Fluid Mechanics: Fundamentals of Fluid Mechanics, 7th Edition,
Bruce R. Munson. Theodore H. Okiishi. Alric P. Rothmayer
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Lecture- 12
Internal Flows
(laminar pipe flow)

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Learning Objectives

- After completing this Lecture, you should be able to:

1. identify and understand various characteristics of the flow in pipes.
2. discuss the main properties of laminar pipe flow.
3. calculate losses in straight portions of pipes as well as those in various pipe system components.
4. apply appropriate equations and principles to analyze a variety of pipe flow situations.
5. predict the flow rate in a pipe by use of common flow meters.

Outline

- Overview of Viscous Pipe Flow
- Laminar Pipe Flow
- Dimensional Analysis of Pipe Flow
- Pressure Gradients Effects
- Some Example Problems

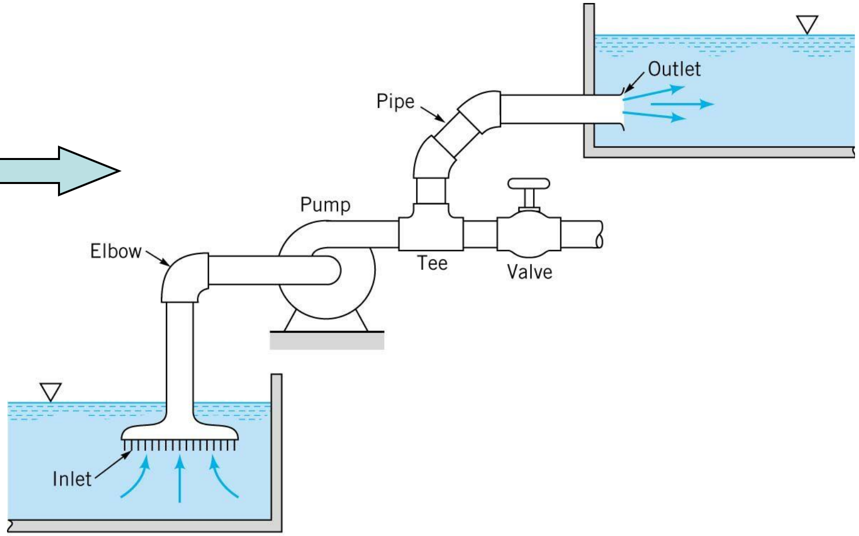
Viscous Pipe Flow: Overview

Pipe Flow is important in daily operations and is described in general as flow in a closed conduit (pipes and ducts). It is also known as an internal flow.

Some common examples are oil and water pipelines, flow in blood vessels, and HVAC ducts.

When real world effects such as viscous effects are considered, it is often difficult to use only theoretical methods. Often theoretical, experimental data, and dimensional analysis is used,

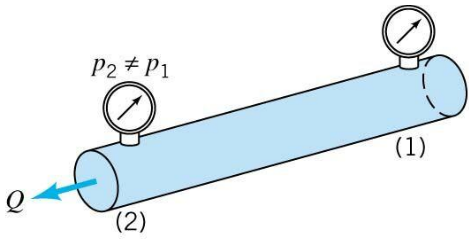
Some common pipe flow components are shown:



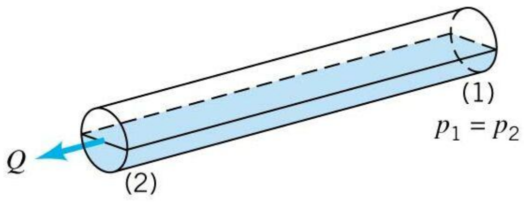
Viscous Pipe Flow: Overview

Pipe flow versus Open-channel flow:

Pipe Flow:



Open-Channel Flow:



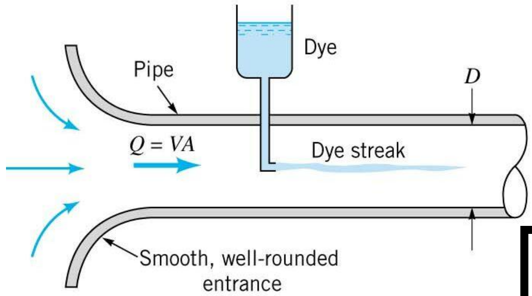
- Pipe is completely filled with fluid
- Pressure Gradients drive the flow
- Gravity can also be important

