Example 2.2

A piston- cylinder device contains $0.2 m^3$ of saturated water vapor at 700kPa pressure. Determine the temperature of the vapor and the mass of the vapor inside the cylinder?

Solution: the state of the saturation water vapor is shown on the P- υ diagram. Since the cylinder contains saturated vapor at 700kPa, the temperature inside must be the saturation temperature at this pressure.

$$T=T_{sat\ at\ 700kPa}=164.97°C$$

The specific volume of the saturated vapor at 700kPa is

$$\upsilon = \upsilon_{g\ at\ 700kPa}=0.2729 m^3/kg$$

then the mass of water vapor inside cylinder becomes

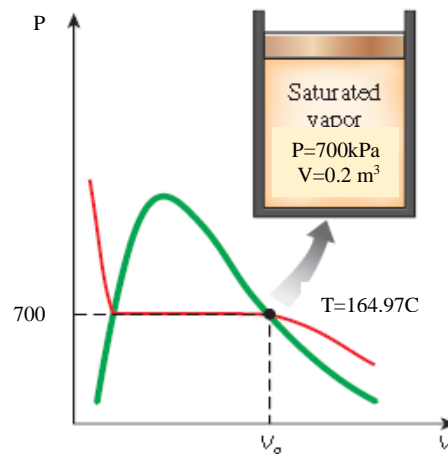$$m = \frac{V}{\upsilon} = \frac{0.2}{0.2729} = 0.7329 kg$$

Figure 26. Schematic and P-v diagram for example 2– 2.

2.5.3. Saturated Liquid-Vapor Mixture

During a vaporization process, a substance exists as part liquid and path vapor. That is, a mixture of saturated liquid and saturated vapor. To analyze this mixture property, we need to know the proportions of liquid and vapor phases in the mixture. This is done by new property called quality (dryness fraction) *x* as the ratio of the mass of vapor to the total mass of the mixture.

$$x = \frac{m_{vapor}}{m_{total}}$$

$$m_{total} = m_{liquid} + m_{vapor} = m_f + m_g$$

consider a tank that contains a saturated liquid-vapor mixture. The volume occupied by saturated liquid V_f, and the volume occupied by vapor is V_g. The total volume V is the sum of these two volumes.

$$V = V_f + V_g$$

$$V = mv \qquad V_f = m_f v_f \qquad V_g = m_g v_g$$

$$mv = m_f v_f + m_g v_g$$

dividing by **m** yields

$$v = \frac{m_f}{m} v_f + \frac{m_g}{m} v_g$$

$$x = \frac{m_g}{m}$$

$$\frac{m_f}{m} = (1 - \frac{m_g}{m}) = 1 - x$$

$$v = (1 - x)v_f + xv_g$$

$$v = v_f + x(v_g - v_f)$$

$$v = v_f + xv_{fg}$$

and also, there two equations similar to that for specific volume can be used to find the internal energy and enthalpy for the mixture:

$$u = u_f + xu_{fg}$$

$$h = h_f + xh_{fg}$$

All the results are of same format, and they can be summarized in a single equation as:

$$Y = Y_f + xY_{fg}$$

the values of the average property of the mixtures are always between the values of the saturated liquid and the saturated vapor properties. That is
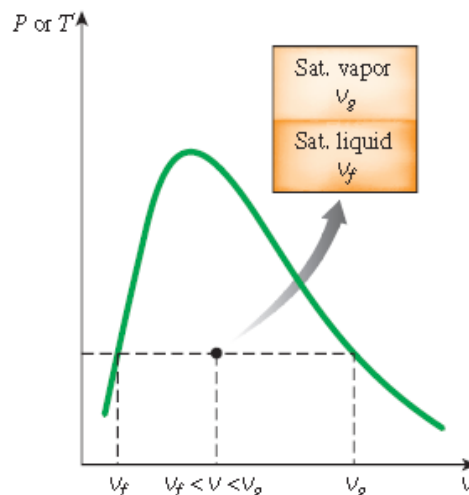
$$Y_f \le Y \le Y_g$$

Figure 27. The **v** value of a saturated liquid–vapor mixture lies between the v_f and v_g values at the specified **T** or **P**.

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Example 2.3

A rigid tank contains 10 kg of water at 90°C. If 8kg of the water is in the liquid form and the rest is in the vapor form. Determine (a) the pressure in the tank and (b) the volume of the tank.

<u>Solution:</u> (a) at T= 90°C there is a saturated mixture of vapor and liquid in equilibrium.

