Channel coding.

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Channel coding.
Seminar topics:

- Block code

Parity check , Binary code space.

- Linear block code

Systematic linear , Hamming single error correction.

- Cyclic code

Systematic cyclic cod , error correction .

Channel coding: communication theory

- Probability of repeat a symbol in a message is P(x).
- The length of code to represent this symbol is:

$I=\log_2(1/p(x))$

- Source Entropy is the average of information in message H(s):

$$\sum_{i=1}^{q} \mathbf{I(xi) * P(xi)}$$

Binary channel: symmetric and non- symmetric.

BSC:

If condition probabilities

Are symmetric:

$P(0|1)=P(1|0)=P$

$P(1|1)=P(0|0)=1-P$

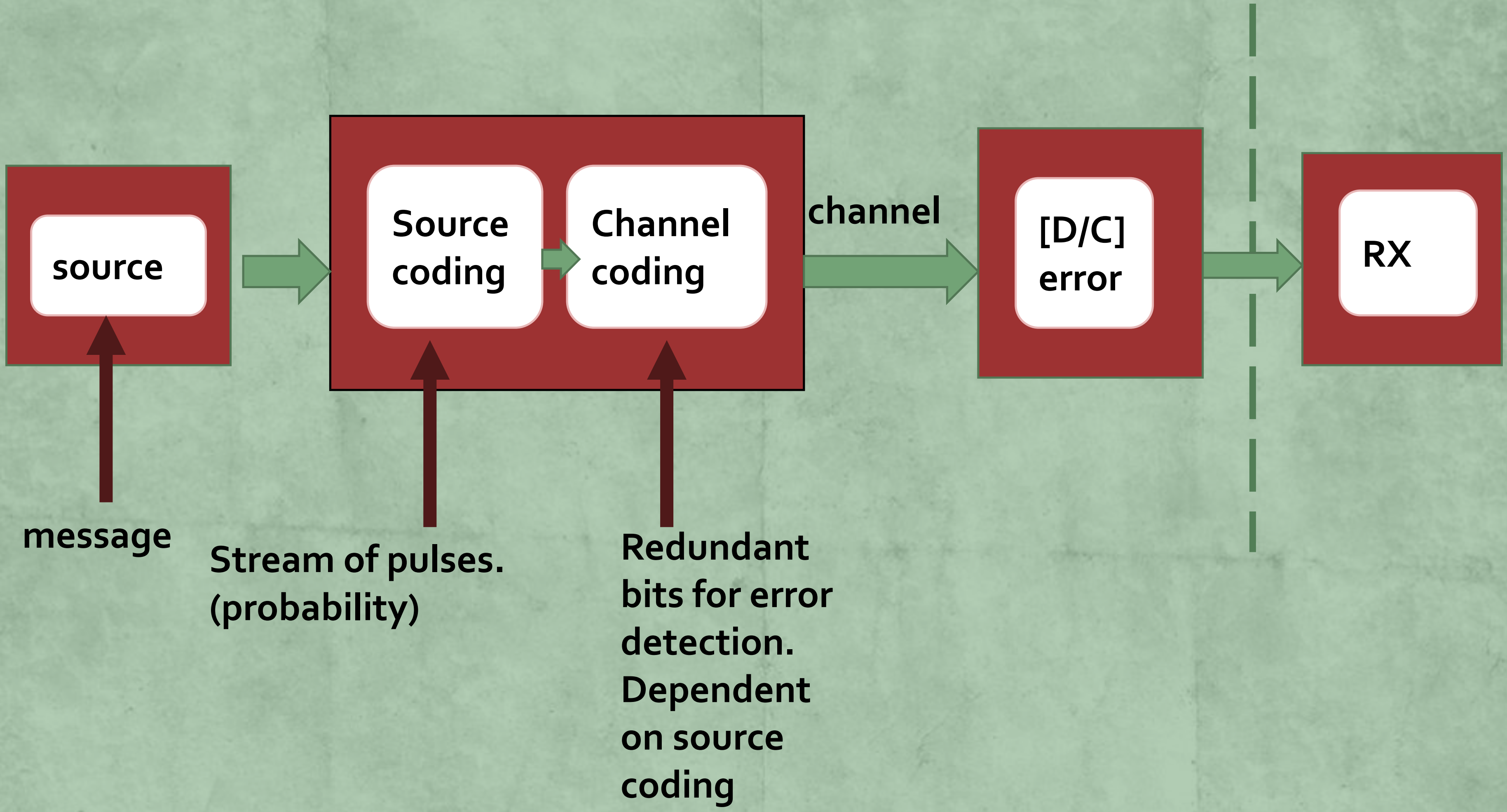
BNC:

