Lecture- 15
External Flows

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Learning Objectives

- After completing this Lecture, you should be able to:

1. identify and discuss the features of external flow.
2. explain the fundamental characteristics of a boundary layer, including laminar, transitional, and turbulent regimes.
3. calculate boundary layer parameters for flow past a flat plate.
4. provide a description of boundary layer separation.
5. calculate the lift and drag forces for various objects.

Outline

- Overview of External Flows
- Boundary Layer Characteristics
- Lift and Drag

External Flows: Overview

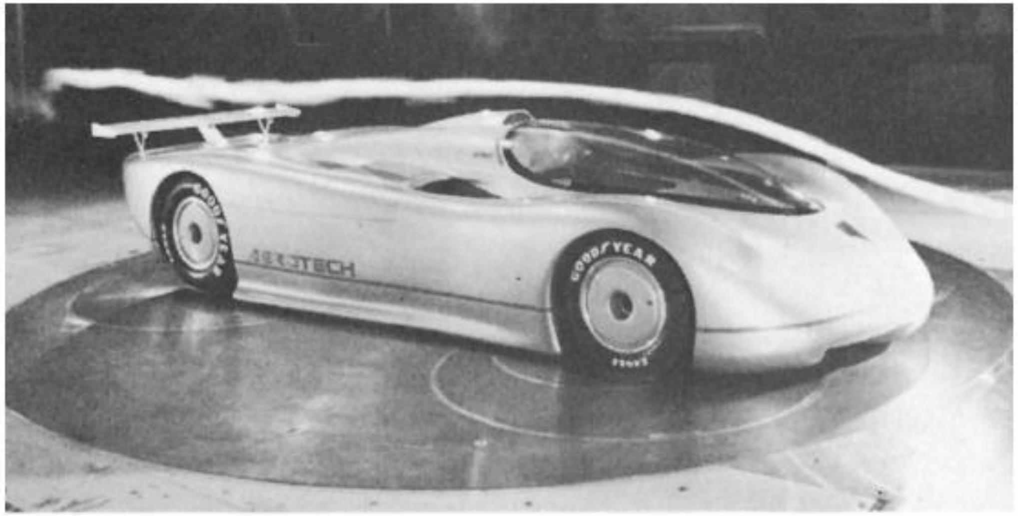
If a body is immersed in a flow, we call it an external flow.

External flows involving air are typically termed aerodynamics.

Some important external flows include airplanes, motor vehicles, and flow around buildings.

Typical quantities of interest are **lift** and **drag** acting on these objects.

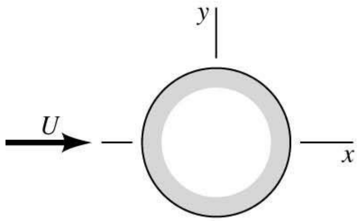
Often flow modeling is used to determine the flow fields in a wind tunnel or water tank.



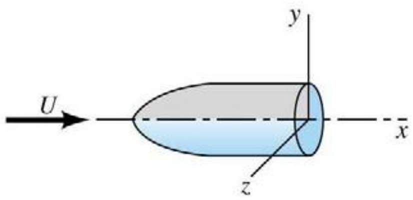
“Lift/Drag:”

External Flows: Overview

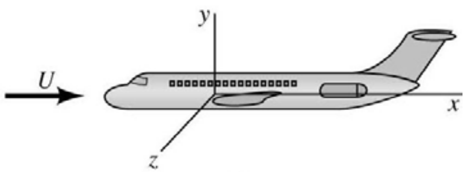
Types of External Flows:



Two-Dimensional: infinitely long and of constant cross-sectional size and shape.



Axisymmetric: formed by rotating their cross-sectional shape about the axis of symmetry.

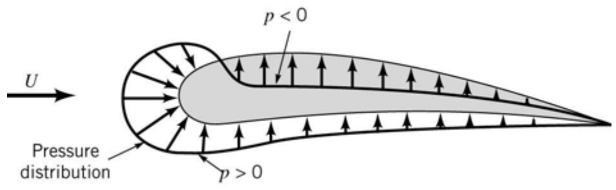


Three-Dimensional: may or may not possess a line of symmetry.

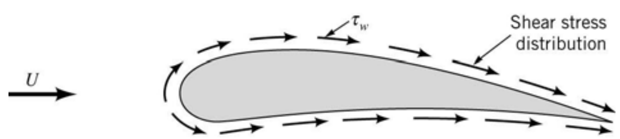
The bodies can be classified as streamlined or blunt. The flow characteristics depend strongly on the amount of streamlining present. Streamlined object typically move more easily through a fluid.

"Shapes:"

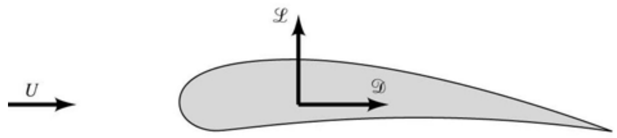
External Flows: Drag and Lift

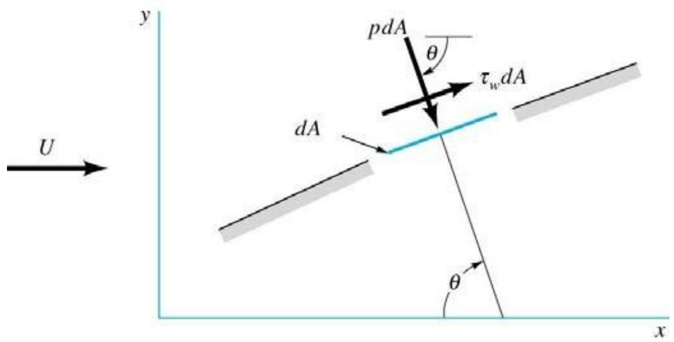


Pressure Distributions around an object (Bluff Body) lead to lift and drag.



Shear Stresses on the surface also lead to lift and drag.





Drag: Aligned with the Flow
Pressure (Form) Drag + Skin Friction Drag

$$dF_x = (p\,dA)\cos\theta + (\tau_w\,dA)\sin\theta$$

$$\mathscr{D} = \int dF_x = \int p \cos\theta\,dA + \int \tau_w \sin\theta\,dA$$

projected area $C_D = \dfrac{\mathscr{D}}{\frac{1}{2}\rho U^2 A}$ **wetted area**

Lift: Normal to the Flow

$$dF_y = -(p\,dA)\sin\theta + (\tau_w\,dA)\cos\theta$$

$$\mathscr{L} = \int dF_y = -\int p \sin\theta\,dA + \int \tau_w \cos\theta\,dA$$

$$C_L = \frac{\mathscr{L}}{\frac{1}{2}\rho U^2 A}$$

External Flows: Friction and Pressure Coefficient

Friction Coefficient: $C_f = \dfrac{\tau_w}{\frac{1}{2}\rho U^2}$

Pressure Coefficient: $C_p = \dfrac{\Delta p}{\frac{1}{2}\rho U^2} = \dfrac{p - p_0}{\frac{1}{2}\rho U^2}$ $\xrightarrow{\text{Applying Bernoulli Eq.}}$ $C_p - 1 = \dfrac{u^2}{U^2}$

Skin Friction Drag Coefficient: $C_{D,f} = \dfrac{D_f}{\frac{1}{2}\rho U^2 \times A_{wetted}}$

Pressure Drag Coefficient: $C_{D,p} = \dfrac{D_p}{\frac{1}{2}\rho U^2 \times A_{projected}}$

Total Drag Coefficient: $C_D = C_{D,f} + C_{D,p}$

EXAMPLE 9.1 Drag from Pressure and Shear Stress Distributions

GIVEN Air at standard conditions flows past a flat plate as is indicated in Fig. E9.1. In case (a) the plate is parallel to the upstream flow, and in case (b) it is perpendicular to the upstream flow. The pressure and shear stress distributions on the surface are as indicated (obtained either by experiment or theory).

FIND Determine the lift and drag on the plate.

