

$P=f(V)$ is simply the equation of the process path on P - V diagram. The quasi-equilibrium expansion process described above is shown on a P - V diagram in figure 34.

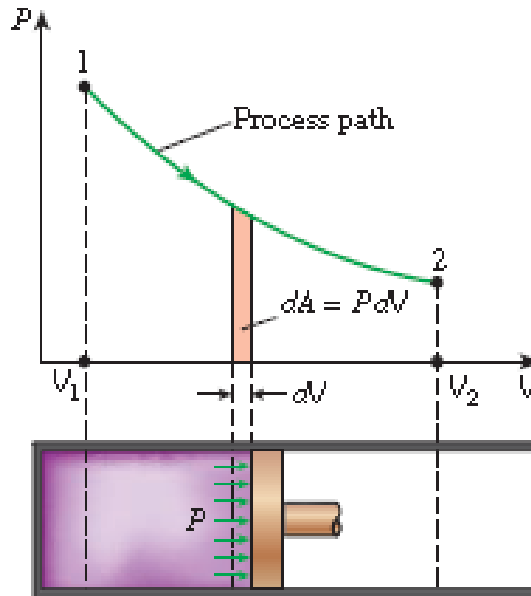


Figure 36. The area under the process curve on a P - V diagram represents the boundary work.

The area under the process curve on a P - v diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.

The gas can follow several different paths as it expands from state 1 to state 2. these paths represent different processes as follows

3.4.1. Constant volume process

In this process

$$V = \text{constant}$$

$$dV = 0$$

$$W = \int_1^2 P dV = 0$$

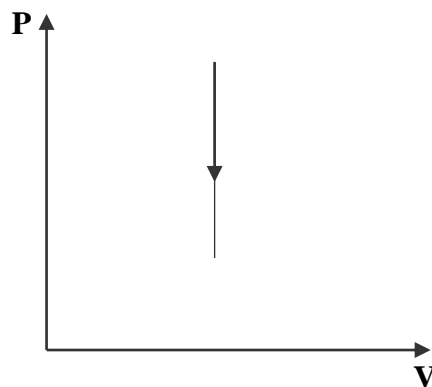


Figure37 . constant volume process



Example 3.4

A rigid tank contains air at 500kPa and 150°C. As a result of the surrounding, the temperature and pressure inside the tank drop to 65°C and 400kPa, respectively. Determine the work done during this process.

Solution: Given $T_1=150^\circ\text{C}$ and $P_1=500\text{kPa}$

$T_2=65^\circ\text{C}$ and $P_2=400\text{kPa}$ with no change in volume because the tank is rigid.

$V=\text{constant}$ and $dv=0$ and so $W=0$

3.4.2. Constant Pressure Process (isobaric process)

$$P = \text{constant}$$

$$P = P_1 = P_2$$

$$W = \int_1^2 P dV = P(V_2 - V_1)$$

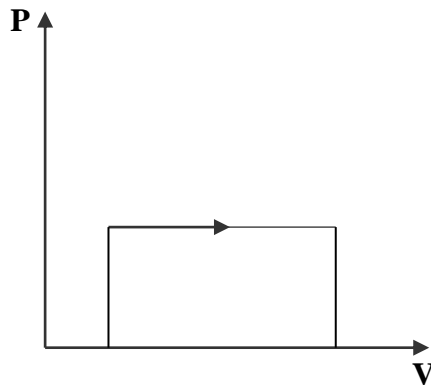


Figure38 . Constant Pressure process

in this process the work of ideal gas is

$$W = P(V_2 - V_1)$$

$$W = MR(T_2 - T_1)$$

and for vapor

$$W = mP(v_2 - v_1)$$

Example 3.5

Five kilograms of saturated vapor water at 1Mpa is contained in a cylinder fitted with a movable piston. This system is now heated at constant pressure until the temperature of the steam is 300°C. Calculate the work done by the steam during the process.

