Fluid Mechanics: Fundamentals of Fluid Mechanics, 7th Edition,
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Lecture- 11
Experimental Modeling (Part B)

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Learning Objectives

- After completing this Lecture, you should be able to:
- discuss the use of dimensionless variables in data analysis.
- apply the concepts of modeling and similitude to develop prediction equations.

Outline

- Dimensionless Groups
- Experimental Data
- Experimental Models
- Examples

Experimental Methods: Dimensionless Groups

A useful physical interpretation can often be given to dimensionless groups:

Some Common Variables and Dimensionless Groups in Fluid Mechanics

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid in important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

[a]The Cauchy number and the Mach number are related and either can be used as an index of the relative effects of inertia and compressibility. See accompanying discussion.

Experimental Methods: Dimensionless Groups

Osborne Reynolds
(1842 – 1912)

Reynolds Number: $\mathrm{Re} = \frac{\rho V \ell}{\mu}$

"Re-number":

Osborne Reynolds, a British Engineer demonstrated that the Reynolds Number could be used as a criterion to distinguish laminar and turbulent flow.

Re << 1, Viscous forces dominate, we neglect inertial effects, creeping flows.

Re large, inertial effects dominate and we neglect viscosity (not turbulent though).

William Froude
(1810 – 1879)

Froude Number: $\mathrm{Fr} = \frac{V}{\sqrt{g\ell}}$

William Froude, a British civil engineer, mathematician, and naval architect who pioneered the use of towing tanks to study ship design.

The Froude number is the only dimensionless group that contains acceleration of gravity, thus indicating the weight of the fluid is important in these flows.

Important to flows that include waves around ships, flows through river or open conduits.

Experimental Methods: Dimensionless Groups

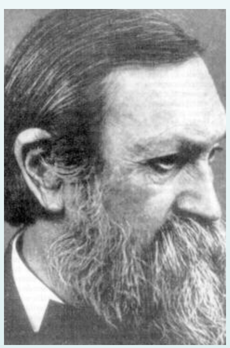
Leonhard Euler
(1707 – 1783)

Euler Number: $$\mathrm{Eu} = \frac{p}{\rho V^2}$$

Leonhard Euler was a Swiss mathematician who pioneered the work between pressure and flow.

Ratio of pressure forces to inertial forces. Sometime called the pressure coefficient.

Euler number is used in flows where pressure differences may play a crucial role.



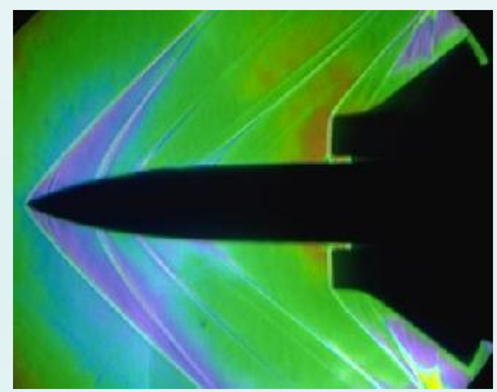
Ernst Mach
(1838 – 1916)

Mach Number: $$\mathrm{Ma} = \frac{V}{c}$$

c is the speed of sound

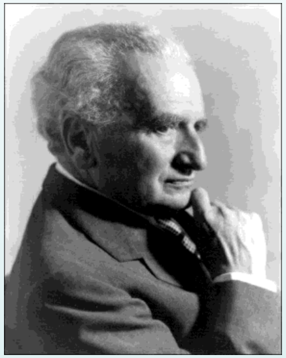
Ernst Mach as Austrian physicist and a philosopher.

The number is important in flows in which there is compressibility.



Experimental Methods: Dimensionless Groups

Vincenz Strouhal
(1850 – 1922)



Theodor von Karman
(1881 – 1963)

Strouhal Number $\mathrm{St} = \frac{\omega \ell}{V}$

Vincenz Strouhal studied "singing wires" which result from vortex shedding.