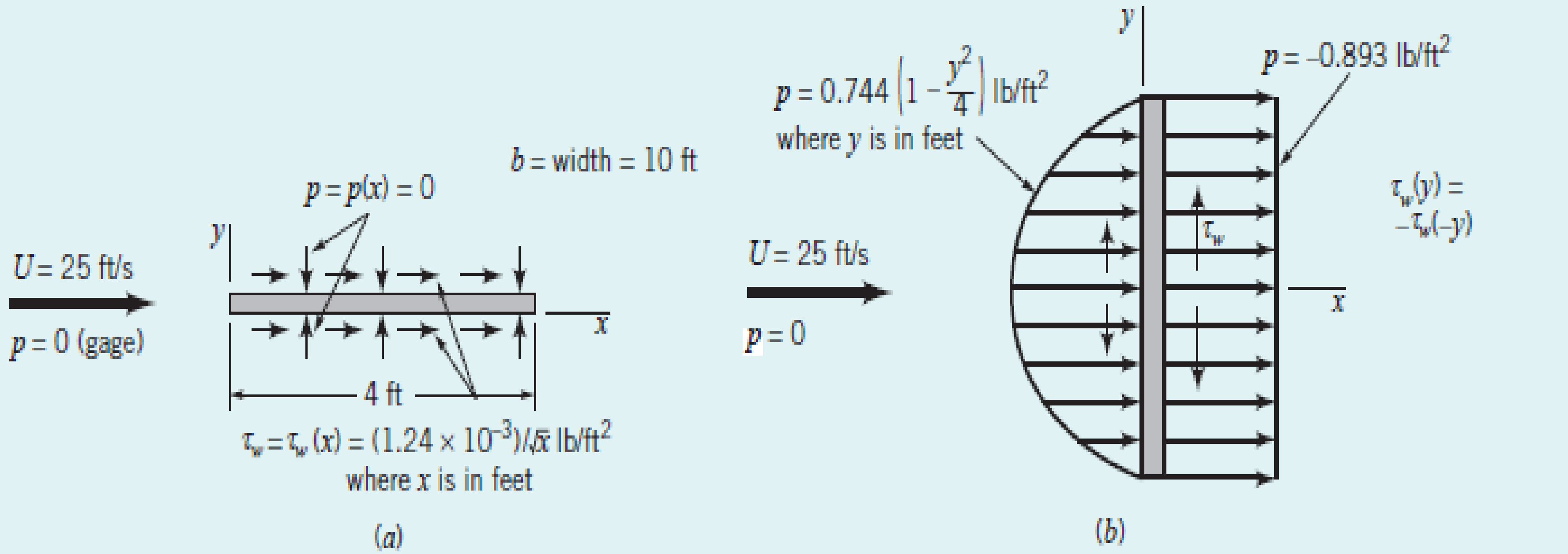


FIGURE E9.1

SOLUTION

For either orientation of the plate, the lift and drag are obtained from Eqs. 9.1 and 9.2. With the plate parallel to the upstream flow we have $\theta = 90°$ on the top surface and $\theta = 270°$ on the bottom surface so that the lift and drag are given by

$$\mathscr{L} = -\int_{\text{top}} p\, dA + \int_{\text{bottom}} p\, dA = 0$$

and

$$\mathscr{D} = \int_{\text{top}} \tau_w\, dA + \int_{\text{bottom}} \tau_w\, dA = 2\int_{\text{top}} \tau_w\, dA \qquad \textbf{(1)}$$

where we have used the fact that because of symmetry the shear stress distribution is the same on the top and the bottom surfaces, as is the pressure also [whether we use gage ($p = 0$) or absolute ($p = p_{\text{atm}}$) pressure]. There is no lift generated—the plate does not know up from down. With the given shear stress distribution, Eq. 1 gives

$$\mathscr{D} = 2\int_{x=0}^{4\text{ ft}} \left(\frac{1.24 \times 10^{-3}}{x^{1/2}}\text{ lb/ft}^2\right)(10\text{ ft})\, dx$$

or

$$\mathscr{D} = 0.0992\text{ lb} \qquad \textbf{(Ans)}$$

With the plate perpendicular to the upstream flow, we have $\theta = 0°$ on the front and $\theta = 180°$ on the back. Thus, from Eqs. 9.1 and 9.2

$$\mathscr{L} = \int_{\text{front}} \tau_w\, dA - \int_{\text{back}} \tau_w\, dA = 0$$

and

$$\mathscr{D} = \int_{\text{front}} p\, dA - \int_{\text{back}} p\, dA$$

Again there is no lift because the pressure forces act parallel to the upstream flow (in the direction of $\mathscr{D}$ not $\mathscr{L}$) and the shear stress is symmetrical about the center of the plate. With the given relatively large pressure on the front of the plate (the center of the plate is a stagnation point) and the negative pressure (less than the upstream pressure) on the back of the plate, we obtain the following drag

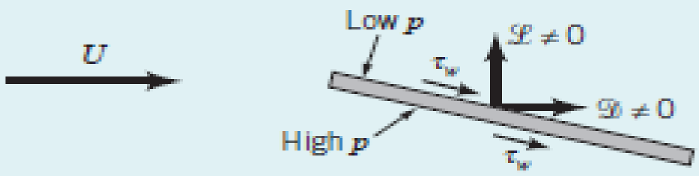
$$\mathscr{D} = \int_{y=-2}^{2\text{ ft}} \left[0.744\left(1 - \frac{y^2}{4}\right)\text{ lb/ft}^2 - (-0.893)\text{ lb/ft}^2\right](10\text{ ft})\, dy$$

or

$$\mathscr{D} = 55.6\text{ lb} \qquad \textbf{(Ans)}$$

COMMENTS Clearly there are two mechanisms responsible for the drag. On the ultimately streamlined body (a zero thickness flat plate parallel to the flow) the drag is entirely due to the shear stress at the surface and, in this example, is relatively small. For the ultimately blunted body (a flat plate normal to the upstream flow) the drag is entirely due to the pressure difference between the front and back portions of the object and, in this example, is relatively large.

If the flat plate were oriented at an arbitrary angle relative to the upstream flow as indicated in Fig. E9.1*c*, there would be both a lift and a drag, each of which would be dependent on both the shear stress and the pressure. Both the pressure and shear stress distributions would be different for the top and bottom surfaces.

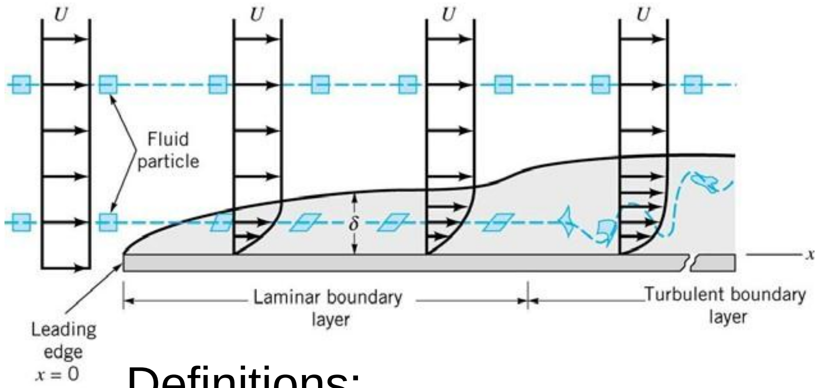


(*c*)

FIGURE E9.1 (*Continued*)

External Flows: Boundary Layers

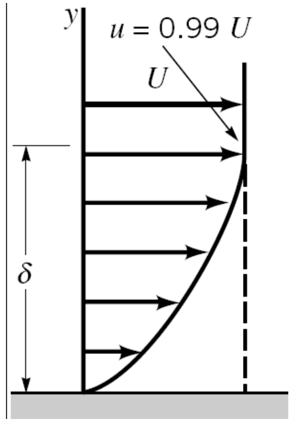
Development of a Boundary Layer:



Particles get distorted in the boundary layer.