- Pipe is not full of fluids
- Pressure gradient is constant
- Gravity is the driving force

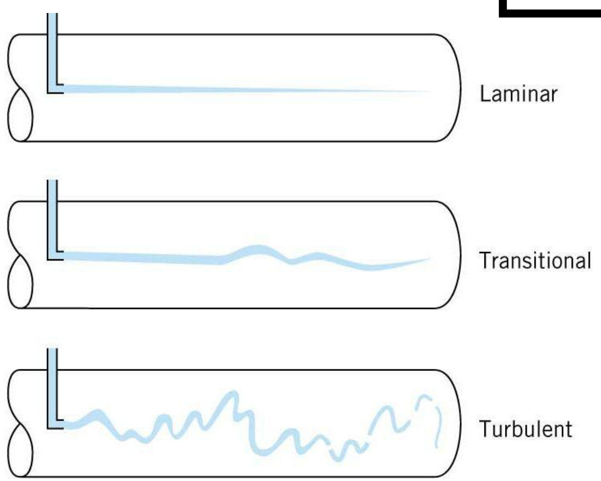
i.e., flow down a concrete spill way.

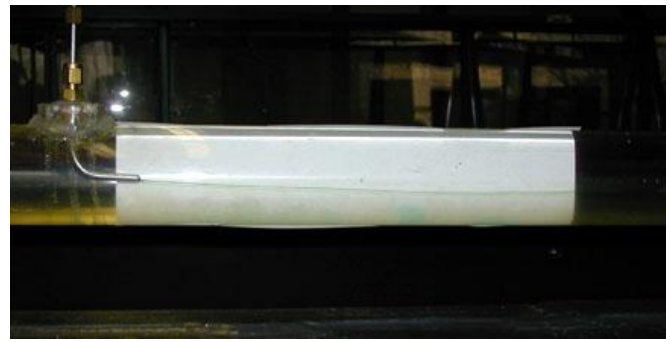
Viscous Pipe Flow: Flow Regime

Osborne Reynolds Experiment to show the three regimes Laminar, Transitional, or Turbulent:

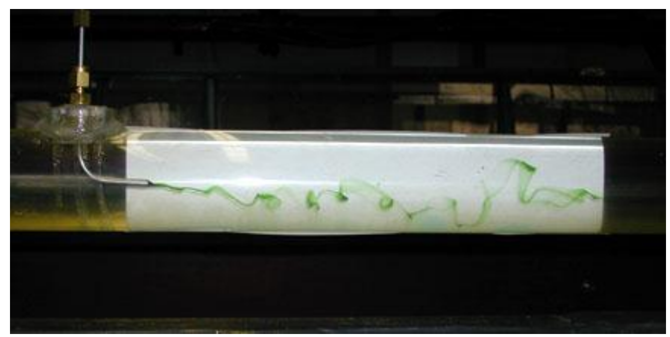


“Experiment”:





Laminar

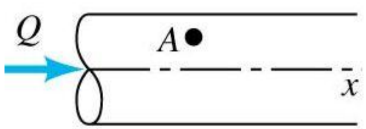


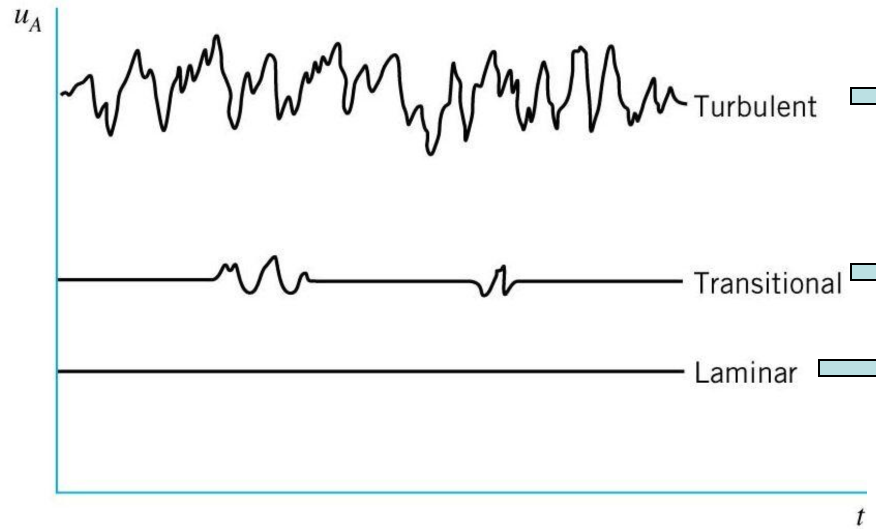
Transitional

Turbulent

Viscous Pipe Flow: Flow Regime

If we measure the velocity at any given point with respect to time in the pipe:





Re > 4000

2100< Re < 4000

Re < 2100

Reynolds Number Dependency:

$$\mathrm{Re} = \rho V D / \mu$$

1. Turbulence is characterized by random fluctuations.
2. Transitional flows are relatively steady accompanied by occasional burst.
3. Laminar flow is relatively steady.

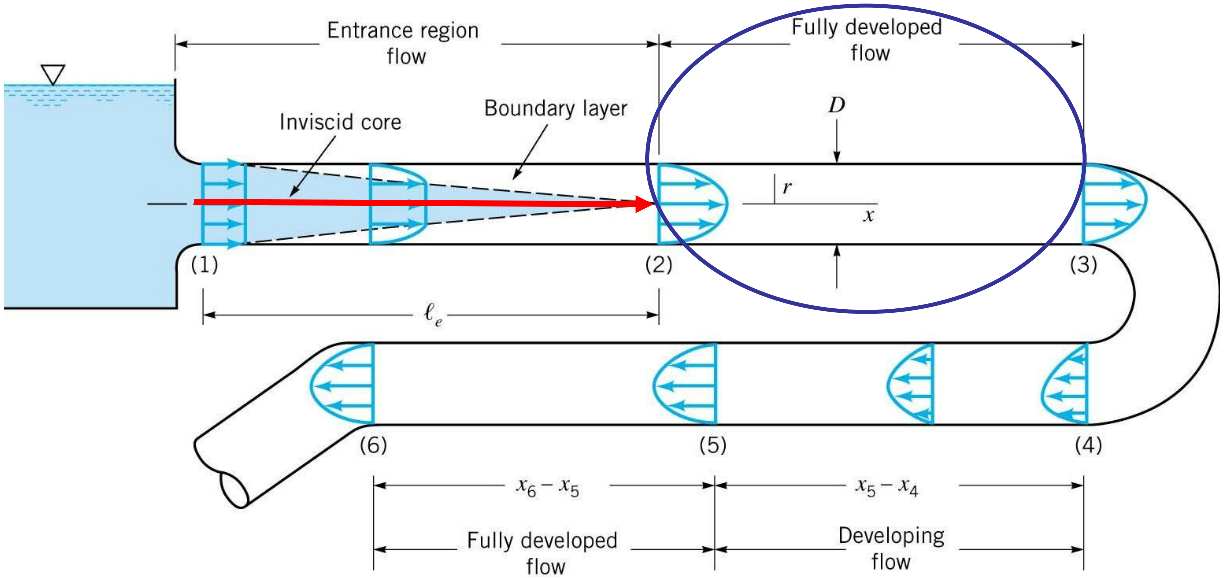
For laminar flow there is only flow direction: $\mathbf{V} = u\hat{\mathbf{i}}$

For turbulent flow, there is a predominate flow direction, but there are random components normal to the flow direction: $\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$

Viscous Pipe Flow: Entrance and Fully Developed

The entrance region in a pipe flow is quite complex (1) to (2):

The fluid enters the pipe with nearly uniform flow.
The viscous effects create a boundary layer that merges.
When they merge the flow is fully developed.



There are estimates for determining the entrance length for pipe flows:

$$\frac{\ell_e}{D} = 0.06\ \mathrm{Re} \text{ for laminar flow}$$ and $$\frac{\ell_e}{D} = 4.4\ (\mathrm{Re})^{1/6} \text{ for turbulent flow}$$

Viscous Pipe Flow: Entrance and Fully Developed

For very low Reynolds numbers (Re = 10), the entrance length is short: $\ell_e = 0.6D$

For large Reynolds number flow the entrance length can be several pipe diameters: $\ell_e = 120D \text{ for Re} = 2000$