This dimensionless group is important in unsteady, oscillating flow problems with some frequency of oscillation ω.

Measure of unsteady inertial forces to steady inertial forces.

In certain Reynolds number ranges, a periodic flow will develop downstream from a cylinder placed in a moving fluid due to a regular pattern of vortices that are shed from the body.

This series of trailing vortices are known as Karman vortex trail named after Theodor von Karman, a famous fluid mechanician.

The oscillating flow is created a a discrete frequency such that Strouhaul numbers can closely be correlated to Reynolds numbers.

"Vortex Shedding":

Experimental Methods: Dimensionless Groups

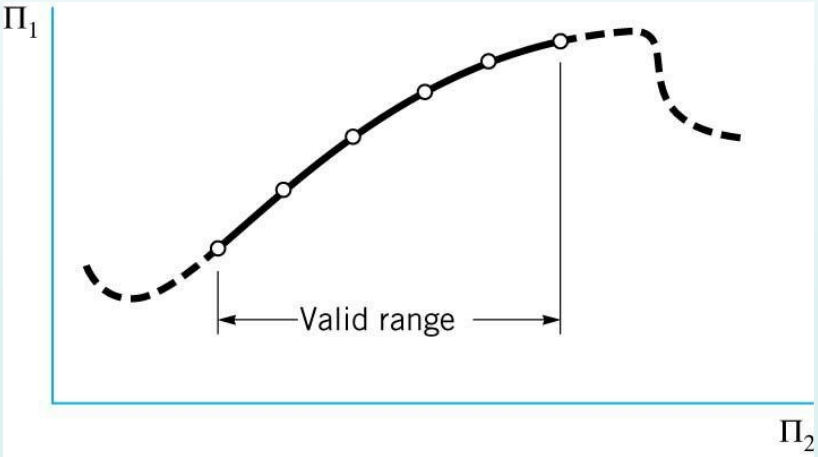
If only one pi variable exists in a fluid phenomenon, the functional relationship must be a constant.

$$\Pi_1 = C$$

The constant must be determine from experiment.

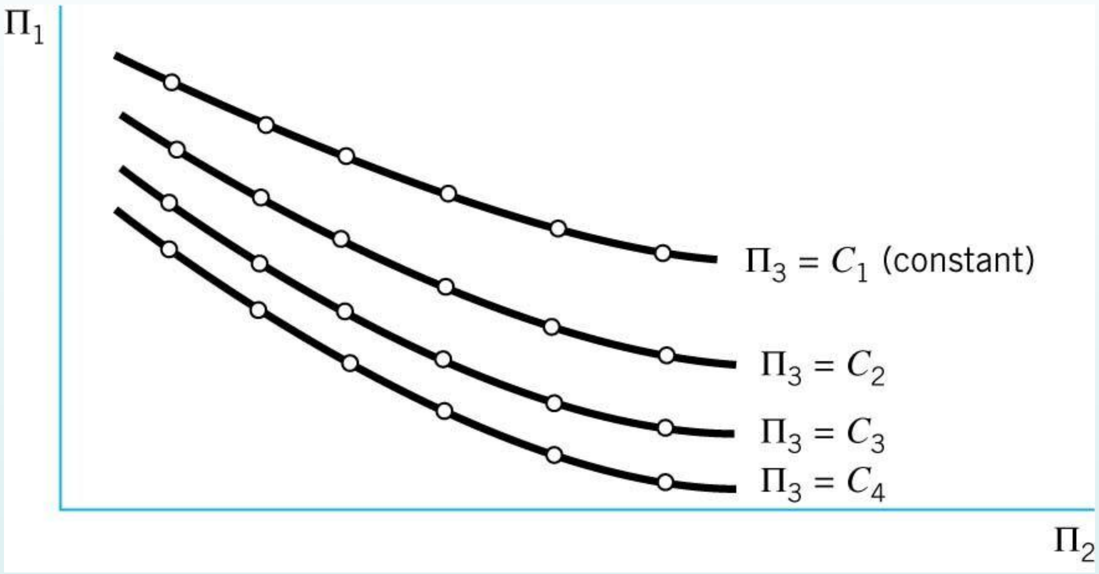
If we have two pi terms, we must be careful not to over extend the range of applicability, but the relationship can be presented pretty easily graphically:

