DEPARTMENT OF MECHANICAL ENGINEERING TECHNIQUES OF POWER

Fluid Mechanics: Fundamentals of Fluid Mechanics, 7th Edition,
Bruce R. Munson. Theodore H. Okiishi. Alric P. Rothmayer
John Wiley & Sons, Inc.l, 2013

Lecture- 13
Internal Flows
(Turbulent Pipe Flow)

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Learning Objectives

- After completing this Lecture, you should be able to:

1. identify and understand various characteristics of the flow in pipes.
2. discuss the main properties of turbulent pipe flow.
3. calculate losses in straight portions of pipes as well as those in various pipe system components.
4. apply appropriate equations and principles to analyze a variety of pipe flow situations.
5. predict the flow rate in a pipe by use of common flow meters.

Outline

- Overview of Viscous Pipe Flow
- Turbulent Pipe Flow
- Some Example Problems

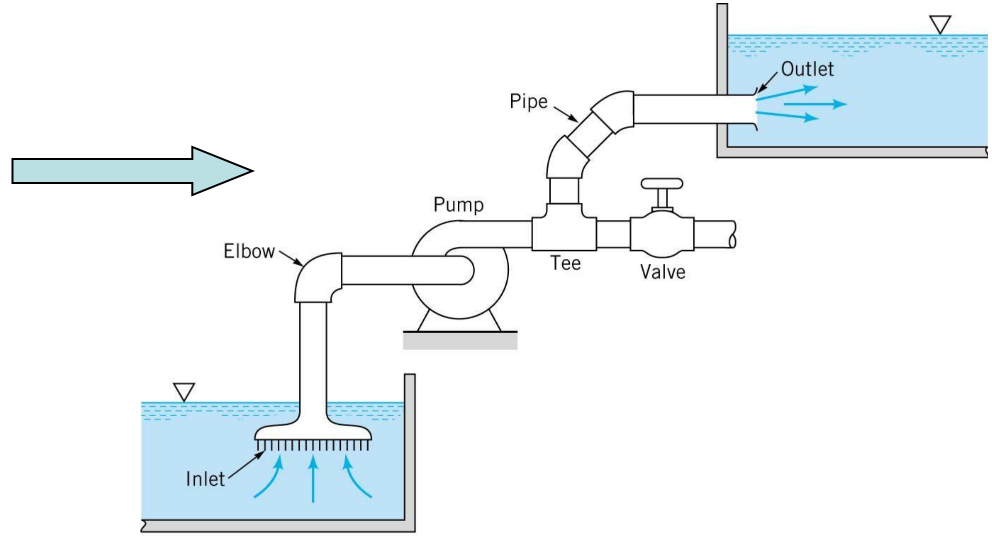
Viscous Pipe Flow: Overview

Pipe Flow is important in daily operations and is described in general as flow in a closed conduit (pipes and ducts). It is also known as an internal flow.

Some common examples are oil and water pipelines, flow in blood vessels, and HVAC ducts.

When real world effects such as viscous effects are considered, it is often difficult to use only theoretical methods. Often theoretical, experimental data, and dimensional analysis is used,

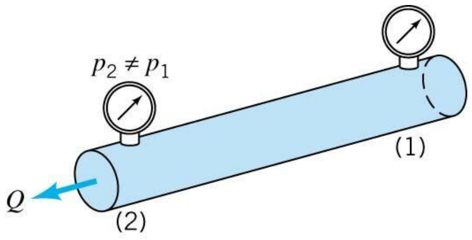
Some common pipe flow components are shown:



Viscous Pipe Flow: Overview

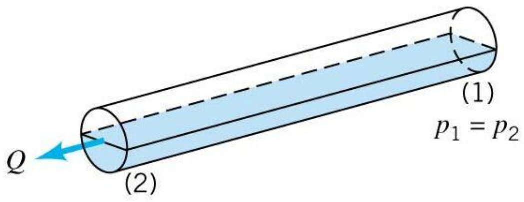
Pipe flow versus Open-channel flow:

Pipe Flow:



•Pipe is completely filled with fluid
•Pressure Gradients drive the flow
•Gravity can also be important

Open-Channel Flow:

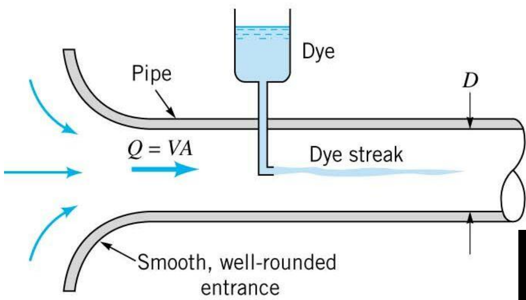


•Pipe is not full of fluids
•Pressure gradient is constant
•Gravity is the driving force

