





# Lecture- 07 MASS, MOMENTUM, AND ENERGY EQUATIONS

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### Learning Objectives

- After completing this Lecture, you should be able to:
- 1. The *mass equation* is an expression of the conservation of mass principle.
- 2. Define mechanical energy

### Outline

- Overview
- Conservation of Mass
- Conservation of Mass Principle
- Mass Balance for Steady-Flow Processes
- Mechanical Energy

This Lecture deals with four equations commonly used in fluid mechanics: the mass, Momentum and energy equations.

•The *mass equation* is an expression of the conservation of mass principle.

• In fluid mechanics, it is found convenient to separate *mechanical energy* from *thermal energy* and to consider the conversion of mechanical energy to thermal energy as a result of frictional effects as *mechanical energy loss*. Then the energy equation becomes the *mechanical energy balance*.

We start this Lecture with an overview of conservation

principles and the conservation of mass relation. This is followed by a discussion of various forms of mechanical energy. Then we derive the Bernoulli equation by applying Newton's second law to a fluid element along a streamline and demonstrate its use in a variety of applications. We continue with the development of the energy equation in a form suitable for use in fluid mechanics and introduce the concept of *head loss*. Finally, we apply the energy equation to various engineering systems.

## **Conservation of Mass**

- The conservation of mass relation for a closed system undergoing a change is expressed as  $m_{sys} = constant$  or  $dm_{sys}/dt = 0$ , which is a statement of the obvious that the mass of the system remains constant during a process.
- For a control volume (CV) or open system, mass balance is expressed in the rate form as

Conservation of mass: 
$$\dot{m}_{\rm in} - \dot{m}_{\rm out} = \frac{dm_{\rm CV}}{dt}$$

- where  $m_{in}$  and  $m_{out}$  are the total rates of mass flow into and out of the control volume, respectively, and  $dm_{CV}/dt$  is the rate of change of mass within the control volume boundaries.
- In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the *continuity equation*.

#### **Conservation of Mass Principle**

• The conservation of mass principle for a control volume can be expressed as: The net mass transfer to or from a control volume during a time interval t is equal to the net change (increase or decrease) in the total mass within the control volume during t. That is,

 $\begin{pmatrix} \text{Total mass entering} \\ \text{the CV during } \Delta t \end{pmatrix} - \begin{pmatrix} \text{Total mass leaving} \\ \text{the CV during } \Delta t \end{pmatrix} = \begin{pmatrix} \text{Net change in mass} \\ \text{within the CV during } \Delta t \end{pmatrix}$  $m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \quad (\text{kg}) \quad (5-10)$ 

or

• where  $\Delta m_{CV=} m_{final} - m_{initial}$  is the change in the mass of the control volume during the process. It can also be expressed in *rate form* as

$$\dot{m}_{\rm in} - \dot{m}_{\rm out} = dm_{\rm CV}/dt$$
 (kg/s)

The Equations above are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

#### **Mass Balance for Steady-Flow Processes**

• During a steady-flow process, the total amount of mass contained within a control volume does not change with time ( $m_{\rm CV}$  = constant). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

Steady flow: 
$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$
 (kg/s)

When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate*  $\dot{m}$ 

It states that the total rate of mass entering a control volume is equal to the total rate of mass leaving it

- Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).
- For these cases, we denote the inlet state by the subscript 1 and the outlet state by the subscript 2, and drop the summation signs

Steady flow (single stream):  $\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ 

#### **EXAMPLE 2–1**: Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 5–12). If it takes 50 s to fill the bucket with water, determine (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.

**Assumptions 1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

**Properties** We take the density of water to be 1000 kg/m3 1 kg/L.

**Analysis** (a) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}} \left( \frac{3.7854 \text{ L}}{1 \text{ gal}} \right) = 0.757 \text{ L/s}$$
  
 $\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = 0.757 \text{ kg/s}$ 

(b) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi (0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2} \left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = 15.1 \text{ m/s}$$

### **MECHANICAL ENERGY**

- Many fluid systems are designed to transport a fluid from one location to another at a specified flow rate, velocity, and elevation difference, and the system may generate mechanical work in a turbine or it may consume mechanical work in a pump or fan during this process.
- These systems do not involve the conversion of nuclear, chemical, or thermal energy to mechanical energy. Also, they do not involve any heat transfer in any significant amount, and they operate essentially at constant temperature.
- Such systems can be analyzed conveniently by considering the *mechanical forms of energy* only and the frictional effects that cause the mechanical energy to be lost (i.e., to be converted to thermal energy that usually cannot be used for any useful purpose).

The **mechanical energy** can be defined as *the form* of energy that can be converted to mechanical work completely and directly by an ideal mechanical device. Kinetic and potential energies are the familiar forms of mechanical energy. Therefore, the mechanical energy of a flowing fluid can be expressed on a unit-mass basis as



In the absence of any changes in flow velocity and elevation, the power produced by an ideal hydraulic turbine is proportional to the pressure drop of water across the turbine. Most processes encountered in practice involve only certain forms of energy, and in such cases it is more convenient to work with the simplified versions of the energy balance. For systems that involve only *mechanical forms of energy* and its transfer as *shaft work*, the conservation of energy principle can be expressed conveniently as

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}}$$

where *E*mech, loss represents the conversion of mechanical energy to thermal energy due to irreversibilities such as friction. For a system in steady operation, the mechanical energy balance becomes

$$E_{\text{mech, in}} = E_{\text{mech, out}} + E_{\text{mech, loss}}$$