Acceleration Diagram

In this lecture we shall discuss the acceleration of points in the mechanisms. The acceleration analysis plays a very important role in the development of machines and mechanisms.

1. Acceleration Diagram for a Link

Consider two points *A* and *B* on a rigid link as shown in Fig. 1 (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link *AB*.

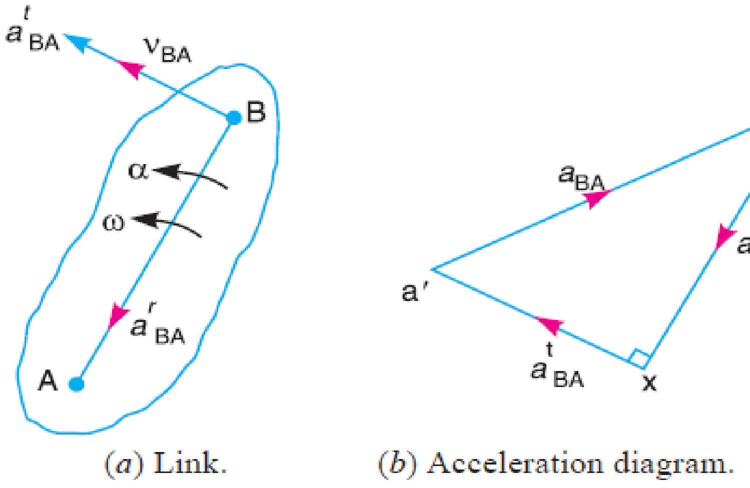


Fig.1. Acceleration for a link.

The radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2 \times AB = \frac{v_{BA}^2}{AB} \qquad \text{(from } B \rightarrow A\text{)}$$

Radial component of acceleration $a^r_{BA} \perp v_{BA}$, In other words, a^r_{BA} // link *AB*.

The tangential component of the acceleration of *B* with respect to *A*,

$$a^t_{BA} = \alpha \times AB$$

This tangential component of acceleration // v_{BA}. In other words, $a^t_{BA} \perp$ link *AB*.

$$a_{BA} = a^r_{BA} + a^t_{BA} \qquad \text{(direction sum)}$$

i. e. $$a_{BA} = \sqrt{a_{BA}^{t2} + a_{BA}^{r2}}$$

2. Acceleration of a Point on a Link

Consider two points *A* and *B* on the rigid link, as shown in Fig. 2 (*a*). Let the acceleration of the point *A i.e.* a_A is known in magnitude and direction and the direction of path of *B* is given. The acceleration of the point *B* is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

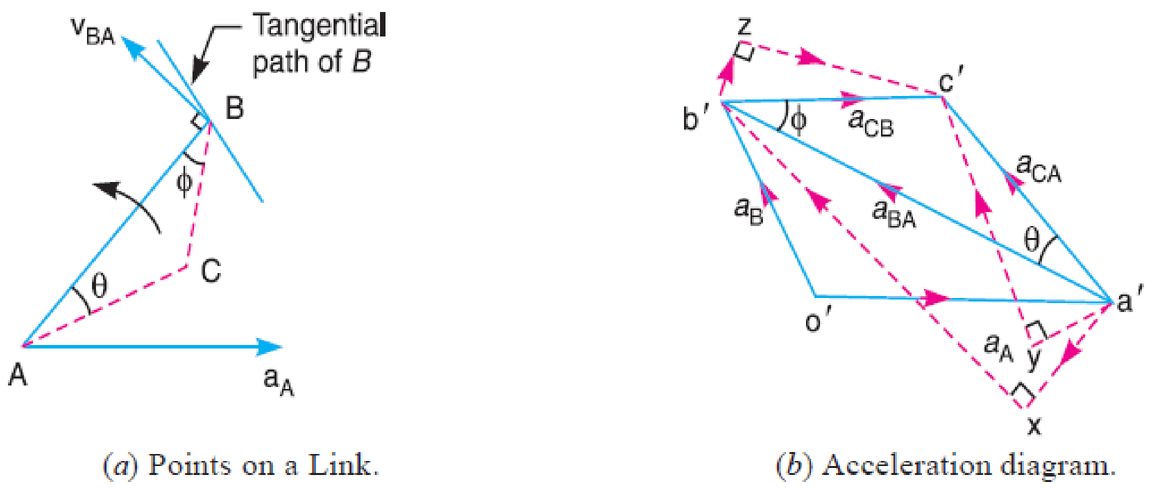


Fig. 2. Acceleration of a point on a link.

1. From any point *o'*, draw vector *o'a'* parallel to the direction of absolute acceleration at point *A i.e.* a_A , to some suitable scale, as shown in Fig. 2 (*b*).
2. The acceleration of *B* with respect to *A i.e.* a_{BA} has two components a^r_{BA} and a^t_{BA}. These two components are mutually perpendicular.
3. Draw vector *a'x* // link *AB*, vector $a'x = a^r_{BA} = \frac{v_{BA}^2}{AB}$
4. From point *x*, draw vector *xb'*⊥ link *AB* or vector *a'x* (because $a^t_{BA} \perp a^r_{BA}$) and through *o'* draw a line parallel to the path of *B* to represent the absolute acceleration of *B i.e.* a_B. The vectors *xb'* and *o'b'* intersect at *b'*. Now the values of a_B and a^t_{BA} may be measured, to the scale.