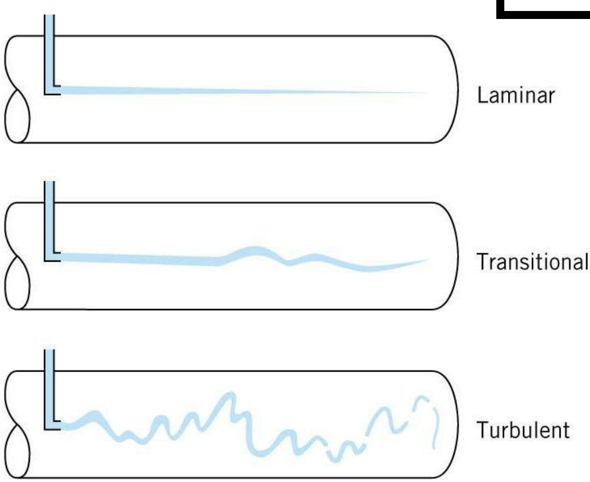
i.e., flow down a concrete spill way.

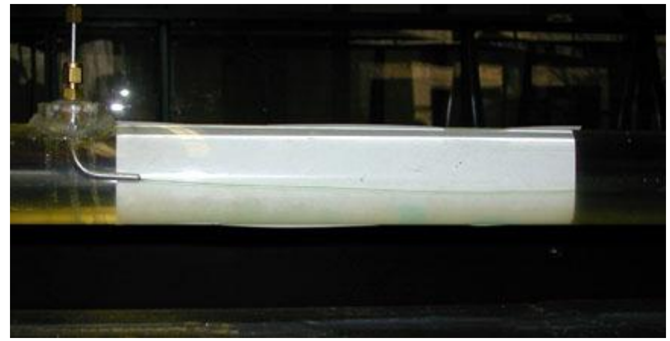
Viscous Pipe Flow: Flow Regime

Osborne Reynolds Experiment to show the three regimes Laminar, Transitional, or Turbulent:

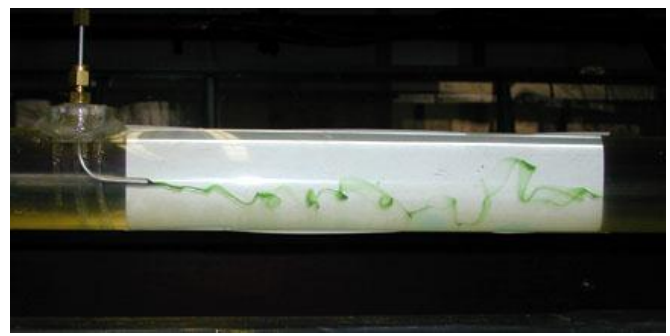


“Experiment”:





Laminar

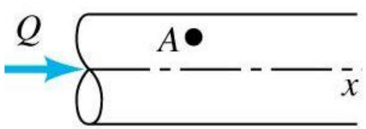


Transitional

Turbulent

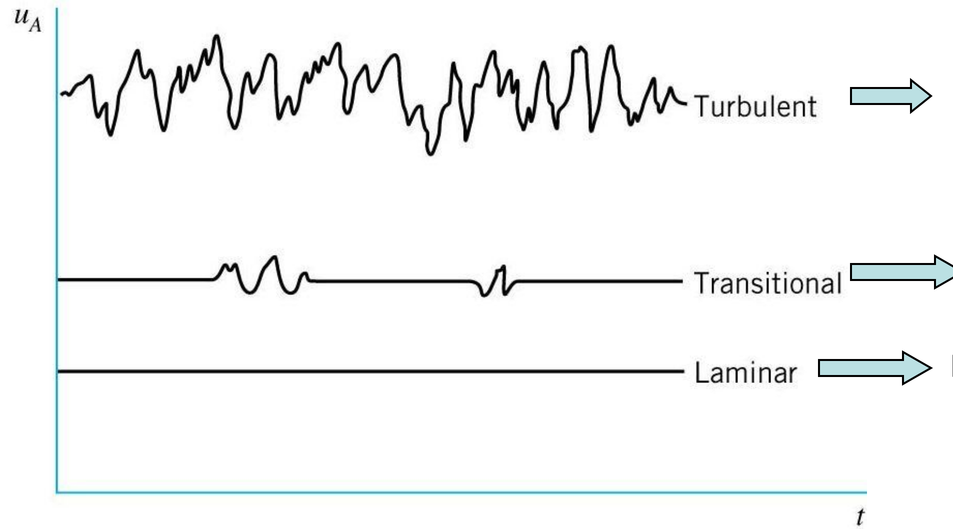
Viscous Pipe Flow: Flow Regime

If we measure the velocity at any given point with respect to time in the pipe:



Reynolds Number Dependency:

$$\mathrm{Re} = \rho V D / \mu$$



Re > 4000

2100< Re < 4000

Re < 2100

1. Turbulence is characterized by random fluctuations.
2. Transitional flows are relatively steady accompanied by occasional burst.
3. Laminar flow is relatively steady.

