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Lecture- 10
Experimental Modeling (Part A)

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Learning Objectives

- After completing this Lecture, you should be able to:
- apply the Buckingham pi theorem.
- develop a set of dimensionless variables for a given flow situation.

Outline

- Dimensional Analysis
- Buckingham Pi Theorem
- Dimensionless Groups

Experimental Methods: Overview

- Many flow problems can only be investigated experimentally
- Few problems in fluids can be solved by analysis alone.
- One must know how to plan experiments.
- Correlate other experiments to a specific problem.
- Usually, the goal is to make the experiment widely applicable.
- Similitude is used to make experiments more applicable.
- Laboratory flows are studied under carefully controlled conditions.

Experimental Methods: Dimensional Analysis

Pipe-Flow Example: Pressure Drop per Unit Length

The pressure drop per unit length that develops along a pipe as the result of friction can not be explained analytically without the use of experimental data.

First, we determine the important variables in the flow related to pressure drop:

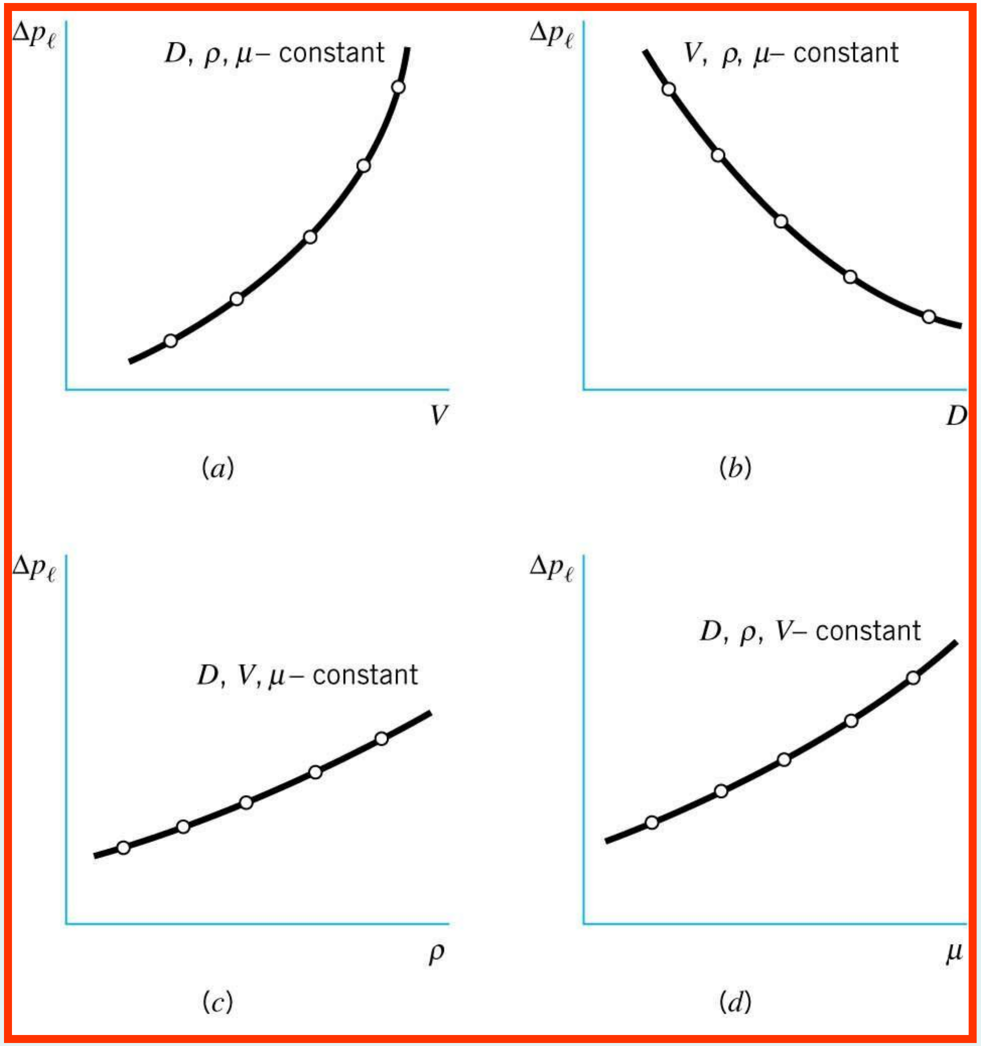
$$\Delta p_\ell = f(D, \rho, \mu, V)$$

D is the diameter of the pipe, ρ is the density of the fluid, μ is the viscosity of the fluid, and V is the flow velocity.

So, how do we approach this problem?

Logically, it seems that we could vary one variable at a time holding the other constants (see the next page).

Experimental Methods: Dimensional Analysis



So, now we have done five experiments for each plot with the other variables held constant (20 total experiments).

What have we gained?

Our analysis is very narrow and specific, not widely applicable.

Now what if do 10 points for each variable, and let the other three variable vary for 10 values.