Solution: Given sat vapor water

$$m=5\text{kg} \quad P_1=P_2=1\text{MPa}$$

$$T_2=300^\circ\text{C}$$

From the saturated water table



$$v_1 = v_{g \text{ at } 1 \text{ MPa}} = 0.19444 \text{ m}^3 / \text{kg}$$

$$T_{sat} = 179.91^\circ \text{C}$$

the second state is a super heated vapor because $T_2 > T_{sat}$

$$v_2 = v_{at \text{ iMPa } 300^\circ \text{C}} = 0.2579 \text{ m}^3 / \text{kg}$$

the process is constant pressure expansion

$$W = mP(v_2 - v_1)$$

$$W = 5 \text{ kg} \times 1000 \text{ kPa} \times (0.2579 \text{ m}^3 / \text{kg} - 0.19444 \text{ m}^3 / \text{kg})$$

$$W = 317.3 \text{ kJ}$$

3.4.3. Hyperbolic Process

In this process

$$PV = \text{Const.} = c \quad \rightarrow \quad P = \frac{c}{V}$$

$$W = \int_1^2 p dV = \int_1^2 \frac{c}{V} dV = c \int_1^2 \frac{dV}{V} = c \ln \frac{V_2}{V_1}$$

$$W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{P_1}{P_2} = P_2 V_2 \ln \frac{P_1}{P_2}$$

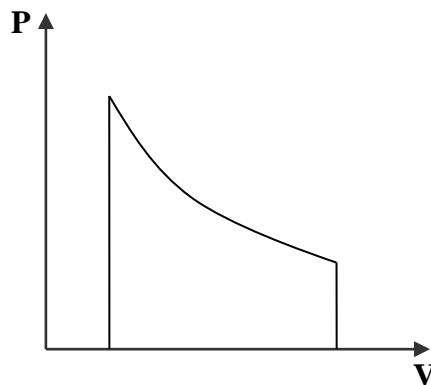


Figure39 . hyperbolic process

this process is called isothermal process for ideal gas(only), because for ideal gases when $PV = \text{const.}$ so $T = \text{const.}$

Example 3.6

One tenth kg of saturated vapor water is at 2MPa is compressed in hyperbolic process to a pressure of 4MPa. Find the final temperature of the water and the work done.

Solution: Given $m=0.1 \text{ kg}$ $P_1=2 \text{ MPa}$ sat water vapor

$P_2=4 \text{ MPa}$ and the process is $PV=\text{constant}$



At the first state $v_1 = v_{g \text{ at } 2\text{MPa}} = 0.09963\text{m}^3 / \text{kg}$

$$P_2 v_2 = P_1 v_1 \quad \text{OR} \quad v_2 = v_1 \frac{P_1}{P_2} = 0.09963\text{m}^3 / \text{kg} \times \frac{2\text{MPa}}{4\text{MPa}} = 0.04982\text{m}^3 / \text{kg}$$

the sat. volume at 4MPa $v_g = 0.04978\text{m}^3 / \text{kg}$

it is found that $v_2 > v_{g \text{ at } 4\text{MPa}}$ so the state is superheated vapor

to find the temperature by using the superheated water table and interpolation as follows

<u>T °C</u>	<u>v m³ / kg</u>
250.4	0.04978
	0.04982
275.0	0.05457

$$T = 250.4 + \frac{(0.04982 - 0.04978)}{(0.05457 - 0.04978)} (275 - 250.4) = 250.6^\circ \text{C}$$

and the work can be calculated by

$$W = m P_1 v_1 \ln \frac{P_1}{P_2} = 0.1 \times 2000 \times 0.09963 \ln \frac{2}{4} = -13.812\text{kJ}$$

3.4.4. Isothermal Process (Constant Temperature Process)

This process can be discussed separately for ideal gas and vapor

1- Ideal gas

when the temperature is constant ($T = \text{constant}$) and from the ideal gas equation of state, with no change in the mass ($PV = \text{constant}$). the process becomes hyperbolic process and

$$W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{P_1}{P_2} = P_2 V_2 \ln \frac{P_1}{P_2}$$

$$PV = mRT$$

$$W = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

Example 3.7

One kilogram of air at 500°C is expanded isothermally from a pressure of 2MPa to a pressure of 0.5MPa, find the work done by the air.

