Control Systems (CS)

Lecture-16
Construction of Root Loci (Part A)

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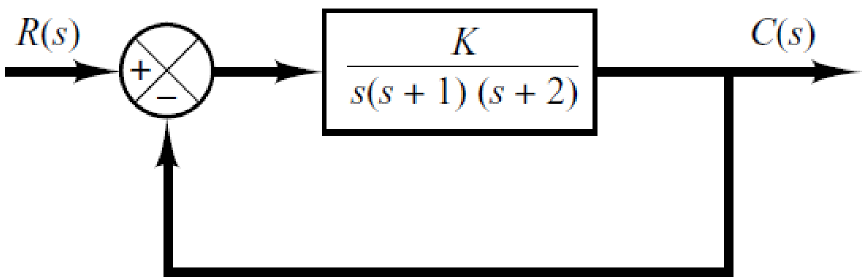
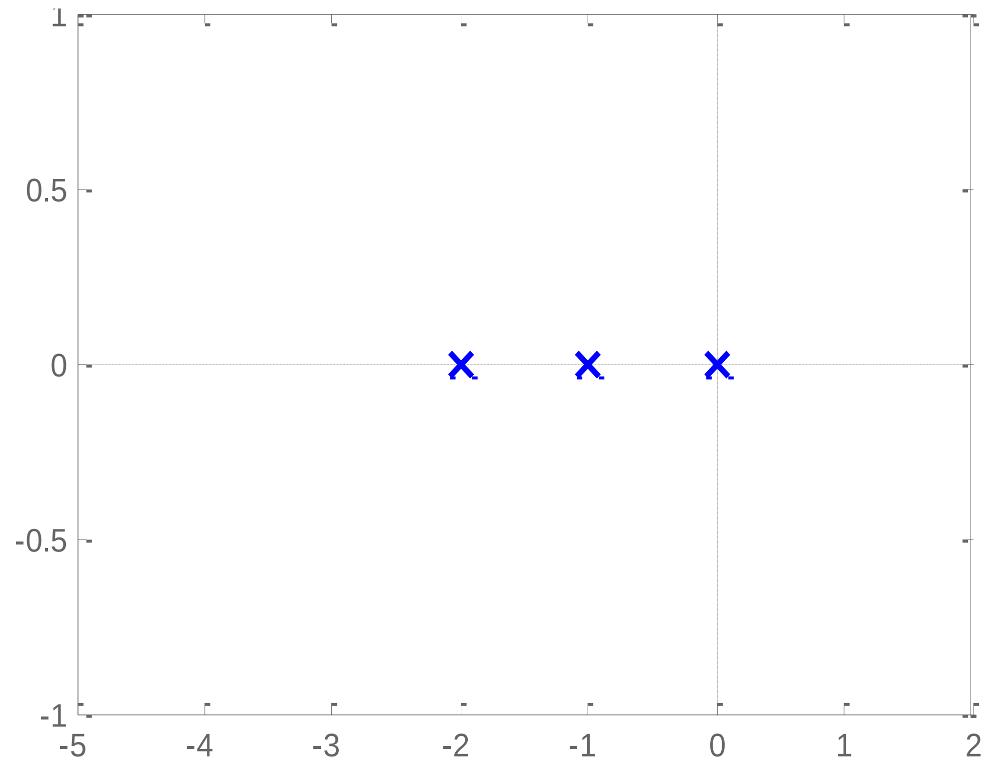
Construction of root loci

- Step-1: The first step in constructing a root-locus plot is to locate the open-loop poles and zeros in s-plane.



