
بسم الله الرحمن الرحيم

Ministry of Higher Education and Scientific Research
Foundation of Technical Education
Technical College / Al-Najaf

Training package in

Signal Energy

For students of second class

Lecturer
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1/ Over view

1 / A –Target population :-

For students of second class in Communications Techniques Engineering Department

1 / B –Rationale :-

Signals may be also be classified as energy and power signals. However, there are some signals can neither be classified as energy signals not power signals.

1 / C –Central Idea :-

-Energy signal

- Parseval’s Theorem

1 / D –Objectives:-

Determine the energy of the signals
Where signals in time domain or frequency domain

2/ Pre test :-

1. Find the energy of X(t) = A [u(t+a) - u(t-a)]

$$E = \int_{\infty}^{\infty} \left| x(t) \right|^2 dt$$

$$E = \int_{-a}^{a} A^2 \, dt$$

$$E = 2aA^2$$

2. State parseval's theorem for energy signals.

$$E = \int_{\infty}^{\infty} \left| x(t) \right|^2 dt = \frac{1}{2\text{®}} \int_{\infty}^{\infty} \left| x(\mu) \right|^2 d\mu$$

3/ Performance Objectives :-

Time/Frequency Domains

- Signal
  - Time domain: waveform, signal duration, waveform decay, causality, periodicity
  - Frequency domain: sinusoidal components, relative amplitudes and phases
- LTI system
  - Impulse response: width indicates system time constant, a.k.a. response time, and amount of dispersion (spreading)
  - Frequency response: bandwidth filter selectively transmits certain frequency components and suppresses the other, transfer functions, stability

Time/Frequency Domains

- Time-Frequency
  - Fourier transform gives a global average of the frequency content of a signal
  - Does not indicate when Fourier components are present in time
  - In speech and audio processing, it is important to know when certain frequencies occurred

Signal Energy

- Energy of signal, ***f*(*t*)**
  - Energy dissipated when voltage *f*(*t*) is applied to 1Ω resistor

$$E_f = \int_{-\infty}^{\infty} \left| f(t) \right|^2 dt < \infty$$

- Instantaneous power: **$|f(t)|^2$**
  - Complex signal: $|f(t)|^2 = f(t)\, f^*(t)$
  - Real-valued signal: $|f(t)|^2 = f^2(t)$
- Parseval's Theorem

$$E_f = \int_{-\infty}^{\infty} \left| f(t) \right|^2 dt < \infty = \frac{1}{2\text{®}} \int_{-\infty}^{\infty} \left| F(\mu) \right|^2 d\mu$$

  - Choose domain in which it is easier to compute energy

Example #1

- ***f*(*t*) = *e***

$$E_f = \int_{-\infty}^{\infty} f^2(t)dt = \int_0^{\infty} e^{-2at}dt = \frac{1}{2a}$$

$$F(\mu) = \frac{1}{j\mu + a}$$

$$|F(\mu)|^2 = F(\mu)F^*(\mu) = \frac{1}{j\mu + a}\,\frac{1}{-j\mu + a} = \frac{1}{\mu^2 + a^2}$$

$$E_f = \frac{1}{2®}\int_{-\infty}^{\infty} |F(\mu)|^2 d\mu = \frac{1}{®}\int_0^{\infty} \frac{1}{\mu^2 + a^2} d\mu$$

$$= \frac{1}{®a}\tan^{-1}\left(\frac{\mu}{a}\right)\Bigg|_0^{\infty} = \frac{1}{®\ a}\left(\frac{®}{2} - 0\right) = \frac{1}{2a}$$

Example #2

- Determine the frequency ***W*** (rad/s) such that energy contributed by frequencies w Č **[0, *W*]** is 95% of the total energy $\boldsymbol{E_f = 1 / (2\,a)}$

$$\frac{1}{®}\int_0^W \frac{1}{µ^2 + a^2}\,dµ = \frac{0.95}{2\,a}$$

$$\frac{1}{®\,a}\tan^{-1}\left(\frac{W}{a}\right) = \frac{0.95}{2\,a}$$

$$\frac{W}{a} = \tan(0.475®) \Rightarrow W = 12.706\,a \text{ rad/s}$$

- ***W*** is known as the effective bandwidth

Quiz /

1. The energy is obtained by $E = \int_{\infty}^{\infty} |x(t)|^2 \, dt$

2. The average power is obtained by--

$$P = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

5/ Post test :-

1. Find the energy of the signal

$$f(t) = \exp(-at)u(t)$$

Solution

$$E_f = \int_0^\infty e^{-2at}\,dt = \frac{1}{2a}$$

References

1. T. R. Ganesh Babu, and G. Srinivasan: “ Communication Theory and systems”, 2006.

2. Sanjay Sharma: “Communication Systems (Analog and Digital) ”

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