Viscous effects are important

“Transition”:



Definitions:

Boundary layer height: $\delta = y \quad \text{where} \quad u = 0.99\,U$

- The boundary layer thickness δ grows continuously from the start of the fluid-surface contact, e.g., the leading edge. It is a function of x, not a constant.
- Velocity profiles and shear stress τ are f(x,y).
- The flow will generally be laminar starting from x = 0.
- The flow will undergo laminar-to-turbulent transition if the streamwise dimension is greater than a distance x_{cr} corresponding to the location of the transition Reynolds number Re_{cr}.
- Outside of the boundary layer region, free stream conditions exist where velocity gradients and therefore viscous effects are typically negligible.

External Flows: Boundary Layers

Local Reynolds Number: $\mathrm{Re_x} = \dfrac{\rho \mathrm{U_\infty x}}{\mu} = \dfrac{\mathrm{U_\infty x}}{\upsilon}$

ρ = fluid density

μ = fluid dynamic viscosity

ν = fluid kinematic viscosity

U_∞ = characteristic flow velocity

x = characteristic flow dimension

Critical Reynolds Number: $\mathrm{Re}_{cr} = \dfrac{\rho U_\infty x_{cr}}{\mu} = 500,000$

$\mathrm{x_{cr}}$ = the value of x where transition from laminar to turbulent flow occurs

$x < x_{cr}$ the flow is laminar

$x \geq x_{cr}$ the flow is turbulent

External Flows: Boundary Layers

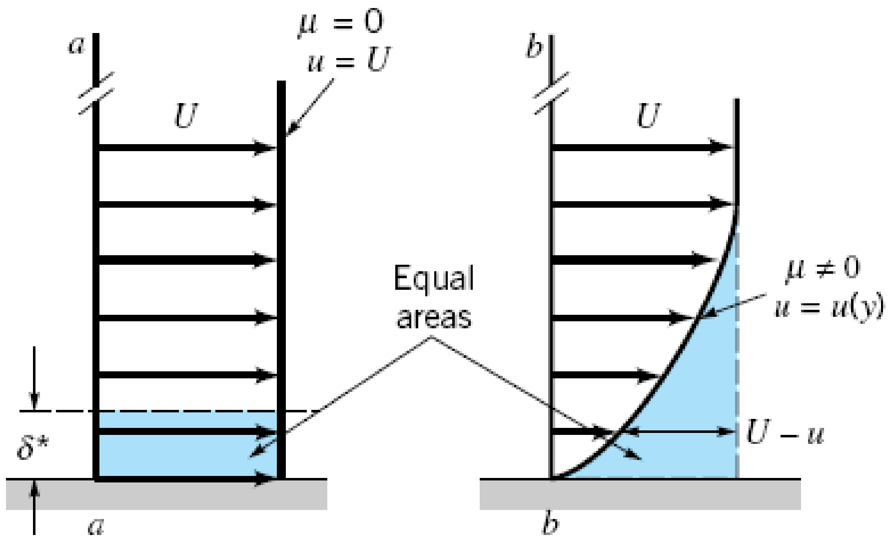
Displacement Thickness:

$\delta^* =$ distance the solid surface would have to be displaced to maintain the same mass flow rate as for non-viscous flow.

"Conservation of Mass"

$$\text{flow reduction} = \int_0^{\delta} (U - u)\, dy = U\delta^*$$

$$\boxed{\delta^* = \int_0^{\infty}\left(1 - \frac{u}{U}\right)dy}$$



Momentum Thickness:

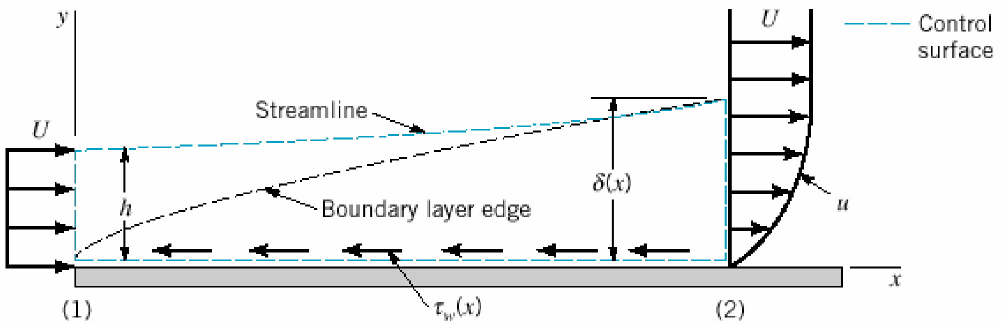
"Momentum Flux"

$$\text{loss of momentum flux} = \int_0^{\delta} \rho\, u(U - u)\, dy = \rho U^2 \theta$$

$$\boxed{\Theta = \int_0^{\infty}\frac{u}{U}\left(1 - \frac{u}{U}\right)dy}$$

External Flows: Boundary Layers

Drag on a Flat Plate: Integral Relationships



b is the width of the plate

$$\sum F_x = -\mathfrak{D} = -\int_{\text{plate}} \tau_w \, dA = -b \int_{\text{plate}} \tau_w \, dx$$

$$-\mathfrak{D} = \rho \int_{(1)} U(-U)\, dA + \rho \int_{(2)} u^2 \, dA$$

$$\mathfrak{D} = \rho U^2 bh - \rho b \int_0^{\delta} u^2 \, dy$$

$$\mathfrak{D} = \rho b \int_0^{\delta} u(U - u)\, dy$$

$$\boxed{\mathfrak{D} = \rho b U^2 \, \Theta}$$

$$\boxed{\tau_w = \rho U^2 \frac{d\Theta}{dx}}$$

Note:

$$Uh = \int_0^{\delta} u \, dy$$

External Flows: Laminar Boundary Layers

Flow over a Flat Plate can be Solved Exactly: **Blasius** Solution In 1908
H. Blasius (1883–1970), one of Prandtl's students

Assumes Steady, 2D Laminar, high Re Flow with negligible gravitational effects.

From Boundary Layer Analysis: $v \ll u \quad \text{and} \quad \dfrac{\partial}{\partial x} \ll \dfrac{\partial}{\partial y}$

conservation of mass: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

momentum conservation: $$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$-\frac{\partial p}{\partial y} = 0$$ pressure is uniform across the boundary layer and is determined by the external flow.

boundary conditions:

at y=0: u=0, v=0, du/dy=constant, $d^2u/dy^2=0$
at y=δ: u=U, du/dy=o

External Flows: Laminar Boundary Layers

After solving, the governing equations with similarity variable:

Boundary Layer Height: $\dfrac{\delta}{x} = \dfrac{5}{\sqrt{\mathrm{Re}_x}}$

Displacement Thickness: $\dfrac{\delta^*}{x} = \dfrac{1.721}{\sqrt{\mathrm{Re}_x}}$

Momentum Thickness: $\dfrac{\Theta}{x} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$

Wall Shear Stress: $\tau_w = 0.332 U^{3/2} \sqrt{\dfrac{\rho\mu}{x}}$

Coefficient of Friction: $c_f = \dfrac{\tau_w}{\frac{1}{2}\rho U^2} \Rightarrow c_f = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$

Coefficient of Drag: $C_{Df} = \dfrac{1.328}{\sqrt{\mathrm{Re}_\ell}}$

note: $C_{D,f} = \dfrac{1}{L}\int_0^L C_{f,x}\, dx$

9.12 Water flows past a flat plate that is oriented parallel to the flow with an upstream velocity of 0.5 m/s. Determine the approximate location downstream from the leading edge where the boundary layer becomes turbulent. What is the boundary layer thickness at this location?