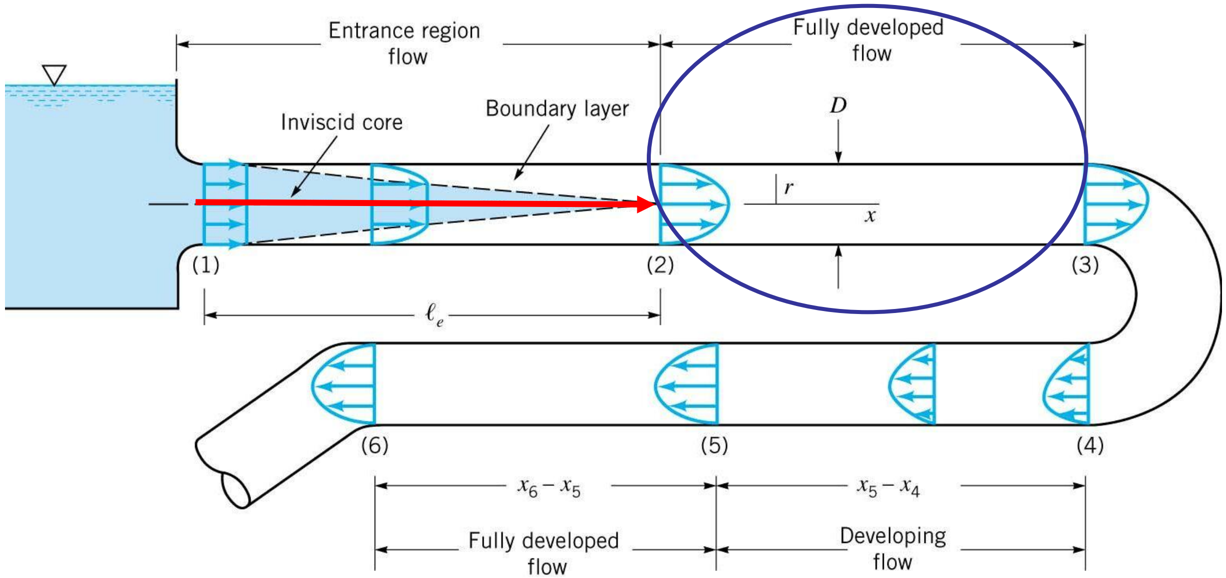
For laminar flow there is only flow direction: $\mathbf{V} = u\hat{\mathbf{i}}$

For turbulent flow, there is a predominate flow direction, but there are random components normal to the flow direction: $\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$

Viscous Pipe Flow: Entrance and Fully Developed

The entrance region in a pipe flow is quite complex (1) to (2):

The fluid enters the pipe with nearly uniform flow.
The viscous effects create a boundary layer that merges.
When they merge the flow is fully developed.



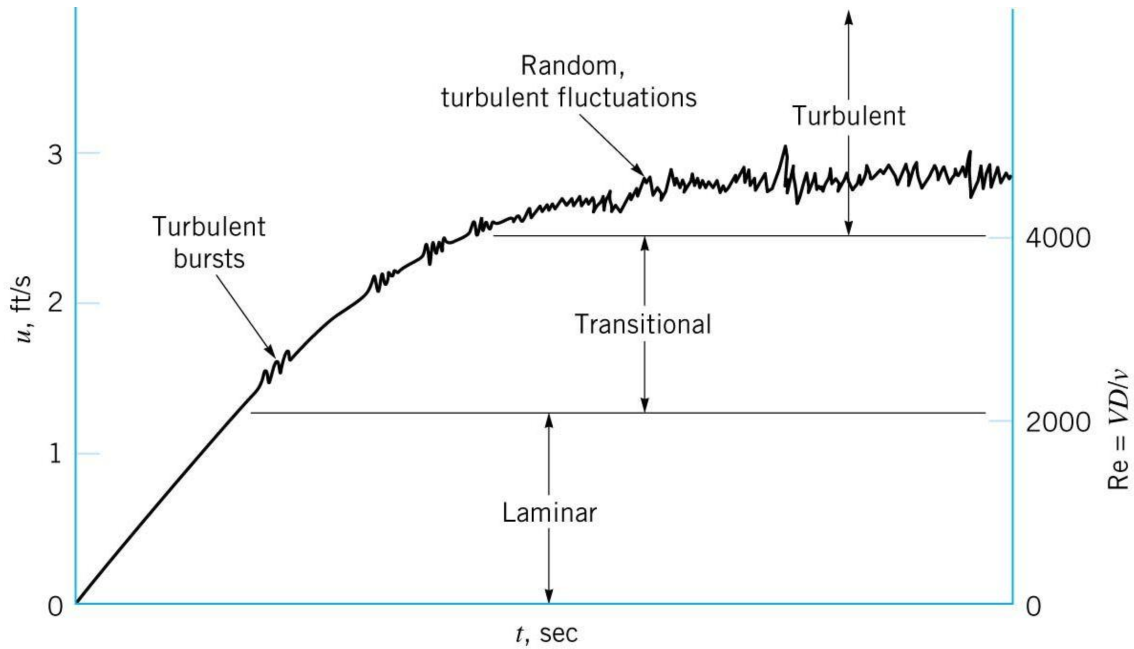
There are estimates for determining the entrance length for pipe flows:

$$\frac{\ell_e}{D} = 0.06\ \text{Re for laminar flow}$$ and $$\frac{\ell_e}{D} = 4.4\ (\text{Re})^{1/6}\ \text{for turbulent flow}$$