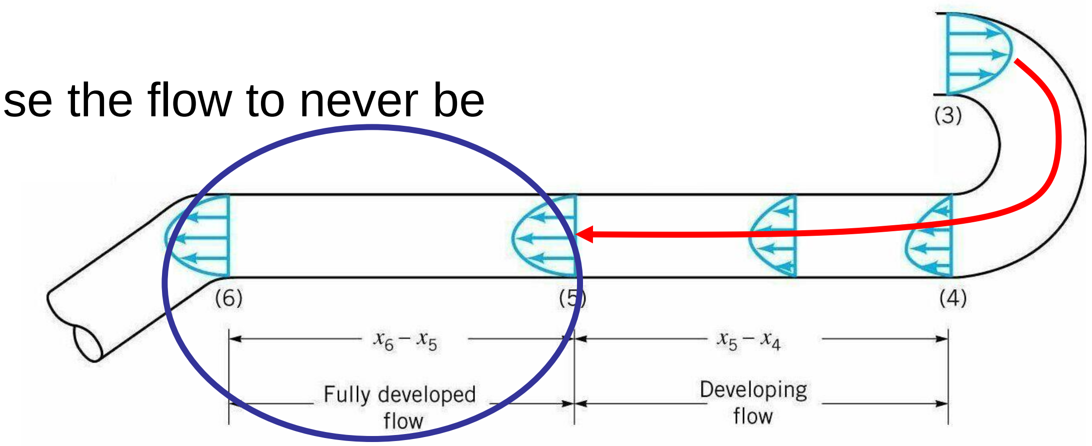
For many practical engineering problems: $10^4 < \text{Re} < 10^5 \text{ so that } 20D < \ell_e < 30D$

Bends and T's affect Fully Developed Flow:

Pipe is fully developed until the character of the pipe changes.

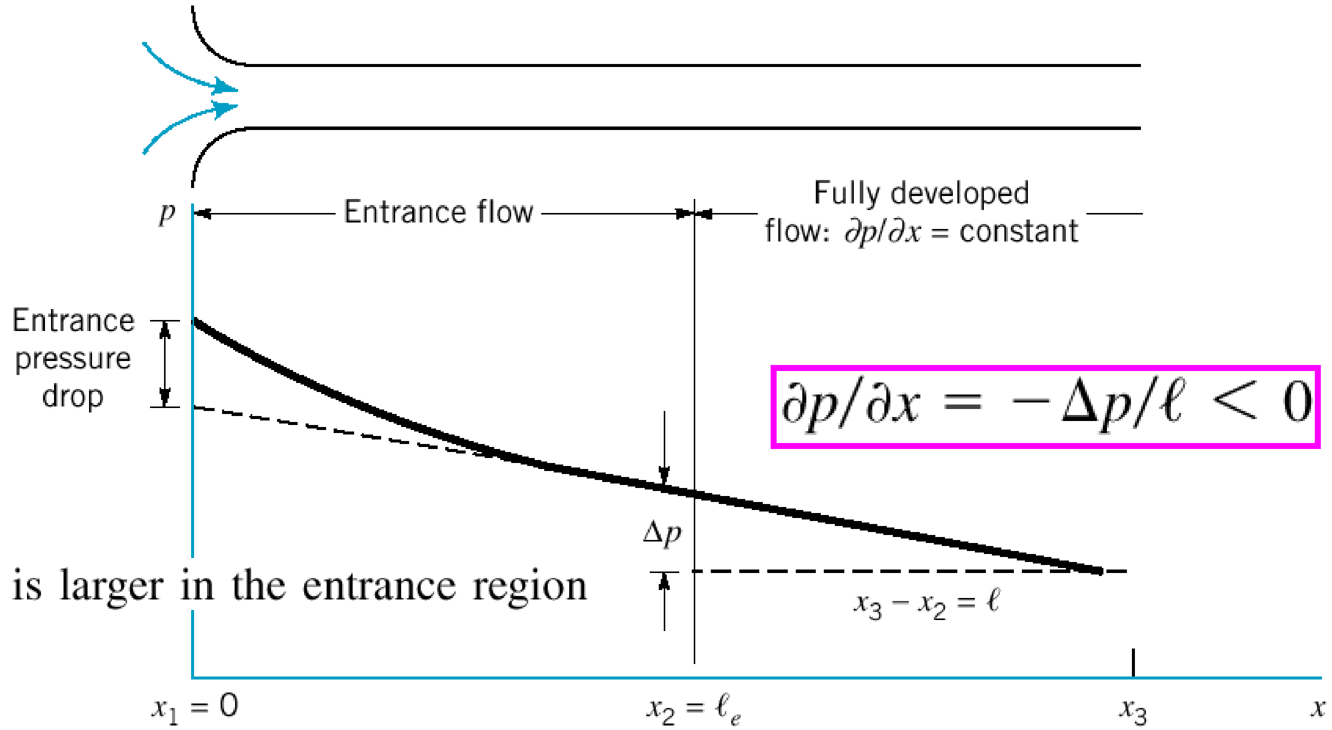
It changes in the bend and becomes fully developed again after some length after the bend.

