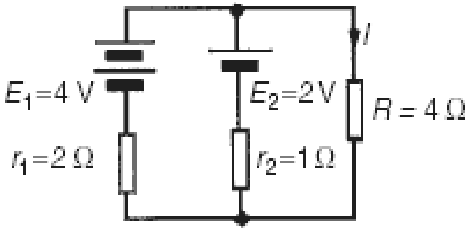


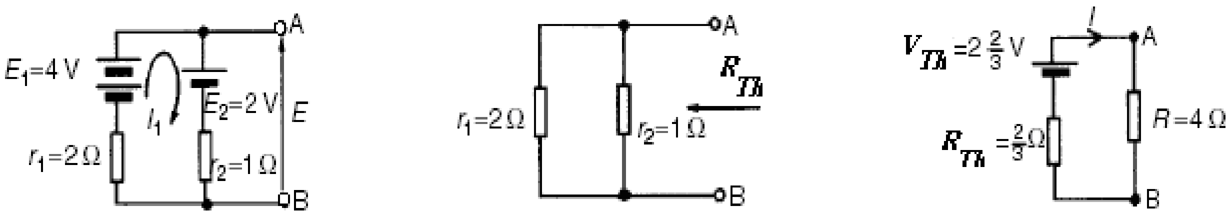
Figure 6.11

Solution

(i) The 4Ω resistor is removed from the circuit as shown in Fig. 6.12(a)

$$I_1 = \frac{E_1 - E_2}{r_1 + r_2} = \frac{4-2}{2+1} = \frac{2}{3}A$$

$$V_{Th} = E_1 - I_1 r_1 = 4 - \frac{2}{3}\times 2 = 2\frac{2}{3}A$$



(a) (b) (c)

Figure 6.12

(iii) Removing the sources of e.m.f. gives the circuit shown in Fig. 6.12 (b), from which, resistance

$$R_{Th} = \frac{2\times 1}{2+1} = \frac{2}{3}\Omega$$

(iv) The equivalent Thevenin's circuit is shown in Fig. 6.12 (c), from which, current,

$$I = \frac{V_{Th}}{R_{Th} + R} = \frac{2\frac{2}{3}}{\frac{2}{3}+4} = 0.571A$$

Power dissipated in the 4Ω resistor, $P=I^2R=(0.571)^2(4)=$**1.304W**

Example 6.3: Use Thevenin's theorem to determine the current flowing in the 3Ω resistance of the network shown in Fig. 6.13.

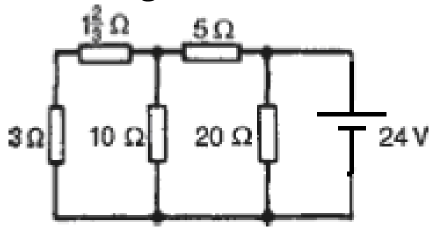


Figure 6.13

Solution

(i) The 3Ω resistance is removed from the circuit as shown in Fig. 6.13(a).

(ii) The $1\frac{2}{3}\Omega$ resistance now carries no current. The voltage across 10Ω resistor,

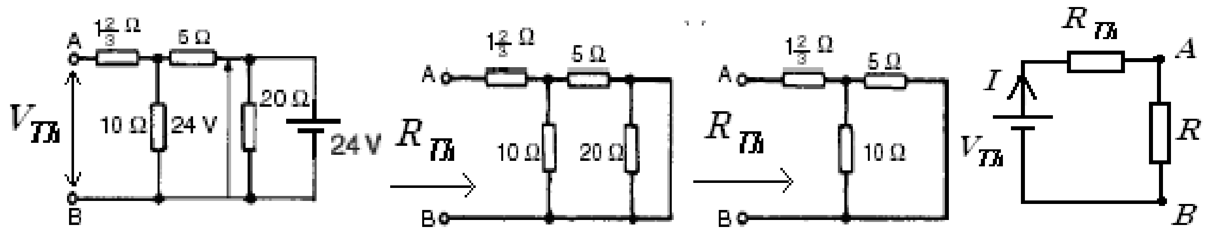
i.e $(V_{Th}) = \frac{10}{10+5} \times 24 = 16V$

(iii) Removing the source of e.m.f. and replacing it short-circuited as shown in Fig. 6.13 (b) . The 20Ω resistance may thus be removed as shown in Fig. 6.13 (c). From Fig. 6.13 (c), resistance,

$R_{Th} = 1\frac{2}{3} + \frac{10 \times 5}{10+5} = 5\Omega$

(iv) The equivalent Thevenin's circuit is shown in Fig. 6.13. (d), from which, current,

$I = \frac{V_{Th}}{R_{Th} + R} = \frac{16}{3+5} = 2A$ = current in the 3Ω resistance



(a) (b) (c) (d)

Figure 6.13

Example6.4: Find the current through the 40Ω resistor (R_L) in the circuit of Fig. 3.14.

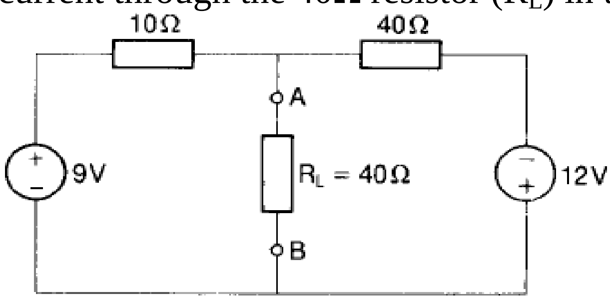


Figure 6.14

Solution

To calculate V_{Th} remove the resistor R_L in Fig.6.14 to give the circuit of Fig. 6.15 (a) and determine the potential difference between the terminals A and B. We therefore need to calculate the current I. Applying KVL to the circuit and taking the clockwise direction to be positive, we have

9 - 10I - 40I + 12 = 0

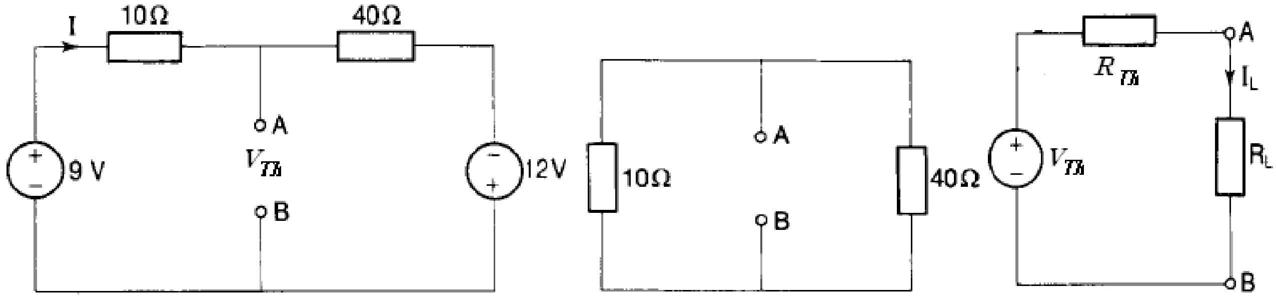
50I = 21

I = 21/50 = 0.42 A

Now, going from A to B via the 9V battery we have that

$-10 - 10 \times 0.42 + 9 = 4.8 = V_{Th}$.

This means that the potential drop is positive so that terminal A is at a higher potential than terminal B and so the current will flow through R_L from A to B.



(a) (b) (c)

Figure 6.15

To calculate R_{Th}, replace the batteries by short circuits to give the circuit of Fig. 6.15(b).

$R_{Th} = (10 \times 40)/(10 + 40) = 8\ \Omega$

Put these values for V_{Th} and R_{Th} in the Thevenin equivalent circuit of Fig. 6.15 (c). Then,

$I_L = E_{Th}/(R_{Th} + R_L) = 4.8/(8 + 40) = 0.1\ A$

Example 6.5: For the Bridge network shown in Fig. 6.16. Calculate the current flowing in the 32Ω resistor, and its direction, using Thevenin's theorem.

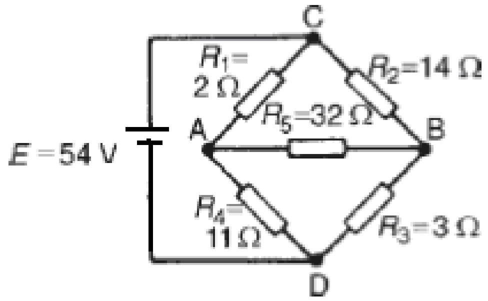


Figure 6.16

Solution

(i) The 32Ω resistor is removed from the circuit as shown in Fig. 6.17 (a).

(ii) The p.d. between A and C,

$$V_{AC} = \left(\frac{R_1}{R_1 + R_4}\right)(E) = \left(\frac{2}{2+11}\right)(54)$$

$$= 8.31\,\mathrm{V}$$

The p.d. between B and C,

$$V_{BC} = \left(\frac{R_2}{R_2 + R_3}\right)(E) = \left(\frac{14}{14+3}\right)(54)$$

$$= 44.47\,\mathrm{V}$$

Hence the p.d. between A and B = 44.47–8.31= **36.16V** = V_{Th}

Point C is at a potential of +54V. Between C and A is a voltage drop of 8.31V. Hence the voltage at point A is 54–8.31=45.69V. Between C and B is a voltage drop of 44.47V. Hence the voltage at point B is 54–44.47=9.53V. Since the voltage at A is greater than at B, current must flow in the direction A to B.

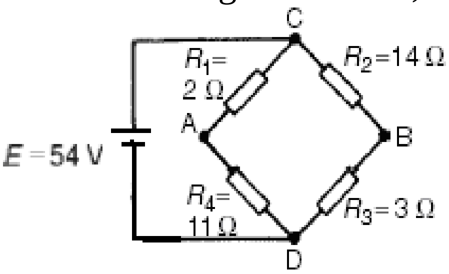
(iii) Replacing the source of e.m.f. with a short-circuit gives the circuit shown in Fig. 6.17 (b). The circuit is redrawn and simplified as shown in Fig. 6.17 (c) and (d), from which the resistance between terminals A and B,

$$R_{Th} = \frac{2 \times 11}{2+11} + \frac{14 \times 3}{14+3} = 4.163\Omega$$

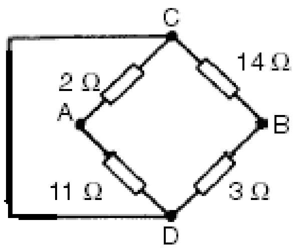
(iv) The equivalent Thevenin's circuit is shown in Fig. 6.17 (e), from which, current

$$I = \frac{36.16}{4.163+32} = 1A$$

Hence the current in the 32Ω resistor of Fig. 6.16 is 1A, flowing from A to B.



a



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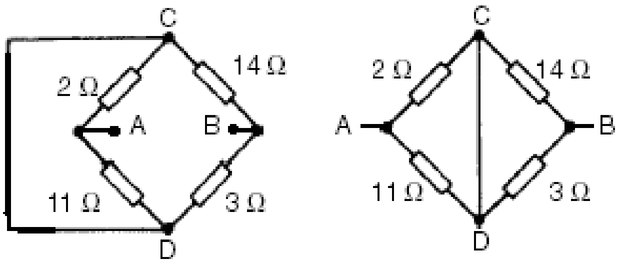
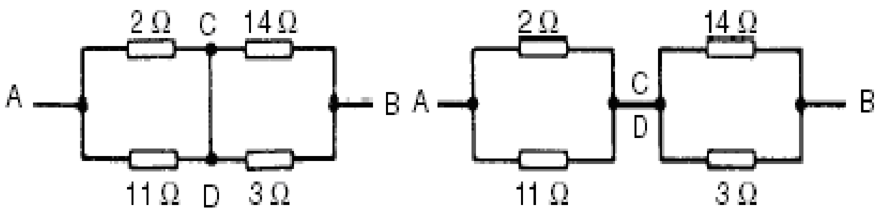
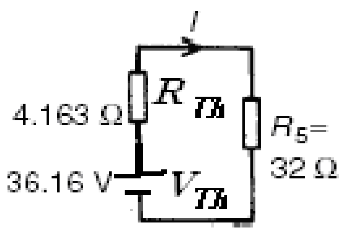


c



d



e

Figure 6.17

6.2 Norton's theorem*

Norton's theorem states: any linear network containing an element connected to two terminals A and B may be represented by an equivalent circuit between those terminals consisting of an current source (I_N) in parallel with a resistor (R_N) as shown in Fig. 6.18.

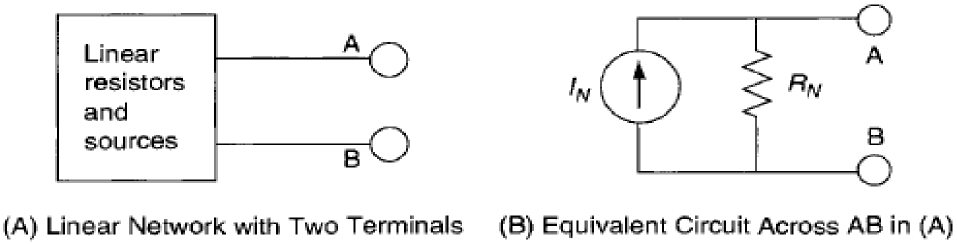


Figure 6.18

The current I_N is that which would flow through a short circuit connected between the terminals A and B, and R_N is the equivalent resistance between the terminals A and B with the element removed and with all sources removed.

The procedure adopted when using Norton's theorem is summarised below.

(i) remove the portion of the network across which the Norton's equivalent circuit is to be found,

(ii) determine the short-circuit current I_{SC} flowing in the branch from where the portion has been removed. It is also called Norton current (I_N),

(iii) Compute the resistance of the network as looked into from these two terminals after all sources have been removed (voltage sources are replaced by a short circuit and current sources are replaced by an open circuit). It is also called Norton resistance (R_N),

(iv) Draw the Norton equivalent circuit and connect back the portion of the circuit previously removed to its terminals,

(v) Finally, calculate the values of the currents or voltages from the equivalent circuit.

Example 6.6: Use Norton's theorem to determine the current flowing in the 10Ω resistance for the circuit shown in Fig. 6.19

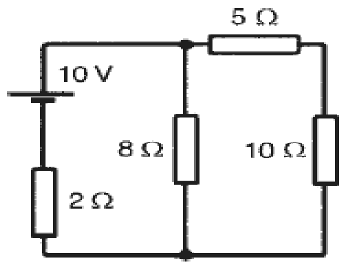


Figure 6.19

Solution

Following the above procedure:

(i)The branch containing the 10Ω resistance is short-circuited as shown in Fig. 6.20(a).