Fully Developed Turbulent Flow: Overview

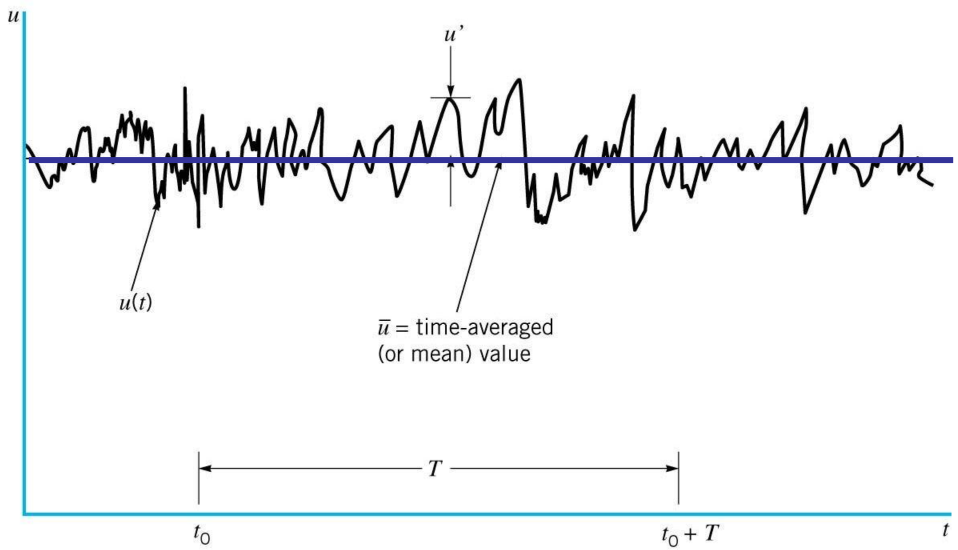
Turbulent flow is the least understood of all flow phenomena, yet is more likely to occur than laminar flow, so we address ways of describing the flow.

Transition from Laminar to Turbulent Flow in a Pipe:



Fully Developed Turbulent Flow: Overview

One see fluctuation or randomness on the macroscopic scale.



$$\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x, y, z, t)\, dt$$

$$u = \bar{u} + u'$$

mean fluctuating

One of the few ways we can describe turbulent flow is by describing it in terms of time-averaged means and fluctuating parts.

Fully Developed Turbulent Flow: Overview

Now consider, the time average of the fluctuating parts:

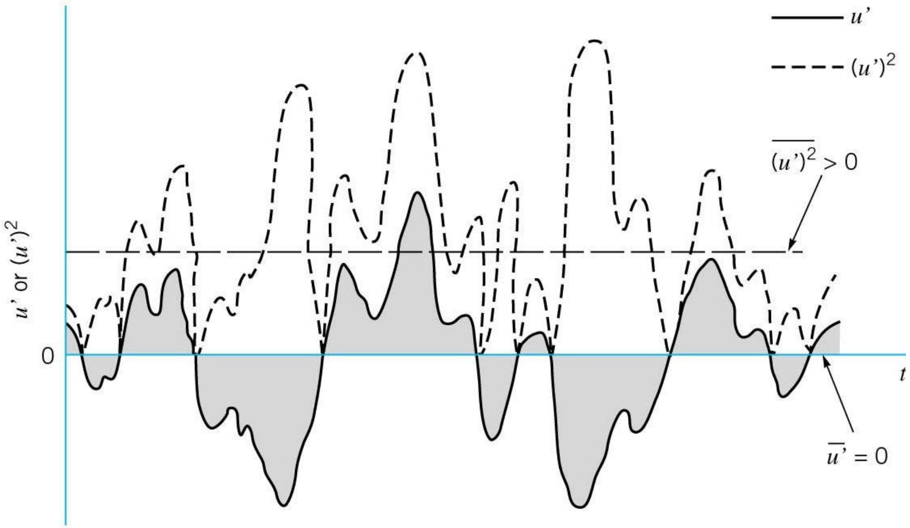
$$u' = u - \overline{u}$$

$$\overline{u'} = \frac{1}{T}\int_{t_0}^{t_0+T}(u - \overline{u})\,dt = \frac{1}{T}\left(\int_{t_0}^{t_0+T} u\,dt - \overline{u}\int_{t_0}^{t_0+T} dt\right) = \frac{1}{T}(T\overline{u} - T\overline{u}) = 0$$

The fluctuations are equally distributed on either side of the average.