Total combinations 10x10x10x10:

10,000 experiments!

More applicable, but very expensive,
At \$50/experiment =\$500,000

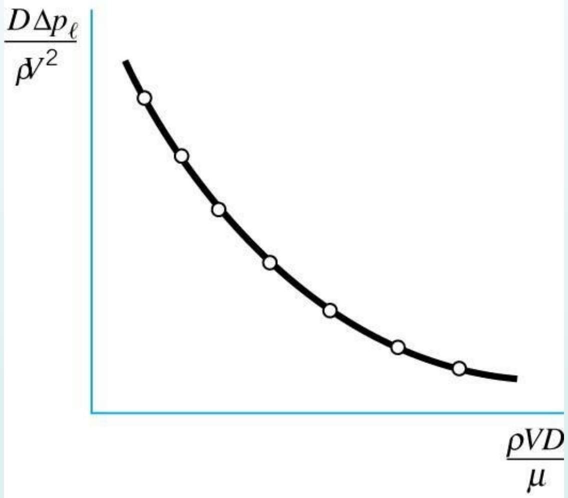
Experimental Methods: Dimensional Analysis

Fortunately, there is a simpler approach: Dimensionless Groups

The original list of variables can be collected into two dimensionless groups.

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Now instead of working with 5 variables, there are only two.



The experiments would consist of varying the independent variable and determining the dependent variable which is related to the pressure drop.

Now, the curve is universal for any smooth walled, laminar pipe flow.

Experimental Methods: Dimensional Analysis

Dimensions are Mass (M), Length (L), Time (T), Force (F or MLT^{-2})

Then, we check our dimensionless groups

$$V \doteq LT^{-1}$$

$$\mu \doteq FL^{-2}T$$

$$\Delta p_\ell \doteq FL^{-3}$$

$$D \doteq L$$

$$\rho \doteq FL^{-4}T^2$$

Substituting, we see no dimensions on our two variables:

$$\frac{D\,\Delta p_\ell}{\rho V^2} \doteq \frac{L(F/L^3)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

$$\frac{\rho VD}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

* Not only have we reduced the number of variables from five to two, but the dimensionless plot is independent of the system of units used.

So, how do we know what groups of dimensionless variables to form?

Experimental Methods: Buckingham Pi Theorem

Buckingham Pi Theorem is a systematic way of forming dimensionless groups:

> If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k - r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

The dimensionless products are referred to as "pi terms".

Requires that equation have dimensional homogeneity:

$$u_1 = f(u_2, u_3, \ldots, u_k)$$

Dimensions on the left side = dimension on the right side

Then if pi terms are formed, they are dimensionless products one each side.

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

*The required number of pi terms is fewer than the original number of variables by r, where r is the minimum number of reference dimensions needed to describe the original set of variables (M, L, T, or F).

Experimental Methods: Buckingham Pi Theorem

Systematic Approach: Example Pipe Flow

Step 1. List all the variables that are involved in the problem:

$$\Delta p_\ell = f(D, \rho, \mu, V)$$

Step 2. Express each of the variables in terms of basic dimensions:

$$\Delta p_\ell \doteq FL^{-3}$$

$$D \doteq L$$

$$\rho \doteq FL^{-4}T^2$$

$$\mu \doteq FL^{-2}T$$

$$V \doteq LT^{-1}$$

Step 3. Determine the require number of pi terms:

The basic dimensions are F,L,T or M,L,T, noting $F = MLT^{-2}$, 3 total

Then the number of pi terms are the number of variables, 5 minus the number of basic dimensions, 3. So there should be two pi terms for this case.

Experimental Methods: Buckingham Pi Theorem

Step 4. Select a number of repeating variables, where the number required is equal to the number of reference dimensions.

We choose three independent variables as the repeating variables—there can be more than one set of repeating variables.

Repeating variables: D, V, and ρ

We note the these three variables by themselves are dimensionally independent; you can not form a dimensionless group with them alone.

Step 5. Form a pi term by multiplying one of the nonrepeating variables by the product of repeating variables each raised to an exponent that will make the combination dimensionless. The first group chosen usually includes the dependent variable.

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c$$

Product should be dimensionless: $(FL^{-3})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$

So, we need to solve for the exponent values.

Experimental Methods: Buckingham Pi Theorem

Step 5 (continued).

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-3 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

Solving the set of algebraic equations, we obtain: a = 1, b = -2, c = -1:

$$\Rightarrow \quad \Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

μ is a remaining nonrepeating variable, so we can form another group:

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

Solving, a = -1, b = -1, and c = -1

$$\Rightarrow \quad \Pi_2 = \frac{\mu}{DV\rho}$$

Experimental Methods: Buckingham Pi Theorem

Step 6. Repeat Step 5. for each of the remaining repeating variables.

We could have chosen D, V and μ as another repeating group (later).

Step 7. Check all the resulting pi terms to make sure they are dimensionless.