Solution: Given Air of $m=1\text{kg}$ at $P_1=2\text{MPa}$ $P_2=0.5\text{MPa}$ $T_1=T_2=T=500^\circ\text{C}$

It is an ideal gas and isothermal process of expansion

$$W = mRT \ln \frac{P_1}{P_2} =$$

$$W = 1\text{kg} \times 0.287 \times (500 + 273.15) \ln \frac{2}{0.5} = 307.61\text{kJ}$$



2-Substance with phase change

i- Saturated region

In saturated region when the temperature is constant the pressure is also constant because the pressure and temperature are dependent properties $P=f(T)$. Therefore, the work in this process is the same to that as in constant pressure process

ii- Superheated region

In this region the temperature and pressure are not dependent properties ($P \neq f(T)$ only). Therefore, the process can be assumed as polytropic process

$$(PV^n = \text{constant})$$

Example 3.8

0.4kg of saturated liquid water at 120°C is vaporized in piston cylinder device isothermally until the volume of liquid becomes one tenth of the total volume. Find the work done by the system.

Solution: Given sat. liquid water $m=0.4$ kg $T=120^\circ\text{C}$ isothermally

$$V_{f2} = \frac{V_2}{10}, \quad V_{g2} = \frac{9V_2}{10}$$

as the water is still in the saturated region the expansion is also constant pressure of $P=P_{\text{sat at } 120^\circ\text{C}}=198.53\text{kPa}$

$$v_f = 0.00106\text{m}^3/\text{kg} \quad v_g = 0.8919\text{m}^3/\text{kg} \quad v_1 = v_f = 0.00106\text{m}^3/\text{kg}$$

$$V_{f2} = m_f v_f = \frac{V_2}{10} = \frac{m v_2}{10} \rightarrow v_2 = 10 \frac{m_f}{m} v_f = 10(1-x)v_f$$

$$V_{g2} = m_g v_g = \frac{9V_2}{10} = \frac{9m v_2}{10} \rightarrow v_2 = \frac{10}{9} \frac{m_g}{m} v_g = \frac{10}{9} x v_g$$

$$v_2 = 10(1-x)v_f = \frac{10}{9} x v_g \rightarrow x = \frac{v_f}{\frac{1}{9}v_g + v_f} = \frac{0.00106}{\frac{0.8919}{9} + 0.00106} = 0.0106$$

$$v_2 = v_f + x(v_g - v_f) = 0.00106 + 0.0106(0.8919 - 0.00106) = 0.0105\text{m}^3/\text{kg} \quad \text{or}$$

$$v_2 = \frac{10}{9} x v_g = \frac{10}{9} \times 0.0106 \times 0.8919 = 0.0105\text{m}^3/\text{kg}$$

$$W = mP(v_2 - v_1) = 0.4 \times 198.53 \times (0.0105 - 0.00106) = 0.75\text{kJ}$$

3.4.5. Polytropic Process

During expansion and compression processes of real gases, pressure and volume are often related by $(PV^n = c)$ where n, and c are constants. A process of this kind is called a polytropic process.

$$W = \int_1^2 P dV$$

$$PV^n = c \rightarrow P = \frac{c}{V^n} = cV^{-n}$$

$$W = \int_1^2 cV^{-n} dV = \frac{cV^{-n+1}}{1-n} \Big|_1^2 = \frac{PV^n V^{1-n}}{1-n} \Big|_1^2 = \frac{PV}{1-n} \Big|_1^2$$

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

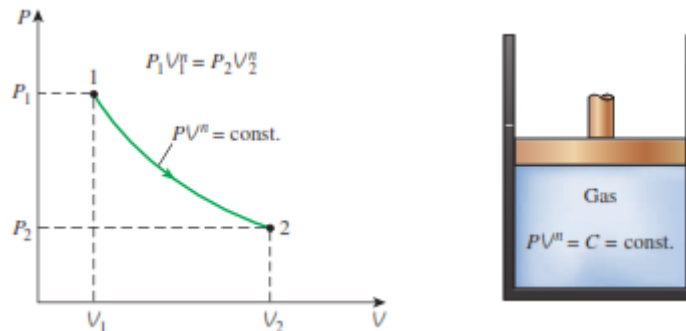


Figure40 . Polytropic process

For change phase substance, the polytropic process

$$W = \frac{m(P_2v_2 - P_1v_1)}{1-n} \quad \text{where } v \text{ is the specific volume}$$