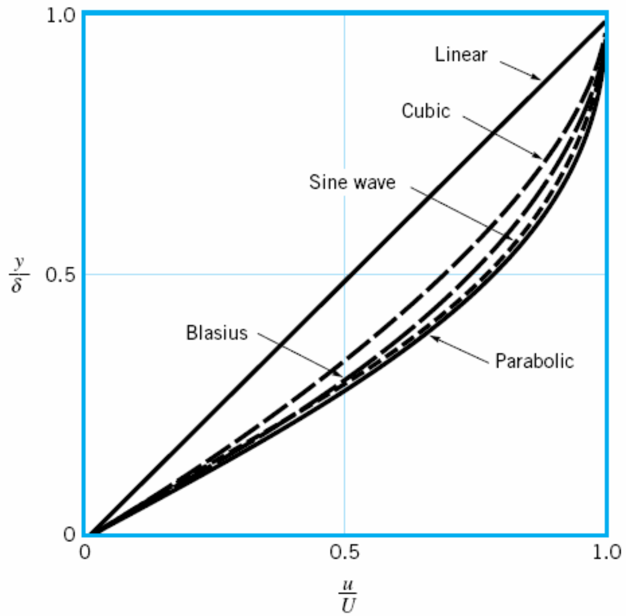
$$Re_{cr} = 5\times10^5 = \frac{U x_{cr}}{\nu}$$

$$x_{cr} = \frac{5\times10^5 \nu}{U} = \frac{5\times10^5 (1.12\times10^{-6}\ m^2/s)}{0.5\ m/s} = \underline{\underline{1.12\ m}}$$

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\ m^2/s)\,1.12\ m}{0.5\ m/s}} = \underline{\underline{7.92\times10^{-3}\ m}}$$

External Flows: Laminar Boundary Layers

If we use various velocity profiles that match the boundary layer conditions of a velocity profile:



Flat Plate Momentum-Integral Results for Various Assumed Laminar Flow Velocity Profiles

Profile Character	$\delta Re_x^{1/2}/x$	$c_f Re_x^{1/2}$	$C_{Df} Re_\ell^{1/2}$
a. Blasius solution	5.00	0.664	1.328
b. Linear $u/U = y/\delta$	3.46	0.578	1.156
c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$	5.48	0.730	1.460
d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$	4.64	0.646	1.292
e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$	4.79	0.655	1.310

9.13 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.

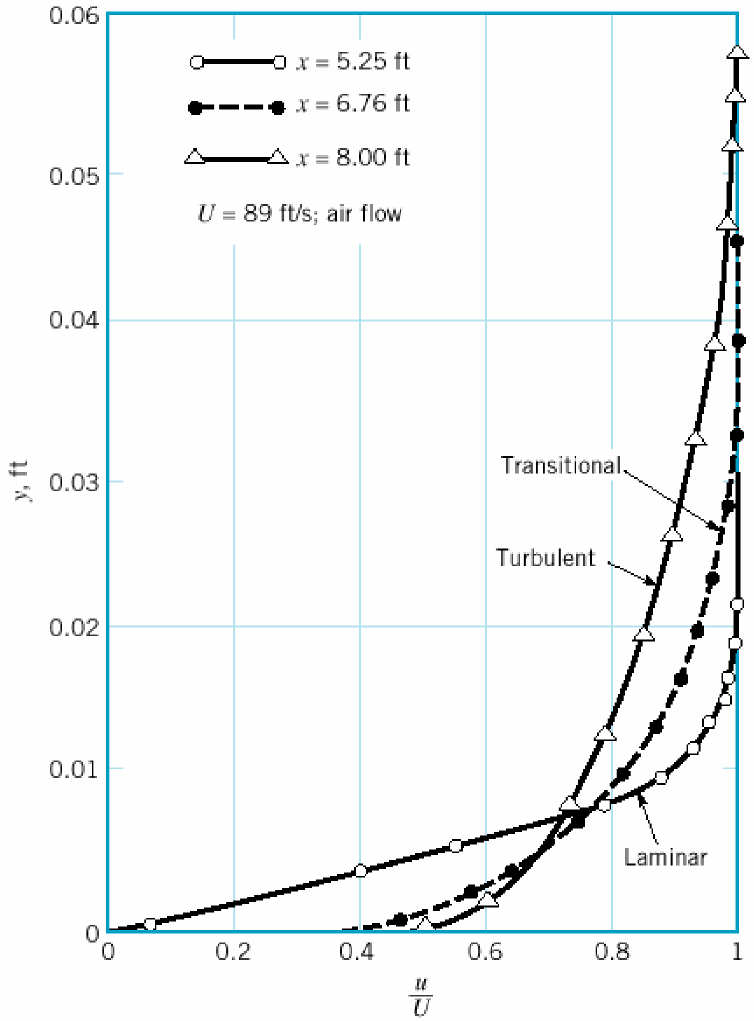
Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3}\,m}{\sqrt{1.3\,m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m,\ \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

External Flows: Transitional and Turbulent Boundary Layers

Turbulent Spots in Transitional Flow



No real theories for transitional boundary layers.

Turbulent boundary layers are very similar to those in pipe flow, and we can use some of those equations and theories.

External Turbulent Flow: Velocity Profile

The velocity profile for turbulent flow is been obtained through experimental analysis, dimensional analysis, and semiempirical theoretical efforts.

Boundary Layer Height: $\frac{\delta}{x} = \frac{0.370}{\mathrm{Re}_x^{1/5}}$

Displacement Thickness: $\delta^* = 0.0463\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$

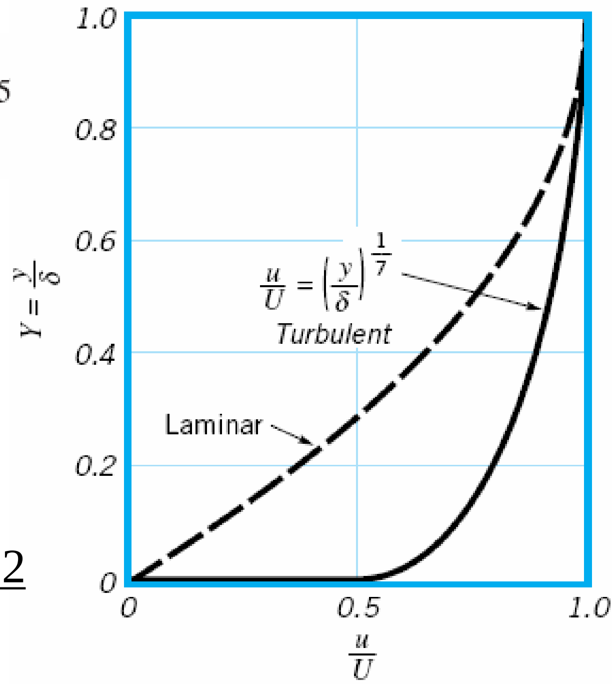
Momentum Thickness: $\Theta = 0.0360\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$

Wall Shear Stress: $\tau_w = \frac{0.0288\rho U^2}{\mathrm{Re}_x^{1/5}}$

Coefficient of Friction: $c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \Rightarrow C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}}$

Coefficient of Drag: $C_{Df} = \frac{0.0720}{\mathrm{Re}_\ell^{1/5}}$

1/7Power Law

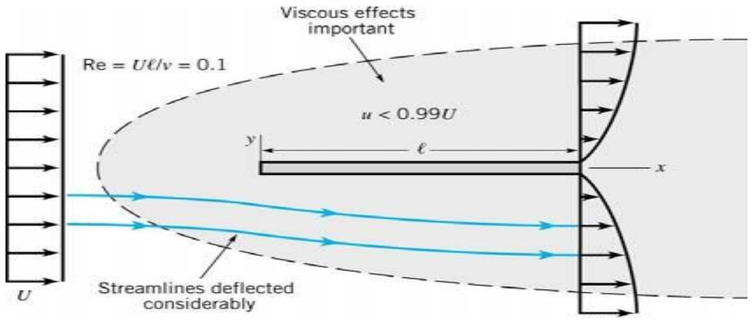


External Flows: Flow Past Objects

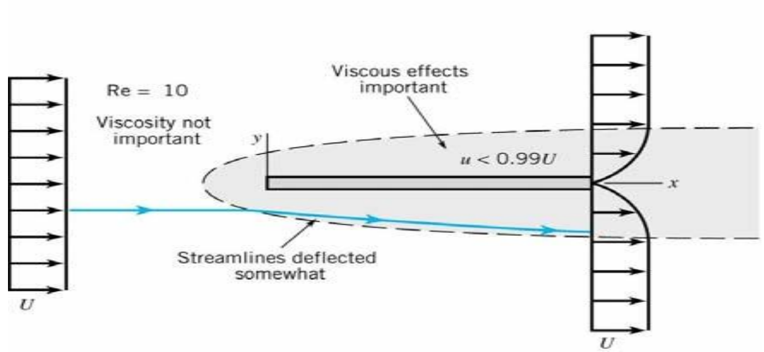
Flat Plate Flow:

Low Reynolds Number: Re = 0.1

Large Boundary Layer

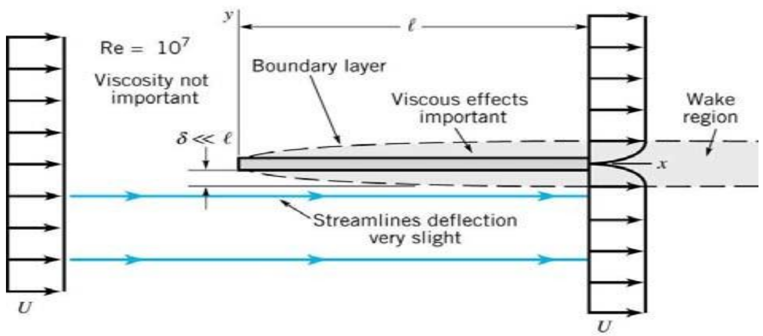


Medium Reynolds Number: Re = 10

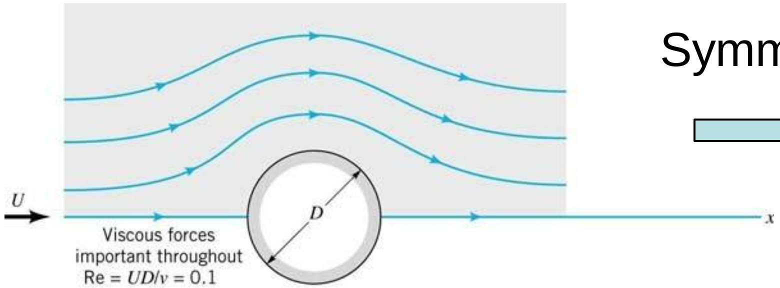


Large Reynolds Number: Re = 10^5

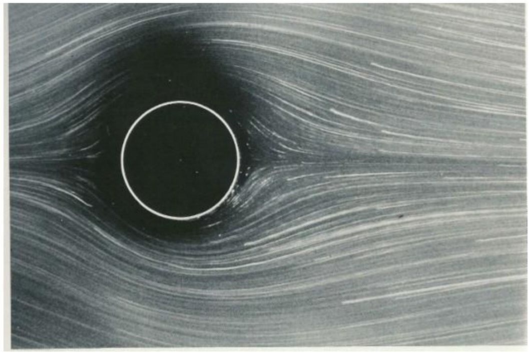
Thin Boundary Layer

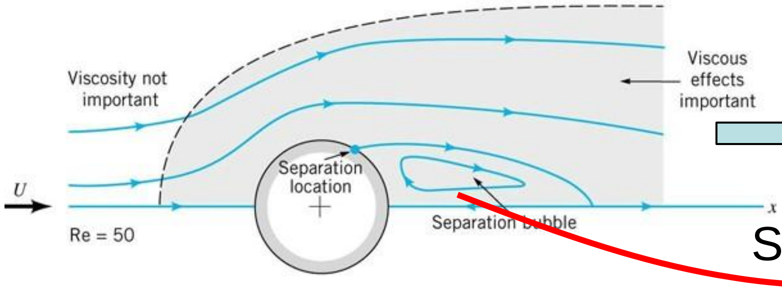


External Flows: Flow Past Objects

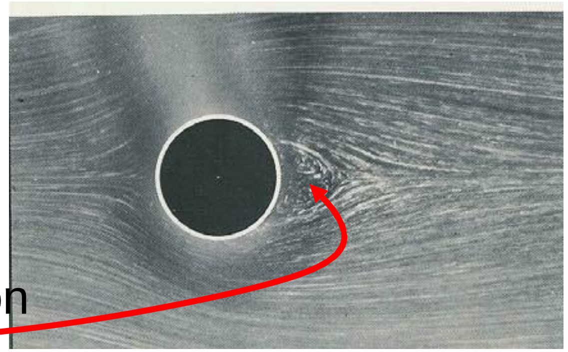


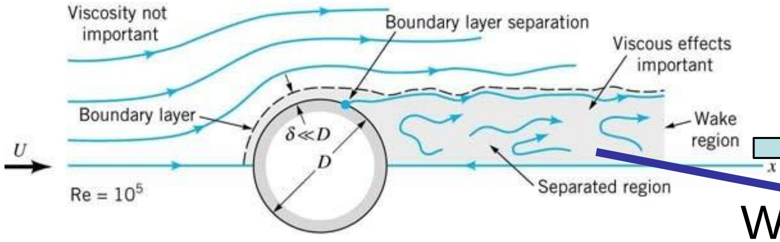
Symmetric



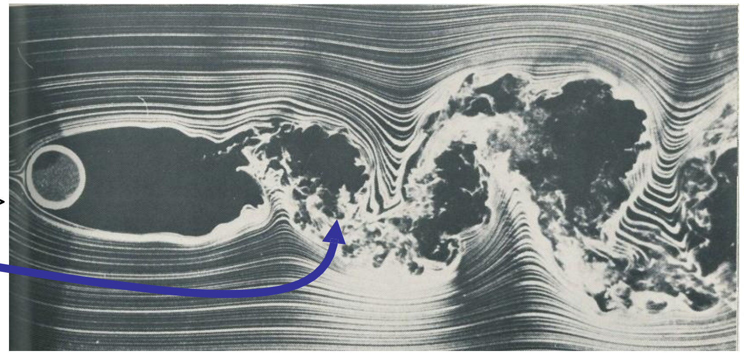


Separation





Wake



External Flows: Drag on Immersed Objects

$$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho U^2 A}$$

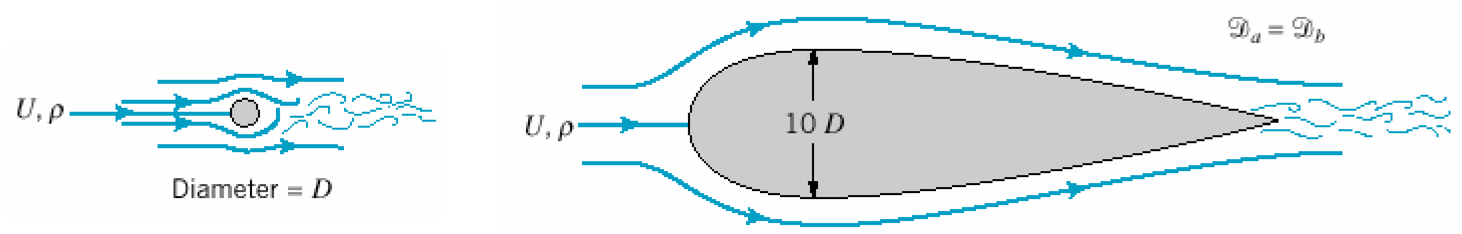
If there were not viscous effects acting on an object there would be no friction drag nor any pressure drag.

Viscosity causes friction and separation which causes pressure drag.