Now, consider the average of the square of the fluctuations: $(u')^2 \geq 0$

$$\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_0+T}(u')^2\,dt > 0$$



Turbulence Intensity:

$$\mathscr{I} = \frac{\sqrt{\overline{(u')^2}}}{\overline{u}} = \frac{\left[\frac{1}{T}\int_{t_0}^{t_0+T}(u')^2\,dt\right]^{1/2}}{\overline{u}}$$

Indication of the “gustiness” of the flow.

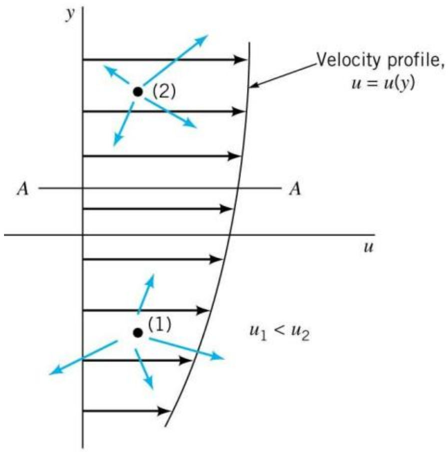
$\mathscr{I} \gtrsim 0.1$ in Atmosphere, $\mathscr{I} \approx 0.01$ In “good” wind tunnel

Fully Developed Turbulent Flow: Overview

Now, shear stress:

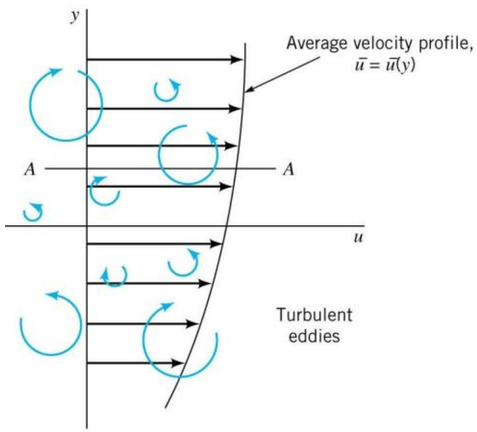
$\tau = \mu \, du/dy$ However, $\tau \neq \mu \, d\bar{u}/dy$ for turbulent flow.

Laminar Flow:



Shear relates to random motion as particles glide smoothly past each other.

Turbulent Flow:



Shear comes from eddy motion which have a more random motion and transfer momentum.

“Experiment”:

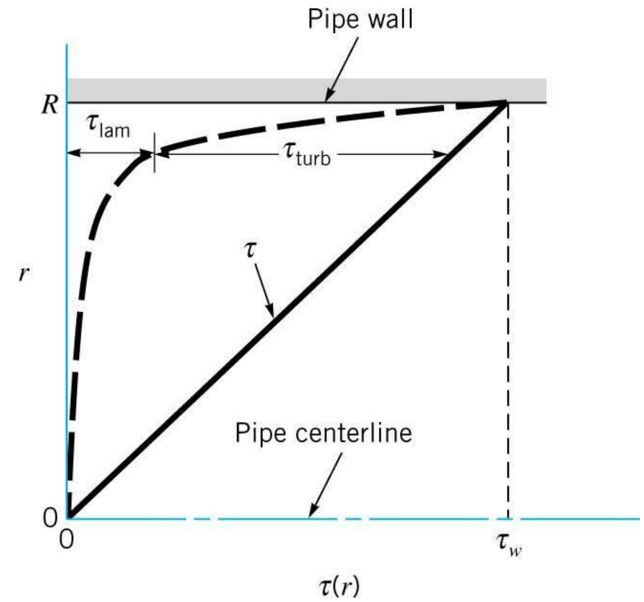
For turbulent flow: $$\tau = \mu \frac{d\bar{u}}{dy} - \rho\overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

Is the combination of laminar and turbulent shear. If there are no fluctuations, the result goes back to the laminar case. The turbulent shear stresses are positive, thus turbulent flows have more shear stress.

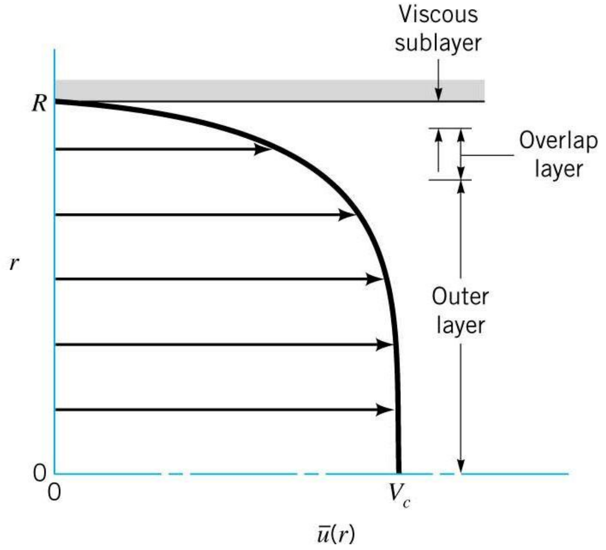
Fully Developed Turbulent Flow: Overview

The turbulent shear components are known as Reynolds Stresses.