the ideal gas polytropic process can be written as

$$W = \frac{P_2V_2 - P_1V_1}{1-n} \quad \text{or} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$$

$$W = \frac{mR(T_2 - T_1)}{1-n} \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

for ideal gas in polytropic process we can drive the following relation:

$$\begin{aligned} P_1V_1 &= mRT_1 & \text{and} & & P_2V_2 &= mRT_2 \\ P_1V_1^n &= C & & & P_2V_2^n &= C \end{aligned}$$



$$P_1 V_1^n = P_2 V_2^n$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

for ideal gas only

Example 3.9

Carbon dioxide with mass of 5kg at 100kPa pressure and 300K temperature is compressed polytropically according to the law $PV^{1.32}=C$ until the pressure of 500kPa. Find (a) initial and final volume (b) the final temperature (c) the work done

Solution: Given CO_2 gas $m = 5kg$ $P_1 = 100kPa$ $T_1 = 300^\circ C$

$P_2 = 500kPa$ for CO_2 the gas constant $R=0.2968kJ/kg.K$

$$V_1 = \frac{mRT_1}{P_1} = \frac{5 \times 0.1889 \times 300}{100} = 2.8335m^3$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 2.8335 \times \left(\frac{100}{500}\right)^{\frac{1}{1.32}} = 0.8371m^3$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300 \times \left(\frac{500}{100}\right)^{\frac{1.32-1}{1.32}} = 443.2K$$

$$W = \frac{mR(T_2 - T_1)}{1 - n} = \frac{5 \times 0.1889(443.2 - 300)}{1 - 1.32} = 422.5kJ \text{ or}$$

$$W = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{500 \times 0.8371 - 100 \times 2.8335}{1 - 1.32} = -422.5$$

3.3 FIRST LAW OF THERMODYNAMICS

It is also known as the conservation of energy principle. It states that energy can be neither created nor destroyed; it can only change forms. The first law of thermodynamics, or the conservation of energy principle for a closed system or a fixed mass, may be expressed as follows:

$$\left[\begin{array}{l} \text{Net energy transfer} \\ \text{to (or from) the system} \\ \text{as heat and work} \end{array} \right] = \left[\begin{array}{l} \text{Net increase (or decrease)} \\ \text{in the total energy} \\ \text{of the system} \end{array} \right]$$

or

$$Q - W = \Delta E \quad (kJ)$$



Where

Q = net heat transfer across system boundary ($= \sum Q_{in} - \sum Q_{out}$)

W = net work done in all forms ($= \sum W_{out} - \sum W_{in}$)

ΔE = net change in the total energy of system ($E_2 - E_1$)

the change in total energy of a system during a process can be expressed as the sum of the change in its internal, kinetic, and potential energies:

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

and by substituting this relation in the energy equation:

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

where

$$\Delta U = m(u_2 - u_1)$$

$$\Delta KE = \frac{m}{2000} (V_2^2 - V_1^2)$$

$$\Delta PE = \frac{m}{1000} (Z_2 - Z_1)$$

Most closed system encountered in practice are stationary i.e. they do not involve any change in their velocity or the elevation of their center of gravity during a process. Thus for stationary closed system the change in kinetic and potential energies are negligible ($\Delta KE = \Delta PE = 0$), and the first – law relation reduces to

$$Q - W = \Delta U \quad (kJ)$$

Sometimes it is convenient to consider the work term in two parts: W_{other} and W_b . where W_{other} represents all forms of work except the boundary work. Then the first law takes the following form:

$$Q - W_{other} - W_b = \Delta U \quad (kJ)$$

It is extremely important that the sign convention be observed for heat and work interactions. Heat flow to a system and work done by a system are positive and heat flow from a system and work done on a system are negative.

Other Forms of the First-Law Relation:

The first law relation for closed system can be written in various forms. It can be represented by unit mass:

$$q - w = \Delta u \quad (kJ/kg)$$

The rate form of the first law is Written as:

$$\dot{Q} - \dot{W} = \frac{dU}{d\tau} \quad (kW)$$

where \dot{Q} is the rate of net heat transfer, \dot{W} is the power, and $\frac{dU}{d\tau}$ is the rate of change of internal energy.