Friction Drag: the part of drag due directly to the shear stress

Pressure Drag/Form Drag: the part of drag due directly to the pressure

The Drag Coefficient is highly dependent on shape and the Reynolds Number:



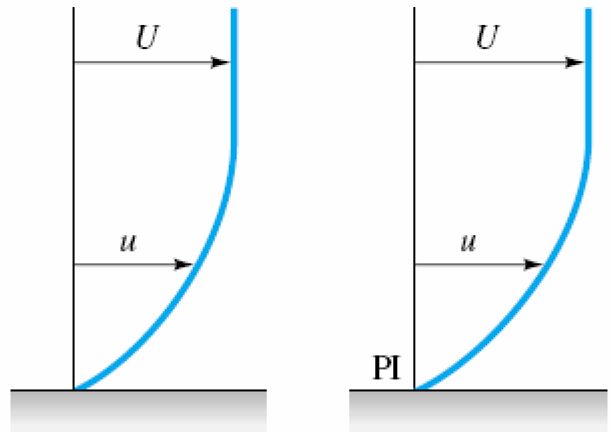
At the same Reynolds number, the above shapes have the same amount of drag.

External Flows: Separation

In a situation where pressure increases down stream the fluid particles can move up against it by virtue of its kinetic energy.

Inside the boundary layer the velocity in a layer could reduce so much that the kinetic energy of the fluid particles is no longer adequate to move the particles against the pressure gradient. This leads to flow reversal.

Since the fluid layer higher up still have energy to mover forward a rolling of fluid streams occurs, which is called **<u>separation</u>**.



(*a*) Favorable gradient:

$$\frac{dU}{dx} > 0$$

$$\frac{dp}{dx} < 0$$

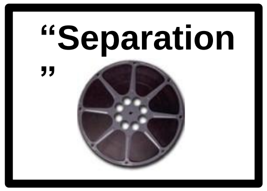
No separation, PI inside wall

(*b*) Zero gradient:

$$\frac{dU}{dx} = 0$$

$$\frac{dp}{dx} = 0$$

No separation, PI at wall



External Flows: Separation

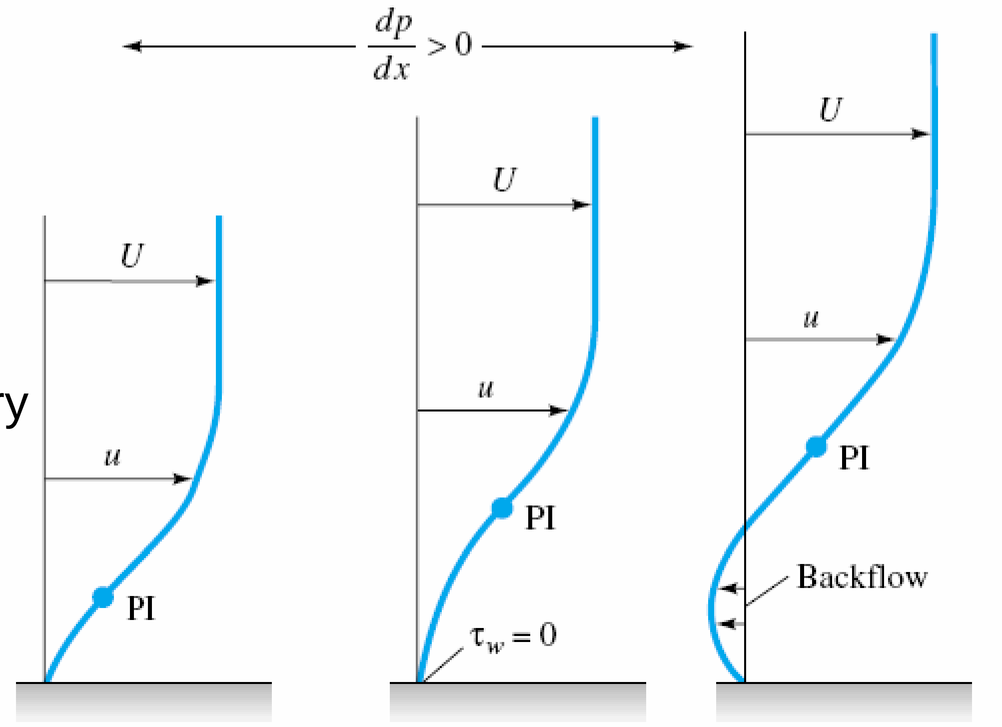
Separation starts with zero velocity gradient at the wall

Flow reversal takes place beyond separation point

Adverse pressure gradient is necessary for separation (*dp/dx* >0)

There is no pressure change after separation So, pressure in the separated region is constant.

Fluid in turbulent boundary layer has appreciably more momentum than the flow of a laminar B.L. Thus a turbulent B.L can penetrate further into an adverse pressure gradient without separation.

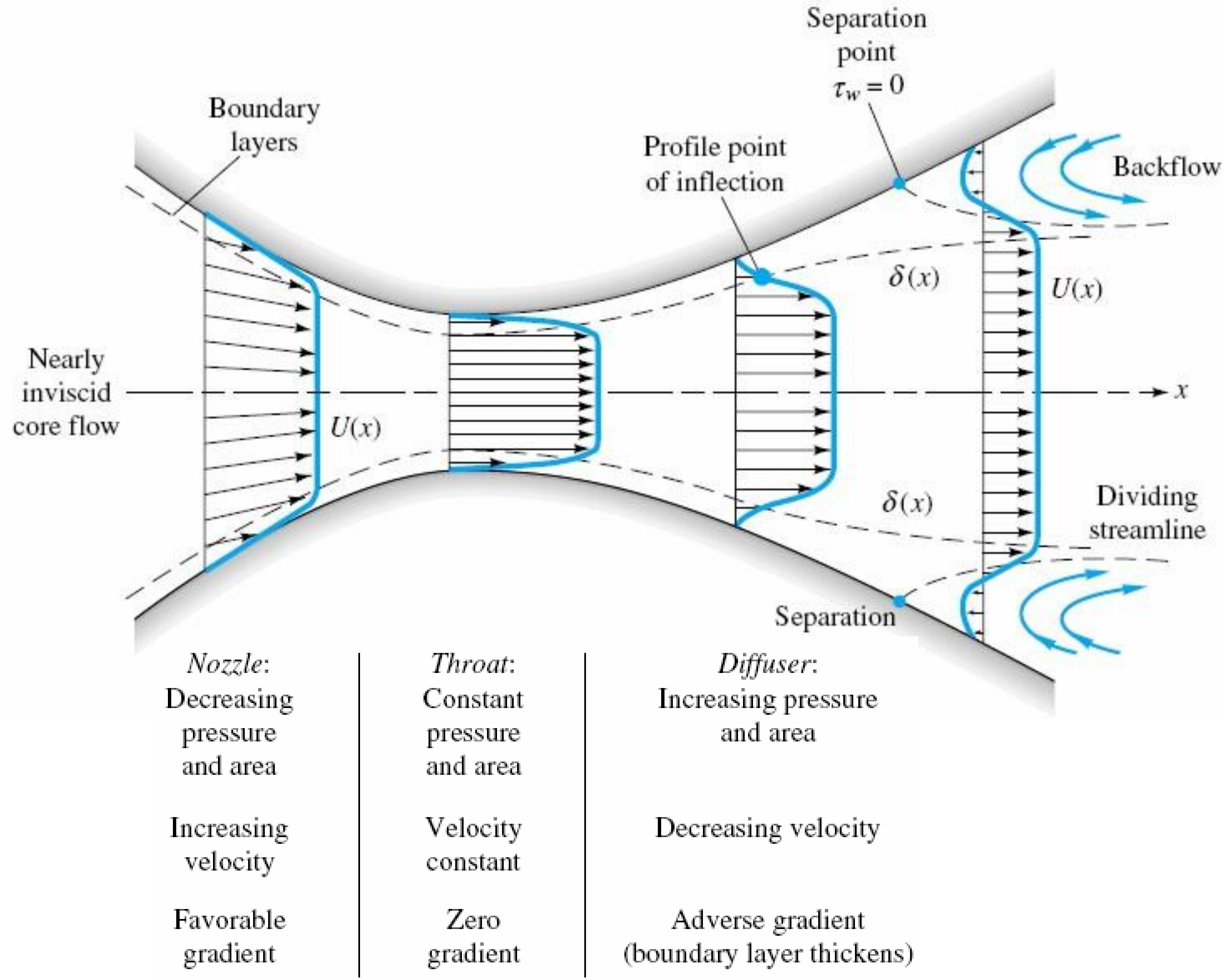


(*c*) Weak adverse gradient: $\frac{dU}{dx} < 0$ $\frac{dp}{dx} > 0$ No separation, PI in the flow

