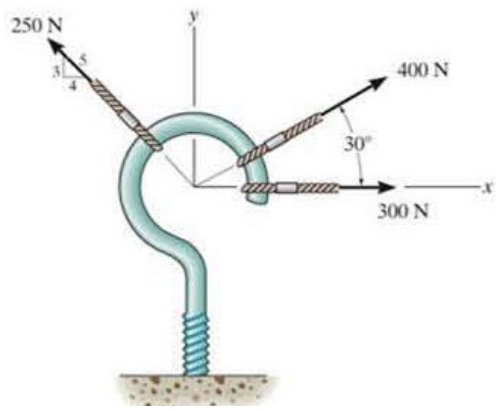
FORCE SYSTEM

Forces

A **force** may be defined as the action of one body on another body which changes or tend to change the motion of the body acted on. Because of the inertia possessed by all material bodies, they react or oppose any force which acts on them (Newton's third low).

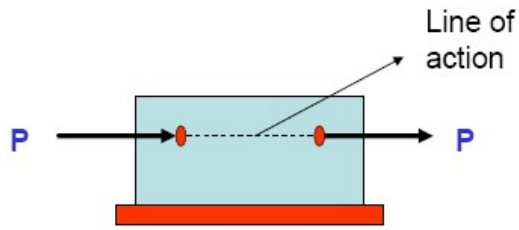
Note1: Forces may be considered as localized vectors and they can not be defined unless all the following characteristics mentioned:

- Magnitude,
- Direction (sense and slope),
- Location of any point on its line of action.



Note2: The third characteristic shows that if two forces have the same direction, they will produce the same external effect on a rigid body. This fact leads to the ***principle of transmissibility.***

So **principle of transmissibility** states that the external effect of a force in a rigid body is independent the point of application of the force along its line of action.



System of Forces

When several forces act in a given situation, they are called ***system of forces*** or ***force system.*** Force systems can be classified according to the arrangement of the lines of action of the forces of the system as follows:

- **Collinear:** All forces of the system have a common line of action.
- **Concurrent, Coplanar:** The action lines of all the forces of the system are in the same plane and intersect at a common point.
- **Parallel, Coplanar:** The action lines of all the forces of the system are parallel and lie in the same plane.
- **Nonconcurrent, Nonparallel, Coplanar:** The action lines of all the forces of the system are in the same plane, but they are not all parallel and they do not intersect at a common point.
- **Concurrent, Noncoplanar:** The action lines of all the forces of the system are intersect at a common point, but they are not all in one plane.
- **Parallel, Noncoplanar:** The action lines of all the forces of the system are parallel and but they are not all in the same plane.
- **Nonconcurent, Nonparallel, Noncoplanar:** The action lines of all the forces of the system do not all intersect at a common point, they are not parallel, and they do not lie in the same plane.

Resultant: The **Resultant** of a force system is the simplest force which can replace the original system without changing its external effect on a rigid body. The resultant of a force system can be:

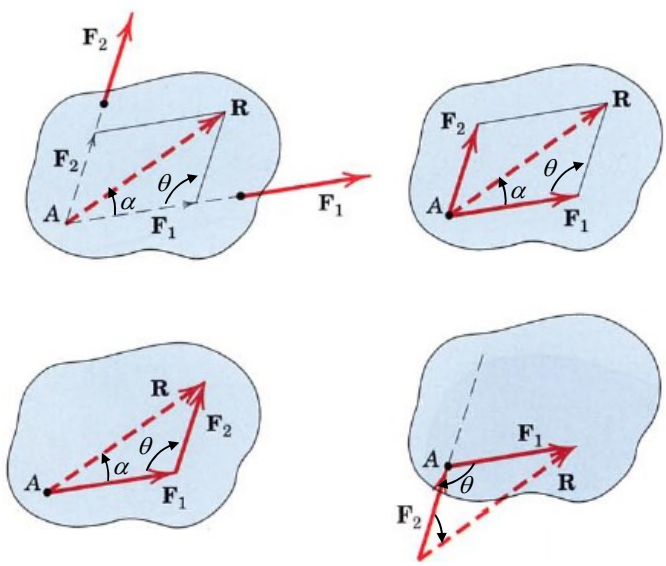
- a single force,
- a pair of parallel forces having the same magnitudes but opposite sense (called a ***couple***), or
- a force and a couple. If the resultant is a force and a couple, the force will not be parallel to the plane containing the couple.