(ii) Fig. 6.20(b) is equivalent to Fig. 6.20(a).

$$I_N = \frac{10}{2} = 5A$$

(iii) If the 10V source of e.m.f. is removed from Fig. 6.20 (a) the resistance 'looking-in' at a break made between A and B is given by:

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* In 1926 Norton, an American engineer, introduced an equivalent circuit which is the dual of Thevenin's

$R_N = \frac{2\times 8}{2+8} = 1.6\Omega$

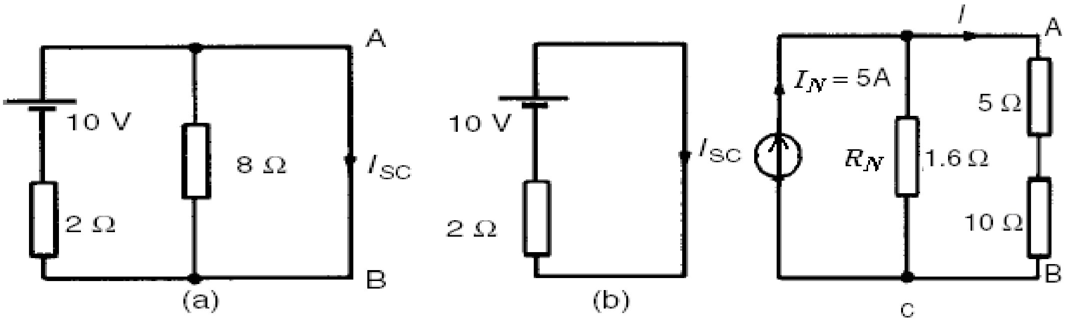

Figure 6.20

(iv) From the Norton equivalent network shown in Fig. 6.20(c) the current in the 10Ω resistance, by current division, is given by:

$$I = \left(\frac{1.6}{1.6+5+10}\right)(5) = \mathbf{0.482\,A}$$

as obtained previously in Example 6.1 using Thevenin's theorem.

Example 6.7: Use Norton's theorem to determine the current *I* flowing in the 4Ω resistance shown in Fig. 6.21.

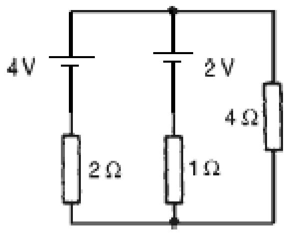

Figure 6.21

Solution

(i) The 4Ω branch is short-circuited as shown in Fig. 6.22(a).

(ii) From Fig. 6.22(a),

$$I_{SC} = I_1 + I_2 = \frac{4}{2} + \frac{2}{1} = 4\,A$$

(iii) If the voltage sources are removed the resistance 'looking-in' at a break made between A and B is given by:

$$R_N = \frac{2\times 1}{2+1} = \frac{2}{3}\Omega$$

(iv) From the Norton equivalent network shown in Fig. 6.20(b) the current in the 4Ω resistance is given by:

$$I = \left(\frac{\frac{2}{3}}{\frac{2}{3}+4}\right)(4) = \mathbf{0.571\,A},$$

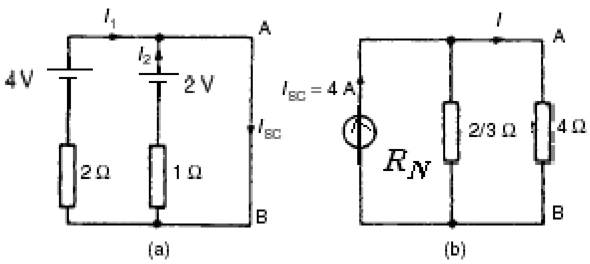

Figure 6.22

Example 6.8: Use Norton's theorem to determine the current flowing in the 3Ω resistance of the network shown in Fig. 6.23.

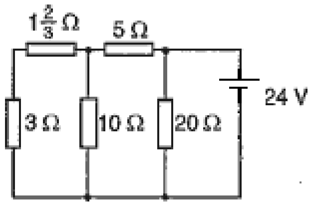

Figure 6.23

Solution

(i) The branch containing the 3Ω resistance is short circuited as shown in Fig. 6.24(a).

(ii) From the equivalent circuit shown in Fig. 6.24(b),

$$I_{SC} = \frac{24}{5} = 4.8\,\text{A}$$

(iii) If the 24V source of e.m.f. is removed the resistance 'looking-in' at a break made between A and B is obtained from Fig. 6.24(c) and its equivalent circuit shown in Fig. 6.24(d) and is given by:

$$R_N = \frac{10\times 5}{10+5} = 3\frac{1}{3}\Omega$$

(iv) From the Norton equivalent network shown in Fig. 6.24(e) the current in the 3Ω resistance is given by:

$$I = \left(\frac{3\frac{1}{3}}{3\frac{1}{3} + 1\frac{2}{3} + 3}\right)(4.8) = \mathbf{2\,A},$$

as obtained previously in Example 6.7 using Thevenin's theorem.

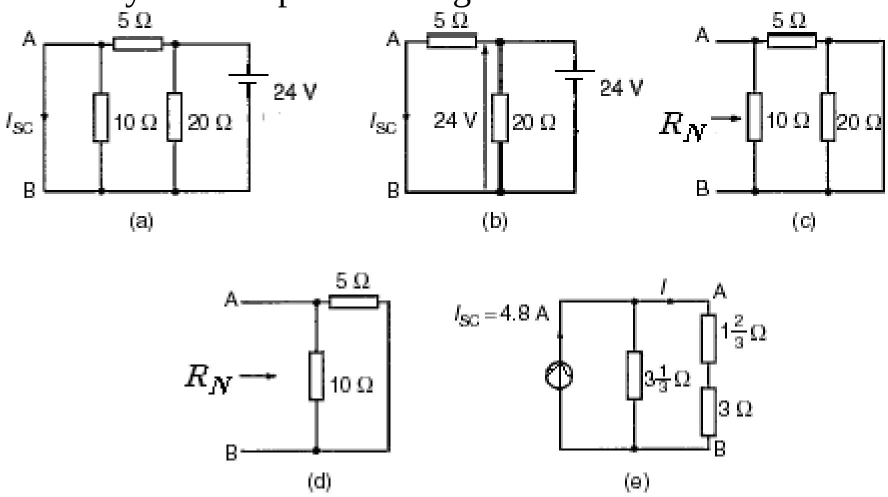

Figure 6.24

6.3 Thevenin and Norton equivalent circuits

The Thevenin and Norton networks shown in Fig. 6.25 are equivalent to each other. The resistance 'looking-in' at terminals AB is the same in each of the networks, i.e. ($R_{Th}= R_N$).

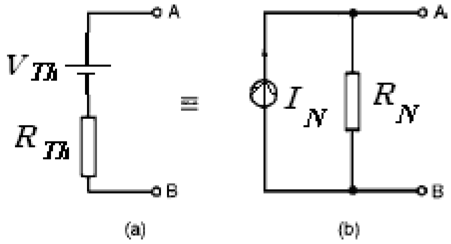

Figure 6.25

If terminals AB in Fig. 6.25 (a) are short-circuited, the short-circuit current is given by V_{Th}/R_{Th}. If terminals AB in Fig. 6.25 (b) are short-circuited, the short-circuit current is I_N. For the circuit shown in Fig. 6.25 (a) to be equivalent to the circuit in Fig. 6.25 (b) the same short-circuit current must flow. Thus $I_N = V_{Th}/R_{Th}$.

Source Transformation

Using a Norton equivalent circuit, a voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor. In a similar manner, using Thevenin theorem, a current source with a parallel resistor can be represented by a voltage source with a series resistor. These transformations are called source transformations. The two sources in Figure 6.26 are equivalent between nodes B and C.

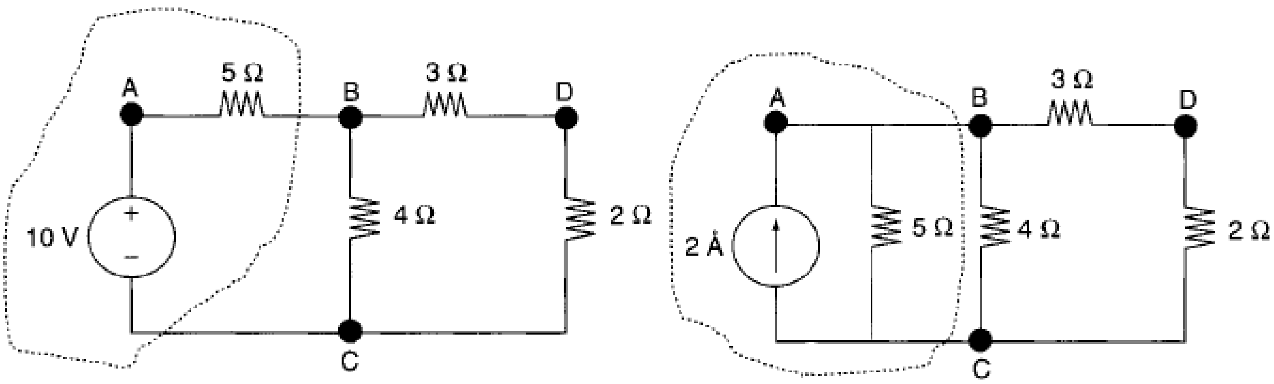

Figure 6.26

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Note: Thevenin's and Norton's theorem is very useful when we wish to determine the current through or the voltage across an element which is variable.

6.4 The maximum power transfer theorem

Fig. 6.27 shows the Thevenin equivalent circuit of a network. The power in the load resistor R_L is given by

$$P_L = I_L^2 R_L .$$

But $I_L = V_{Th}/(R_{Th} + R_L)$, so that

$$P_L = V_{Th}^2 RL/(R_{Th} + R_L)^2$$

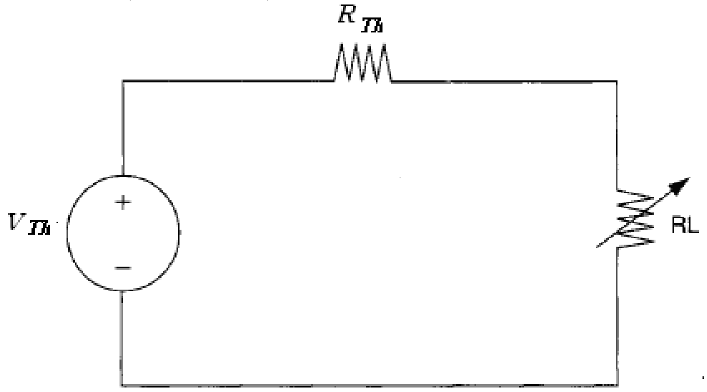

Figure 6.27

As R_L varies, with V_{Th} and R_{TH} being constant, this will be a maximum (or a minimum) when $dP_L/dR_L = 0$. Using the technique for differentiating a quotient we get

$dP_L/dR_L = [(R_{Th} + R_L)^2 V_{Th}^2 - V_{Th}^2 R_L(2(R_{Th} + R_L))]/(Ro + RL)^4$

This will be zero when the numerator is zero, i.e. when

$(R_{Th} + R_L)^2 V_{Th}^2 = 2V_{Th}^2 R_L(R_{Th} + R_L)$

$R_{Th} + R_L = 2R_L$

$R_{Th} = R_L$

This can be confirmed as a maximum, rather than a minimum, by showing that d^2pL/dRL^2 is negative.

The power delivered to the load is therefore a maximum *when the resistance of the load is equal to the internal resistance of the network*, and this is called the maximum power transferred theorem. The actual value of the maximum power transferred is obtained by putting $R_L = R_{Th}$ into the equation for P_L. This gives

$P_{max} = V_{Th}^2 R_L /(R_L + R_L)^2 = V_{Th}^2 R_L /(2R_L)^2$

$P_{max} = V_{Th}^2 /4R_L$

Typical practical applications of the maximum power transfer theorem are found in stereo amplifier design, seeking to maximise power delivered to speakers, and in electric vehicle design, seeking to maximise power delivered to drive a motor.

Example 6.9: Find the value of the load resistor R_L shown in Fig. 6.27 that gives maximum power dissipation and determine the value of this power.

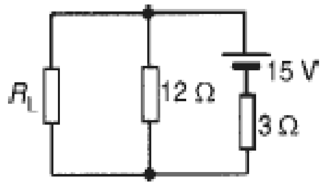

Figure 6.27

Solution

Using the procedure for Thevenin's theorem:

(i) Resistance R_L is removed from the circuit as shown in Fig. 6.28 (a).

(ii) The p.d. across AB is the same as the p.d. across the 12Ω resistor. Hence V_{Th}

$V_{Th} = \frac{12 \times 15}{12+3} = 12V$

(iii) Removing the source of e.m.f. gives the circuit of Fig. 6.28 (b), from which, resistance,

$R_{Th} = \frac{12 \times 3}{12+3} = 2.4\Omega$

(iv)The equivalent Thevenin's circuit supplying terminals AB is shown in Fig. 6.28 (c), from which,

$I = \frac{V_{Th}}{R_{Th} + R_L}$

For maximum power, $R_L = R_{Th} = 2.4\Omega$

Thus current, $I = \frac{12}{2.4 + 2.4} = 2.5\,A$

Power, P, dissipated in load R_L,

$P = I^2 R_L = (2.5)^2(2.4) = \mathbf{15\,W}$.

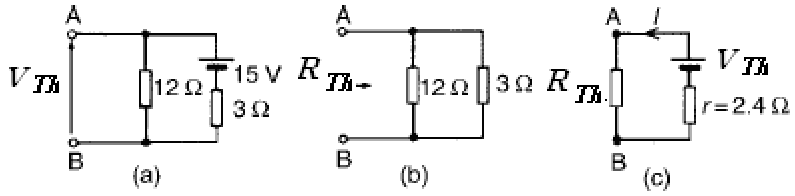

Figure 6.28

Example 6.9: Consider the circuit shown in Figure 6.29. Reduce the portion of the circuit to the left of terminals a–b to (a) a Thevenin equivalent and (b) a Norton equivalent. Find the current through R = 16Ω, and comment on whether resistance matching is accomplished for maximum power transfer.

Solution

(a) KVL: $144 - 24I_L - 48I_L - 96 = 0$, or $72I_L = 48$, or $I_L = 2/3$ A

$$V_{oc} = 144 - 24(2/3) = 128\ \text{V}$$

$$R_{Th} = 48 \| 24 = \frac{48 \times 24}{48 + 24} = 16\ \Omega$$

$I = 128/32 = 4$ A

(b) $I_{sc} = 8$ A

I=8x16/(16+16)=4A

Since $R_L = R_{Th}$

Maximum power transfer is accomplished.