So   P=P$_{sat\ at\ 90^{o}C}$=70.14kPa   (Table A– 4)

      (b) At 90°C, v_f=0.001036m³/kg and v_g=2.361m³/kg   (Table A– 4)

one way to find the volume of the tank is to determine volume occupied by each phases

$$V = V_f + V_g = m_f v_f + m_g v_g$$

$$V = 8 \times 0.001036 + 2 \times 2.361 = 4.73 m^3$$

another way is to determine the quality x.

$$x = \frac{m_g}{m} = \frac{2}{10} = 0.2$$

$$v = v_f + x(v_g - v_f) = 0.001036 + 0.2(2.361 - 0.001036)$$

$$v = 0.473 m^3 / kg$$

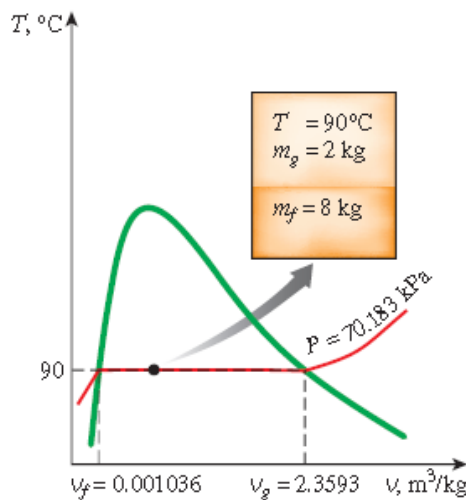$$V = mv = 10 \times 0.473 = 4.73 m^3$$

Figure 28. Schematic and T-v diagram for example 2–3.

2.5.4. Superheated Vapor:

In the region to the right of the saturated vapor line a substance exits as superheated vapor. Since the superheated region is a single-phase region (vapor phase only), the temperature and pressure are no longer dependent properties and these can conveniently be used as the two independent properties in the table.

In these tables, the properties are listed versus temperature for selected pressures starting from saturated vapor data. The saturated temperature is given in parentheses following the pressure value.

Superheated vapor is characterized by :

Lower pressure   ($P<P_{sat}$ at given T)
Higher temperature   ($T>T_{sat}$ at given P)
Higher specific volume   ($v > v_g$ at given P or T)
Higher internal energy   ($u> u_g$ at given P or T)
Higher enthalpy   ($h> h_g$ at given P or T)

Example 2.5

Determine the temperature of water at a state of P=0.5MPa and h=3064.2kJ/kg.

Solution: given water   at P=0.5MPa   ,   h=3064.2kJ/kg

The enthalpy of saturated water vapor is hg=2748.7kJ/kg

And since h>hg it is superheated vapor, and from the superheated table it is found that   T=300ºC.

2.5.5. Compressed Liquid:

There is only date for compressed liquid water and the data of this table is not take a wide range of pressures and temperatures

In the absence of the compressed liquid data, a general approximation is to treat compressed liquid as saturated liquid at the given temperature. This is because the compressed liquid properties depend on temperature more strongly than they do on pressure.

Thus

$$v \cong v_{f \ at \ T}$$

$$u \cong u_{f \ at \ T}$$

$$h \cong h_{f \ at \ T}$$

This approximation gives a negligible error in v and u but the error in h may reach undesirable levels. However the error in h at very high pressure can be reduced significantly by evaluating it from

$$h = h_{f \ at \ T} + v_f (P - P_{sat})$$

In general compressed liquid is characterized by
Higher pressure   (P > P_{sat} at a given T)
Lower temperature   (T < T_{sat} at a given P)
Lower specific volume   ($v < v_f$ at a given P or T)
Lower internal energy   (u < u_f at a given P or T)
Lower enthalpy   (h < h_f at a given P or T)

Example 2.6:

Determine the internal energy of compressed liquid water at 80ºC and 5MPa using (a) data from compressed liquid table and (b) saturated liquid data

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depending saturated temperature (c) saturated liquid data depending saturated pressure. What is the error involved in the second and third cases.

Solution: given compressed liquid at   T=80°C   P=5MPa
        We can first check if the case is compressed liquid as the problem said or not.
        At T=80°C P_{sat}= 47.39kPa   so P>P_{sat} it is compressed liquid or
        At P=5MPa T_{sat}=263.99°C   so T< T_{sat} it is compressed liquid
        (a) from compressed liquid table (Table A-7)   at P=5MPa and T=80°C

$$u=333.72kJ/kg$$

        (b) by using the data depending the temperature (Table A-4):
$$u=u_f \text{ at } 80°C =334.86kJ/kg$$
        (c) by using the data depending the pressure:
$$u=u_f \text{ at } 5MPa =1147.81kJ/kg$$

        error in second case on the first case is
$$error\% = \left|\frac{334.86-333.72}{333.72}\right| \times 100 = 0.3416\%$$
        error in third case on the first case is
$$error\% = \left|\frac{1147.81-333.72}{333.72}\right| \times 100 = 243.94\%$$