Composition and Resolution of Forces

The process of replacing a force system by its resultant is called ***composition.***

The **resultant** of **pair of concurrent forces** can be determined by means of *parallelogram law*.

$$R = \sqrt{(F_1)^2 + (F_2)^2 - 2(F_1).(F_2)\cos\theta} \qquad \textbf{...(2-1)}$$

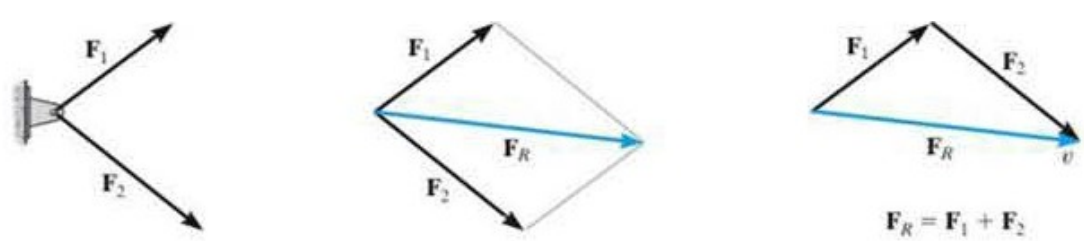


The angle the resultant makes with either force can be determined by ***law*** of ***sines***, for example:

$$\frac{\sin\alpha}{F_2} = \frac{\sin\theta}{R} \qquad \textbf{...(2-2)}$$

Resolution: The process of replacing a force by its components is called **resolution.**

A **component of a force** is any one of two or more forces having the given forces as a resultant. So the term "***component***" is used to mean either one of two concurrent forces or any one of three noncoplanar concurrent forces having the given force as a resultant

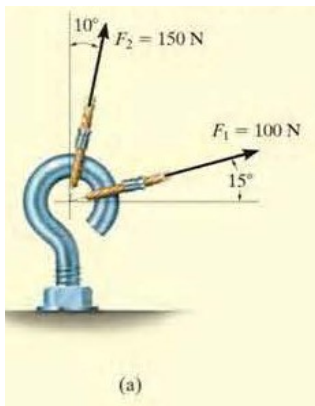


Resultant of concurrent forces:

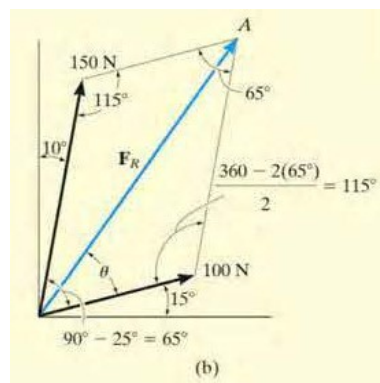
a) Resultant of concurrent, coplanar forces:

The resultant of concurrent, coplanar forces is force only, while as we will be seeing later, the resultant of noncocurrent forces is force or moment or both.

Example 1: The screw in Figure (a) is subjected to two forces, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$. Determine the magnitude and direction of the resultant force.