5. By joining the points a' and b' we may determine the total acceleration of B with respect to A *i.e.* a_{BA}. The vector $a'\,b'$ is known as ***acceleration image*** of the link AB.

6. For any other point C on the link, draw triangle $a'\,b'\,c'$ similar to triangle ABC.

$$\text{vector } b'c' = a_{CB}, \quad \text{and} \quad \text{vector } a'c' = a_{CA}.$$

a_{CB} and a_{CA} will each have two components as follows :

i. a_{CB} has two components; a^r_{CB} and a^t_{CB} as shown by triangle $b'zc'$ in Fig. 2 (*b*), in which $b'z \parallel BC$ and $zc' \perp b'z$ or BC.

ii. a_{CA} has two components; a^r_{CA} and a^t_{CA} as shown by triangle $a'yc'$ in Fig. 2 (*b*), in which $a'y \parallel AC$ and $yc' \perp a'y$ or AC.

7. Angular acceleration of the link AB,

$$\alpha_{AB} = \frac{a^t_{BA}}{AB}$$

3. Acceleration in the Slider Crank Mechanism

A slider crank mechanism is shown in Fig. 3 (*a*). Let the crank OB makes an angle θ with horizontal and rotates in a clockwise direction about the fixed point O with uniform angular velocity ω_{BO} rad/s.

$$v_{BO} = v_B = \omega_{BO} \times OB$$

Radial acceleration of B (because O is a fixed point),

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{OB}$$

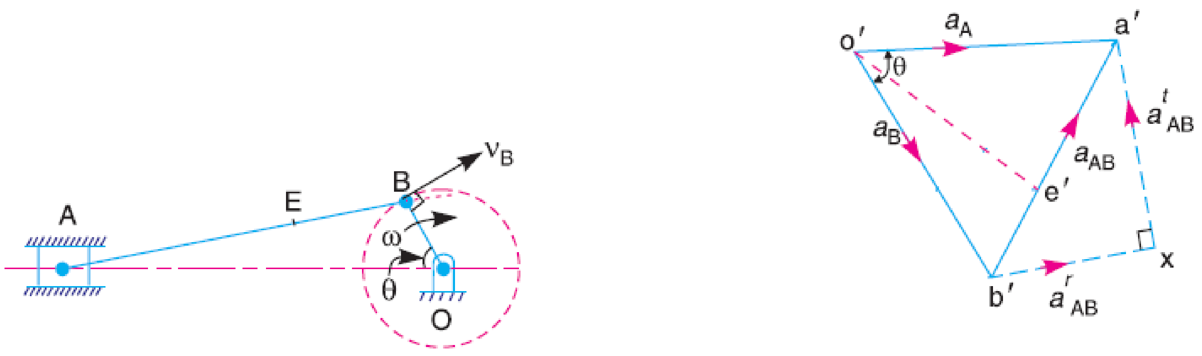
Note: A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration, $a^t_{BO} = 0$.

The acceleration diagram, as shown in Fig. 3(*b*), may now be drawn as discussed below:

1. Draw vector $o'\,b' \parallel BO$ represent $a^r_{BO} = a_B$, to some suitable scale.

2. From point b', draw vector $b'x \parallel BA$,

$$\text{vector } b'x = a^r_{BA} = \frac{v^2_{BA}}{AB}$$



(*a*) Slider crank mechanism. (*b*) Acceleration diagram.

Fig. 3. Acceleration in the slider crank mechanism.

3. From point *x*, draw vector $xa' \perp b'x$ (or *AB*). The vector *xa'* represents a^t_{AB} .

4. From *o'*, draw *o'a'* // *AO*, intersecting the vector *xa'* at *a'*.

Now the acceleration of the piston or the slider *A* (a_A) and a^t_{AB} may be measured to the scale.

5. a_{AB} = vector $b'a' = b'x + xa'$ (vectors sum)

6. The acceleration of any other point on *AB* such as *E* can be obtained by

$$\frac{a'b'}{AB} = \frac{a'e'}{AE}$$

7. Angular acceleration of the connecting rod *AB*,

$$\alpha_{AB} = \frac{a^t_{AB}}{AB}$$ (Clockwise about *B*)

Example: 1

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :

1. linear velocity and acceleration of the midpoint of the connecting rod, and

2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

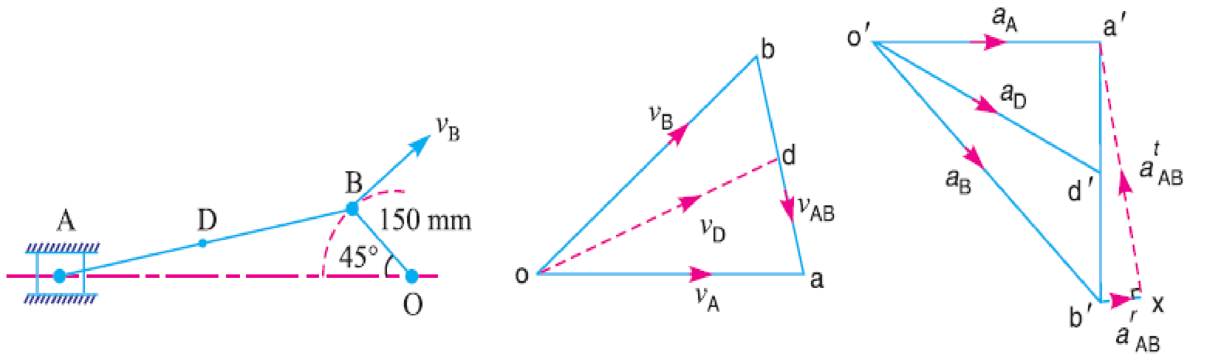
Solution:

1. Draw the mechanism, to some suitable scale, as shown in Fig.(*a*).
let 100 mm = 1 cm in paper

2. Find $v_{BO} = v_B = \omega_{BO} \times OB$

N_{OB} = 300 r.p.m. , $\omega_{BA} = 2\pi \times 300/60 = 31.42$ rad/s

$v_{BO} = v_B = 31.42 \times 150 = 4.713$ m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

3. Draw the velocity diagram, as shown in Fig. (*b*), since the point *O* is fixed. Now from point *o*, draw vector *ob* ⊥ *OB*, to some suitable scale.

let 4.713 m/s = 5 cm in paper

vector *ob* = v_B

4. From point *b* draw vector *ba* ⊥ *BA* to represent v_{AB}, and from point *o*, draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent v_A. The vectors *ba* and *oa* intersect at *a*.

By measurement, we find that

$$v_{AB} = \text{vector } ab = 3.6 \times \frac{4.713}{5} = 3.4 \text{ m/s}$$

Angular velocity of the connecting rod $= \omega_{AB} = \frac{v_{AB}}{AB} = \frac{3.4}{0.6} = 5.67$ rad/s.

(Anticlockwise about *B*) **Ans.**

$$v_A = \text{vector } oa = 4.2 \times \frac{4.713}{5} = 4 \text{ m/s}$$

5. Since the point *D* lies on *AB*,

$$\frac{ba}{BA} = \frac{bd}{BD}$$

Note: Point *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

By measurement, we find that

$$v_D = \text{vector } od = 4.3 \times \frac{4.713}{5} = 4.1 \text{ m/s}$$ **Ans.**

Draw the acceleration diagram, as shown in Fig. (*c*)

1. $$a^r_{BO} = a_B = \frac{v_{BO}^2}{OB} = \frac{4.713}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v_{AB}^2}{OB} = \frac{3.4}{0.6} = 19.3 \text{ m/s}^2$$

Scale for acceleration diagram: let 148.1 m/s^2 = 5 cm in paper

Note: Since the crank *OB* rotates at a constant speed, therefore $a^t_{BO} = 0$

3. The acceleration of *A* with respect to *B* (a_{AB}) has two components:

$$a_{AB} = a^r_{AB} + a^t_{AB}$$

from point *b'*, draw vector *b' x* // *AB* to represent a^r_{AB} = 19.3 m/s = 0.7 cm in paper (from $A \rightarrow B$), and from point *x* draw vector *xa'* ⊥ vector *b'x* whose magnitude is yet unknown.

4. Now from *o'*, draw vector *o'a'* parallel to the path of motion of *A* (which is along *AO*) to represent a_A . The vectors *xa'* and *o'a'* intersect at *a'*. Join *a' b'*.
5. Point *D* is the midpoint of *AB*, therefore *d'* is also midpoint of vector *b'a'*,

$$\frac{\acute{b}\acute{a}}{BA} = \frac{\acute{b}\acute{d}}{BD}$$

Vector $\acute{o}\acute{d}$ = 3.95 cm in paper

$$a_D = \text{vector } \acute{o}\acute{d} = 3.95 \times \frac{148.1}{5} = 117 \text{ m/s}^2$$ **Ans.**

$\acute{b}\acute{a}$ = 3.45 cm in paper

$$a^t_{AB} = \text{vector } \acute{b}\acute{a} = 3.45 \times \frac{148.1}{5} = 103 \text{ m/s}^2$$

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ m/s}^2 \text{ (Clockwise about B)}$$ **Ans.**

Example :2

In the mechanism, as shown in Figure below, the crank *OA* rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks *B* and *D*. The dimensions of the various links are *OA* = 300 mm; *AB* = 1200 mm; *BC* = 450 mm and *CD* = 450 mm. For the given configuration, determine :

1. velocities of sliding at *B* and *D*,
2. Angular velocity of *CD*,
3. linear acceleration of *D*, and
4. angular acceleration of *CD*.