Many disruptions can cause the flow to never be fully developed.



In many flows, the fully developed region is greater than the developing region.

Viscous Pipe Flow: Pressure and Shear Stress



$$\boxed{\partial p/\partial x = -\Delta p/\ell < 0}$$

$\partial p/\partial x$, is larger in the entrance region

The shear stress in laminar flow is a direct result of momentum transfer along the randomly moving molecules (microscopic).

The shear stress in turbulent flow is due to momentum transfer among the randomly moving, finite-sized bundles of fluid particles (macroscopic).

The physical properties of shear stress are quite different between the two.

Fully Developed Laminar Flow: Overview

Both turbulent and laminar flows become fully developed in long enough straight pipes. However, the details of the two flows are quite different.

Some important quantities that we calculate: velocity profiles, pressure drop, head loss, and flow rate.

Although most flows are turbulent rather than laminar, and many pipes are not long enough to allow the attainment of fully developed flow, a fully understanding of fully developed laminar flow is important.

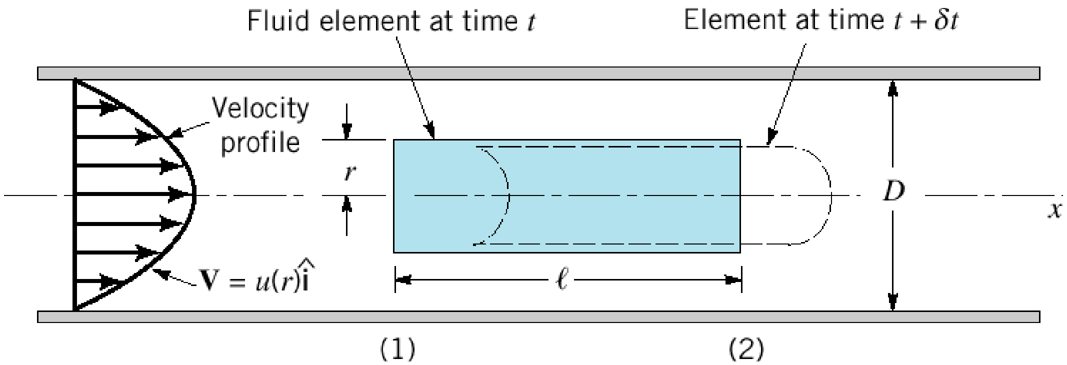
This study is the basis for more complex analysis, and there are some cases where these assumption are good.

The equations or a description can be obtained in three different ways (1) Momentum applied to a fluid element, (2) Navier-Stokes equations, and (3) dimensional analysis methods.

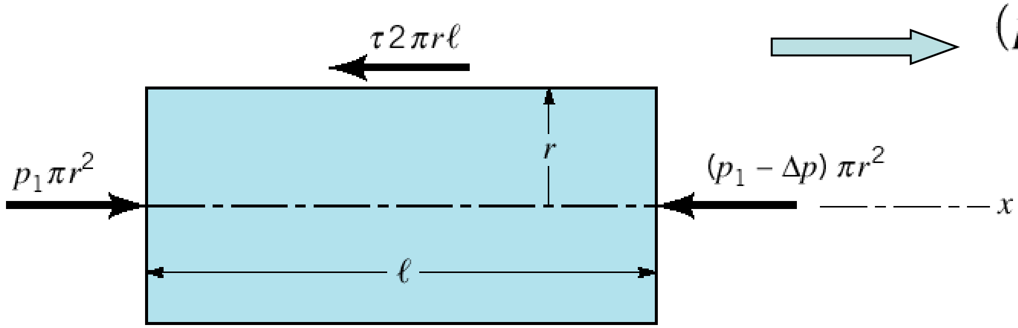
Fully Developed Laminar Flow: Fluid Element Method

Basic Pipe flow is governed by a balance of viscous and pressure forces.

