Where the constant of proportionality R is called the gas constant. This equation is called the ideal-gas equation of state, or simply the ideal-gas relation, and a gas which obeys this relation is called an ideal gas.

In this equation P is the absolute pressure, T is the absolute temperature, v is the specific volume.

The gas constant R is different for each gas and is determined from

$$R = \frac{R_u}{M} \text{ [kJ/kg.K or kPa.m}^3\text{/kg.K]}$$

where R_u is the universal gas constant and M is the molar mass (also called molecular weight) of the gas. Constant R_u is the same for all substance, and its given as

$$R_u = \begin{cases} 8.314 kJ/(kmol.K) \\ 8.314 kPa.m^3/(kmol.K) \\ 0.08314 bar.m^3/(kmol.K) \\ 1.986 Btu/(lbmol.R) \\ 10.73 Psia.ft^3/(lbmol.R) \\ 1545 ft.lbf/(lbmol.R) \end{cases}$$

The molar mass M can simply be defined as the mass of one mole of a substance in grams or, the mass of one kmole in kilograms. The mass of a system is equal to the product of its molar mass M and the mole number N.

$$m = MN \quad [kg]$$

The values R and M for several substances are given in tables.

The ideal-gas equation of state can be written in several different forms:

$$V = mv \rightarrow PV = mRT$$

$$mR = (MN)R = N(MR) = NR_u \rightarrow PV = NR_uT$$

$$V = N\bar{v} \rightarrow P\bar{v} = R_uT$$

Where $\bar{v}$ is the molar specific volume, i.e., the volume per unit mole (m^3/kmol or ft^3/mol). A bar above a property will denote values on a unit-mole basis through this text.

For a fixed mass, the property of an ideal gas at two different states are related to each other by:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Or in general $\frac{PV}{T} = Const.$

Example 2.8

Determine the mass of air in a room whose dimensions are 4mx5mx7m at 100kPa and 25°C.

Solution: the given P=100kPa T=25°C V=4x5x7 m^3

Air at specified condition can be treated as an ideal gas. The gas constant is R=0.287kJ/kg.K

and the absolute temperature is T= 25=273=298K

the volume of the room V=4x5x7=140m^3

the mass of air in the room $m = \frac{PV}{RT} = \frac{100 \times 140}{0.287 \times 298} = 163.7kg$

2.7. Compressibility Factor:

It is measure of deviation from ideal-gas behavior. This deviation from ideal-gas behavior at a given temperature and pressure can accurately be form by the introduction of a correction factor called the compressibility factor Z. it is defined as:

$$Z = \frac{Pv}{RT}$$

Or $$pv = ZRT$$

it can also be expressed as

$$Z = \frac{v_{actual}}{v_{ideal}}$$

where $v_{ideal} = \frac{RT}{p}$, obviously, Z=1 for ideal gas for real gas Z can be greater than or less than unity. The farther away Z is from unity, the more the gas deviates from ideal-gas behavior.

Gases behave differently at a given temperature and pressure, but they behave very much the same at temperatures and pressures normalized with respect to their critical temperatures and pressures. The normalization is done as

$$P_r = \frac{P}{P_{cr}} \quad \text{and} \quad T_r = \frac{T}{T_{cr}}$$

here P_r is called the reduced pressure and T_r the reduced temperature.

The Z factor for all gases is approximately the same at the same reduced pressure and temperature. This is called the principle of corresponding states. By curve fitting all the data, we obtain the generalized compressibility chart which can be used for all gases. The use of a compressibility chart requires a knowledge of critical-point data, and the results obtained are accurate to within a few percent.

The following observation can be made from the generalized compressibility chart:

1- at very low pressures (P_r<<1), the gases behave an ideal gas regardless of temperature.
2- At high temperatures (T_r>>1), ideal gas behavior can be assumed with good accuracy regardless of pressure (except when P_r>>1).
3- The deviation of a gas from ideal gas behavior is greatest in the vicinity of the critical point.

2.8. Other Equation of State:

The van der Waals equation of state was proposed in1873, and it has two constants which are determined from the behavior of a substance at the critical point. The van der Waals equation of state is given by

$$\left(P+\frac{a}{v^2}\right)(v-b)=RT$$

The term $\frac{a}{v^2}$ accounts for the intermolecular attraction force, and b accounts for the volume occupied by gas molecules.

The determination of the two constants appearing in this equation based on the observation that the critical isotherm on a P-υ diagram has a horizontal inflection point at the critical point. Thus the first and the second derivatives of P with respect to υ at the critical point must be zero. That is

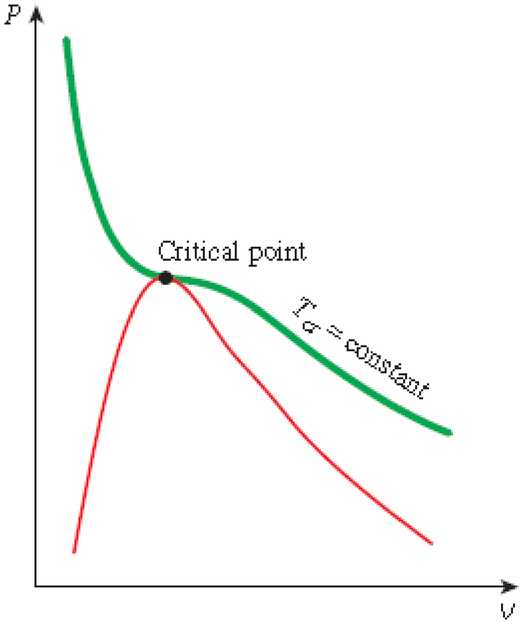

Figure 31. Critical isotherm of a pure substance has an inflection point at the critical state