Shear Stress in Turbulent Flows:



Turbulent Velocity Profile:



In viscous sublayer: $\tau_{laminar}$ > τ_{turb} 100 to 1000 times greater.

In the outer layer: τ_{tirb} > $\tau_{laminar}$ 100 to 1000 time greater.

The viscous sublayer is extremely small.

Fully Developed Turbulent Flow: Velocity Profile

The velocity profile for turbulent flow is been obtained through experimental analysis, dimensional analysis, and semiempirical theoretical efforts.

In the viscous sublayer: $\frac{\overline{u}}{u^*} = \frac{yu^*}{\nu}$ for a smooth wall, "Law of the Wall"

$u^* = (\tau_w/\rho)^{1/2}$ is the friction velocity, and $y = R - r$

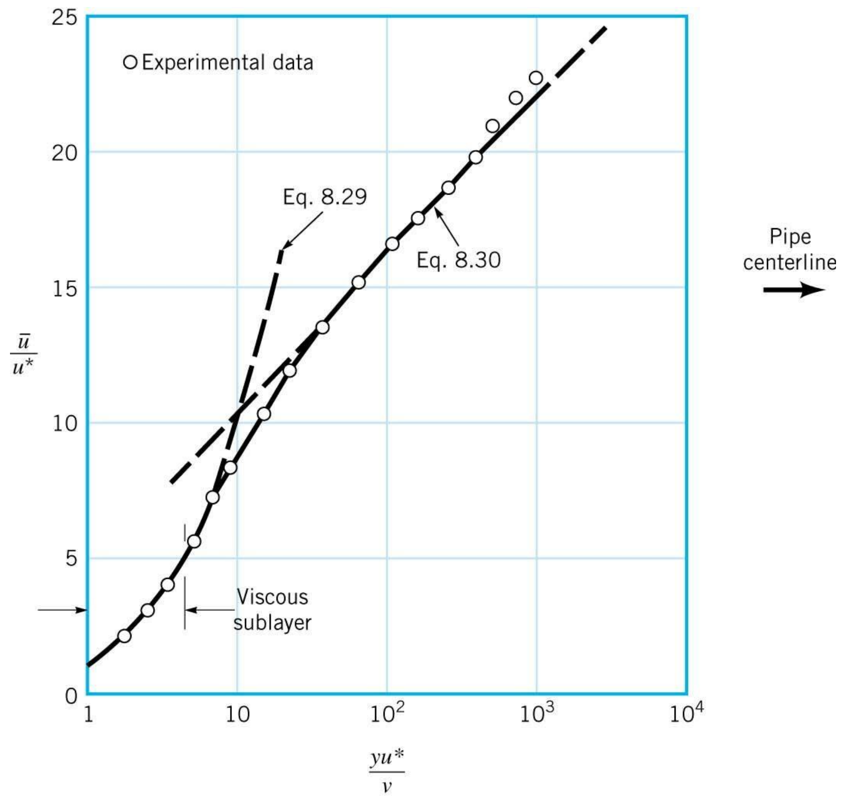
In the overlap region:

$$\frac{\overline{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0$$

From dimensional analysis arguments

Possible outer region approximation:

$$(V_c - \overline{u})/u^* = 2.5 \ln(R/y)$$



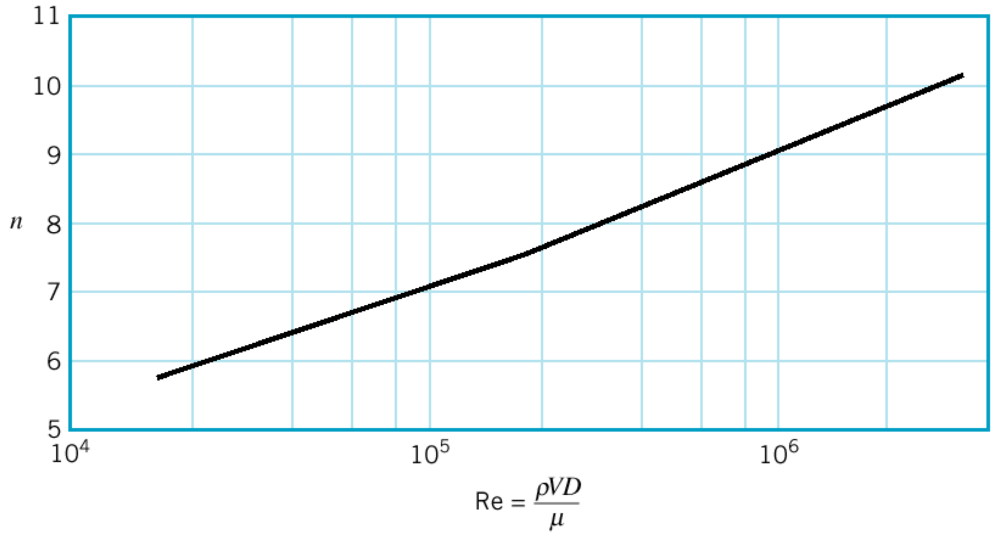
Fully Developed Turbulent Flow: Velocity Profile

