Chapter four: First law of thermodynamic control volume

4.1. Thermodynamic analysis of control volume

A large number of engineering problems involve mass flow in and out of a system and, therefore, are modeled as control volumes. A water heater, car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open system) instead of as control mass (closed system). In general, any arbitrary region in space can be selected as a control volume.

The boundaries of a control volume are called a control surface, and they can be real or imaginary (Fig 4.1).

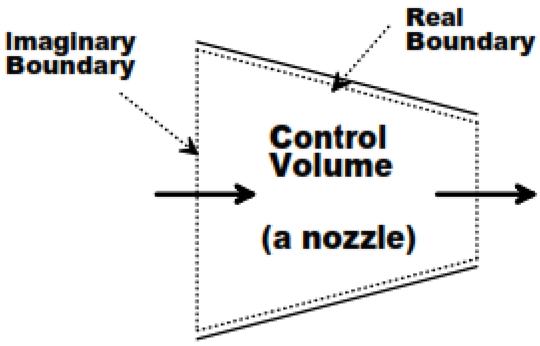

Fig 4.1 The Nozzle

A control volume can be fixed in size and shape, as in the case of a nozzle, or it may involve moving boundaries. Most control volumes, however, have fixed boundaries and thus do not involve any moving boundary work. A control volume may also involve heat and work interactions just as a closed volume, in addition to mass interaction.

A large variety of thermodynamic problems may be solved by the control volume analysis. Even it is possible to drive the relevant equations for the most general case and simplify them for special cases.

The terms steady and uniform used extensively in this chapter, and thus it is important to have a clear understanding of their meanings. The term steady implies no change with time. The opposite of steady is unsteady or transient. The term uniform, however implies no change with location over a specified region.

4.1.1. Mass and Volume Flow rate

The amount of mass flowing through a cross section per unit time is called the ***mass flow rate*** and denoted $\dot{m}$. As before, the dot over a symbol is used to indicate a quantity per unit time.

The mass flow rate of fluid flowing in a pipe or duct is proportional to the cross-sectional ***A*** of the pipe or duct, the density ρ, and the velocity ***C*** of the fluid. The mass flow rate through a differential area ***dA*** can be expressed as: -

$$d\dot{m} = \rho C_n dA$$

Where C_n is the velocity component normal to ***dA.***

The mass flow rate through the entire cross-section area of pipe or duct is obtained by integration.

$$\dot{m} = \int_A \rho C_n dA \quad \text{(kg/sec)}$$

In most practical application, the flow of a fluid through pipe or duct can be approximated to be one dimensional flow. That is, the properties can be assumed to vary in one dimension only (the direction of flow). As a result, all properties are uniform at any cross section normal to the flow direction, and properties are assumed to have bulk average values over the cross section. But the values of the properties at a cross section may change with time.

The integration result of the upper equation will give: -

$$\dot{m} = \rho C_{av} A \qquad \text{(kg/sec)}$$

or

$$\dot{m} = \rho C A \qquad \text{(kg/sec)}$$

Where:

ρ= density, kg/m^3 (=$1/\upsilon$)
C=C_{av} = average fluid velocity normal to ***A***, m/sec
A= cross-sectional area normal to flow direction, m^2

The volume flow rate $\dot{V}$ through a cross-section per unit time

$$\dot{V} = \int C_{av} dA = CA \qquad \text{(m}^3\text{/sec)}$$

The mass and volume flow rates are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{\upsilon}$$

4.1.2. Conservation of Mass Principle

The conservation of mass is one of the most fundamentals principles in nature. Mass, like energy, is a conserved property, and it can not be created or destroyed.

For closed systems, the conservation of mass principle is implicitly used by requiring that the mass of the system remain constant during a process. For control volume, however, mass can cross the boundaries, that amount of mass entering and leaving the control volume (Fig 4.2).

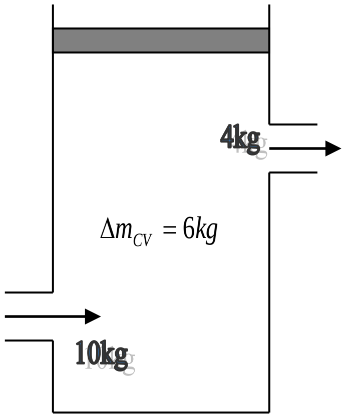

Fig. 4.2. Conservation of mass principle

The conservation of mass principle for a control volume ***CV*** undergoing a process can be expressed as

$$(Total\ mass\ entring\ CV) - (Total\ mass\ leaving\ CV) = (Net\ Chane\ in\ Mass\ Within\ CV)$$

or

$$\sum m_i - \sum m_e = \Delta m_{CV}$$

where the subscripts ***i***, ***e***, and ***CV*** stand for inlet, exit, and control volume, respectively.

The conservation of mass equation could also expressed in the rate form by expressing the equation per unit time. The conservation of mass equation is often referred to as the continuity equation in fluid mechanics.