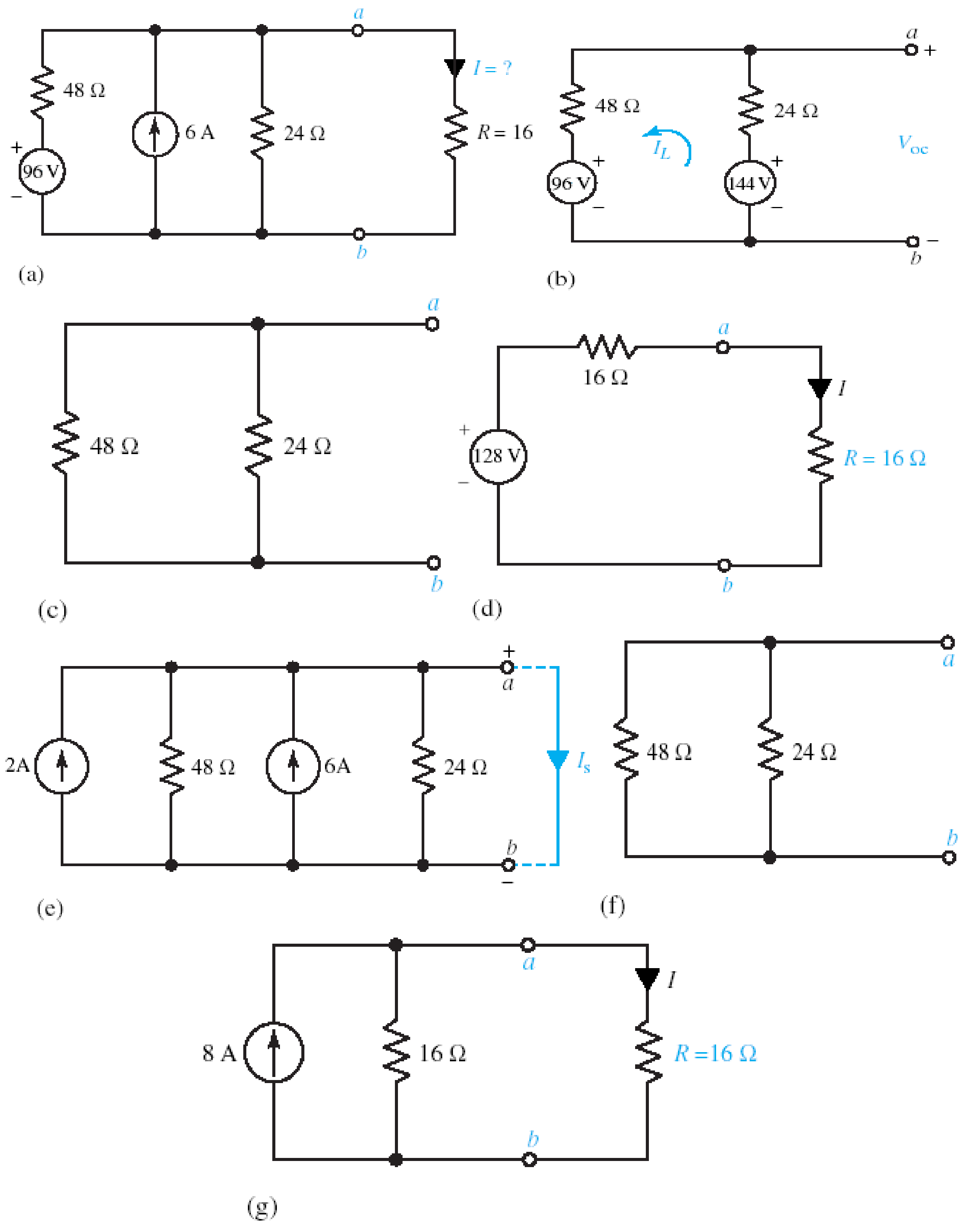

Figure 6.29

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Note: When the independent sources in the network are deactivated. The R_{TH} can also be determined as $R_{Th} = V_{oc}/I_{sc}$, where V_{oc} is the open circuit voltage across terminals A and B and I_{sc} is the short circuit current that will flow from A to B through short circuit. It is easy to see the following relation between Thevenin equivalent circuit parameters and the Norton equivalent circuit parameters:

$R_N = R_{Th}$ *and* $I_N = V_{Th} / R_{Th}$

Example 6.10: Consider the circuit of Figure E2.1.2(a), Obtain the Thevenin equivalent at terminals *a–b*.

Solution:

KCL at node a: $I + 9I = I_1$, or $I_1 = 10I$

KVL for the left-hand mesh: $2000I + 200I_1 = 10$, or $4000I = 10$, or $I = 1/400$ A

$$V_{oc} = 200I_1 = 200(1/400) = 0.5 \text{ V}$$

Because of the presence of a dependent source, in order to find R_{Th}, one needs to determine I_{sc} after shorting terminals a–b, as shown in Figure E2.1.2(b).

Note that $I_1 = 0$, since $V_{ab} = 0$.

KCL at node a: $I_{sc} = 9I + I = 10I$

KVL for the outer loop: $2000I = 10$, or $I = 1/200$ A

$$I_{sc} = 10(1/200) = 1/20 \text{ A}$$

Hence the equivalent Thévenin resistance R_{Th} viewed from terminals a–b is

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{0.5}{1/20} = 10\Omega$$

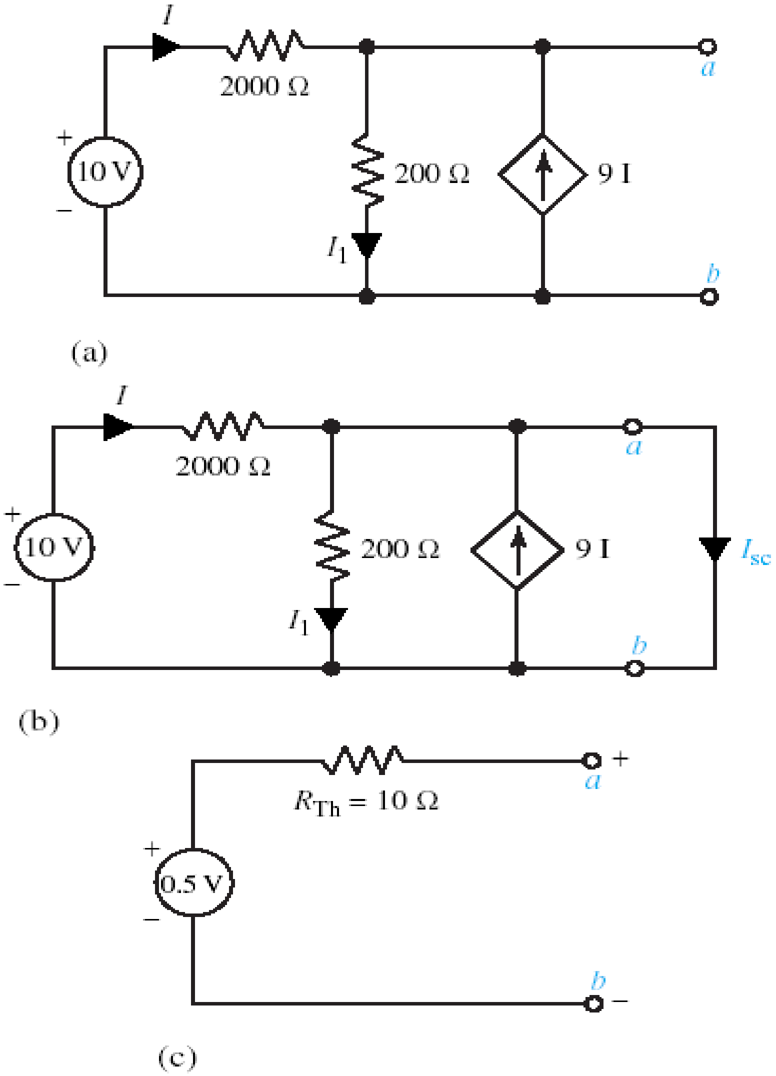

Figure 6.29

Problems

1. Determine the voltage across the 20Ω resistor in the following circuit of Figure 2.30 with the application of superposition. (Ans. 32.25V)

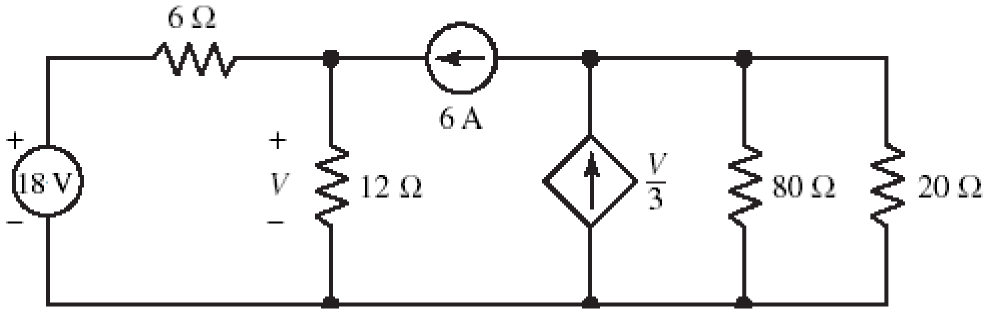

Figure 6.30

2.Find the Thevenin and Norton equivalent circuit seen by the load R_L in the circuit of Figs. 2.31.

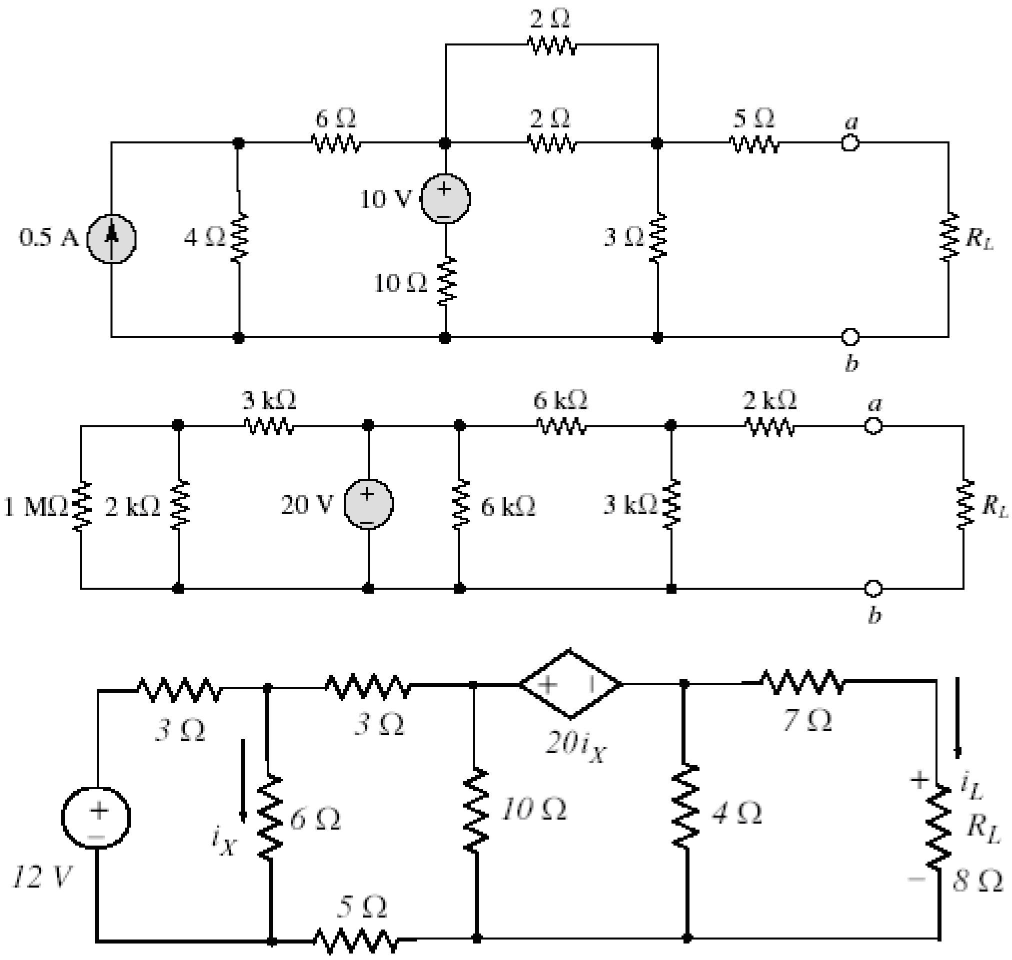

Figure 6.31

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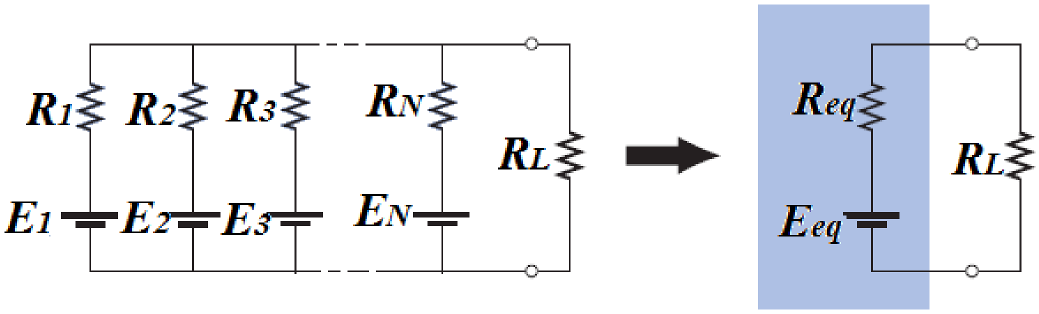
Millman's theorem

أي عدد من مصادر الفولتية (معها مقاومة على التوالي)، والمرتبطة على التوازي، كما في الشكل، يمكن اختصارها الى مصدر فولتية واحد مكافئ مع مقاومة على التوالي. وبذلك يمكن حساب الفولتية أو التيار في المقاومة (R_L) دون الحاجة الى تطبيق أي نظرية للحل.



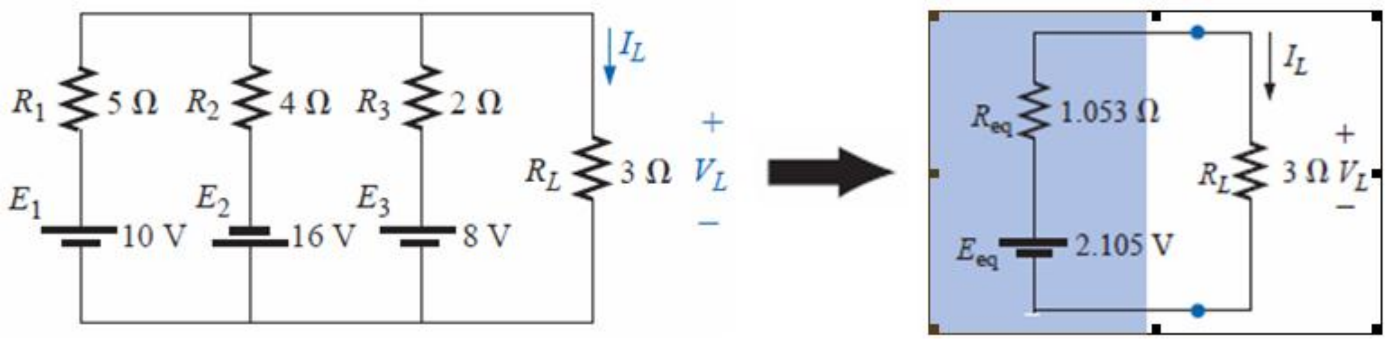
تستخدم المعادلتين التاليتين للحل

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

$$E_{eq} = \frac{\pm\frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \cdots \pm \frac{E_N}{R_N}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

والإشارة السالبة تأخذ بنظر الاعتبار حالة اختلاف اتجاه تجهيز التيار من مصادر الفولتية.