$$\Pi_1 = \phi(\Pi_2)$$



Experimental Methods: Dimensionless Groups

If we have three pi groups, we can represent the data as a series of curves, however, as the number of pi terms increase the problem becomes less tractable, and we may resort to modeling specific characteristics.



Experimental Methods: Similitude

Often we want to use models to predict real flow phenomenon.

"Model":

We obtain similarity between a model and a prototype by equating pi terms. In these terms we must have geometric, kinematic, and dynamic similarity.

Geometric similarity: A model and a prototype are geometrically similar if and only if all body dimensions in all three coordinates have the same linear scale ratio.

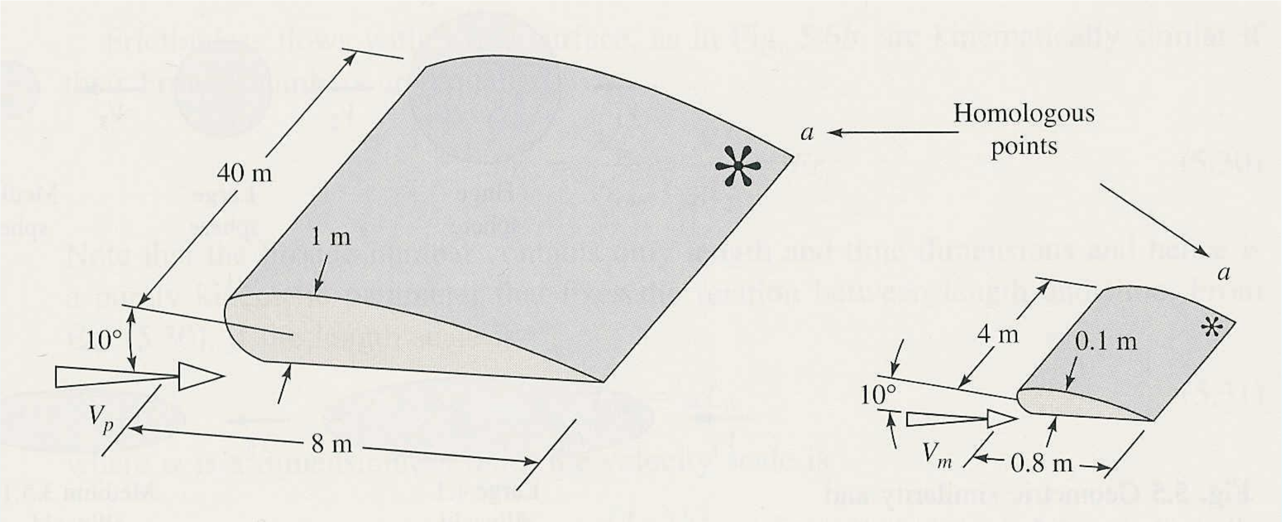
$$\frac{\ell_1}{\ell_2} = \frac{\ell_{1m}}{\ell_{2m}} \Longrightarrow \frac{\ell_{1m}}{\ell_1} = \frac{\ell_{2m}}{\ell_2}$$

All angles are preserved.
All flow directions are the same.
Orientations must be the same.

*Things that must be considered that are over-looked: roughness, scale of fasteners protruding.

Experimental Methods: Similitude

Geometric Similarity: Scale 1/10th



Experimental Methods: Similitude

Kinematic Similarity: Same length scale ratio and same time-scale ratio. The motion of the system is kinematically similar if homolgous particles lie at homolgous locations at homologous times.