(*d*) Critical adverse gradient: Zero slope at the wall: *Separation*

(*e*) Excessive adverse gradient: Backflow at the wall: Separated flow region

External Flows: Separation



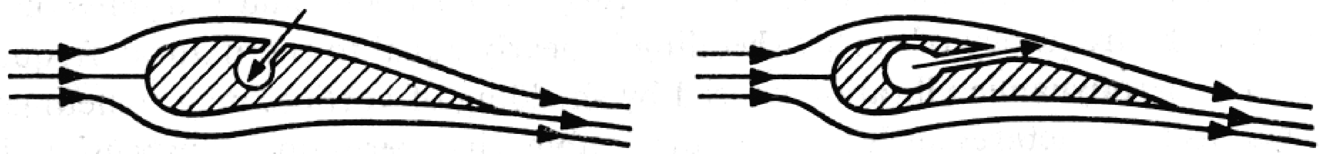
External Flows: Separation

Streamlining reduces adverse pressure gradient beyond the maximum thickness and delays separation

Fluid particles lose kinetic energy near separation point. So these are either removed by suction or higher energy

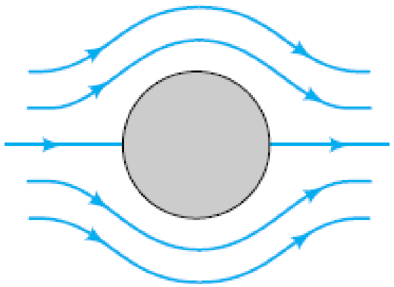
High energy fluid is blown near separation point

Roughening surface to force early transition to turbulent boundary layer



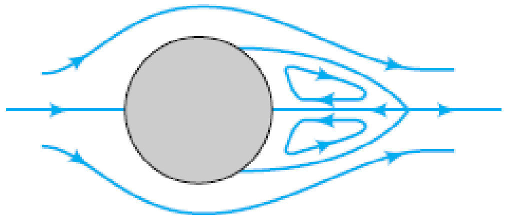
(a) Suction (b) Injection

External Flows: Drag on Immersed cylinder



No separation

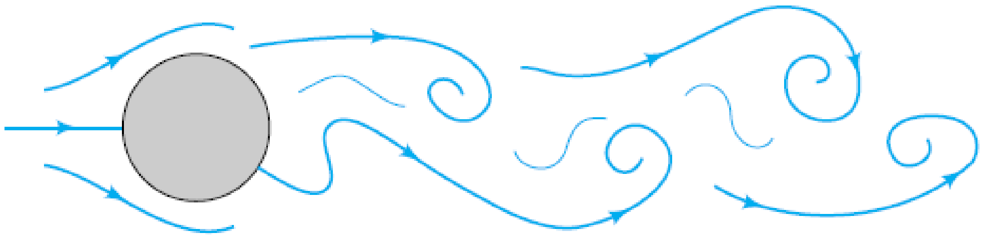
(*A*)



Steady separation bubble

(*B*)

at very small velocities (Re<0.5) the fluid sticks to the cylinder all the way round and never separate from cylinder. This produces a streamline pattern similar to that of an ideal fluid.



Oscillating Karman vortex street wake

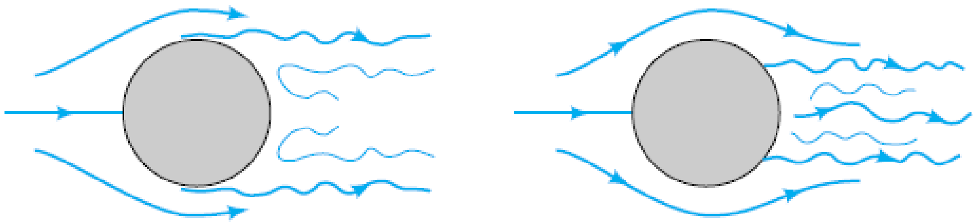
(*C*)

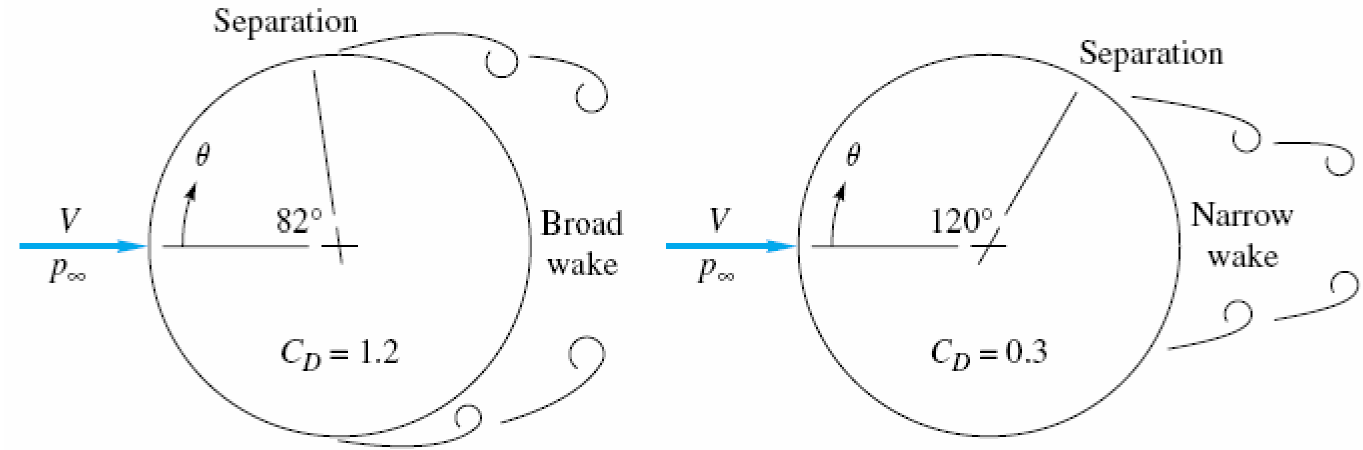
as velocity increases the boundary layer breaks away and eddies are formed behind. Further increases in velocity cause the eddies to elongate.

at Re number of around 90 the vortices break away alternatively form the top and bottom of the cylinder producing a vortex street in the wake region called **Karman vortex street**.

Note: in the laminar flow as Re number increases, the separation point moves to front.

External Flows: Drag on Immersed cylinder





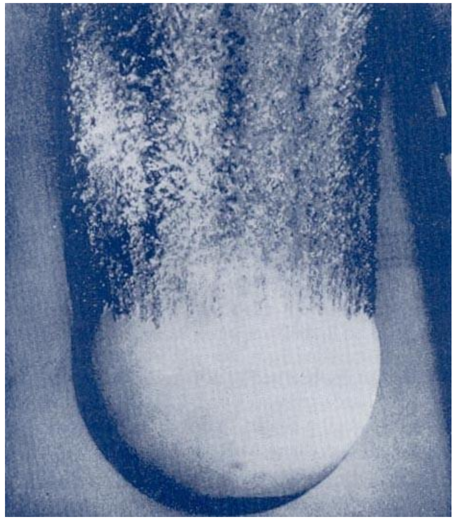
as flow within the boundary later becomes turbulent, the point of separation moves back producing a narrow wake since fluid particles have more kinetic energy (momentum) due to the nature of the turbulent flow (eddies existence.(

the friction drag is higher in the turbulent flow, but since pressure drag dominates, the net result is a significant reduction in the total drag.