Example: Using Millman's theorem, find the current through and voltage across the resistor (R_L)



Solution:

$$E_{eq} = \frac{+\frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

الإشارة السالبة للمصدر (E_2) لان قطبيته معاكسة لقطبية المصدرين (E_1&E_3)

$$E_{eq} = \frac{+\frac{10\text{ V}}{5\,\Omega} - \frac{16\text{ V}}{4\,\Omega} + \frac{8\text{ V}}{2\,\Omega}}{\frac{1}{5\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{2\,\Omega}} = \frac{2}{0.95} = \boxed{2.105\text{ V}}$$

$$R_{eq} = \frac{1}{\frac{1}{5\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}} = \frac{1}{0.95} = \mathbf{1.053\ \Omega}$$

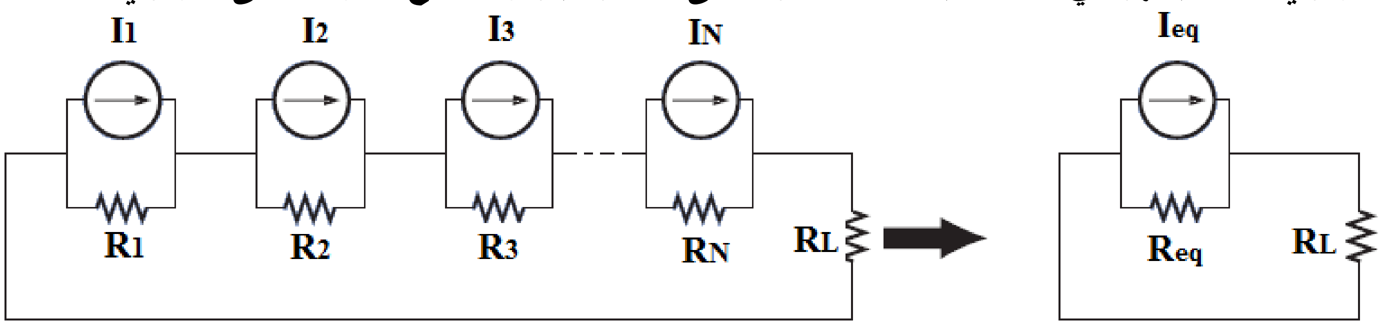
From equivalent circuit

$$I_L = \frac{2.105\ \text{V}}{1.053\ \Omega + 3\ \Omega} = \frac{2.105\ \text{V}}{4.053\ \Omega} = \mathbf{0.519\ A}$$

$$V_L = I_L R_L = (0.519\ \text{A})(3\ \Omega) = \mathbf{1.557\ V}$$

Dual Millman's theorem:

أي مجموعة متوالية من مصادر التيار (current sources) والتي مع كل منها مقاومة على التوازي ، كما يظهر في الشكل، يمكن اختصارها الى مصدر تيار واحد مع مقاومة على التوازي.



وتحسب (R_{eq}) و (I_{eq}) من المعادلتين:

$$I_{eq} = \frac{\pm I_1R_1 \pm I_2R_2 \pm I_3R_3}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

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AC power

EXAMPLE 15.8 Determine the input impedance to the series network of Fig. 15.23. Draw the impedance diagram.

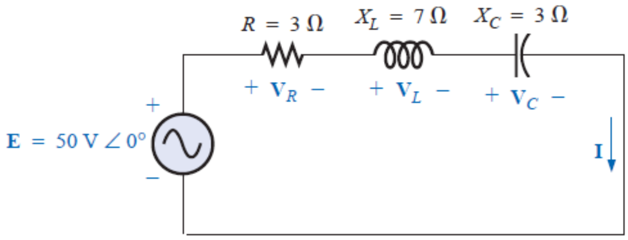


Fig. 15.23.

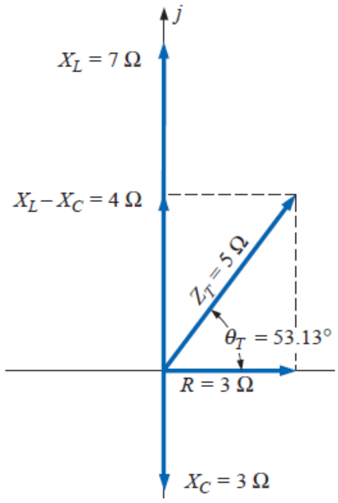


Fig. 15.37.

Solution:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\,\angle 0^\circ + X_L\,\angle 90^\circ + X_C\,\angle -90^\circ$$
$$= 3\,\Omega + j\,7\,\Omega - j\,3\,\Omega = 3\,\Omega + j\,4\,\Omega$$
$$\mathbf{Z}_T = \mathbf{5\,\Omega\,\angle 53.13^\circ}$$

Impedance diagram: As shown in Fig. 15.37.

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\,\angle 0^\circ}{5\,\Omega\,\angle 53.13^\circ} = \mathbf{10\text{ A}\,\angle -53.13^\circ}$$

$$\mathbf{V}_R = \mathbf{IZ}_R = (I\,\angle\theta)(R\,\angle 0^\circ) = (10\text{ A}\,\angle -53.13^\circ)(3\,\Omega\,\angle 0^\circ)$$
$$= \mathbf{30\text{ V}\,\angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{IZ}_L = (I\,\angle\theta)(X_L\,\angle 90^\circ) = (10\text{ A}\,\angle -53.13^\circ)(7\,\Omega\,\angle 90^\circ)$$
$$= \mathbf{70\text{ V}\,\angle 36.87^\circ}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I\,\angle\theta)(X_C\,\angle -90^\circ) = (10\text{ A}\,\angle -53.13^\circ)(3\,\Omega\,\angle -90^\circ)$$
$$= \mathbf{30\text{ V}\,\angle -143.13^\circ}$$

The phasor diagram of Fig. 15.38 indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90°, and leads the voltage across the capacitor by 90°.

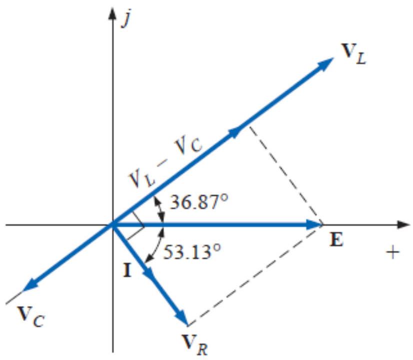


FIG. 15.38

VOLTAGE DIVIDER RULE

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$\boxed{\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T}}$$

where $\mathbf{V}_x$ is the voltage across one or more elements in series that have total impedance $\mathbf{Z}_x$, **E** is the tota voltage appearing across the series circuit, and $\mathbf{Z}_T$ is the total impedance of the series circuit.

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit of Fig. 15.41.

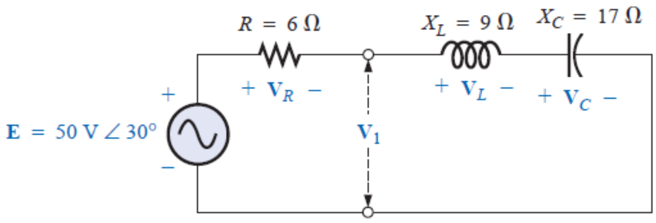


FIG. 15.41

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\,\Omega\,\angle 0^\circ)(50\text{ V}\,\angle 30^\circ)}{6\,\Omega\,\angle 0^\circ + 9\,\Omega\,\angle 90^\circ + 17\,\Omega\,\angle -90^\circ} = \frac{300\,\angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\,\angle 30^\circ}{6 - j\,8}$$
$$= \frac{300\,\angle 30^\circ}{10\,\angle -53.13^\circ} = \mathbf{30\text{ V}\,\angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\,\Omega\,\angle 90^\circ)(50\text{ V}\,\angle 30^\circ)}{10\,\Omega\,\angle -53.13^\circ} = \frac{450\text{ V}\angle 120^\circ}{10\,\angle -53.13^\circ}$$
$$= \mathbf{45\text{ V}\,\angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V} \angle -60^\circ}{10\ \angle -53^\circ}$$
$$= \mathbf{85\ V\ \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$
$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ}$$
$$= \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$

CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ as shown in Fig. 15.76,

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

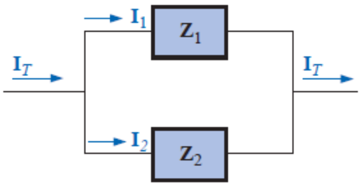


FIG. 15.76

EXAMPLE 15.15 Using the current divider rule, find the current through each impedance of Fig. 15.77.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\ \angle 90^\circ)(20\ \text{A}\ \angle 0^\circ)}{3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ} = \frac{80\ \text{A} \angle 90^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{16\ A\ \angle 36.87^\circ}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\ \angle 0^\circ)(20\ \text{A}\ \angle 0^\circ)}{5\ \Omega\ \angle 53.13^\circ} = \frac{60\ \text{A} \angle 0^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{12\ A\ \angle -53.13^\circ}$$

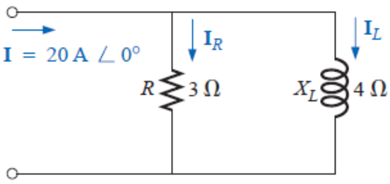


FIG. 15.77

EXAMPLE 15.16 Using the current divider rule, find the current through each parallel branch of Fig. 15.78.

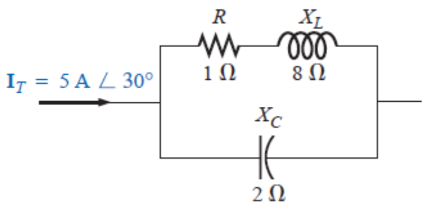


FIG. 15.78

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega\ \angle -90^\circ)(5\ \text{A}\ \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j\,6}$$
$$= \frac{10\ \text{A} \angle -60^\circ}{6.083\ \angle 80.54^\circ} \cong \mathbf{1.644\ A\ \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L}\mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A}\ \angle 30^\circ)}{6.08\ \Omega\ \angle 80.54^\circ}$$
$$= \frac{(8.06\ \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08\ \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083\ \angle 80.54^\circ}$$
$$= \mathbf{6.625\ A\ \angle 32.33^\circ}$$

EXAMPLE : For the network of Fig. 16.

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.

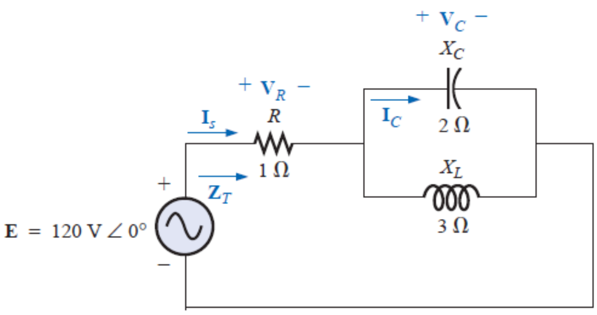


FIG. 16.1

Solutions:

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

$$\mathbf{Z}_1 = R \angle 0° = 1\ \Omega \angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \| \mathbf{Z}_L = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j X_C + j X_L} = \frac{(2\ \Omega \angle -90°)(3\ \Omega \angle 90°)}{-j\, 2\ \Omega + j\, 3\ \Omega}$$

$$= \frac{6\ \Omega \angle 0°}{j\, 1} = \frac{6\ \Omega \angle 0°}{1 \angle 90°} = 6\ \Omega \angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\, 6\ \Omega = \mathbf{6.08\ \Omega \angle -80.54°}$$

b. $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V} \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54°}$$

c. Referring to Fig. 16.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\ \text{A} \angle 80.54°)(1\ \Omega \angle 0°) = \mathbf{19.74\ V \angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\ \text{A} \angle 80.54°)(6\ \Omega \angle -90°)$$

$$= \mathbf{118.44\ V \angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\ \text{V} \angle -9.46°}{2\ \Omega \angle -90°} = \mathbf{59.22\ A \angle 80.54°}$$

EXAMPLE For the network of Fig. 16.5:

a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.