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \doteq \frac{(FL^{-3})(L)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

$$\Pi_2 = \frac{\mu}{DV\rho} \doteq \frac{(FL^{-2}T)}{(L)(LT^{-1})(FL^{-4}T^2)} \doteq F^0L^0T^0$$

Step 8. Express the final form as relationship among the pi terms and think about what it means.

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

For our case, $\dfrac{\Delta p_\ell D}{\rho V^2} = \tilde{\phi}\left(\dfrac{\mu}{DV\rho}\right)$ or $\dfrac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\dfrac{\rho VD}{\mu}\right)$

Pressure drop depends on the Reynolds Number.

Reynolds Number

Experimental Methods: Choosing Variables

One of the most important aspects of dimensional analysis is choosing the variables important to the flow, however, this can also prove difficult.

We do not want to choose so many variables that the problem becomes cumbersome.

Often we use engineering simplifications, to obtain first order results sacrificing some accuracy, but making the study more tangible.

Most variables fall in to the categories of geometry, material property, and external effects:

Geometry: lengths and angles, usually very important and obvious variables.

Material Properties: bind the relationship between external effects and the fluid response. Viscosity, and density of the fluid.

External Effects: Denotes a variable that produces a change in the system, pressures, velocity, or gravity.

Experimental Methods: Choosing Variables

We must choose the variables such that they are independent:

$$f(p, q, r, \ldots, u, v, w, \ldots) = 0$$

$$q = f_1(u, v, w, \ldots)$$

Then, u, v, and w are not necessary in f if they only enter the problem through q, or q is not necessary in f.

Experimental Methods: Choosing Variables

In summary, the following points should be considered in the selection of variables:

1. Clearly define the problem. What is the main variable of interest (the dependent variable)?
2. Consider the basic laws that govern the phenomenon. Even a crude theory that describes the essential aspects of the system may be helpful.
3. Start the variable selection process by grouping the variables into three broad classes: geometry, material properties, and external effects.
4. Consider other variables that may not fall into one of the above categories. For example, time will be an important variable if any of the variables are time dependent.
5. Be sure to include all quantities that enter the problem even though some of them may be held constant (e.g., the acceleration of gravity, g). For a dimensional analysis it is the dimensions of the quantities that are important—not specific values!
6. Make sure that all variables are independent. Look for relationships among subsets of the variables.

Experimental Methods: Uniqueness of Pi Terms

Now, back to our example of pressure drop, but choose a different repeating group (D, V, μ).

If we evaluate, we find $\frac{\Delta p_\ell D^2}{V\mu}$ The other pi term remains the same.

$$\frac{\Delta p_\ell D^2}{V\mu} = \phi_1\left(\frac{\rho V D}{\mu}\right)$$

But, we note that the L.H.S, is simply what we had before multiplied by the Reynolds Number.

$$\left(\frac{\Delta p_\ell D}{\rho V^2}\right)\left(\frac{\rho V D}{\mu}\right) = \frac{\Delta p_\ell D^2}{V\mu}$$

There is not a unique set of pi terms, but rather a set number of pi terms. In this case there are always two.

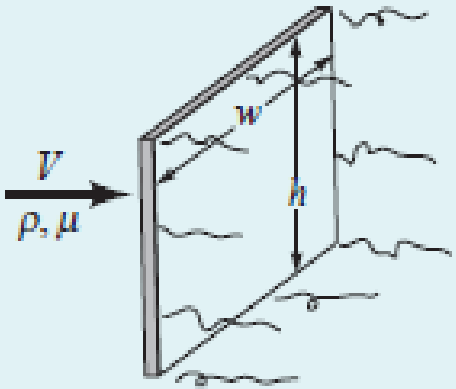
If we three pi terms, we can form another by multiplying $\Pi_1 = \phi(\Pi_2, \Pi_3)$

$$\Pi_2' = \Pi_2^a\,\Pi_3^b \Longrightarrow \Pi_1 = \phi_1(\Pi_2', \Pi_3) \text{ or } \Pi_1 = \phi_2(\Pi_2, \Pi_2')$$

*Often the set of pi terms chosen is based on previous flow analysis.

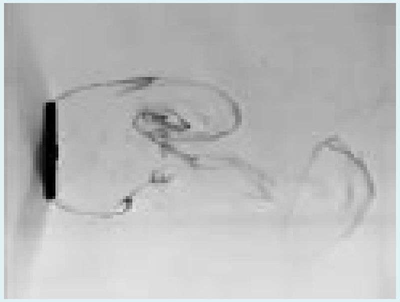
EXAMPLE 7.1 Method of Repeating Variables

GIVEN A thin rectangular plate having a width w and a height h is located so that it is normal to a moving stream of fluid as shown in Fig. E7.1. Assume the drag, $\mathfrak{D}$, that the fluid exerts on the plate is a function of w and h, the fluid viscosity and density, μ and ρ, respectively, and the velocity V of the fluid approaching the plate.



■ FIGURE E7.1

V7.2 Flow past a flat plate



FIND Determine a suitable set of pi terms to study this problem experimentally.