

GENERAL PRINCIPLES

Mechanics: is a branch of physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces.

In general, this subject can be divided into three branches:

- Rigid-body mechanics,
- Deformable-body mechanics, and
- Fluid mechanics.

Rigid-body mechanics is divided into two areas:

1. **Static:** deals with the equilibrium of bodies, that is, those that are at rest or move with a constant velocity.
2. **Dynamics:** is concerned with the accelerated motion of bodies.

Note: We can consider static special case of dynamics, in which the acceleration is zero.

Fundamental Concepts

Basic Quantities: The basic quantities which used throughout mechanics are:

Length, Time, Mass, and Force

Force: *force* is the action of one body to another. A force tends to move a body in the direction of its action.

Note: A force is completely characterized by its magnitude, direction and point of application.

Idealizations: Models or idealization are used in mechanics in order to simplify application of theory.

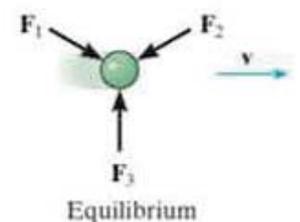
Particle: A **particle** is a body of negligible dimensions. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body *will not be involved* in the analysis of the problem.

Rigid body: A **rigid body** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying load. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and rigid-body assumption is suitable for analysis.

Concentrated Force: A **concentrated force** represents the effect of a loading which is assumed to act at a point on a body.

Newton's Three Laws of Motion:

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is *not* subjected to an unbalanced force.

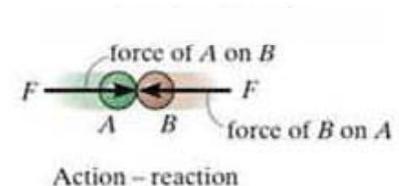


Second Law: A particle acted upon by an unbalanced Force F experiences an acceleration a that has the same direction as the force and magnitude that is directly proportional to the force. If F is applied to a particle of mass m , this law may be mathematically as

$$F = ma \quad \dots(1-1)$$



Third Law: The forces of action and reaction between particles are equal, in magnitude, opposite in direction and collinear (they lie in the same direction).



Newton's Law of Gravitational Attraction:

Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically:

$$F = G \frac{m_1 m_2}{r^2} \quad \dots(1-2)$$

Where F = force of gravitation between the two particles,

G = universal constant of gravitational according experimental evidence, $G = 66.73(10^{-12}) \text{ m}^3/(\text{kg}\cdot\text{s}^2)$,

m_1, m_2 = mass of each of the two particles, and

r = distance between the two particles.

Weight: The **weight** of body is the gravitational force acting on it.

From Eq. 1-2, we can develop an approximate expression for finding the weight W of a particle having a mass $m_1 = m$. If we assume the earth to be a nonrotating sphere of constant density and having a mass $m_2 = M_e$, then if r is the distance between the earth's center and the particle, we have

$$F = G \frac{mM_e}{r^2} \quad \dots(1-3)$$

Letting $g = GM_e/r^2$ yields:

$$W = mg \quad \dots(1-4)$$

By comparison with $F = ma$, we can see that g is the acceleration due to gravity. Since it depends on r , then weight of a body is *not* an absolute quantity. Instead, its magnitude is determined from where the measurement was made. For most

engineering calculations, however, g is determined at sea level and at latitude of 45° , which is considered the "standard location".

Units of Measurement

The four basic quantities (length, time, mass, and force) are not all independent from one another: in fact they are related by Newton's second law of motion, $F = ma$.

SI Units: The **I**nternational **S**ystem (SI) (from French, System International d'units) defines length in meter (m), time in seconds (s), and mass in kilogram. The unit of force, called a Newton (N), is derived from $F = ma$.

Thus **1 Newton** is equal to a force required to give 1 kilogram of mass an acceleration of 1 m/s^2 ($\text{N}=\text{kg.m/s}^2$). The value of g , in the standard location, was found to be 9.80665 m/s^2 ; however for calculations the value $g = 9.81 \text{ m/s}^2$ will be used.

U.S. Customary: In the U. S. Customary system of units (or British System of units) (FPS) length is measured in feet (ft), time in seconds, and force in pounds (lb). The unit of mass, called a *slug*, is derived from $F = ma$.

Hence, **1 slug** is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1lb ($\text{slug} = \text{lb.s}^2/\text{ft}$). Therefore if measurements are made at the "standard location" where $g = 32.2 \text{ ft/s}^2$, then from Eq. 1-3,

$$m = \frac{W}{g} \quad (g = 32.2 \text{ ft/s}^2) \quad \dots(1-5)$$

TABLE 1-1 Systems of units

Name	Length	Time	Mass	Force
International System (SI)	meter	second	Kilogram	Newton*
	m	s	Kg	$\text{N}=\text{kg.m/s}^2$
U. S. Customary system (FPS)	foot	second	Slug	pound
	ft	s	$\text{slug}=\text{lb.s}^2/\text{ft}$	lb

* Derived unit

Conversion of Units:

In the FPS system, 1 ft = 12 in. (inches),
 5280 ft = 1 mi (mile),
 1000 lb = 1 kip (kilo-pound), and
 2000 lb = 1 ton.