This requires equivalence of dimensionless groups:

Reynolds Number, Froude Number, Mach numbers, etc.

For a flow in which Froude Number and Reynolds Number is important:

<u>Length scale:</u>

Froude Number similarity: $\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}} \Rightarrow \frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\lambda_\ell}$

Reynolds Number similarity: $\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu} \Rightarrow \frac{V_m}{V} = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{\ell}{\ell_m}$

Then, $\frac{\mu_m/\rho_m}{\mu/\rho} = \frac{\nu_m}{\nu} = (\lambda_\ell)^{3/2}$ Might relax condition.

<u>Time scale:</u> $\frac{t_m}{t} = \frac{l_m/v_m}{l/v} = \sqrt{\lambda_l}$

Experimental Methods: Similitude

Dynamic Similarity: the same length scale, time-scale, and force scale is required.

First, satisfy geometric, and kinematic similarity. Dynamic similarity then exists if the force and pressure coefficient are the same.

In order to ensure that the force and pressure coefficients are the same:

For compressible flow: Re#, Mach#, and specific heat ratio must be matched.

For incompressible flow with no free surface: Re# matching only.

For incompressible flow with a free surface: Re#, Froude#, and possibly Weber number (surface tension effects), and cavitation number must be matched.

7.10 The excess pressure inside a bubble (discussed in Chapter 1) is known to be dependent on bubble radius and surface tension. After finding the pi terms, determine the variation in excess pressure if we (a) double the radius and (b) double the surface tension.

Given $\Delta p = f(R, \sigma)$, where $\Delta p \doteq \frac{F}{L^2} = \frac{M}{LT^2}$, $R \doteq L$, and $\sigma \doteq \frac{F}{L} = \frac{M}{T^2}$

Consider the (MLT) units so that
$k - r = 3 - 3 = 0$ since there 3 variables and 3 dimensions.
According to this, there should be $k-r=0$ pi terms!?
However, if we consider the (FLT) units we see that it takes only F and L, T is not needed, so that $r = 2$.
Hence, $k - r = 3 - 2 = 1$, so only 1 pi term is needed.

That is, $\Pi_1 = \text{constant}$
To determine Π_1, consider

$$\Pi_1 = \Delta p\, R^a \sigma^b \quad \text{or}$$

$$\Delta p\, R^a \sigma^b \doteq \frac{F}{L^2} L^a \left(\frac{F}{L}\right)^b = F^{1+b} \; L^{-2+a-b}$$

Thus:

F: $1 + b = 0$

L: $a - b - 2 = 0$

or $b = -1$ and $a = b + 2 = 1$

Hence $\underline{\underline{\Pi_1 = \frac{\Delta p\, R}{\sigma}}}$ or $\frac{\Delta p\, R}{\sigma} = C$, where $C = \text{constant}$.

or
$$\Delta p = \frac{C\sigma}{R} \qquad (1)$$

(a) If R is doubled, $\underline{\underline{\Delta p \text{ is reduced by half.}}}$ (See Eq. (1))

(b) If σ is doubled, $\underline{\underline{\Delta p \text{ is doubled.}}}$ (See Eq. (1))

7.16 The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2} \qquad D \doteq L \qquad \rho \doteq FL^{-4}T^{2} \qquad \omega \doteq T^{-1} \qquad Q \doteq L^{3}T^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p \, D^a \rho^b \omega^c$$

and

so that $$(FL^{-2})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$1 + b = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-2$, $b=-1$, $c=-2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2 \rho \omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = Q D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$b = 0 \qquad \text{(for } F\text{)}$$
$$3 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$-1 + 2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-3$, $b=0$, $c=-1$, and therefore

$$\Pi_2 = \frac{Q}{D^3 \omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3 \omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\underline{\underline{\frac{\Delta p}{D^2 \rho \omega^2} = \phi\left(\frac{Q}{D^3 \omega}\right)}}$$