External Flows: Drag on Immersed cylinder

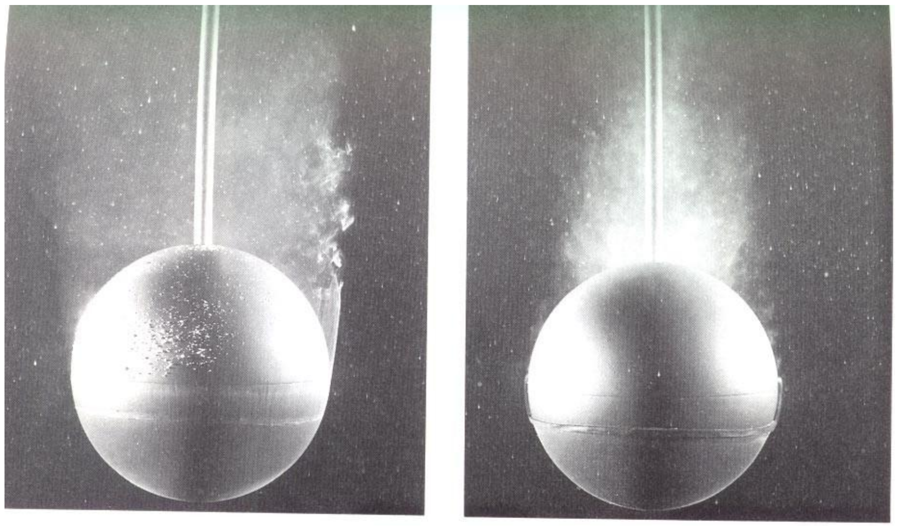
Roughness Effect:

smooth ball

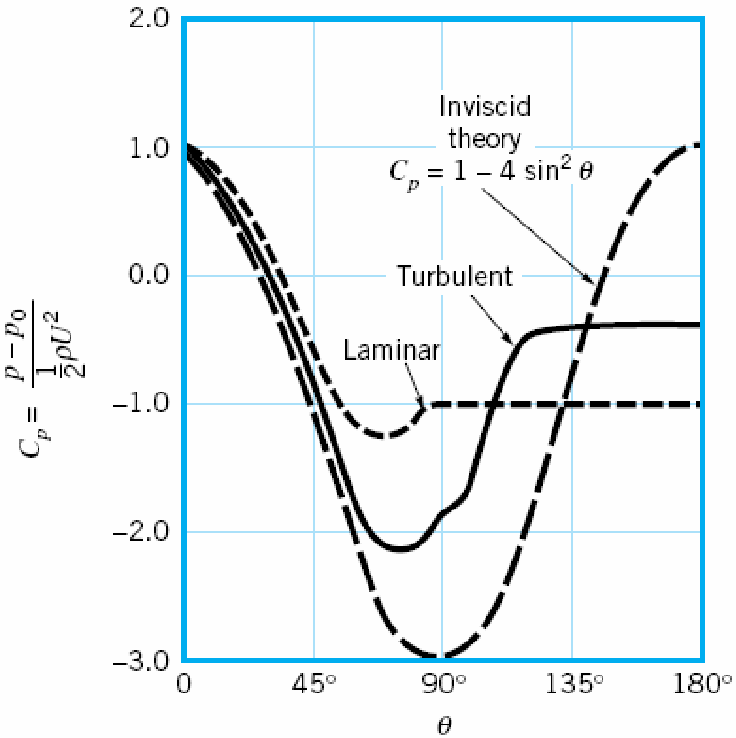


rough ball (et. Golf ball(

Wire Ring Effect:



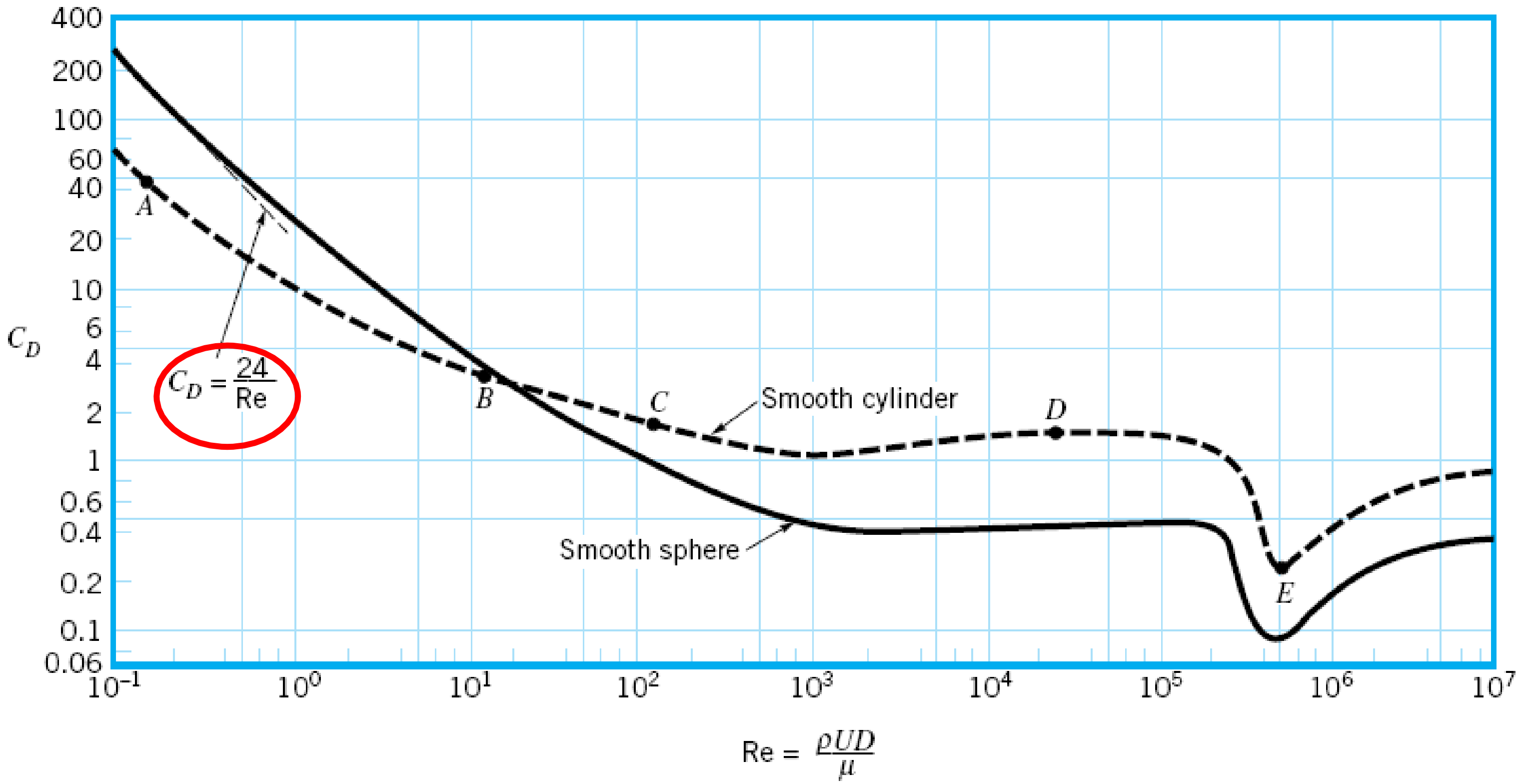
External Flows: Drag on Immersed cylinder



the front to rear pressure difference is greater for laminar flow, thus grater drag.

External Flows: Drag on Immersed Objects

Drag on a Smooth Sphere and Cylinder:



at Re<0.5 the drag coefficient is at its highest and is mainly due to skin friction.
as boundary layer becomes turbulent, a pronounced drop in the drag coefficient is produced.

External Flows: Lift on Immersed Objects

$$C_L = \frac{\mathscr{L}}{\frac{1}{2}\rho U^2 A}$$

Most all lift comes from pressure forces and not viscous forces.

Most lift generating devices are not symmetrical.

Lift can be generated by adjusting the angel of attack of the object.

Lift and drag coefficients of wings are dependent on angle of attack.

At large angles of attack, the boundary layer separates and the wing stalls.