TABLE 1-2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.304 8 m

Prefixes: When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix.

Note: The SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of metric system. Except for some volume and area measurements, the use of these prefixes is to be avoided in science and engineering.

TABLE 1-3 Prefixes

	Exponential form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	<i>G</i>
1 000 000	10^6	mega	<i>M</i>
1 000	10^3	kilo	<i>K</i>
Submultiple			
0.001	10^{-3}	milli	<i>m</i>
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	<i>N</i>

* The kilogram is the only base unit that is defined with a prefix

Example 1: Convert 2km/h to m/s. How many ft/s is this?

Solution: Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of units can be applied;

$$\begin{aligned}
 2\text{km} / \text{h} &= \frac{2\cancel{\text{km}} \left(\frac{1000\cancel{\text{m}}}{\cancel{\text{km}}} \right) \left(\frac{1\cancel{\text{h}}}{2600\cancel{\text{s}}} \right)}{\cancel{\text{h}}} \\
 &= \frac{2000\text{m}}{3600\text{s}} = 0.556\text{m} / \text{s}
 \end{aligned}$$

From Table 1-2, 1 ft = 0.304 8m. Thus,

$$\begin{aligned}
 0.556\text{m} / \text{s} &= \left(\frac{0.556\cancel{\text{m}}}{\cancel{\text{s}}} \right) \left(\frac{1\cancel{\text{ft}}}{0.3048\cancel{\text{m}}} \right) \\
 &= 1.82\text{ft} / \text{s}
 \end{aligned}$$

Note: Remember to round off the final answer to three significant figures.

Example 2: Convert the quantities 300 lb.s and 52 slug/ft³ to appropriate SI units.

Solution: Using Table 1-2, 1 lb = 4.448 2 N

$$\begin{aligned}
 300\text{ lb}\cdot\text{s} &= 300\cancel{\text{lb}}\cdot\text{s} * \left(\frac{4.448\text{N}}{1\cancel{\text{lb}}} \right) \\
 &= 1334.5\text{N}\cdot\text{s} = 1.33\text{kN}\cdot\text{s}
 \end{aligned}$$

Since 1 slug = 14.593 8 kg and 1 ft = 0.304 8 m, then

$$\begin{aligned}
 52\text{slug} / \text{ft}^3 &= \frac{52\cancel{\text{slug}}}{\cancel{\text{ft}}^3} * \left(\frac{14.59\text{kg}}{1\cancel{\text{slug}}} \right) \left(\frac{1\cancel{\text{ft}}}{0.3048\cancel{\text{m}}} \right)^3 \\
 &= 26.8(10)^3\text{kg} / \text{m}^3 \\
 &= 26.8\text{Mg} / \text{m}^3
 \end{aligned}$$

Example 3: Evaluate each of the following and express with SI units having an appropriate prefix:

- (a) (50 mN)(6 GN),
- (b) (400 mm)(0.6 MN)²,
- (c) 45mN³/900Gg.

Solution: First convert each number to base units, perform the indicated operations, and then choose an appropriate prefix.

Part (a)

$$\begin{aligned}
 (50 \text{ mN})(6 \text{ GM}) &= [50(10^{-3})\text{N}][6(10^9)\text{N}] \\
 &= 300(10^6) \text{ N}^2 \\
 &= 300(10^6) \cancel{\text{N}^2} \left(\frac{1\text{kN}}{10^3 \cancel{\text{N}}} \right) \left(\frac{1\text{kN}}{10^3 \cancel{\text{N}}} \right) \\
 &= 300 \text{ kN}^2
 \end{aligned}$$

Part (b)

$$\begin{aligned}
 (400 \text{ mm})(0.6 \text{ MN})^2 &= [300(10^{-3})\text{m}][0.6(10^6)\text{N}]^2 \\
 &= [300(10^{-3})\text{m}][0.36(10^{12})\text{N}^2] \\
 &= 144(10^9) \text{ m.N}^2 \\
 &= 144 \text{ Gm.N}^2
 \end{aligned}$$

We can also write

$$\begin{aligned}
 144(10^9) \text{ m.N}^2 &= 144(10^9)\text{m} \cancel{\text{N}^2} \left(\frac{1\text{MN}}{10^6 \cancel{\text{N}}} \right) \left(\frac{1\text{MN}}{10^6 \cancel{\text{N}}} \right) \\
 &= 0.144 \text{ m. GN}^2
 \end{aligned}$$

Part (c)

$$\begin{aligned}
 \frac{45\text{MN}^3}{900\text{Gg}} &= \frac{45(10^6 \text{ N})^3}{900(10^6)\text{kg}} = 50(10^9)\text{N}^3/\text{kg} \\
 &= 50(10^9) \cancel{\text{N}^3} \left(\frac{1\text{kN}}{10^3 \cancel{\text{N}}} \right)^3 \frac{1}{\text{kg}} = 50 \text{ kN}^3/\text{kg}
 \end{aligned}$$