It can also be expressed in the differential form as



$$\delta Q - \delta W = dU$$

$$\delta q - \delta w = du$$

for a cyclic process, the initial and final state are identical, and therefore $\Delta U = U_2 - U_1 = 0$. then the first law relation for a cycle simplifies to

$$Q - W = 0$$

that is, the net heat transfer and network done during a cycle must be equal.

Example 3.10

A rigid vessel of .1m³ volume contains refrigerant-12 5% liquid and 95% vapor by volume at 24°C. the vessel is heated until the refrigerant exit as saturated vapor. Find (a) the initial pressure in the vessel (b) the mass of Refrigerant-12, (c) the final pressure and temperature, and (d) heat transfer occurs during the process:

Solution: Given $V=0.1\text{m}^3$ 10% liquid 90% vapor $T_1=24^\circ\text{C}$ second state is sat. vapor.

$$V_f = 0.05V = 0.05 \times 0.1 = 0.005\text{m}^3, \quad V_g = 0.95V = 0.95 \times 0.1 = 0.095\text{m}^3$$

and from the sat. R-12 table we find that the following properties at 24°C

$$P_{sat} = 634.05\text{kPa}, \quad v_f = 0.0007607\text{m}^3/\text{kg}, \quad v_g = 0.02759\text{m}^3/\text{kg},$$

$$u_f = 58.25\text{kJ}/\text{kg}, \quad u_g = 179.85\text{kJ}/\text{kg}$$

The initial pressure: because the initial state is saturated mixture at 24°C, then $P_1 = P_{sat \text{ at } 24^\circ\text{C}} = 634.05 \text{ kPa}$

The mass of R-12 in the vessel

$$m_f = \frac{V_f}{v_f} = \frac{0.005}{0.0007607} = 6.573\text{kg}, \quad m_g = \frac{V_g}{v_g} = \frac{0.095}{0.02759} = 3.443\text{kg}$$

$$m = m_f + m_g = 6.573 + 3.443 = 10.016\text{kg}$$

(c) The final pressure and temperature of the R-12 in the vessel: the final state is saturated vapor with $v_2 = \frac{V}{m} = \frac{0.1}{10.016} = .01\text{m}^3/\text{kg}$

and at the second state $v_g = v_2 = 0.01\text{m}^3/\text{kg}$

From the pressure table, we find the following data and using extrapolation we can find the data at the $v_g = 0.01\text{m}^3/\text{kg}$

P kPa	T °C	$v_f \text{ m}^3/\text{kg}$	$v_g \text{ m}^3/\text{kg}$	$u_f \text{ kJ}/\text{kg}$	$u_g \text{ kJ}/\text{kg}$
1400	56.09	0.0008448	0.01222	90.28	191.11
1600	62.19	0.0008660	0.01054	96.80	192.95
1664.4	64.15	0.0008728	0.01	98.90	193.54

So $P_2 = 1664.4\text{kPa}$, $T_2 = 64.15^\circ\text{C}$

(d)The heat transfer: because there is no change in volume so $W=0$



$$Q = m(u_2 - u_1)$$

$$u_1 = u_f + x(u_g - u_f) = 58.25 + 0.344(179.85 - 58.25) = 100.08 \text{ kJ/kg}$$

$$u_2 = u_{g \text{ at second state}} = 193.54 \text{ kJ/kg}$$

$$Q = 10.016 \times (193.54 - 100.08) = 2940.9 \text{ kJ}$$

Example 3.11

A rigid insulated tank of 0.5 m^3 , contains 5 kg of water at 100°C . An electric heater is passing through the tank with a voltage of 200V and a current of 5A for 30 minutes. Find the final state of water.

Solution: Given $V=0.5 \text{ m}^3$, rigid, insulated $Q=0$, $m=5 \text{ kg}$ water, $T=100^\circ\text{C}$, electric heater $V=200 \text{ Volt}$, $I=5 \text{ A}$, $\text{time}=30 \text{ minutes}=1800 \text{ sec}$.