Consider and cylindrical fluid element within a pipe:



Free-Body Diagram:



$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

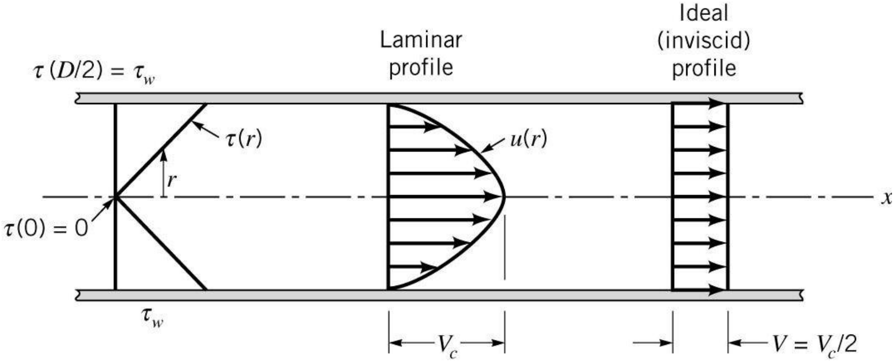
Fully Developed Laminar Flow: Fluid Element Method

Now since neither the pressure gradient nor the length depend on r, the R.H.S. must also be independent of r.

Then, $\tau = Cr$. Then at r = 0, τ =0, and at r = D/2, τ is the wall shear stress.

Now, $\tau = \dfrac{2\tau_w r}{D}$

The shear profile is linear.



$$\Rightarrow \quad \Delta p = \frac{4\ell\tau_w}{D}$$

A small shear stress can produce a large pressure difference if the pipe is relatively long.

The shear stress for laminar Newtonian Flow: $\tau = -\mu \dfrac{du}{dr}$

$\tau > 0$

$du/dr < 0$ Velocity decreases from the center-line.

Fully Developed Laminar Flow: Fluid Element Method

Now, recall $$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

Substitute, the shear stress definition, and rearrange: $$\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell}\right) r$$

Integrate, $$\int du = -\frac{\Delta p}{2\mu\ell} \int r\, dr$$

$$u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

Apply the boundary conditions, no-slip, u =0 at r = D/2, and solve for C_1:

$$C_1 = (\Delta p / 16\mu\ell) D^2$$

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

V_c = centerline velocity

Also, we can write in terms of shear stress: $$\boxed{u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]}$$

Fully Developed Laminar Flow: Fluid Element Method

Find the Volumetric Flow Rate:

$$Q = \int u \, dA = \int_{r=0}^{r=R} u(r) 2\pi r \, dr = 2\pi \, V_c \int_0^R \left[1 - \left(\frac{r}{R} \right)^2 \right] r \, dr$$

$$Q = \frac{\pi R^2 V_c}{2}$$

$$V = Q/A = Q/\pi R^2$$ The average velocity is V.

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32 \mu \ell}$$ The average velocity is $1/2V_c$

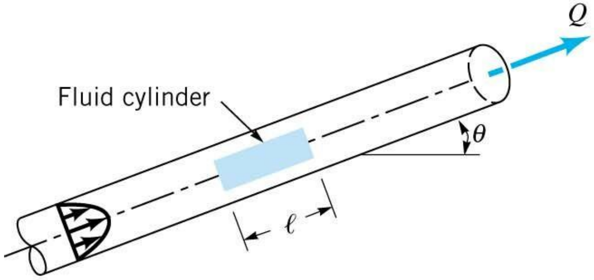
or, $$\boxed{Q = \frac{\pi D^4 \, \Delta p}{128 \mu \ell}}$$ Hagen-Poiseuille Flow

Fully Developed Laminar Flow: Fluid Element Method

Some general remarks:

1. The flowrate is directly proportional to the pressure drop.
2. The flowrate is inversely proportional to the viscosity.
3. The flowrate is inversely proportional to the pipe length.
4. The flowrate is directly proportional to the pipe diameter to the 4th power.

We could adjust the equations for non-horizontal pipes:



$$\frac{\Delta p - \gamma \ell \sin\theta}{\ell} = \frac{2\tau}{r}$$

Mean Velocity: $$V = \frac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell}$$

Volumetric Flow: $$Q = \frac{\pi(\Delta p - \gamma \ell \sin\theta) D^4}{128 \mu \ell}$$

Lastly, we could develop these flows from Navier-Stokes as in Lecture 8.