If condition probabilities

Are non- symmetric:

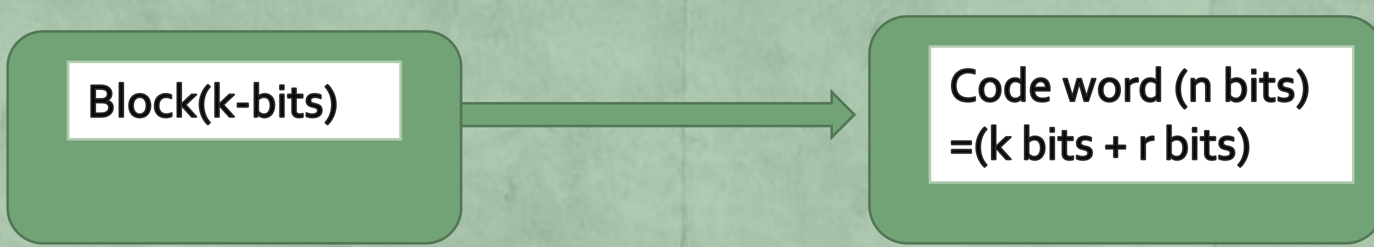
$P(0|1)\neq P(1|0)$

$P(1|1)\neq P(0|0)$


source
Source
coding
Channel
coding
channel
[D/C]
error
RX
message
Stream of pulses.
(probability)
Redundant
bits for error
detection.
Dependent
on source
coding

Channel coding.
1. Block code:

Block Code meaning that segmentation the data into segments called block, and each block consist of (k bits) encoded into a new block (n bits) called a Code word, if the redundant bits added to beginning Of code word called systematic code, **n>k**



1.Block code:
Parity check code.

The Block code with check code of one Bit , if the

number of *one* bits in the message is even it is:

Even parity check code. And if the number of *one* bits

in the Message is odd it is: *odd parity check code*.

Note that if single error occurs in received message can be detect but, can't be corrected.

Ex: 111 it is odd parity(r=1)

11011 it is even parity (r=0)

1.Block code:
Binary code space.

Hamming weight: defined as the number of non-zero bits in a code word.

Ex:

00000 (weight=0)

00111 (weight=3)

11011 (weight=4)

Binary space code:

Hamming distance : error control capability, the hamming distance between two code words is equal to the number of difference between each bit of them…

Ex:10011011

11010010 the hamming distance is (3)

The minimum distance of a Block code is the smallest distance between any pair of code words in the code.

In general if (n) minimum distance of block code then:

1. $\frac{n-1}{2}$ Error can be corrected if (n) is odd.
2. $\frac{n-2}{2}$ error can be corrected and, $\frac{n}{2}$ can be detected if(n) is even.

Binary space code:

The relation between the number of bit error correction (t) and the minimum distance $d_{\min}$ should be: $d_{\min} \geq (2t+1)$

Relation between $d_{\min}$ and error detection capability (q) ,(where q number of bit error detection possible) is: $d_{\min} \geq (q+1)$

to find the number of maximum possible code word of the length (n) and minimum distance $d_{\min}$ this is given by ***B(n,d)*** and hamming give the following values:

B(n,1) = 2^n

B(n,2)= 2^{n-1}

B(n,3)= $2^m \leq \dfrac{2^n}{n+1}$

Binary code space:

B(n,4)= $2^m \leq \frac{2^{n-1}}{n}$

B(n,2k)= B$(n-1, 2k-1)$

B(n,2k+1)= $2^m \leq \frac{2^n}{1+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{k}}$

The equality in ***B(n,3)*** $2^m \leq \frac{2^n}{n+1}$ *is* valid when $\frac{2^n}{n+1}$ is an integer

such codes are referred to as:

Close packed codes, These are obtained by selecting$\left(n = 2^k - 1\right)$

K is positive integer ,for example :k=2,3,4 and n=3,7,15 resulting in block packed codes are: B(3,3)= $\frac{2^3}{3+1} = 2$

Binary block code:

B(7,3)= $\frac{2^7}{7+1} = 16$,B(15,3)= $\frac{2^{15}}{15+1} = 2048$

When $(n \neq 2^k - 1)$ the number of maximum possible code words is found out from : $B(n,3) < \frac{2^n}{n+1}$

For n=5:

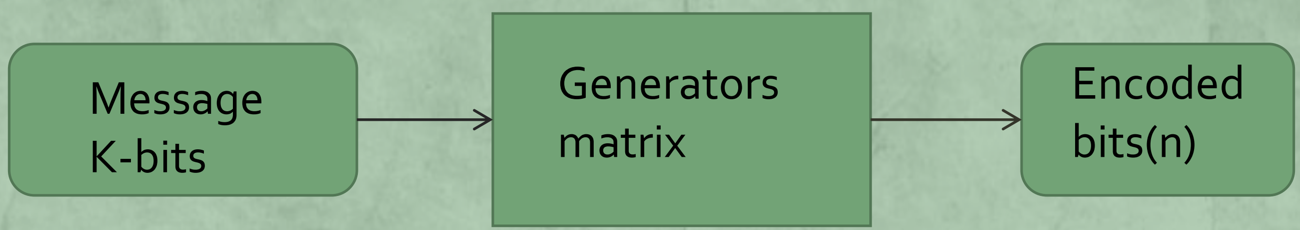
$B(5,3) = 2^m \leq \frac{2^5}{5+1}$ gives: $2^m = 5.33 \rightarrow m = 2$

B(5,3) (m=2)	B(6,3) (m=3)	
00000	000000	100110
01101	010101	110011
10110	111000	011110
11011	101101	001011

2. Linear block code:

If we have a stream of bits we segment it to blocks(k) and added to them a redundant bits (r), we get the encoded bits(n).

The conversion operation from message(k-bits) to encoder(n-bits) by using Generators matrix.



n>k, if n=k mean that no encoding

r=n-k

2.Linear block code: Generators Matrix.

G= $[\boldsymbol{P}_{k*r} \quad \boldsymbol{I}_{k*k}]$,P: parity check, I: identity matrix.

G= $\left[\begin{bmatrix} \mathrm{v}_{11} & \cdots & \mathrm{v}_{1r} \\ \vdots & \ddots & \vdots \\ \mathrm{v}_{k1} & \cdots & \mathrm{v}_{kr} \end{bmatrix} \quad \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}\right]$

Where v_{kr} dependent on original message (k-bits)

Ex:(7,4) linear block code and you have m=(1101)
solution: n=7 , k=4 , then r=7-4=3 and G(k*n)

$$G= \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

2. Linear block code: systematic code

$$v_{\text{n bits}} = [\mathrm{G}] * \mathrm{m}_{\text{k bits}}$$

$$\mathrm{v}_{1*\mathrm{n}} = [1101] * \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$v = [1001101]$$

V: represent the code word for message (m),

At receiver the received message (r) may be equal to the transmitted message(v) or not,

2. Linear block code:
at Rx

To detect the error in received message and correcting it:

R (n bits) → H^T → S(r bits) (syndrome)

If vector S=0 meaning that no error occurred during the transmission in the channel.

If S≠0 meaning that the error occurred.

If we received (R=1001001) to find the error must be find the matrix H?

$$H = [I_{r*r} \quad P^T]$$

2. Linear block code: at Rx error correction.

S=R* H^T = [1001001] $\begin{bmatrix}1 & 0 & 0 & 1 & 1 & 0 & 1\\0 & 1 & 0 & 1 & 1 & 1 & 0\\0 & 0 & 1 & 0 & 1 & 1 & 1\end{bmatrix}^T$

S= [1001001]* $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\\1 & 1 & 1\\0 & 1 & 1\\1 & 0 & 1\end{bmatrix}$ =[111] if we compare this data with H^T

we found it in fifth row then the

error occurred in the fifth bit into

the received message.

The correction is be : [1001101]

2.Linear code:
Hamming single error correction code:

Hamming code is a Linear block code (n,k) bits.

The code word $v = [p_1 \quad p_2 \quad d_3 \quad p_4 \quad d_5 \quad d_6 \quad d_7]$

$$p_1 = d_3 \quad d_5 \quad d_7$$
$$p_2 = d_3 \quad d_6 \quad d_7$$
$$p_4 = d_5 \quad d_6 \quad d_7$$

Where d3,d5,d6,d7 is data message bits.