$$\left(\frac{\partial P}{\partial v}\right)_{T=Tcr}=0$$

and

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=Tcr}=0$$

to find the value of the constants a and b the following procedure be followed:

$$P=\frac{RT}{v-b}-\frac{a}{v^2}$$

$$\left(\frac{\partial P}{\partial v}\right)_T=\frac{-RT}{(v-b)^2}+\frac{2a}{v^3}=0 \quad \text{--------(1)}$$

$$\left(\frac{\partial^2 P}{\partial^2 v}\right)_T=\frac{2RT(v-b)}{(v-b)^4}-\frac{3a}{v^4}=0 \quad \text{-------(2)}$$

from eq.(1) $\frac{a}{v^3}=\frac{RT}{2(v-b)^2}$

substituting in eq.(2) and it become

$$\frac{2}{v-b}=\frac{3}{2v} \longrightarrow b=\frac{v}{3}$$

substituting in eq. (1)

$$\frac{a}{v^3}=\frac{RT}{2(v-\frac{v}{3})^2}\rightarrow a=\frac{9RTv}{8}$$

by substituting in equation of van der Waals

$$P=\frac{RT}{v-\frac{v}{3}}-\frac{9RT}{8v}$$

and at critical point

$$v=\frac{3RT}{8P}$$

$$a=\frac{27RT_{cr}}{64P_{cr}}$$

$$b=\frac{RT_{cr}}{8P_{cr}}$$

The other equation of state is the Beattic-Bridgeman equation of state. It is based on the five experimentally determined constants. It was proposed in the form of

$$P=\frac{R_uT}{\bar{v}^2}\left(1-\frac{C}{\bar{v}T^3}\right)(\bar{v}-B)-\frac{A}{\bar{v}^2}$$

and $A=A_o\left(1-\frac{a}{\bar{v}}\right)$ and $B=B_o\left(1-\frac{b}{\bar{v}}\right)$ and these constants are given in table for varies substances.

The equation of can be expressed in a series form as

$$P=\frac{RT}{v}+\frac{a(T)}{v^2}+\frac{b(T)}{v^3}+\frac{c(T)}{v^4}+---$$

this similar equations are called the virial equation of state, and the coefficient, equation of state, and the coefficient, a(T), b(T), c(T) are called virial coefficients.

Example 2.10

Predict the pressure of nitrogen gas at T=175K and v=0.00375m^3/kg on the basis of (a) the ideal-gas equation, (b) the van der Waals equation. (c) the Beattic-Bridgeman equation of state (d) the generalized chart. Compare the value obtained to the experimentally value of 1000kPa.

Solution: a given N_2 T=175K υ=0.00375m^3/kg and R=0.2968kJ/kg.K

(a) ideal-gas equation $P=\frac{RT}{v}=\frac{0.2968\times 175}{0.00375}=13850kPa$

(b) Van der Waals equation

$$a=\frac{27R^2T_{cr}^2}{64P_{cr}}=\frac{27\times(0.2968)^2\times 126.2^2}{64\times 3390}=0.1749$$

$$b=\frac{RT_{cr}}{8P_{cr}}=\frac{0.2968\times 126.2}{8\times 3390}=1.38\times 10^{-3}$$

$$P=\frac{RT}{v-b}-\frac{a}{v^2}=\frac{0.2968\times 175}{0.00375-1.38\times 10^{-3}}-\frac{0.1749}{(0.00375)^2}=9488.7kPa$$

(c) Beattic-Bridgemen equation

$$A=A_o\left(1-\frac{a}{\bar{v}}\right)=136.2315\left(1-\frac{0.02617}{28\times 0.00375}\right)=102.27$$

$$B=B_o\left(1-\frac{b}{\bar{v}}\right)=0.05046\left(1-\frac{-0.00691}{28\times 0.00375}\right)=0.0538$$

$$C=4.2\times 10^4$$

$$P=\frac{R_uT}{\bar{v}^2}\left(1-\frac{C}{\bar{v}T^3}\right)(\bar{v}-B)-\frac{A}{\bar{v}^2}=10110kPa$$

(d) generalized chart

$$T_r=\frac{T}{T_{cr}}=\frac{175}{126.2}=1.3867$$

$$v_r=\frac{v}{RT_{cr}/P_{cr}}=\frac{0.00375}{\frac{0.2968\times 126.2}{3390}}=0.3372$$

and from the chart $P_r=2.95$

$$P=2.95\times 3390=10000.5kPa$$

the value of P=10000kPa that taken from the table

Error in (a) $error\% = \left|\frac{13850-10000}{10000}\right| \times 100 = 38.5\%$

Error in (b) $error\% = \left|\frac{9488.7-10000}{10000}\right| \times 100 = 5.113\%$

Error in (c) $error\% = \left|\frac{10110-10000}{10000}\right| \times 100 = 1.1\%$

Error in (d) $error\% = \left|\frac{10000.5-10000}{10000}\right| \times 100 = 0.005\%$

It is found that the generalized chart give the more accurate value than the other methods.