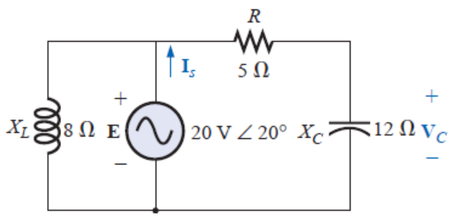


FIG. 16.5

Solutions:

a. The network is redrawn as shown in Fig. 16.6, with

$$\mathbf{Z}_1 = 5\ \Omega = 5\ \Omega \angle 0° \quad , \quad \mathbf{Z}_2 = -j\, 12\ \Omega = 12\ \Omega \angle -90°$$

$$\mathbf{Z}_3 = +j\, 8\ \Omega = 8\ \Omega \angle 90°$$

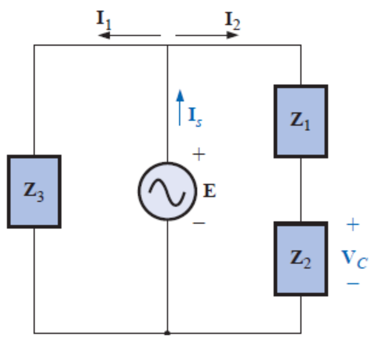


FIG. 16.6

$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega \angle -90°)(20\ \text{V} \angle 20°)}{5\ \Omega - j\, 12\ \Omega}$$

$$= \frac{240\ \text{V} \angle -70°}{13 \angle -67.38°} = \mathbf{18.46\ V \angle -2.62°}$$

b. $$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\ \text{V} \angle 20°}{8\ \Omega \angle 90°} = 2.5\ \text{A} \angle -70°$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V} \angle 20°}{13\ \Omega \angle -67.38°} = 1.54\ \text{A} \angle 87.38°$$

and

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 = 2.5\ \text{A} \angle -70° + 1.54\ \text{A} \angle 87.38°$$

$$= (0.86 - j\, 2.35) + (0.07 + j\, 1.54) = 0.93 - j\, 0.81 = \mathbf{1.23\ A \angle -41.05°}$$

EXAMPLE For the network of Fig. 16.14:

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

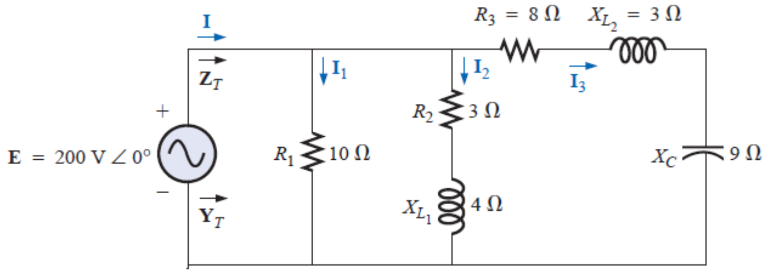
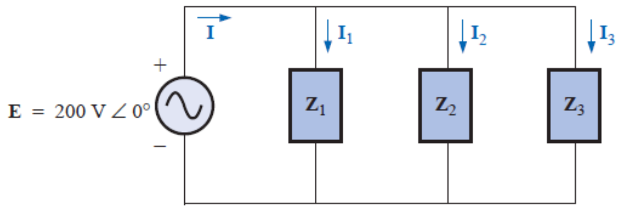


FIG. 16.14

Solutions:

a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10\ \Omega\ \angle 0^\circ$$
$$\mathbf{Z}_2 = R_2 + j X_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + j X_{L_2} - j X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$



The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\
&= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega} \\
&= 0.1\ \mathrm{S} + \frac{1}{5\ \Omega\ \angle 53.13^\circ} + \frac{1}{10\ \Omega\ \angle -36.87^\circ} \\
&= 0.1\ \mathrm{S} + 0.2\ \mathrm{S}\ \angle -53.13^\circ + 0.1\ \mathrm{S}\ \angle 36.87^\circ \\
&= 0.1\ \mathrm{S} + 0.12\ \mathrm{S} - j\,0.16\ \mathrm{S} + 0.08\ \mathrm{S} + j\,0.06\ \mathrm{S} \\
&= 0.3\ \mathrm{S} - j\,0.1\ \mathrm{S} = 0.316\ \mathrm{S}\ \angle -18.435^\circ
\end{aligned}$$

The current **I**:

$$\begin{aligned}
\mathbf{I} = \mathbf{E}\mathbf{Y}_T &= (200\ \mathrm{V}\ \angle 0^\circ)(0.316\ \mathrm{S}\ \angle -18.435^\circ) \\
&= \mathbf{63.2\ A\ \angle -18.435^\circ}
\end{aligned}$$

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\ \mathrm{V}\ \angle 0^\circ}{10\ \Omega\ \angle 0^\circ} = \mathbf{20\ A\ \angle 0^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\ \mathrm{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{40\ A\ \angle -53.13^\circ}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\ \mathrm{V}\ \angle 0^\circ}{10\ \Omega\ \angle -36.87^\circ} = \mathbf{20\ A\ \angle +36.87^\circ}$$

c.
$$\begin{aligned}
\mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 \\
60 - j\,20 &= 20\ \angle 0^\circ + 40\ \angle -53.13^\circ + 20\ \angle +36.87^\circ \\
&= (20 + j\,0) + (24 - j\,32) + (16 + j\,12) \\
60 - j\,20 &= 60 - j\,20 \quad \text{(checks)}
\end{aligned}$$

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\ \mathrm{S}\ \angle -18.435^\circ}$$

1 For the network of Fig. 16.46:

a. Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.

b. Find the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

c. Verify Kirchhoff's current law by showing that $\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$.

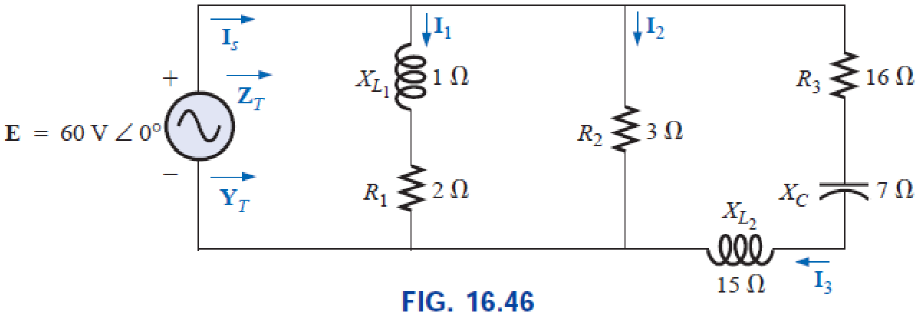


FIG. 16.46

2: For the network of Fig. 16.47:

a. Find the total admittance $\mathbf{Y}_T$.

b. Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.

c. Find the current $\mathbf{I}_3$.

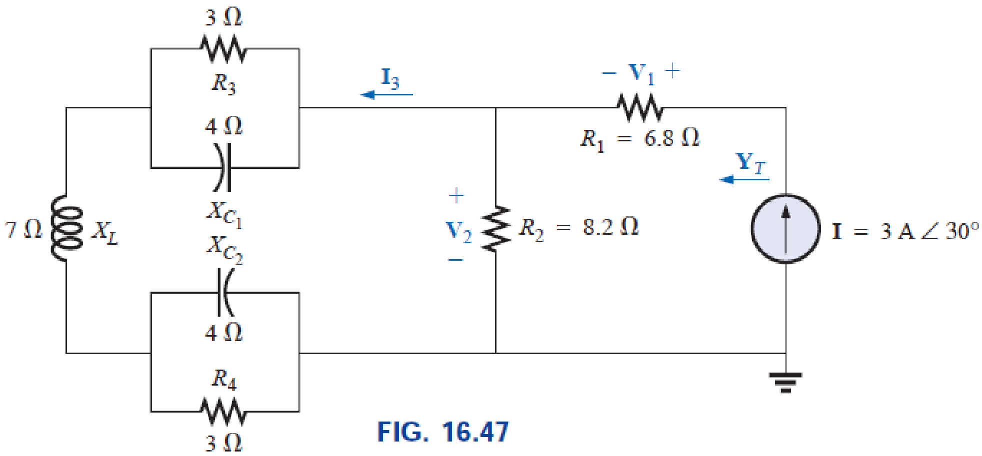


FIG. 16.47

3: Find the current $\mathbf{I}_5$ for the network of Fig. 16.50. Note the effect of one reactive element on the resulting calculations.

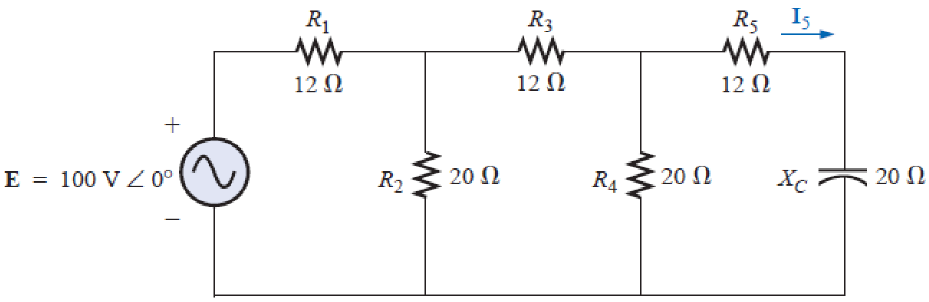
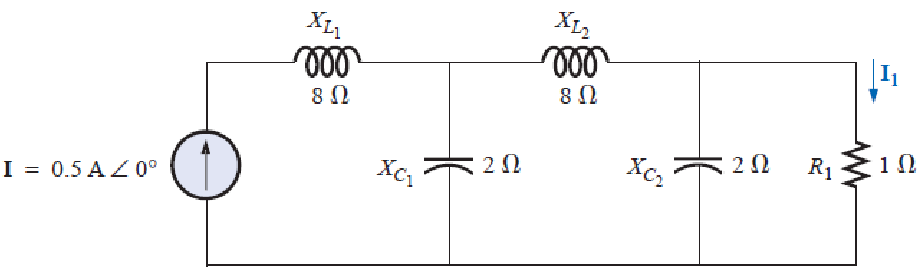


FIG. 16.50

4: Find the current $\mathbf{I}_1$ for the network of Fig. 16.52.



دوائر

THE TOTAL *P*, *Q*, AND *S*

The total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of any system can be found using the following procedure:

1. *Find the real power and reactive power for each branch of the circuit.*
2. *The total real power of the system (P_T) is then the sum of the average power delivered to each branch.*
3. *The total reactive power (Q_T) is the difference between the reactive power of the inductive loa and that of the capacitive loads.*
4. *The total apparent power is $S_T = \sqrt{P_T^2 + Q_T^2}$.*
5. *The total power factor is P_T/S_T.*

EXAMPLE: Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p of the network in Fig. 19.17. Draw the power triangle and find the current in phasor form.

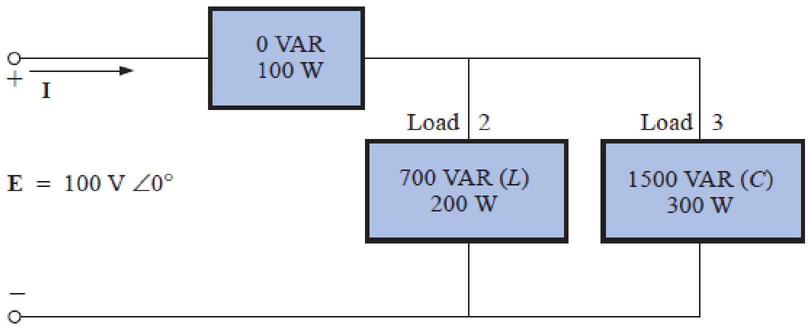


FIG. 19.17

Solution: Construct a table such as shown in Table

Load	W	VAR	VA
1	100	0	100
2	200	700 (*L*)	$\sqrt{(200)^2 + (700)^2} = 728.0$
3	300	1500 (*C*)	$\sqrt{(300)^2 + (1500)^2} = 1529.71$
	$P_T = \mathbf{600}$ Total power dissipated	$Q_T = \mathbf{800\ (C)}$ Resultant reactive power of network	$S_T = \sqrt{(600)^2 + (800)^2} = \mathbf{1000}$ (Note that $S_T \neq$ sum of each branch: $1000 \neq 100 + 728 + 1529.71$)

Thus,

$$F_p = \frac{P_T}{S_T} = \frac{600\ \text{W}}{1000\ \text{VA}} = \mathbf{0.6\ leading\ (C)}$$

The power triangle is shown in Fig. 19.18.

Since $S_T = VI = 1000$ VA, $I = 1000$ VA/100 V $= 10$ A; and since θ of $\cos\theta = F_p$ is the angle between the input voltage and current:

$$\mathbf{I = 10\ A\ \angle +53.13°}$$

The plus sign is associated with the phase angle since the circuit is predominantly capacitive.

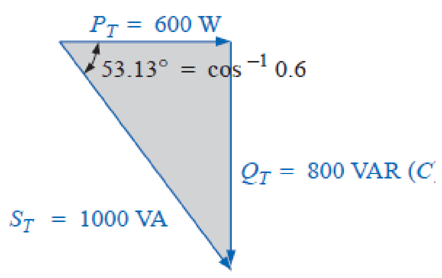


FIG. 19.18

EXAMPLE

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p for the network of Fig. 19.19.

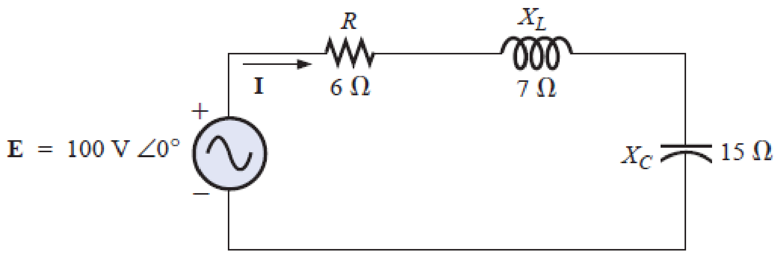


FIG. 19.19

b. Sketch the power triangle.

c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.

d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

Solutions:

a. $$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V} \angle 0°}{6\ \Omega + j\,7\ \Omega - j\,15\ \Omega} = \frac{100 \text{ V} \angle 0°}{10\ \Omega \angle -53.13°} = 10 \text{ A} \angle 53.13°$$

$$\mathbf{V}_R = (10 \text{ A} \angle 53.13°)(6\ \Omega \angle 0°) = 60 \text{ V} \angle 53.13°$$
$$\mathbf{V}_L = (10 \text{ A} \angle 53.13°)(7\ \Omega \angle 90°) = 70 \text{ V} \angle 143.13°$$
$$\mathbf{V}_C = (10 \text{ A} \angle 53.13°)(15\ \Omega \angle -90°) = 150 \text{ V} \angle -36.87°$$

$$P_T = EI\cos\theta = (100 \text{ V})(10 \text{ A})\cos 53.13° = \mathbf{600\ W}$$
$$= I^2R = (10 \text{ A})^2(6\ \Omega) = \mathbf{600\ W}$$
$$= \frac{V_R^2}{R} = \frac{(60 \text{ V})^2}{6} = \mathbf{600\ W}$$

$$S_T = EI = (100 \text{ V})(10 \text{ A}) = \mathbf{1000\ VA}$$
$$= I^2Z_T = (10 \text{ A})^2(10\ \Omega) = \mathbf{1000\ VA}$$
$$= \frac{E^2}{Z_T} = \frac{(100 \text{ V})^2}{10\ \Omega} = \mathbf{1000\ VA}$$

$$Q_T = EI\sin\theta = (100 \text{ V})(10 \text{ A})\sin 53.13° = \mathbf{800\ VAR}$$
$$= Q_C - Q_L$$
$$= I^2(X_C - X_L) = (10 \text{ A})^2(15\ \Omega - 7\ \Omega) = \mathbf{800\ VAR}$$

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150 \text{ V})^2}{15\ \Omega} - \frac{(70 \text{ V})^2}{7\ \Omega} = 1500 \text{ VAR} - 700 \text{ VAR} = \mathbf{800\ VAR}$$

$$F_p = \frac{P_T}{S_T} = \frac{600 \text{ W}}{1000 \text{ VA}} = \mathbf{0.6\ leading\ (\mathit{C})}$$

b. The power triangle is as shown in Fig. 19.20.