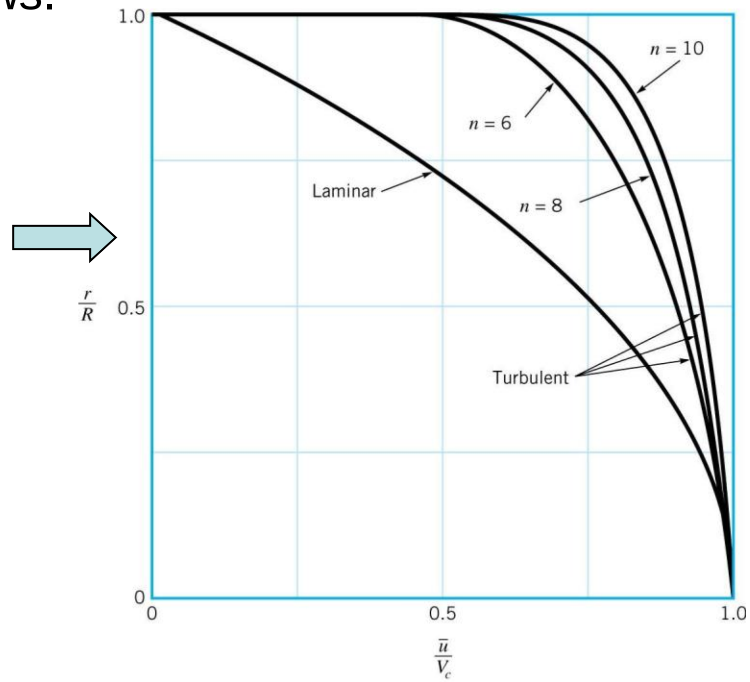
Some alternative, approach include the Power-Law equation:

$$\frac{\overline{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

n = 7 for many practical flows.

n, chosen based on the Reynolds number.





Turbulent velocity profiles are relatively flat in a pipe flow.

The power-law equation is not valid near the wall, since that would give an infinite velocity gradient.

Also, the shear does not go to zero at the center-line.

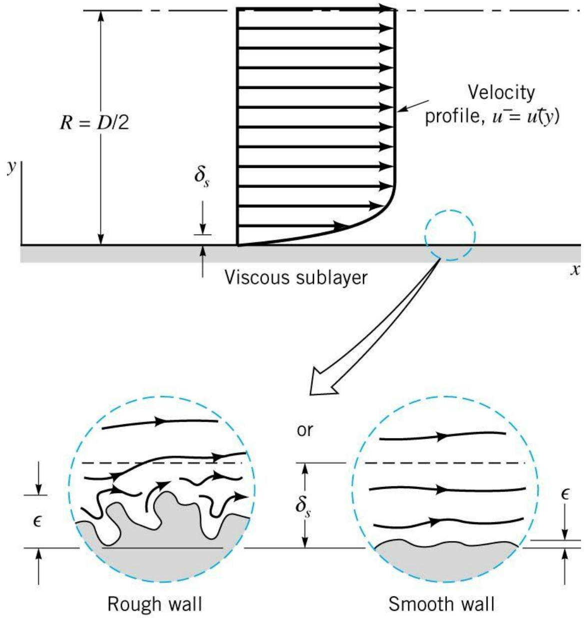
"Profiles":

Dimensional Analysis of Pipe Flow: Moody Chart

Most turbulent pipe flow data is based on experiments. In turbulent flow, in order to do dimensional analysis we consider the roughness of the pipe, as well as density which relates to momentum.

Variables: $\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$

roughness



Roughness is important in the viscous sub-layer in turbulent flows, if it protrudes sufficiently in this layer.

The viscous layer in laminar flow is so large, that small roughness does not play a role.

Then range of roughness for validity of this analysis is for: $0 \leq \varepsilon/D \lesssim 0.05$

Then, the dimensionless groups are the following:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

Dimensional Analysis of Pipe Flow: Moody Chart

As for laminar flow, the pressure drop must be proportional to the pipe length:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

Recalling the definition of the friction factor: $$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

Then the friction factor is one of our dimensionless groups: $$f = \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

Then using experiments, we can find the above relationship with various manufactured pipe roughness values:

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

→ "Moody Chart"