Ex: data(1011)
solution: d3=1 , d5=0 ,d6=1 , d7=1

P1= 1 0 1 =0 , p2=1 1 1=1 , p4= 0 1 1=0

$$G = \begin{array}{c} \\ d_3 \\ d_5 \\ d_6 \\ d_7 \end{array} \begin{array}{ccccccc} p_1 & p_2 & d_3 & p_4 & d_5 & d_6 & d_7 \end{array} \atop \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow v = [0110011]$$

2. Linear code:
Hamming code:

$$H = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \left[\begin{array}{ccccccc} p_1 & p_2 & d_3 & p_4 & d_5 & d_6 & d_7 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array} \right]$$

At receiver use H^T to detect the error and correction it.

S= H^T * R if results of parity bits in syndrome equal to parity bits in the received message that meaning no error occurred , if not equal the error occurred.

Ex: R=**1001110** , to detect the error calculate parity bits:

P1=0 1 0=1, p2= 0 1 0=1 , p4= 1 1 0=0 $\longrightarrow$ p=[1 1 0] not equal to parity bits in R then the error occurred in the sixth bit in R,

The correct cod word is [1001100]

2. Linear block code: Cyclic code.

The code word form of cyclic code is: $v_{(x)} = D_{(x)}\, g_{(x)}$

where D is data $D_{(x)} = d_0x^0 + d_1x^1 + d_2x^2 + d_3x^3 + \cdots + d_{k-1}x^{k-1}$

$g_{(x)}$ is polynomial of the degree (n-k) and its factor (n-k).

Ex: D= 1101 , $g_{(x)=1+x+x^3}$

Solution: $D_{(x)} = 1x^0 + 1x^1 + 0x^2 + 1x^3 \longrightarrow D_{(x)} = 1 + x + x^3$

$v_{(x)} = D_{(x)}\, g_{(x)} \longrightarrow v_{(x)} = 1 + x + x^3 + x + x^2 + x^4 + x^3 + x^4 + x^6$

$v_{(x)} = 1 + x^2 + x^6$

V=1010001

Systematic Cyclic code:

$$v_{(x)} = x^{n-k} D_{(x)} + r_{(x)}$$

$r_{(x)}$ is a remainder of division $x^{n-k} D_{(x)} / g_{(x)}$

Ex: D= 1101 $\longrightarrow$ $D_{(x)} = 1 + x + x^3$, $g_{(x)=1+x+x^3}$

1. $x^{n-k} D_{(x)} = x^3 D_{(x)} = x^3 + x^4 + x^6$

2.Long division:

$q_{(x)}$

$$\begin{array}{r|l} & x^3 + x^2 \\ \hline x^3 + x^2 + 1 & x^6 + x^4 + x^3 \\ & x^6 + x^5 + x^3 \\ \hline & x^5 + x^4 \\ & x^5 + x^4 + x^2 \\ \hline & x^2 \end{array}$$

$r_{(x)}$

$r_{(x)}$ = 001

$v_{(x)} = r_{(x)} D_{(x)}$ = 0011101

Error correction in systematic cyclic code:

To find Syndrome from received message R(x) from the remainder of division $R_{(x)}/g_{(x)}$, if S(x) =0 meaning no error occurred ,and if S(x)≠0 will correct the error by shifting the received message with [n] repeating shift and correct [n-1-i] from shift.

Ex: $R_{(x)=1+x^2+x^5}$ $g_{(x)} = 1 + x^2 + x^3$

S(x)= rem $R_{(x)}/g_{(x)}$

$$
\begin{array}{r|l}
 & x^2 + x \quad + 1 \\
\hline
x^3 + x^2 + 1 & x^5 + x^2 + 1 \\
 & x^5 + x^4 + x^2 \\
\cline{2-2}
 & x^4 + 1 \\
 & x^4 + x^3 + x^1 \\
\cline{2-2}
 & x^3 + x \quad + 1 \\
 & x^3 + x^2 + 1 \\
\cline{2-2}
 & x^2 + x
\end{array}
$$

The remainder not equal to zero ,

To correct the error make a cyclic shift:

Cyclic code error correction:

R(X)= 1 0 1 0 0 1 0 after shift (n-1-i) correct the error.

1^{st} shift= 0 1 0 1 0 0 1

2^{nd} shift= 1 0 1 0 1 0 0

3^{rd} shift= 0 1 0 1 0 1 0

4^{th} shift= 0 0 1 0 1 0 1 (n-1-i)=7-1-2=4

5^{th} shift= 0 0 0 1 0 1 0 7-1-1 =5

6^{th} shift = 1 0 0 0 1 0 1

7^{th} shift= 1 1 0 0 0 1 0, the correction is [1100010]

Reference :

1.elements of digital communication (sarkar)
2.youtube – channel coding.

Thank you..