(a)



(b)

Solution: By parallelogram, the resultant F_R is:

$$F_R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115^o}$$

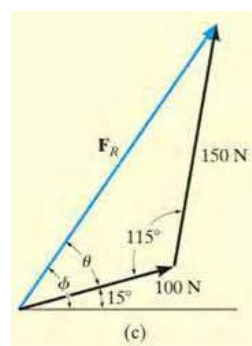
$$= \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$= 212.6\,N \cong 213 \text{ N}$$

Applying the law of sines to determine θ,

$$\frac{150}{\sin 115^o} = \frac{212.6}{\sin\theta} \quad \Longrightarrow \quad \sin\theta = \frac{150}{212.6}(\sin 115^o)$$

$$\theta = 39.8^o$$

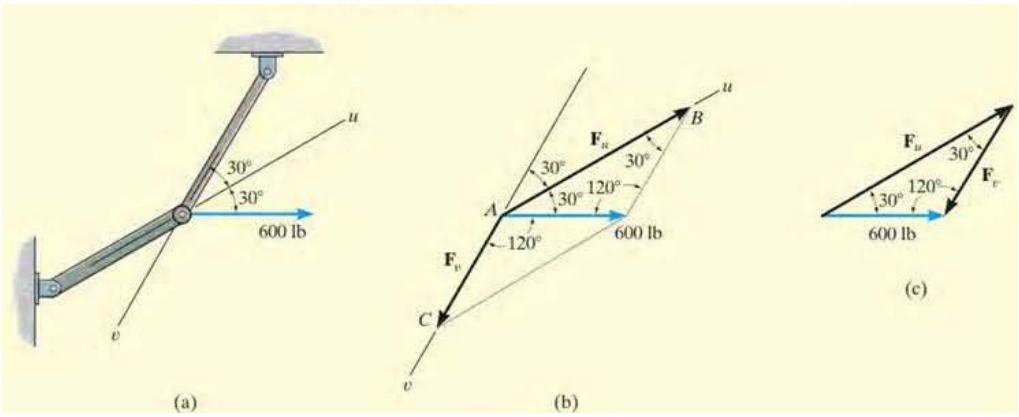


(c)

Thus, the direction ϕ of $\boldsymbol{F_R}$, measured from the horizontal, is

$$\phi = 39.8^o + 15.0^o = 54.8^o$$

Example 2: Resolve the horizontal 600-lb force in Figure (a) into components acting along u and v axes and determine the magnitude of these components.



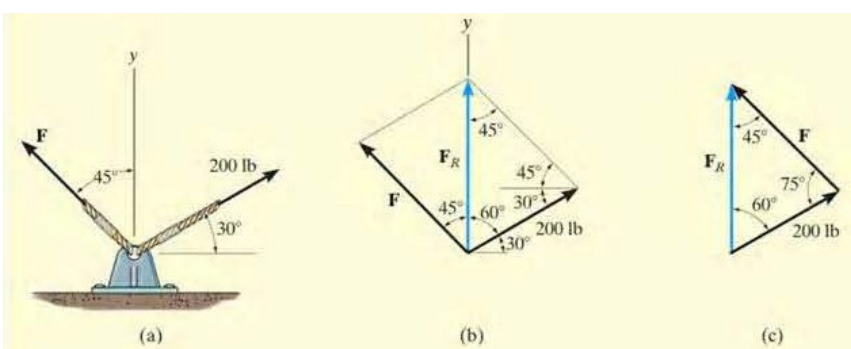
Solution: Applying the law of sines,

$$\frac{F_u}{\sin 120^o} = \frac{600}{\sin 30^o} \quad \Longrightarrow \quad F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30^o} = \frac{600}{\sin 30^o} \quad \Longrightarrow \quad F_v = 600 \text{ lb}$$

Note: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

Example 3: Determine the magnitude of the components force $\boldsymbol{F}$ in Figure (a) and the magnitude of the resultant force $\boldsymbol{F_R}$ if $\boldsymbol{F_R}$ is directed along the positive y-axis.

