Tractive effort and tractive resistance

A vehicle has a torque of 480 N m applied to its driving wheels. Rolling resistance is 320 N. If the vehicle has a mass of 1780 kg what propelling force will be available to accelerate the vehicle while it is climbing a gradient of 1 in 20. The rolling radius of the road wheels is 0.3 m.

$$N = mg$$

$$= 1780 \times 9.81$$

$$= 17461.8 \text{ N}$$

$$\text{gradient resistance } R_g = \frac{mg}{X} = \frac{17462}{20} = 873 \text{ N}$$

$$\text{TE available} = \frac{T_r}{r} = \frac{480}{0.3} = 1600 \text{ N}$$

$$\text{total resistance } R_T = R_r + R_g$$

$$= 320 + 873$$

$$= 1193 \text{ N}$$

therefore surplus TE

$$= 1600 - 1193$$

$$= 407 \text{ N}$$

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An engine producing 12 kW power at 3000 rev/min is fitted to a vehicle with a mass of 1700 kg. If the rolling resistance is 130 N determine the maximum gradient this vehicle could climb in top gear with a 4.5:1 final drive ratio, a transmission efficiency of 70% and wheel radius 0.28 m.

$$P_b = \frac{T2\pi N}{60}$$

Therefore

$$T = \frac{P_b \times 60}{2\pi N} = \frac{12\,000 \times 60}{2\pi \times 3000}$$

$$= 38.2 \text{ N}$$

$$\text{TE} = \frac{Tnfe}{r} = \frac{38.2 \times 1 \times 4.5 \times 0.7}{0.28} = 429.75 \text{ N}$$

$$\text{surplus TE} = \text{TE (total)} - R_r$$
$$= 429.75 - 130$$
$$= 299.75 \text{ N}$$

$$\text{gradient resistance } R_g = \frac{mg}{X}$$

Therefore

$$X = \frac{mg}{R_g} \text{ or } \frac{mg}{\text{surplus TE}}$$

Maximum gradient vehicle could negotiate in top gear

$$= \frac{mg}{\text{surplus TE}} = \frac{1700 \times 9.81}{299.75} = 55.63 \text{ to } 1$$

Set the final drive and the gear ratios for a vehicle, given the following data: Maximum engine torque is 117 N m at 2000 rev/min. Maximum engine power is produced at 4200 rev/min, which gives a maximum road speed of 144 km/h. The maximum tractive resistance (R_t) is 6230 N and the transmission efficiency in top gear (e) is 80% Rolling radius of wheels (r) is 0.365 m.

$$N = \frac{2.652\, nf \times \text{km/h}}{r}$$

and

$$f = \frac{Nr}{2.652\, n \times \text{km/h}} = \frac{4200 \times 0.365}{2.652 \times 1 \times 144} = 4.014{:}1$$

$$\text{maximum TE in top gear} = \frac{Tfe}{r} = \frac{117 \times 4.014 \times 0.8}{0.365} = 1029 \text{ N}$$

$$n_1 = \frac{\text{maximum } R_T}{\text{maximum TE in top}} = \frac{6230}{1029} = 6.05{:}1$$

$$\text{engine speed ratio } (Z) = \frac{\text{rev/min of maximum torque}}{\text{rev/min of maximum power}}$$
$$= \frac{2000}{4200}$$
$$= 0.476$$

thus

$$n_2 = n_1 Z = 6.05 \times 0.476$$
$$= 2.879{:}1$$

and

$$n_3 = n_2 Z = 2.879 \times 0.476$$
$$= 1.370{:}1$$

Final drive ratio = 4.014:1

first gear = 6.05:1

second gear = 2.879:1

third gear = 1.370:1

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An engine producing a torque of 352 N m is fitted to a vehicle of mass 1670 kg which has a 4.35 final drive ratio giving a transmission efficiency of 80% in top gear. The rolling resistance is 356 N and the effective wheel diameter is 0.62 m. If this vehicle is negotiating a 1-in-10 gradient in top gear, what force is available to accelerate the vehicle on the gradient, and what value would this acceleration be? If the vehicle was driven down the gradient with the same torque applied, what would be the acceleration?