4.1.3. Conservation of Energy Principle

For control volume, an additional mechanism can change the energy of a system: mass flow in and out the control volume. When mass enters a control volume, the energy of the control volume increases because the entering mass carries some energy with it. Likewise, when some mass leaves the control volume, the energy contained within the control volume decreases because the leaving mass takes out some energy with it.

Then the conservation of energy equation for control volume undergoing a process can be expressed as

$$\begin{pmatrix} Total\ \ energy \\ crossing\ \ boundary \\ as\ \ heat\ \ and\ \ work \end{pmatrix} + \begin{pmatrix} Total\ \ energy \\ of\ \ mass \\ entering\ \ CV \end{pmatrix} - \begin{pmatrix} Total\ \ energy \\ of\ \ mass \\ leaving\ \ CV \end{pmatrix} = \begin{pmatrix} Net\ \ change \\ in\ \ energy \\ of\ \ CV \end{pmatrix}$$

Or

$$Q - W + \sum E_{in} - \sum E_{out} = \Delta E_{CV}$$

Heat transfer to or from a control volume should not be confused with the energy transported with mass into and out of a control volume. Remember that heat is form of energy transferred as a result of a temperature difference between the control volume and the surroundings.

4.1.4. Flow Work

The energy required to push fluid into or out of a control volume is called the flow work, or flow energy. It is considered to be part of the energy transported with the fluid.

To obtain a relation for flow work, consider a fluid element of volume ***V*** as shown in fig. 4.3. The fluid immediately upstream will force this fluid element to enter the control volume; thus, it can be regarded as an imaginary piston. The fluid element can be chosen to be sufficiently small so that it has uniform properties throughout.

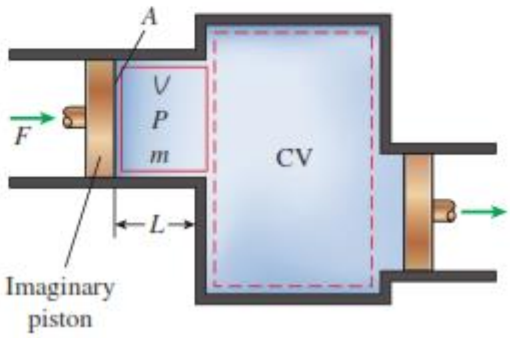

Fig.4.1.3. Schematic of flow work

If the fluid pressure is ***P*** and the cross-sectional area of the fluid element is ***A*** (fig. 4.4), the force applied on the fluid element by the imaginary piston is

$F = PA$

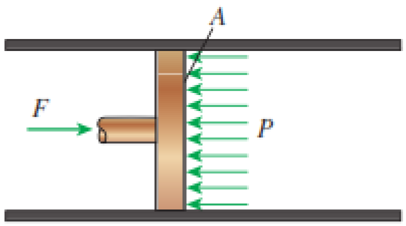

Fig. 4.1.4 The force applied on the piston by the fluid

To push the entire fluid element into the control volume, this force must act through a distance ***L***. Thus, the work done in pushing the fluid element across the boundary (i.e., the flow work) is

$W_{flow} = FL = PAL = PV$ (kJ)

The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

$w_{flow} = P/\upsilon$ (kJ/kg)

4.1.5. Total Energy of a Flowing Fluid

As we discussed in Chapter 1, the total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies. On a unit mass basis, it is expressed as

$$e = u + ke + pe = u + \frac{C^2}{2000} + \frac{gz}{1000} \ \text{(kJ/kg)}$$

where ***C*** is the velocity and ***z*** is the elevation of the system relative to some external reference frame.

The fluid entering or leaving a control volume possesses an additional form of energy- the flow energy ***Pv***, as discussed above. Then the total energy of a flowing fluid on a unit-mass basis (denoted by θ) becomes

$$\theta = P\upsilon + e = P\upsilon + (u + ke + pe)$$

But the combination $P\upsilon + u$ has been previously defined as the enthalpy ***h***, so the above relation reduces to

$$\theta = h + ke + pe = h + \frac{C^2}{2000} + \frac{gz}{1000} \qquad (kJ/kg)$$

Professor J. Kestin proposed in 1966 that the term ***θ*** be called methalpy (from metaenthalpy, which means beyond enthalpy)

4.2. The steady- flow process

A large number of engineering devices such as turbines, compressors, and nozzles operate for long time under the same conditions, and they are classified as steady-flow devices.

The processes involving steady- flow devices can be represented reasonably well by a somewhat idealized process, called the steady-flow processes. A ***steady-flow process*** can be defined as a process during which a fluid flows through a control volume steadily. That is, the fluid properties can change from point to point within control volume, but at any fixed point they remain the same during the entire process. (Remember, steady means no change with time.) A steady- flow process is characterized by the following.

No properties (intensive or extensive) within the control volume change with time. Thus, the volume ***V,*** and the mass m, and the total energy ***E*** of the control volume remain constant during a steady-flow process (fig. 4.5). As a result, the boundary work is zero for steady-flow system (since V_{CV}=constant), and the total mass or energy entering the control volume must equal to the total mass or energy leaving it (since m_{CV}=constant and E_{CV}=constant). These observations greatly simplify the analysis.