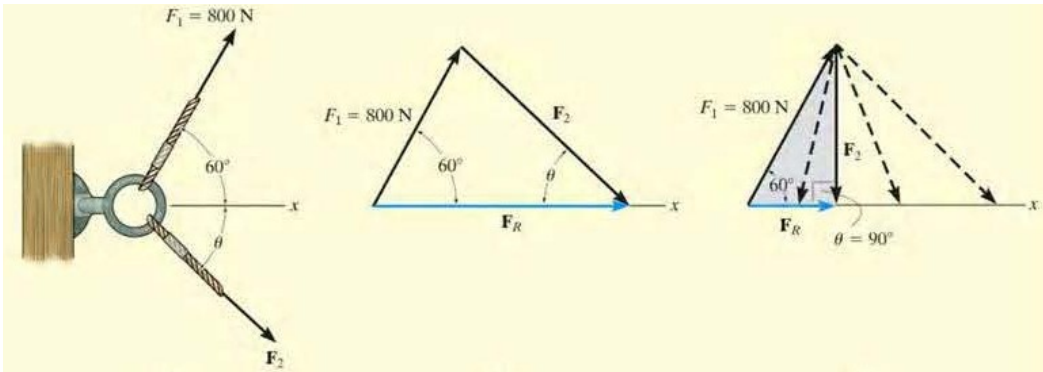


Solution: The magnitude of $\boldsymbol{F_R}$ and $\boldsymbol{F}$ can be determined by applying the law of sines,

$$\frac{F}{\sin 60^o} = \frac{200}{\sin 45^o} \quad \Longrightarrow \quad F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^o} = \frac{200}{\sin 45^o} \quad \Longrightarrow \quad F_R = 273 \text{ lb}$$

Example 4: It is required that the resultant force acting on the eyebolt in Figure (a) be directed along the positive *x*-axis and that $\boldsymbol{F_2}$ has a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.



Solution: The magnitude of $\boldsymbol{F_2}$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\boldsymbol{F_R}$, that is, when,

$$\theta = 90$$

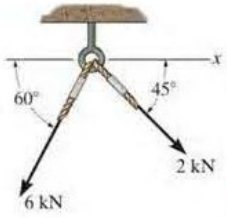
Since the vector addition now forms a right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = 800 \cos 60^o = 400 \text{ N}$$

$$F_2 = 800 \sin 60^o = 693 \text{ N}$$

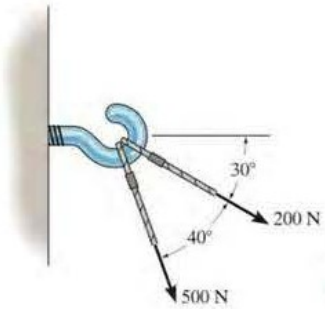
FUNDAMENTAL PROBLEMS*

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



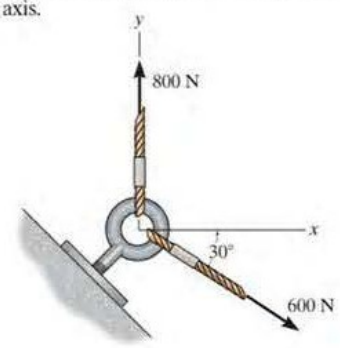
F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



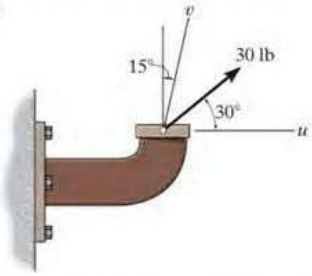
F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



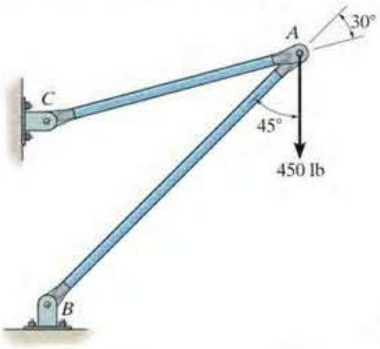
F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



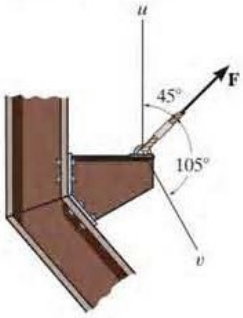
F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



F2–5

F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the v axis.



F2–6