$$\text{TE} = \frac{Tnfe}{r} = \frac{352 \times 1 \times 4.35 \times 0.8}{0.31} = 3951 \text{ N}$$

$$\text{gradient resistance } R_g = \frac{mg}{X} = \frac{1670 \times 9.81}{10} = 1638 \text{ N}$$

$$\text{total resistance} = R_r + R_g = 356 + 1638 = 1994 \text{ N}$$

$$\text{TE available for acceleration} = \text{total TE} - \text{total } R_T$$
$$= 3951 - 1994 = 1957 \text{ N}$$

$$F = ma$$

therefore

$$a = \frac{F}{m} = \frac{\text{TE}}{m} = \frac{1957}{1670} = 1.17 \text{ m/s}^2$$

Acceleration down gradient

$$\text{TE available} = (\text{TE} - R_r) + R_g$$
$$= (3951 - 356) + 1638$$
$$= 5233 \text{ N}$$

Then a (m/s²) down the gradient

$$= \frac{TE}{m} = \frac{5233}{1670} = 3.134 \text{ m/s}^2$$

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A vehicle of 1200 kg mass is capable of accelerating in top gear on a level road at 3 m/s². If the total resistance to the vehicle is 2587 N and the rolling wheel radius 0.27 m, determine the engine torque in newton metres, assuming a final drive ratio of 4.6 to 1 and transmission efficiency of 76%.

$$TE = ma = 1200 \times 3 = 3600 \text{ N}$$

TE available to accelerate vehicle = total TE = 3600 + 2587

$$(TE_a + R_T) = 6187 \text{ N}$$

and

$$TE = \frac{Tnfe}{r}$$

therefore

$$T \text{ (N m)} = \frac{TEr}{nfe} = \frac{6187 \times 0.27}{1 \times 4.6 \times 0.76}$$

$$= 477.8 \text{ N m}$$

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A car of total mass 1140 kg has roadwheels of effective diameter 0.7 m and attains a speed of 120.7 km/h at an engine speed of 3600 rev/min. The rolling resistance is 209 N and air resistance 1264 N. (a) Calculate the top gear ratio, and, assuming a transmission efficiency of 90%, the brake power developed by the engine. (b) If the engine develops its maximum torque of 176.28 N m at 1800 rev/min determine the road speed in top gear at this engine speed.

(a) Total resistance on level road

$$R_T = R_r + R_a$$

$$= 209 + 1264$$

$$= 1473 \text{ N}$$

Power at the roadwheels

$$P_r = P_b e = TEv$$

and

$$P_b = \frac{TEv}{e}$$

$$= \frac{1473 \times 120.7}{0.9 \times 3.6}$$

= 54 873 W or 54.87 kW

$$P_b = \frac{T2\pi N}{60} \text{ and } T = \frac{P_b 60}{2\pi N} = \frac{54\,873 \times 60}{2\pi\, 3600}$$

= 145 N m torque

$$TE = \frac{Tnfe}{r}$$

therefore top gear = final drive ratio

$$f = \frac{TEr}{Tne}$$

Final drive and top gear ratio

$$f = \frac{1473 \times 0.35}{145 \times 1 \times 0.9}$$

= 3.95:1

(b)

$$TE = \frac{Tnfe}{r} = \frac{176.28 \times 3.95 \times 0.9}{0.35} = 1790.5 \text{ N}$$

$$\text{engine rev/min (N)} = \frac{\text{km/h} \times nf}{r0.377}$$

$$\text{and km/h} = \frac{N \times r0.377}{f}$$

$$= \frac{1800 \times 0.35 \times 0.377}{3.95}$$

= 60.1 km/h

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A van having a mass of 1360 kg is travelling at a speed of 80.5 km/h and then accelerates at the rate of 1.5 m/s² for a period of 8 s before reaching a gradient of 1 in 20, 600 m in length. What is the speed of the van in km/h as it reaches the gradient? If this speed is maintained up the incline by a force of 890 N, determine the total work done against gravity, the work done on the gradient, and the power necessary to maintain both the tractive force and the road speed.