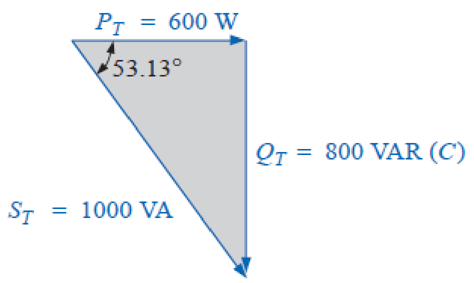


FIG. 19.20

c. $$W_R = \frac{V_RI}{f_1} = \frac{(60 \text{ V})(10 \text{ A})}{60 \text{ Hz}} = \mathbf{10\ J}$$

d. $$W_L = \frac{V_LI}{\omega_1} = \frac{(70 \text{ V})(10 \text{ A})}{(2\pi)(60 \text{ Hz})} = \frac{700 \text{ J}}{377} = \mathbf{1.86\ J}$$

$$W_C = \frac{V_CI}{\omega_1} = \frac{(150 \text{ V})(10 \text{ A})}{377 \text{ rad/s}} = \frac{1500 \text{ J}}{377} = \mathbf{3.98\ J}$$

EXAMPLE

For the system of Fig. 19.21,

a. Find the average power, apparent power, reactive power, and F_p for each branch
b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.
c. Find the source current I.

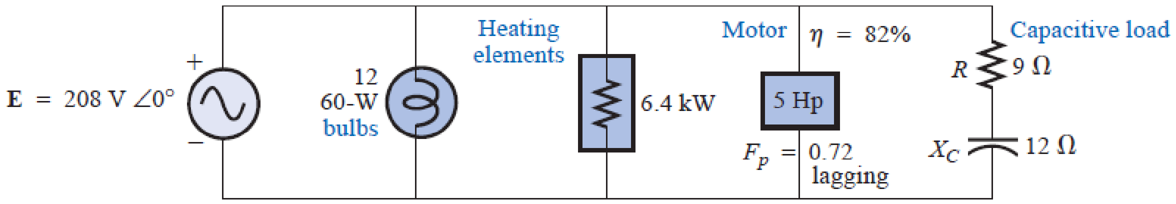


FIG. 19.21

Solutions:

Bulbs:

$P_1 = 12(60\text{ W}) = \mathbf{720\text{ W}}$, $Q_1 = \mathbf{0\text{ VAR}}$, $S_1 = P_1 = \mathbf{720\text{ VA}}$, $F_{p_1} = \mathbf{1}$

Heating elements:

$P_2 = \mathbf{6.4\text{ kW}}$, $Q_2 = \mathbf{0\text{ VAR}}$, $S_2 = P_2 = \mathbf{6.4\text{ kVA}}$, $F_{p_2} = \mathbf{1}$

Motor:

$$\eta = \frac{P_o}{P_i} \longrightarrow P_i = \frac{P_o}{\eta} = \frac{5(746\text{ W})}{0.82} = \mathbf{4548.78\text{ W}} = P_3$$

$$F_p = \mathbf{0.72\text{ lagging}}$$

$$P_3 = S_3\cos\theta \longrightarrow S_3 = \frac{P_3}{\cos\theta} = \frac{4548.78\text{ W}}{0.72} = \mathbf{6317.75\text{ VA}}$$

Also, $\theta = \cos^{-1} 0.72 = 43.95°$, so that

$$Q_3 = S_3\sin\theta = (6317.75\text{ VA})(\sin 43.95°) = (6317.75\text{ VA})(0.694) = \mathbf{4384.71\text{ VAR }(L)}$$

Capacitive load:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{208\text{ V}\angle 0°}{9\ \Omega - j\,12\ \Omega} = \frac{208\text{ V}\angle 0°}{15\ \Omega\angle -53.13°} = 13.87\text{ A}\angle 53.13°$$

$$P_4 = I^2R = (13.87\text{ A})^2\cdot 9\ \Omega = \mathbf{1731.39\text{ W}}$$

$$Q_4 = I^2X_C = (13.87\text{ A})^2\cdot 12\ \Omega = \mathbf{2308.52\text{ VAR }(C)}$$

$$S_4 = \sqrt{P_4^2 + Q_4^2} = \sqrt{(1731.39\text{ W})^2 + (2308.52\text{ VAR})^2}$$

$$= \mathbf{2885.65\text{ VA}}$$

$$F_p = \frac{P_4}{S_4} = \frac{1731.39\text{ W}}{2885.65\text{ VA}} = \mathbf{0.6\text{ leading}}$$

b. $P_T = P_1 + P_2 + P_3 + P_4$

$$= 720\text{ W} + 6400\text{ W} + 4548.78\text{ W} + 1731.39\text{ W} = \mathbf{13{,}400.17\text{ W}}$$

$$Q_T = \pm Q_1 \pm Q_2 \pm Q_3 \pm Q_4$$

$$= 0 + 0 + 4384.71\text{ VAR }(L) - 2308.52\text{ VAR }(C) = \mathbf{2076.19\text{ VAR }(L)}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}400.17\text{ W})^2 + (2076.19\text{ VAR})^2} = 13{,}560.06\text{ VA}$$

$$F_p = \frac{P_T}{S_T} = \frac{13.4\text{ kW}}{13{,}560.06\text{ VA}} = \mathbf{0.988\text{ lagging}}$$

$$\theta = \cos^{-1} 0.988 = 8.89°$$

c. $S_T = EI \longrightarrow I = \dfrac{S_T}{E} = \dfrac{13{,}559.89\text{ VA}}{208\text{ V}} = 65.19\text{ A}$

Lagging power factor: **E** leads **I** by 8.89°, and

$$\mathbf{I} = \mathbf{65.19\text{ A}\angle -8.89°}$$

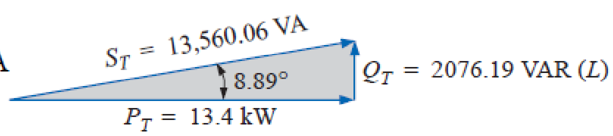


FIG. 19.22

POWER-FACTOR CORRECTION

The design of any power transmission system is very sensitive to the magnitude of the current in the lines as determined by the applied loads. Increased currents result in increased power losses (by a squared factor since $P = I^2R$) in the transmission lines due to the resistance of the lines. Heavier currents also require larger conductors, increasing the amount of copper needed for the system, and, quite obviously, they require increased generating capacities by the utility company.

Every effort must therefore be made to keep current levels at a minimum. Since the line voltage of a transmission system is fixed, the apparent power is directly related to the current level. In turn, the smaller the net apparent power, the smaller the current drawn from the supply. Minimum current is therefore drawn from a supply when $S = P$ and $Q_T = 0$. Note the effect of decreasing levels of Q_T on the length (and magnitude) of S in Fig. 19.24 for the same real power. Note also that the power-factor angle approaches zero degrees and F_p approaches 1, revealing that the network is appearing more and more resistive at the input terminals.

The process of introducing reactive elements to bring the power factor closer to unity is called **power-factor correction.** Since most loads are inductive, the process normally involves introducing elements with capacitive terminal characteristics having the sole purpose of improving the power factor.

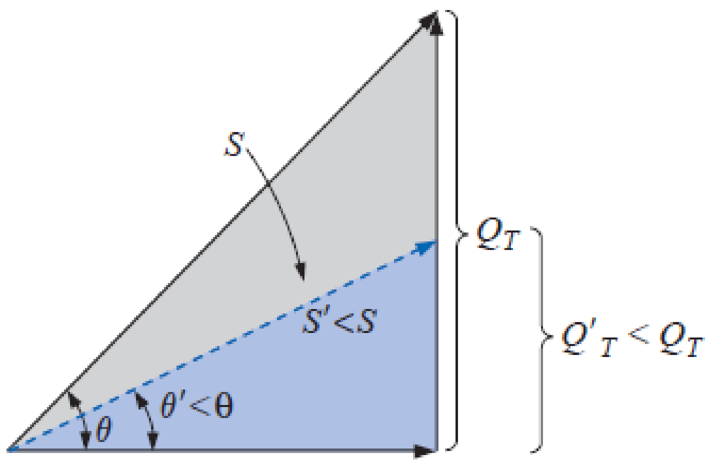


FIG. 19.24

Demonstrating the impact of power-factor correction on the power triangle of a network.

EXAMPLE A 5-hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208-V, 60-Hz supply.

a. Establish the power triangle for the load.
b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.

Solutions:

a. Since 1 hp = 746 W,

$$P_o = 5\text{ hp} = 5(746\text{ W}) = 3730\text{ W}$$

and P_i (drawn from the line) $= \dfrac{P_o}{\eta} = \dfrac{3730\text{ W}}{0.92} = 4054.35\text{ W}$

Also, $F_P = \cos\theta = 0.6$

and $\theta = \cos^{-1} 0.6 = 53.13°$

Applying $\tan\theta = \dfrac{Q_L}{P_i}$

we obtain $Q_L = P_i \tan\theta = (4054.35\text{ W})\tan 53.13° = 5405.8\text{ VAR }(L)$

and

$$S = \sqrt{P_i^2 + Q_L^2} = \sqrt{(4054.35\text{ W})^2 + (5405.8\text{ VAR})^2} = 6757.25\text{ VA}$$

The power triangle appears in Fig. 19.26.

b. A net unity power-factor level is established by introducing a capacitive reactive power level of 5405.8 VAR to balance Q_L. Since

$$Q_C = \frac{V^2}{X_C}$$

then $X_C = \dfrac{V^2}{Q_C} = \dfrac{(208\text{ V})^2}{5405.8\text{ VAR }(C)} = 8\ \Omega$

and $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{(2\pi)(60\text{ Hz})(8\ \Omega)} = \mathbf{331.6\ \mu F}$

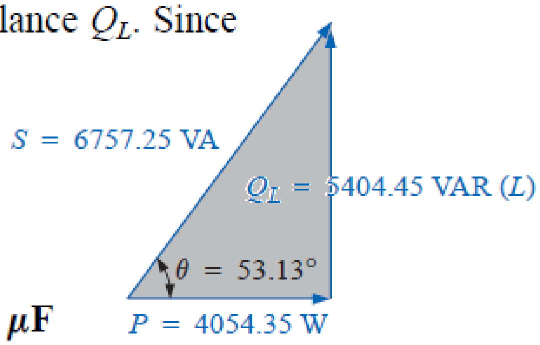


FIG. 19.26

EXAMPLE 19.6

A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($F_p = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.

Solutions:

For the induction motors,

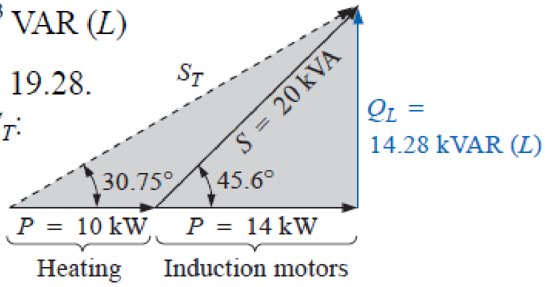
$$S = VI = 20 \text{ kVA}$$

$$P = S\cos\theta = (20 \times 10^3 \text{ VA})(0.7) = 14 \times 10^3 \text{ W}$$

$$\theta = \cos^{-1} 0.7 \cong 45.6°$$

$$Q_L = VI\sin\theta = (20 \times 10^3 \text{ VA})(0.714) = 14.28 \times 10^3 \text{ VAR } (L)$$

The power triangle for the total system appears in Fig. 19.28. Note the addition of real powers and the resulting S_T:



$$S_T = \sqrt{(24 \text{ kW})^2 + (14.28 \text{ kVAR})^2} = 27.93 \text{ kVA}$$

with $$I_T = \frac{S_T}{E} = \frac{27.93 \text{ kVA}}{1000 \text{ V}} = \mathbf{27.93 \text{ A}}$$

The desired power factor of 0.95 results in an angle between S and P of

$$\theta = \cos^{-1} 0.95 = 18.19°$$

changing the power triangle to that of Fig. 19.29:

with $$\tan\theta = \frac{Q'_L}{P_T} \longrightarrow Q'_L = P_T\tan\theta = (24 \times 10^3 \text{ W})(\tan 18.19°)$$

$$= (24 \times 10^3 \text{ W})(0.329) = 7.9 \text{ kVAR } (L)$$

The inductive reactive power must therefore be reduced by

$$Q_L - Q'_L = 14.28 \text{ kVAR } (L) - 7.9 \text{ kVAR } (L) = 6.38 \text{ kVAR } (L)$$

Therefore, $Q_C = 6.38$ kVAR, and using $Q_C = \dfrac{E^2}{X_C}$

$$X_C = \frac{E^2}{Q_C} = \frac{(10^3 \text{ V})^2}{6.38 \times 10^3 \text{ VAR}} = 156.74\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60 \text{ Hz})(156.74\ \Omega)} = \mathbf{16.93\ \mu F}$$

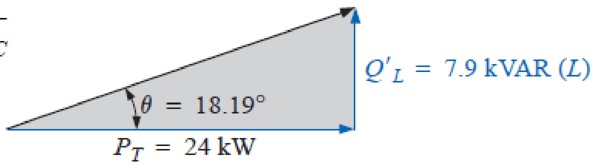


FIG. 19.29

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Q: The lighting and motor loads of a small factory establish a 10-kVA power demand at a 0.7 lagging power factor on a 208-V, 60-Hz supply.

a. Establish the power triangle for the load.

b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.

Q: The load on a 120-V, 60-Hz supply is 5 kW (resistive), 8 kVAR (inductive), and 2 kVAR (capacitive).

a. Find the total kilovolt-amperes.

b. Determine the F_p of the combined loads.

c. Find the current drawn from the supply.

d. Calculate the capacitance necessary to establish a unity power factor.

e. Find the current drawn from the supply at unity power factor, and compare it to the uncompensated level.

Q1: For the system of Fig. 19.42:

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p.

b. Draw the power triangle.

c. Find the current $\mathbf{I}_s$.

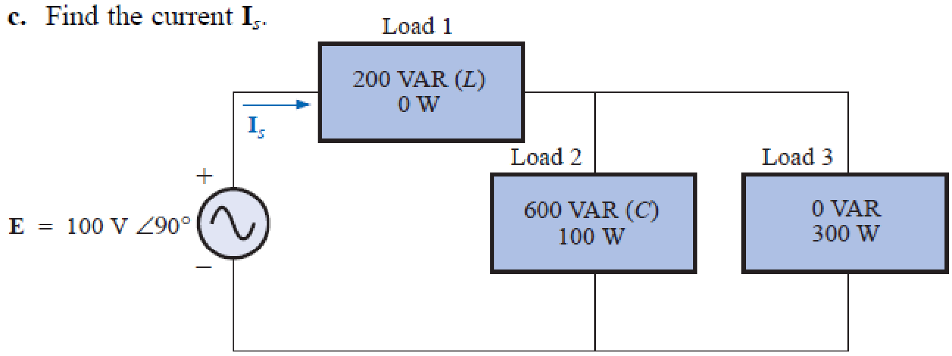


FIG. 19.42

Q2: For the system of Fig. 19.44:

a. Find P_T, Q_T, and S_T. **b.** Find the power factor F_p.

c. Draw the power triangle. **d.** Find $\mathbf{I}_s$.