The energy equation can be written as:

$$Q - W_e - W_b = \Delta U$$

where $Q=0$ for insulated tank, $W_b=\text{boundary work}=0$ rigid tank

$W_e=\text{electric work} = V \times I \times \text{time} / 1000 = 200 \times 5 \times 1800 / 1000 = 1800 \text{ kJ}$

This work is negative because it is done in the system.

$$-W_e = m\Delta u$$

$$-(-1800) = 5\Delta u$$

$$\Delta u = 360 \text{ kJ}$$

from the first state $T=100^\circ\text{C}$, and $v_1 = \frac{V}{m} = \frac{0.5}{5} = 0.1 \text{ m}^3/\text{kg}$

and it is shown that the state is saturated mixture because $v_f < v_1 < v_g$

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.1 - 0.001044}{1.6729 - 0.001044} = 0.06$$

$$u_1 = u_f + x_1 u_{fg} = 418.94 + 0.06 \times 2087.6 = 542.5 \text{ kJ/kg}$$

$$u_2 = u_1 + \Delta u = 542.5 + 360 = 902.5 \text{ kJ/kg}$$

$$v_2 = v_1 = 0.1 \text{ m}^3/\text{kg}$$

It is shown from the values of the internal energy and specific volume that the water is still saturated mixture. And by trial and error we can get the temperature or pressure.

$$T_2 = 134.9^\circ\text{C}, \quad P_2 = 312.3 \text{ kPa}, \quad x = 0.1696 = 16.96\%$$

Example 3.12

A piston cylinder device contains water at 300kPa, and 250°C with a volume of 0.4 m^3 . If the weight of the piston is required a pressure of 300kPa to rise it. The heat is transfer until the water become saturated mixture with quality of 80%. (a) prove that the heat transfer in a constant pressure process equal to the change in enthalpy. (b) the work done. (c) heat transfer during the process.

Solution: Given $P_1 = 300 \text{ kPa}$, $V_1 = 0.4 \text{ m}^3$, $T_1 = 250^\circ\text{C}$ constant pressure process.

$$x_2 = 0.8$$



for constant pressure process $W = P(V_2 - V_1) = PV_2 - PV_1$

the energy equation for closed system $Q = W + \Delta U$

$$Q = PV_2 - PV_1 + U_2 - U_1 = (P_2V_2 + U_2) - (P_1V_1 + U_1)$$

$$Q = H_2 - H_1 = \Delta H$$

For $P_1 = 300kPa$, $T_1 = 250^\circ C$, from superheated water table

$$v_1 = 0.7964m^3/kg, \quad u_1 = 2728.7kJ/kg, \quad h_1 = 2967.6kJ/kg$$

at state 2 $P_2 = 300kPa$, $x_2 = 0.8$

$$v_2 = v_f + x(v_g - v_f) = 0.001073 + 0.8(0.6058 - 0.001073) = 0.4849m^3/kg$$

$$u_2 = u_f + xu_{fg} = 561.15 + 0.8 \times 1982.4 = 2147.07kJ/kg$$

$$h_2 = h_f + xh_{fg} = 561.47 + 0.8 \times 2163.8 = 2292.51kJ/kg$$

$$m = \frac{V_1}{v_1} = \frac{0.4}{0.7964} = 0.5023m^3/kg$$

$$W = mP(v_2 - v_1) = 0.5023 \times 300 \times (0.4849 - 0.7964) = -46.94kJ$$

(c) the heat transfer: it can be calculated by two ways,

$$(i) \quad Q = W + m\Delta u = -46.94 + 0.5023 \times (2147.07 - 2728.7) = -339.1kJ$$

$$(ii) \quad Q = m\Delta h = 0.5023 \times (2292.5 - 2967.6) = -339.1kJ$$

3.5. Specific heats:

The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

We are interested in two kinds of specific heats.

3.5.1 Specific Heat at Constant Volume C_v :

It is the energy required to raise the temperature of the unit mass of substance by one degree as the volume maintained constant.

$$C_v = \frac{\delta Q}{dT}$$

$$\delta Q = C_v dT$$

In a closed system at a constant volume process of heating the energy equation can be written as:

$$\delta Q - \delta W = dU \quad \text{and for constant volume}$$

$$\delta W = 0$$

so

$$\delta Q = dU$$

$$C_v dT = dU$$

$$\text{and} \quad C_v = \left(\frac{dU}{dT} \right)_v$$