velocity of van as it reaches incline = $v = u + at$

$$= 22.36 + (1.5 \times 8)$$

$$= 34.36 \text{ m/s}$$

gradient resistance $R_g = \dfrac{mg}{X}$

$$= \frac{1360 \times 9.81}{20} = 667 \text{ N}$$

work done against gravity = R_g x distance travelled

$$= \frac{667 \times 30}{10^3} = 20 \text{ kJ}$$

work done on gradient = force x distance = $\dfrac{890 \times 600}{10^3} = 534$ kJ

power at the roadwheels P_r = TEv = 890 x 34.36

$$= 30\,580 \text{ W or } 30.58 \text{ kW}$$

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Power and acceleration

A motor car of 1318 kg mass is fitted with an engine developing 28.348 kW at 2000 rev/min. The top gear ratio is 4.8:1 with an effective wheel radius of 0.33 m and the transmission efficiency is 88%. The rolling resistance is 173.5 N and air resistance is 230 N. The inertia of the car, including that of the engine, transmission and road wheels, may be assumed to be equal to 17 800 N. From this information calculate (a) the forward speed of the car at 2000 engine rev/min, (b) the power available for hill climbing at this speed, and (c) the maximum acceleration possible.

(a) $N = \dfrac{\text{km/h} \times nf\, 2.652}{r}$

therefore

$$\text{km/h} = \frac{Nr}{nf\,2.652}$$

$$= \frac{2000 \times 0.33}{1 \times 4.8 \times 2.652}$$

$$= 51.84 \text{ km/h}$$

(b) total resistance $R_T = R_r + R_a = 173.5 + 230 = 403.5$ N

$$P_b = \frac{T2\pi N}{60}$$

therefore

$$T = \frac{P_b \times 60}{2\pi N} = \frac{28.348 \times 10^3 \times 60}{2 \times \pi \times 2000} = 135 \text{ N m}$$

$$\text{TE} = \frac{Tnfe}{r} = \frac{135 \times 4.8 \times 0.88}{0.33} = 1728 \text{ N}$$

TE available for gradient = 1728 − 403.5 = 1324.5 N

$P_b = \text{TE}v$

therefore

$$P_b = 1324.5 \times 14.39$$

$$= 19\,059 \text{ W} = 19 \text{ kW}$$

(c) $F = ma$

therefore

$$a = \frac{F}{m} = \frac{\text{TE}}{m} = \frac{1324.5}{1318} = 1.005 \text{ m/s}^2$$

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A truck engine is developing 269 kW power at 2800 rev/min. The drive passes via a gearbox which has a ratio of 5.78:1 engaged, a final drive reduction of 6.72:1, and a hub reduction to the driving wheels of 3.62:1. Determine the speed, torque and power delivered to a single driving wheel if the truck is moving in a straight line and the overall transmission efficiency is 78%.

$$\text{brake power} = P_b = \frac{T2\pi N}{60}$$

therefore

$$T(\text{N m}) = \frac{P_b \times 10^3 \times 60}{2\pi N}$$

$$= \frac{269 \times 10^3 \times 60}{2\pi \times 2\,800}$$

$$= 917 \text{ N m}$$

$$\text{torque at a roadwheel} = \frac{Tnfhe}{2}$$

where T is the engine torque (N m); n the gearbox ratio; f the final drive ratio; h the hub reduction ratio; e the transmission efficiency and T_r the torque at the roadwheel (kN m).