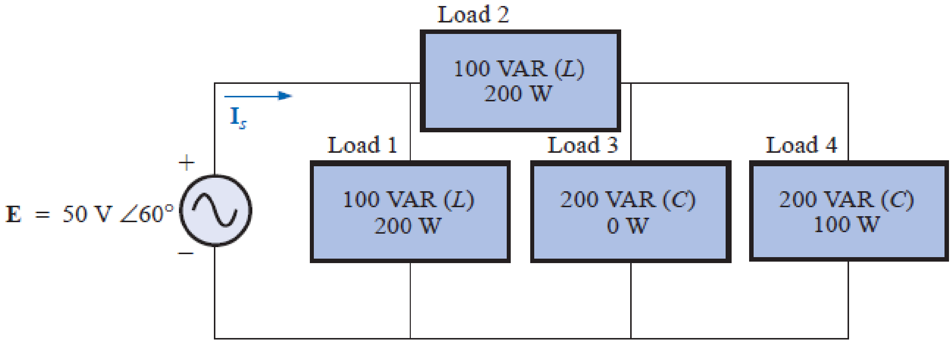


FIG. 19.44

Q3: For the circuit of Fig. 19.45:

a. Find the average, reactive, and apparent power for the 20-Ω resistor.

b. Repeat part (a) for the 10-Ω inductive reactance.

c. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p.

d. Find the current $\mathbf{I}_s$.

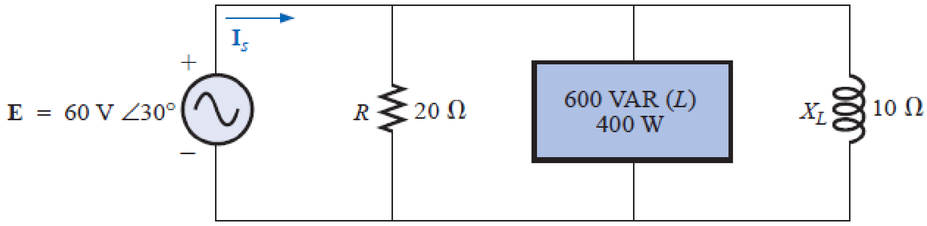


FIG. 19.45

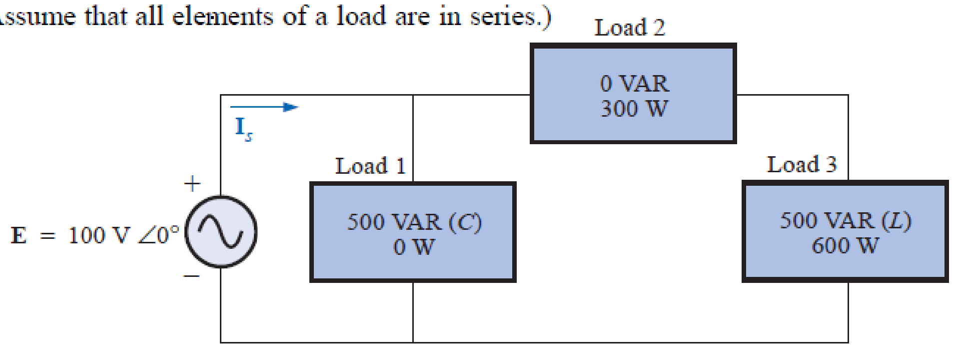
Q4: For the system of Fig. 19.49:

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and F_p.

b. Find the current $\mathbf{I}_s$.

c. Draw the power triangle.

d. Find the type of elements and their impedance in ohms within each electrical box. (Assume that all elements of a load are in series.)



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الدائرة يمكن حلها كما يلي

- المقاومة (6&12) على التوزي نختزلهما الى مقاومة واحدة

6//12=4

- فولتية المقاومة (12 ohm) معلومة وتساوي (Vth=15)

ولذلك نستطيع حساب تيارها من قاون اوم

$I12= 15/12=1.25A$

- التيار المار بالمقاومة (4) يساوي (I-1.25)

في هذا السؤال مطلوب ايجاد (Is) وهو التيار الرئيسي بالدائرة وكما تعرفون ان التيار الكلي في دوائر التيار المتناوب يمكن ايجاده من احدى العلاقات التالية

$It=Vt/Zt$

$It=St/Vt$

حيث الرمز (t) يعني الكلي وهومختصر لكلمة (total)

هنا الممانعة الكلية (Zt) غير موجودة ولكن يمكن حسابها ولكن هذا يدخلنا في طريق طويل للحل لذلك من الاسهل استخدام العلاقة الثانية لايجاد التيار

لان حساب (St)سهل جدا..... كما ان الفولتية الكلية معلومة

ناتي الان الى حساب (St) ونستخدم العلاقة التالية

$St=Pt \pm jQt=\sqrt{}P^2 + Q^2$

حيث ان

$Pt= P1+P2+P3$ (القدرة الحقيقية الكلية تساوي مجموع القدرات الحقيقية في الدائرة)

$Pt=0 +300+600=900$ watt

وكذلك

Qt= Q1+Q2+Q3 (القدرة المتفاعلة الكلية تساوي مجموع القدرات المتفاعلة في الدائرة)

ولانه حدد في السؤال ان الحمل الاول الذي يستهلك (Q) هو متسعة فان قيمتها تكون سالبة والحمل الثالث الذي يستهلك (Q) هو ملف ...لذلك تكون قيمتها موجبة وبالتالي يصبح حساب (Qt) كالتالي

Qt= -500+ 0 +600=100VAR

ونغوض عن القيم لايجاد (St)

St=900+j100= 905.5 6.34

بعد ذلك نحسب (It) من العلاقة التالية

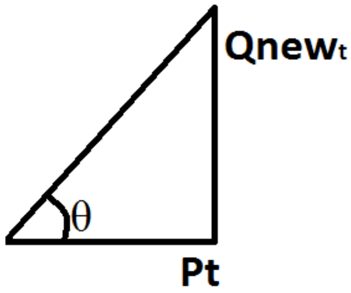
It=St/Vt= 905.5 6.34/100 0= 90.5 6.34

المطلب الثاني بالسؤال والذي دائما يطلب فيه ايجاد قيمة المتسعة التي تحسن معامل القدرة الى القيمة التي يحددها في السؤال له خطوات واحدة لجميع الاسئلة ويمكن وضع تسلسل لها كما يلي:

اولا: معامل القدرة (power factor) بعد الاضافة وهو يساوي (cosθ) يعطى بالسؤال...اي ان في هذه الحالة يكون معلوم عندنا...الزاوية (θ)...وظل الزاوية (tanθ) وهي البيانات التي نحتاجها في الحل:

$\theta = \cos^{-1} \mathbf{PF} =$

ثانيا: من مثلث القدرة يمكننا الاستفادة من العلاقة التالية لحساب مقدار القدرة المتفاعلة الكلية الجديدة ($Qnew_t$)



$$\tan\theta = \frac{\text{المقابل}}{\text{المجاور}} = \frac{Qnew_t}{Pt}$$

وان (Pt) هي نفس قيمتها قبل وبعد الاضافة لان المتسعة المضافة ليس لها تاثير على القدرة الحقيقية

ثالثا: نحسب قيمة (Qc) وهي القدرة المتفاعلة التي تم اضافتها من المتسعة للدائرة وهي تساوي الفرق بين القدرة المتفاعلة الكلية قبل الاضافة (Qt) والقدرة المتفاعلة الجديدة ($Qnew_t$)...اي ان :

$Qc = Qt - Qnew_t$

رابعا: نحسب قيمة الرادة السعوية (Xc) من العلاقة التالية:

$Xc = V^2 / Qc$

حيث ان فولتية المتسعة هي نفسها فولتية المصدر لانها تربط معه على التوازي معه مباشرة

خامسا: نحسب قيمة المتسعة (C) المطلوبة من القانون المعروف:

$Xc = 1 / (2 \pi f C)$

وغالبا يعطى التردد في السؤال

1) $\theta = \cos^{-1} PF = \cos^{-1} 1 = 0$, $\tan\theta = \tan 0 = 0$

2) $\tan\theta = Qnew_t / Pt$ ===➔ $Qnew_t = 0 * 900 = 0$ VAR

3) $Qc = Qt - Qnew_t = 100 - 0 = 100$ VAR

4) $Xc = V^2 / Qc = (100)^2 / 100 = 100\ \Omega$

5) $Xc = 1 / (2 \pi f C)$ ===➔ $C = 1/ (2 \pi * 100 * 50) =$ 0.000031F

6) $= 300\ \mu F$

++

Three phase circuit

قبل حل السؤال نعطي مقدمة بسيطة على الدوائر ثلاثية الطور (three phae)....

يوجد هناك نوعين من الربط سواء للمولد (generator) او للحمل (load) وهما:

اولا: ربط الدلتا (delta) ونستخدم له العلاقات التالية للفولتية والتيار:

$I_L = \sqrt{3}\ I_{ph}$ ……………………(1) (معادلة التيار)

حيث (I_L) يسمى تيار الخط وهو التيار المار في خط النقل....بينما (I_{ph}) هو التيار الذي يمر بداخل ملفات المولد او التيار المار بالحمل (الممانعات)

ودائما تيار الطور يتقدم ب (30) درجة عن تيار الخط

Ex: delta connection, $I_L = 20\ \ 20^0$, find I_{ph}

$I_{ph} = I_L/\sqrt{3} = 20/\sqrt{3} = 11.547$ A ==➔ $I_{ph} = 11.547\ \ 50^0$ A

$V_L = V_{ph}$ (في ربط الدلتا فولتية الخط والطور متساويتان) ……..(2) (معادلة الفولتية)

ثانيا: ربط الدلتا (star) ونستخدم له العلاقات التالية للفولتية والتيار:

$I_L = I_{ph}$ (في ربط الستار تيار الخط والطور متساويتان) …….(3) (معادلة التيار)

$V_L = \sqrt{3}\ V_{ph}$ …………………..(4) (معادلة الفولتية)

ودائما فولتية الخط تتقدم ب (30) درجة عن فولتية الطور

Ex: star connection, $V_{ph}=220 \quad 30^0$, find V_L

$V_L = V_{ph} * \sqrt{3} = 220 * \sqrt{3} = 381$ V ==➔ $V_L = 381 \quad 60^0$ V

اما القدرة في دوائر التيار المتناوب فلها قانون واحد لكلا الربطين:

$P_{3ph}= \sqrt{3} *V_L * I_L * \cos\theta$

$Q_{3ph}= \sqrt{3} *V_L * I_L * \sin$

$S_{3ph}= \sqrt{3} *V_L * I_L$

$PF_{3ph}= \cos\theta = P_{3ph} / S_{3ph}$

حيث (θ) تمثل زاوية الطور بين (V_L) و (I_L)

بعد هذه المقدمة ناتي الى حل السؤال حيث نلاحظ ان الربط للحمل والمولد هو (star)

لذلك نستخدم العلاقات الخاصة بالربط وهي

$$I_L = I_{ph}$$
$$V_L = \sqrt{3}\, V_{ph}$$

المطلوب الاول بالسؤال هو حساب (V_{ph}) و (I_{ph}) للثلاثة اطور أي مطلوب حساب مايلي

Ia, Ib, Ic (تيارات الطور الثلاثة)

Van, Vbn, Vcn (فولتيات الطور الثلاثة)

المعلومات الموجودة بالسؤال كما موضحة بالرسم هي فولتيات الخط الثلاثة لذلك سنحسب فولتية الطور مباشرة:

$V_{ph}= V_L / \sqrt{3}= 400/ \sqrt{3} = 231$ V

وهذا يعني لدينا ثلاث فولتيات للطور هي:

$Van = 231 \quad -30^0$ V

$Vbn = 231 \angle 90^0$ V

$Vcn = 231 \angle -150^0$ V

نلاحظ ان الزوايا لفولتية الطور قد تم استنتاجها من الملاحظة التي ذكرت سابقا وهي ان فولتية الخط تتقدم على فولتية الطور المتنظرة لها بالتسلسل ب (30^0)

بعد حساب فولتية الطور يصبح من الممكن حساب تيار الطور من قانون اوم ..(I=V/Z)

ولان الممانعات متساوية (متوازنة) لذلك يكون التيار متساوي للاطوار الثلاثة ولكن بفرق طور بينهما مقداره (120^0)

نحول الممانعة الى صيغة (polar) لان عندنا عملية قسمة

$Zan = Zbn = Zcn = 10 + j5 = 11.18 \angle 26^0 \ \Omega$

$Ia = Van / Zan = 231 \angle -30^0 / 11.18 \angle 26^0 = 20.66 \angle -56^0$

$Ib = Vbn / Zbn = 231 \angle 90^0 / 11.18 \angle 26^0 = 20.66 \angle 64^0$

$Ic = Vcn / Zcn = 231 \angle -150^0 / 11.18 \angle 26^0 = 20.66 \angle -176^0$

المطلب الثاني هو اثبات ان تيار النيوترل (In) يساوي صفر

وبما ان تيار النيوترل يساوي حاصل جمع التيارات للاطوار الثلاثة (In = Ian+Ibn + Icn)

نعوض عن التيارات الثلاثة سنجد ان (In=0)

$In = 20.66 \angle -56^0 + 20.66 \angle 64^0 + 20.66 \angle -176^0 =$

وهنا لان عندنا عملية جمع نحتاج الى تحويل الى (rectangular) وبعد ذلك نجد ان المحصلة تساوي صفر

المطلب الثالث نطبق قوانين القدرة مباشرة لايجاد القدرة بانواعها الثلاث

$P_{3ph} = \sqrt{3} * V_L * I_L * \cos\theta = \sqrt{3} * 400 * 20.66 * \cos 26^0 = 12865$ W

$Q_{3ph} = \sqrt{3} * V_L * I_L * \sin\theta = \sqrt{3} * 400 * 20.66 * \sin 26^0 = 6274$ VAR

$S_{3ph} = \sqrt{3} * V_L * I_L = 14313$ VA

$PF_{3ph} = \cos\theta = P_{3ph} / S_{3ph} = 12865 / 14313 = 0.8988$

ملاحظة١: الربط هو (star) لذلك يكون ($I_L = I_{ph}$)

ملاحظة ٢: يمكن حساب (PF_{3ph}) مباشرة من ($\cos 26^0 = 0.898$)