Fully Developed Laminar Flow: Dimensional Analysis

Important Variables: $\Delta p = F(V, \ell, D, \mu)$

Density is not important because of "fully developed"

Number of dimensionless groups: $k - r = 5 - 3 = 2$

Two dimensional groups: $\dfrac{D\,\Delta p}{\mu V} = \phi\left(\dfrac{\ell}{D}\right)$

If we state that the pressure drop has to be directly proportional to l:

$$\frac{D\,\Delta p}{\mu V} = \frac{C\ell}{D} \Rightarrow \frac{\Delta p}{\ell} = \frac{C\mu\, V}{D^2}$$

Or, solving for V, and substituting in Q: $Q = AV = \dfrac{(\pi/4C)\,\Delta p D^4}{\mu \ell}$

C in this case must be determined by experimental analysis. It is 32 for circular pipes.

Now, return to $\Delta p = 32\mu\ell V/D^2$

and divide by dynamic pressure $\rho V^2/2$.

$$\Rightarrow \frac{\Delta p}{\frac{1}{2}\,\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\,\rho V^2}$$

Fully Developed Laminar Flow: Dimensional Analysis

Now, simplifying, $$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{\ell}{D}\right) = \frac{64}{\mathrm{Re}}\left(\frac{\ell}{D}\right)$$

Now, rewriting, $$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$ f is the Darcy Friction Factor (dimensionless).

$$f = \Delta p(D/\ell)/(\rho V^2/2) \Longrightarrow f = \frac{64}{\mathrm{Re}} \Longrightarrow f = \frac{8\tau_w}{\rho V^2}$$

EXAMPLE 8.2 Laminar Pipe Flow

GIVEN An oil with a viscosity of $\mu = 0.40 \ \mathrm{N \cdot s/m^2}$ and density $\rho = 900 \ \mathrm{kg/m^3}$ flows in a pipe of diameter $D = 0.020$ m.

FIND **(a)** What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5} \ \mathrm{m^3/s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m?

(b) How steep a hill, θ, must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$?

(c) For the conditions of part (b), if $p_1 = 200$ kPa, what is the pressure at section $x_3 = 5$ m, where x is measured along the pipe?

SOLUTION

(a) If the Reynolds number is less than 2100 the flow is laminar and the equations derived in this section are valid. Since the average velocity is $V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3/\text{s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637\ \text{m/s}$, the Reynolds number is $\text{Re} = \rho VD/\mu = 2.87 < 2100$. Hence, the flow is laminar and from Eq. 8.9 with $\ell = x_2 - x_1 = 10$ m, the pressure drop is

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4}$$

$$= \frac{128(0.40\ \text{N}\cdot\text{s/m}^2)(10.0\ \text{m})(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(0.020\ \text{m})^4}$$

or

$$\Delta p = 20{,}400\ \text{N/m}^2 = 20.4\ \text{kPa} \qquad \textbf{(Ans)}$$

(b) If the pipe is on a hill of angle θ such that $\Delta p = p_1 - p_2 = 0$, Eq. 8.12 gives

$$\sin\theta = -\frac{128\mu Q}{\pi\rho g D^4} \qquad \textbf{(1)}$$

or

$$\sin\theta = \frac{-128(0.40\ \text{N}\cdot\text{s/m}^2)(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.020\ \text{m})^4} \qquad \textbf{(Ans)}$$

Thus, $\theta = -13.34°$.

8.44 Oil ($SG = 0.9$), with a kinematic viscosity of 0.007 ft^2/s, flows in a 3-in.-diameter pipe at 0.01 ft^3/s. Determine the head loss per unit length of this flow.

$h_L = f \frac{l}{D} \frac{V^2}{2g}$ where $l = 1\,ft$
for "per unit length of pipe".

Determine friction factor based on Re & ε/D

$Q = 0.01\ ft^3/s = VA$

$V = \dfrac{0.01}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 0.20\ ft/s$

$Re = \dfrac{VD}{\nu} = \dfrac{0.20(3/12)}{0.007} = 7.14$

Since Re is below 2100, the flow is laminar.

The friction factor can be determined from

$f = 64/Re = 64/7.14 = 8.96$

$h_L = (8.96)\dfrac{1}{(3/12)}\dfrac{(0.2)^2}{2(32.2)} = \underline{\underline{0.022\,ft}}$
per ft of pipe