$$T_r(\text{kN m}) = \frac{917 \times 5.78 \times 6.72 \times 3.62 \times 0.78}{2 \times 10^3}$$

$$= 50.28 \text{ kN m}$$

$$\text{roadwheel rev/min} = \frac{\text{engine rev/min}}{\text{overall MR}}$$

$$= \frac{2800}{5.78 \times 6.72 \times 3.62} = 19.9 \text{ rev/min}$$

power at one roadwheel

$$\text{kW} = \frac{T_r 2\pi N_r}{60} = \frac{50.28 \times 2 \times \pi \times 19.9}{60}$$

$$= 104.7 \text{ kW}$$

Rev/min, torque and power at *one* roadwheel are 19.9 rev/min; 50.28 kN m and 104.7 kW, respectively.

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EXERCISES

1. A vehicle of mass 1.3 tonnes is travelling at a speed of 114 km/h on a level road. The rolling resistance is 167 N per tonne, and the streamline constant is 0.053. The cross-sectional frontal area of the vehicle is 1.36 m^2. The final drive ratio is 4.77:1 and transmission efficiency is 79% in top gear. The rolling radius of the wheels is 30 cm. Determine the necessary torque required at the propellor shaft to maintain a steady road speed of 114 km/h.

91.85 Nm

2. The tractive resistance opposing a vehicle of 7.9 Mg mass is 330 N at 40 km/h. If the brake power of the engine is increased to 26.8 kW, calculate the acceleration possible and the road speed which could be reached in 20 s if the transmission efficiency is 84%. Assume that the tractive resistance remains constant during the period.

3. A vehicle of 1400 kg mass travelling at 72 km/h on a level road is put into neutral and allowed to coast to rest. The distance covered is 320 m. Determine the resistance during the retardation, which may be assumed constant throughout the period. If from rest the acceleration is 1.66 m/s^2, how long and in what distance will the vehicle again attain 72 km/h?

4. A car weighing 11 772 N is being accelerated up a gradient of 1:23. Power of 56 kW is produced at 3800 rev/min. Rolling resistance is 160 N per tonne. The rear axle ratio is 4.79 to 1. The rolling diameter of the wheels is 0.7 m and transmission efficiency 82%. Determine the acceleration at the given engine speed neglecting air resistance.

5. The rolling resistance of a vehicle of mass 1700 kg is 216 N, and the air resistance is 372 N when the vehicle is travelling at 72 kM/h. It has a transmission efficiency of 76% with a rolling wheel diameter of 0.68 m and final drive ratio of 4.73:1. Find the engine power in kW required to maintain the speed on a level road surface, and the increase necessary to climb a 1-in-9 gradient at a steady 20 km/h.

6. A six cylinder four-stroke engine with a compression ratio of 8:1, and total clearance volume of 250 cm^3 produces a brake mean effective pressure of 690 kN/m^2 at 3000 rev/min. First gear ratio is 3.2:1, and final drive 4.75:1. The transmisison efficiency is 88%. On a left-hand curve in first gear the nearside roadwheel travels at 40% of the crownwheel speed. Determine the torque, revolutions per minute and the power at the offside wheel.

7. Power of 82 kW is produced at 5750 rev/min, which represents an engine's maximum power, and at the point of maximum torque the power is 38.5 kW. The engine speed ratio is 0.368. If the overall reduction ratio between engine and roadwheels is 13.65 to 1, determine the maximum torque and power at the roadwheels and their revolutions per minute if the transmission efficiency is 79%. What torque is available at the roadwheels at maximum engine power, and what power is available at the roadwheels at point of maximum engine torque?

8. A vehicle of mass 1.9 tonnes is fitted with an engine capable of providing 52 kW of power at 3800 rev/min. If the rolling resistance is 170 N per tonne mass, the transmission efficiency 78%, the final drive ratio 5.37:1 and the wheel rolling radius is 0.39 m, neglecting air resistance, calculate the maximum acceleration in top on a level road and the max. gradient