        Note: it is shown from the example that the value taken at temperature is more accurate than that taken at the pressure.
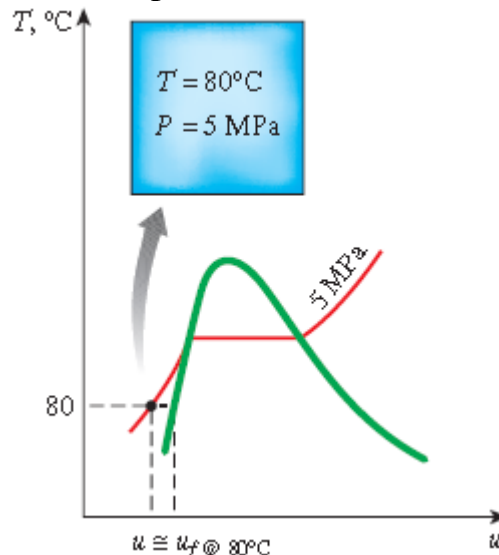

Figure 29. Schematic and *T-u* diagram for Example 3–6.

2.5.6. Linear Interpolation:

        It is assumed that the relation between any two values followed each other in the table is a linear relation. The equation of a straight line can be used to find the values of any properties against any given values having no value in

the table. The method is as following. From the fig.(30) on the x,y coordinate and assuming that, there are two point (y_1,x_1) and (y_2,x_2) connected by a straight line. The slop of this line R is equal

$$R = \frac{y_2 - y_1}{x_2 - x_1}$$

the equation of the line

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = R$$

hence $y = y_1 + R(x - x_1)$

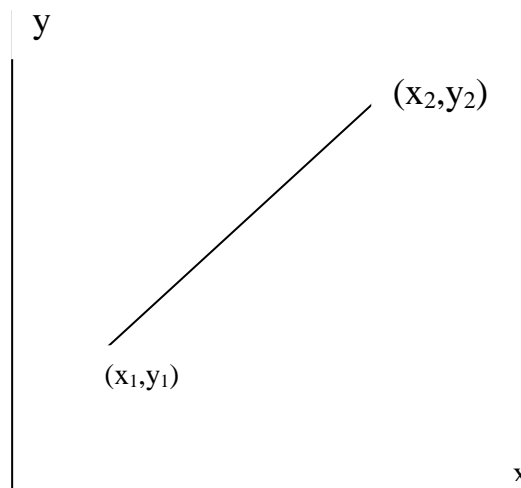$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Figure 30. the interpolation

This equation can be used to find the values by substituting the known property instead of x and the needed property instead of y.

Example 2.7:

Determine the temperature of a superheated water vapor at a pressure of 0.5MPa and enthalpy 2890kJ/kg.

Solution: the given P=0.5MPa   h=2890kJ/kg superheated vapor
From the superheated vapor water at 0.5MPa we read:

TºC	h [kJ/kg]
200	2855.4
250	2960.7

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The value h=2890kJ/kg is greater than 2855.4kJ/kg at 200ºC, and less than 2960.7kJ/kg at 250ºC, thus the temperature value must be between 200 and 250ºC and by using the linear interpolation.

	T	h
(1)	200	2855.4
		2890
(2)	250	2960.7

$$T = T_1 + \frac{h - h_1}{h_2 - h_1}(T_2 - T_1)$$

$$T = 200 + \frac{2890.0 - 2855.4}{2960.7 - 2855.4}(250 - 200)$$

$$= 216.4^o C$$

2.6.   Ideal-Gas Equation of State:

Any equation that relates the pressure, temperature, and specific volume of a substance is called an equation of state. Property relations which involve other properties of a substance at equilibrium states are also referred to as equation of states. There are several equations of states, some simple and other very complex. The simplest and the best known equation of state for substances in the gas phase is the ideal gas equation of state. This equation predicts the P-υ-T behavior of a gas quite accurately within some properly selected region.

In 1662 Robert Boyle, an Englishman, observed during his experiments with vacuum chamber that the pressure of gases is inversely proportional to their volume.

$$P \propto \frac{1}{v} \qquad at \quad T = cons \tan t$$

$$P = \frac{K}{v} \qquad K = proportionalty \quad cons \tan t$$

In 1802, J. Charles and J.Gay-Lussase, Frenchmen, experimentally determined that at low pressures the volume of a gas is proportional to its temperature.

$$v \propto T$$

$$v = kT$$

that is $$v \propto \frac{T}{P}$$

or $$Pv \propto T$$

and $$Pv = RT$$

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