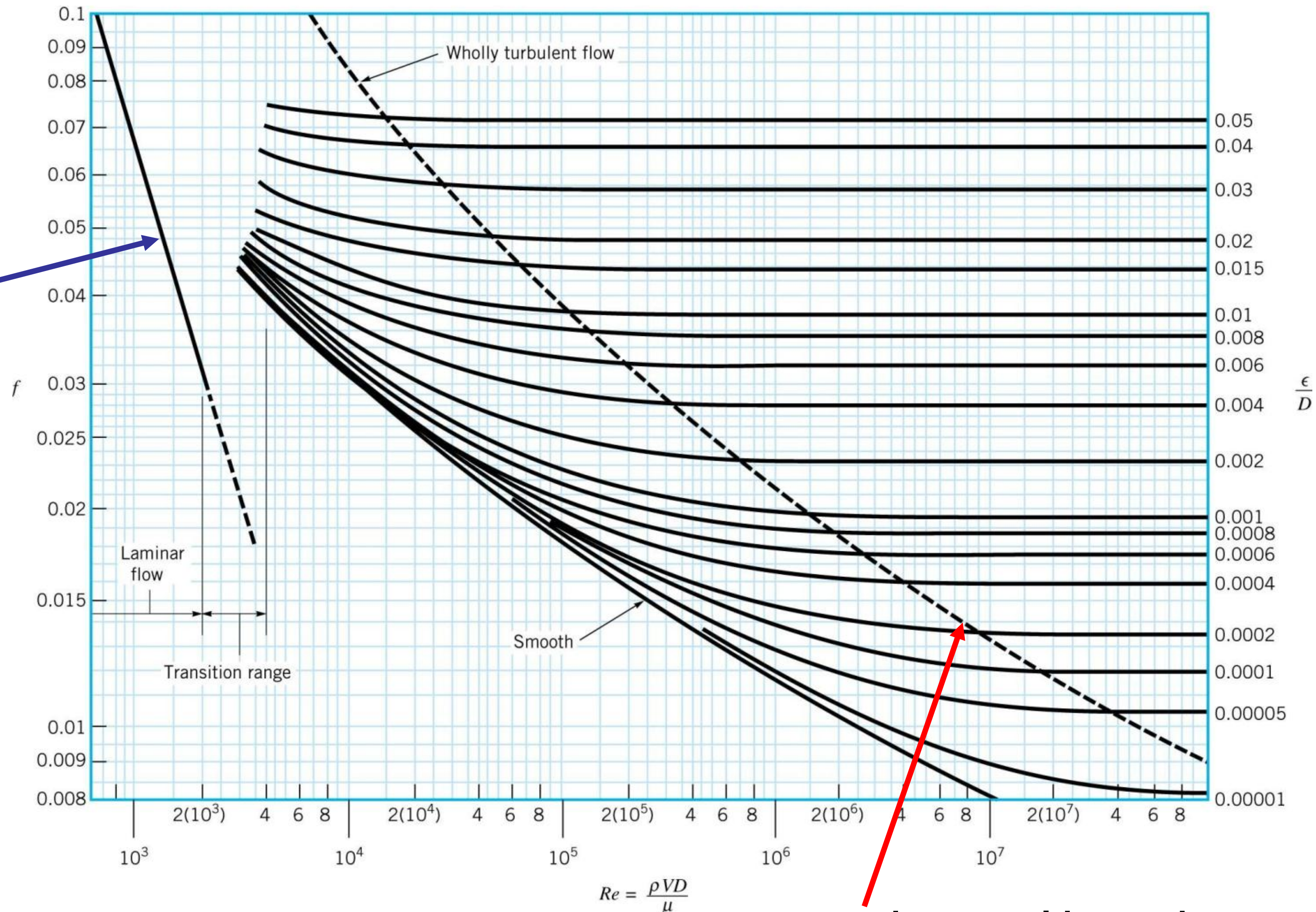
Colebrook Relation for Non-Laminar part of the Moody Chart (curve fit):

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Dimensional Analysis of Pipe Flow: Moody Chart

$$f = \frac{64}{\mathrm{Re}}$$

Laminar



Marks Reynolds Number independence

Dimensional Analysis of Pipe Flow: Moody Chart

Energy Equation relation to Pipe Flow:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

α's account for non-uniform velocity profiles.

For fully developed pipe flow in a horizontal pipe: $\Delta p = p_1 - p_2 = \gamma h_L$

And, $\Delta p = f \frac{\ell}{D} \frac{\rho V^2}{2}$ ⟹ Darcy-Weisbach Equation: $h_L = f \frac{\ell}{D} \frac{V^2}{2g}$

8.42 Water flows through a horizontal plastic pipe with a diameter of 0.2 m at a velocity of 10 cm/s. Determine the pressure drop per meter of pipe using the Moody chart.

The pressure drop in the pipe can be fo[und]
from

$$\Delta P = f \frac{l}{D} \frac{\rho V^2}{2}$$

The friction factor is determined from the Moody chart.

$$Re = \frac{\rho V D}{\mu} = \frac{(999)(0.1)(0.2)}{1.12 \times 10^{-3}} = 1.8 \times 10^4$$

For plastic pipe, $\varepsilon = 0.0\,mm$

$$\varepsilon/D = 0.0/0.2 = 0.0$$

From the Moody chart

$$f = 0.026$$

So ΔP per meter ($l = 1m$)

$$\Delta P = (0.026)\left(\frac{1}{0.2}\right)\left[\frac{999(0.1)^2}{2}\right]$$

$\underline{\Delta P = 0.649}$ Pa per meter