++

Q1: For the Y-Δ system of Figure,

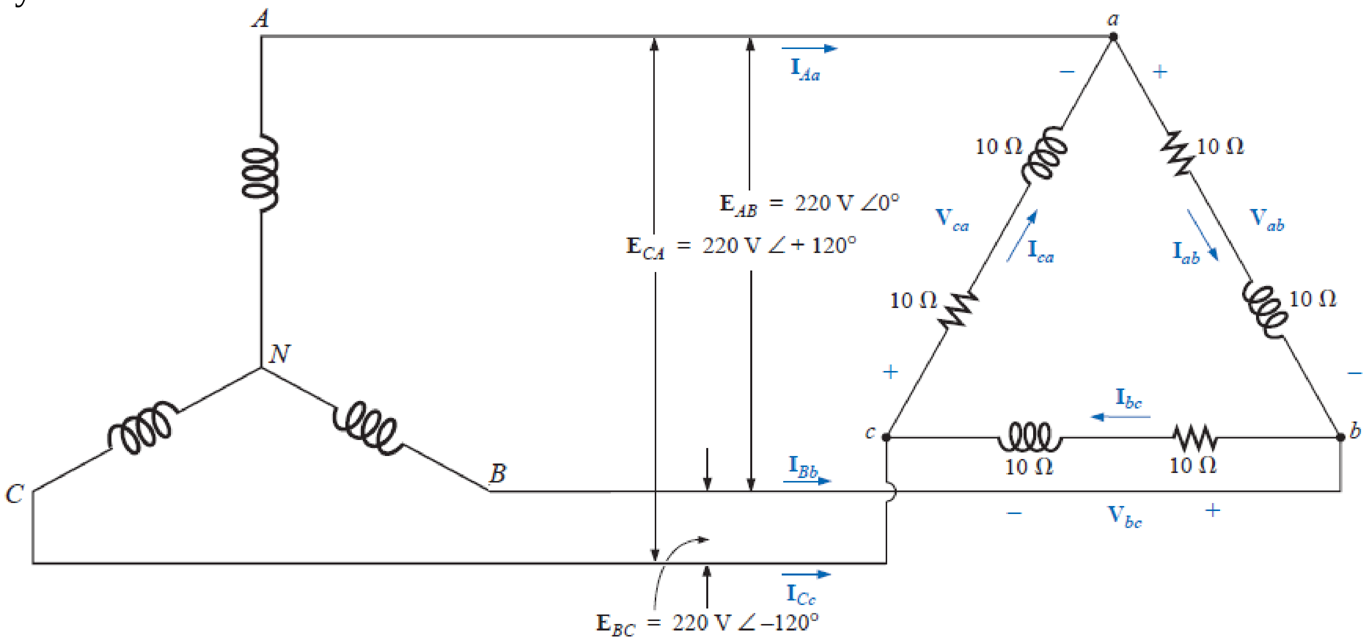
a. Find the phase voltages in phasor form.

b. Draw the phasor diagram of the voltages found in part (a), and show that their sum is zero around the closed loop of the Δ load.

c. Find the phase current through each impedance in phasor form.

d. Find the line currents and generator phase voltages.

e. Find the total watts, volt-amperes reactive, volt-amperes, and *Fp* of the three-phase system



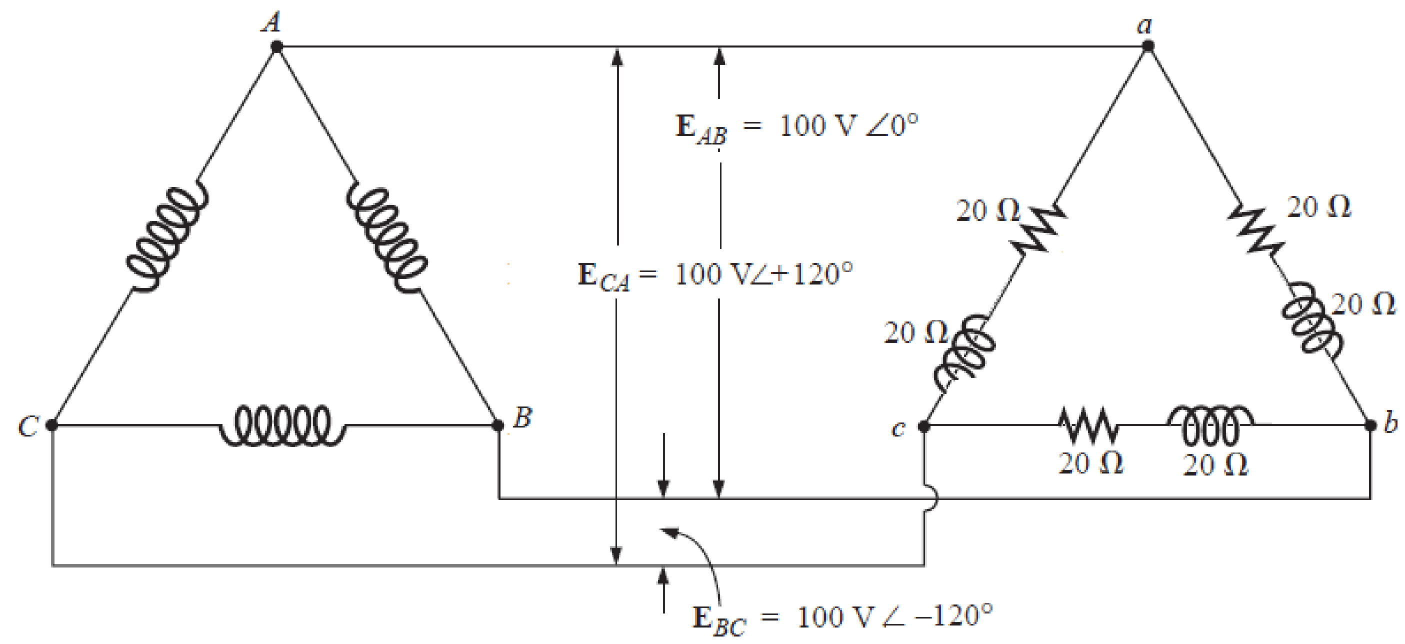
Q2: For the Δ-Δ system of Figure,

a. Find the phase currents in phasor form.

b. Draw the phasor diagram of the currents found in part (a), and show that their sum is zero around the closed loop of the Δ load.

d. Find the line currents.

e. Find the total active, reactive , apparent, and *Fp* of the three-phase system

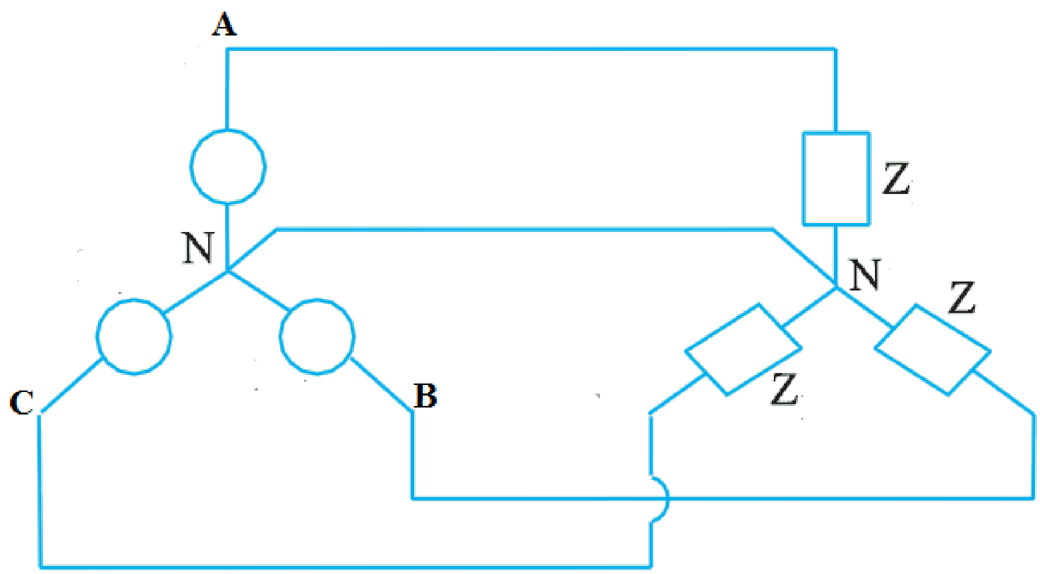


Q3: A balanced star-connected load of (Z=8 + j6 Ω) per phase is connected to a balanced 3-phase 400V, star- generator as shown in Figure.

a. Find the load phase voltages.

b. Find the line currents and show that their sum is zero through the neutral line.

e. Find the total watts, volt-amperes reactive, volt-amperes, and *Fp* of the three-phase system



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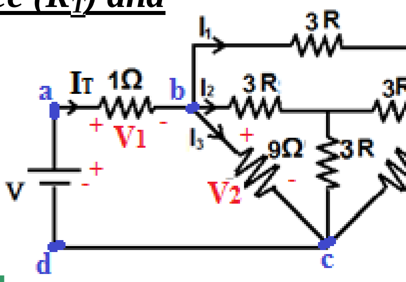
Q1: :- If the currents; I_1=10A, I_2=6A, I_3=2A, find the total resistance (R_T) and source voltage (V).



$I_T = I_1+I_2+I_3$ =18Amp

$V_1=1*I_T$=18volt, $V_2=9*I_1$=90volt (Ohm's law)

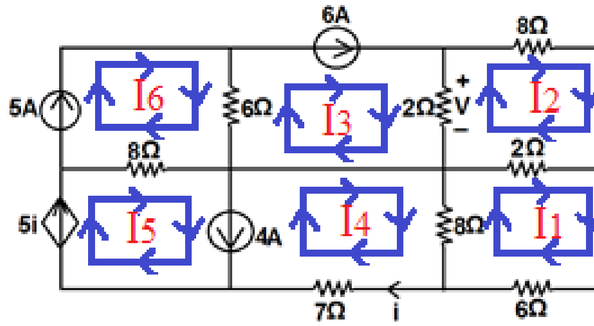
KVL to loop (abcda) $-V_1-V_2+V=0$ ➡ $-18-90+V=0$ ➡ **V=108**

$R_T = V / I_T = 108/ 18 =$ **6Ω**

ملاحظة: هناك طريقة اخرى للحل تتم بايجاد المقاومة الكلية للدائرة باستخدام تحويل Δ الى Y وبالتالي نحصل على المقاومة الكلية والتي تساوي {RT=(3R//9)+1} ...

+++

Q2: Use loop current method to find V_1 and I.



Find "V"

I_1= -6 ……(1), I_3 = 6 ….. (2) , $12 I_2 -2 I_1 -2 I_3 =4V_1$ …...(3)

Substitute eqs. (1) & (2) in eq. (3) yields;

$12I_2-2*(-6)-2*6=4V_1$ ➡ $12 I_2 +12 -12 =4V_1$,

➡ $12 I_2=4V_1$ ➡ $3I_2 = V_1$ ➡ $I_2 = V_1 /3$ .....(4)

From ohm's law:

$V_1=(I_3 - I_2)*2$ ➡ $V_1= (6- I_2)*2$ ➡ $V_1= 12-2 I_2$ ..(5)

Substitute eq. (4) in eq. (5) yields;

$V_1=12-2*(V_1 /3)$, **V_1 =36 /5 volt**

Find "I"

I_5=5I , I_4=I, $I_5- I_4$= 4 ➡ 5I-I=4 ➡ **I=1 Amp**

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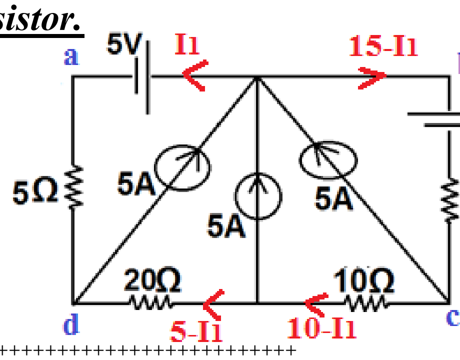
Q3: Use Kirchhoff's laws to find the power absorbed by 20Ω resistor.



KVL to loop (abcda)

$5-20-10(15-I_1) - 10(10-I_1) -20(5-I_1)+ 5I_1 =0$

$45 I_1=365$

I_1 =365/45 Amp

$P=I^2R$

$P=(5-I_1)^2*20=$ **193.5 watt**

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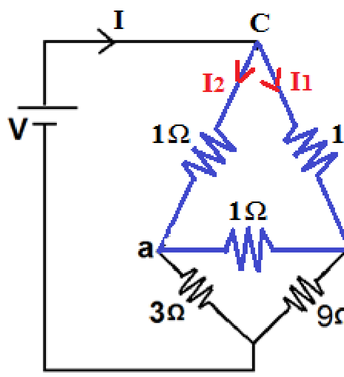
Q4: If the voltage , V_{ab}=6 V, find the total current (I).



First method:

Convert Y (3,3,3) → Δ

V_{ab}=6 volt

$|V_{ac}|= |V_{cb}|= 1/2V_{ab}$ = 3volt

Apply ohm's Law

$I_1=I_2=V_{ac}/1$ = 3Amp

$I = I_1+I_2$ = 6A

+++

Second method: use Nodal analysis

$V_a(1/3 + 1/3)-1/3V_C=0$ ➡ $2/3 V_a - 1/3 V_C =0$ ➡ $2/3 V_a=1/3 V_C$ (1)

$V_b(1/3 + 1/9)-1/3V_C=0$ ➡ $4/9 V_b - 1/3 V_C=0$ ➡ $4/9 V_b= 1/3 V_C$ (2)

equation (1) = equation (2) ➡ $2/3 V_a= 4/9 V_b$

$\Rightarrow V_a = 2/3\ V_b \Rightarrow V_a - 2/3\ V_b = 0$ (3)

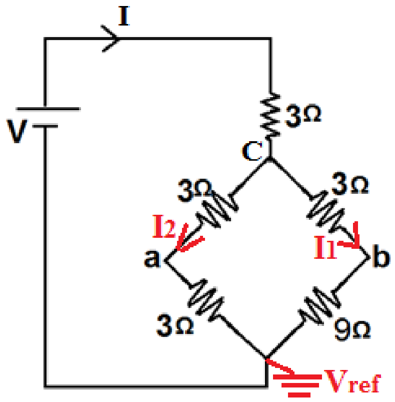
But; $V_{ab} = V_a - V_b = 6 \Rightarrow V_a - V_b = 6$ (4)

Solving equation (3) and equation (4)

$1/3\ V_b = -6 \rightarrow V_b = -18$ volt $\rightarrow V_a = -12$volt

$I_1 = |V_b|/9 = 2A$, $I_2 = |V_a|/3 = 4A$

$I = I_{1+} I_2 = 6$Amp



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