$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$



Construction of root loci

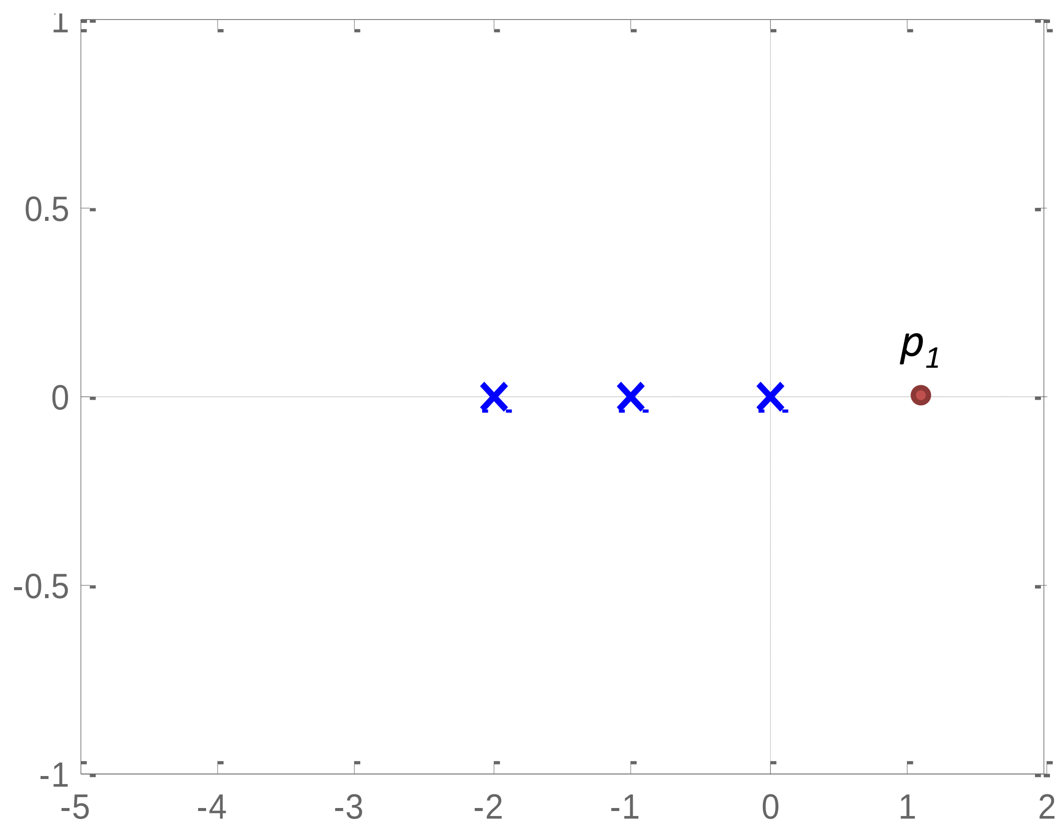
- Step-2: Determine the root loci on the real axis.

- To determine the root loci on real axis we select some test points.
- e.g: p_1 (on positive real axis).

$$\underline{/s} = \underline{/s+1} = \underline{/s+2} = 0°$$

- The angle condition is not satisfied.

- Hence, there is no root locus on the positive real axis.



Construction of root loci

- Step-2: Determine the root loci on the real axis.

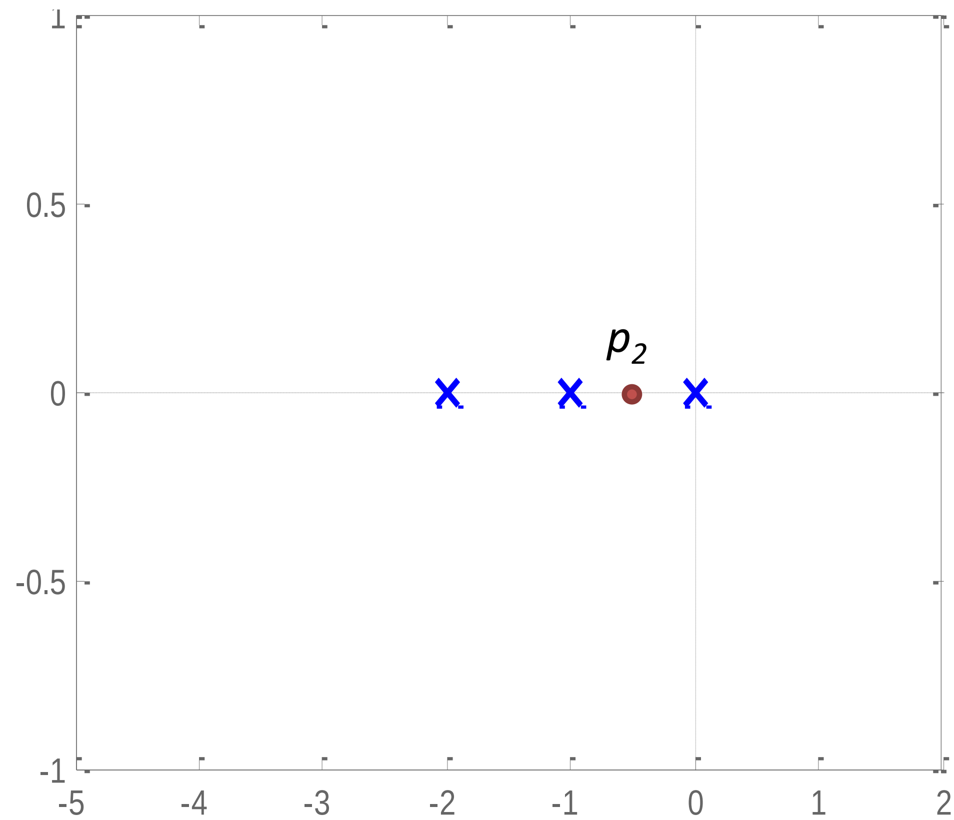
- Next, select a test point on the negative real axis between 0 and –1.
- Then

$$\underline{/s} = 180^\circ, \qquad \underline{/s+1} = \underline{/s+2} = 0^\circ$$

- Thus

$$-\underline{/s} - \underline{/s+1} - \underline{/s+2} = -180^\circ$$

- The angle condition is satisfied. Therefore, the portion of the negative real axis between 0 and –1 forms a portion of the root locus.



Construction of root loci

- Step-2: Determine the root loci on the real axis.

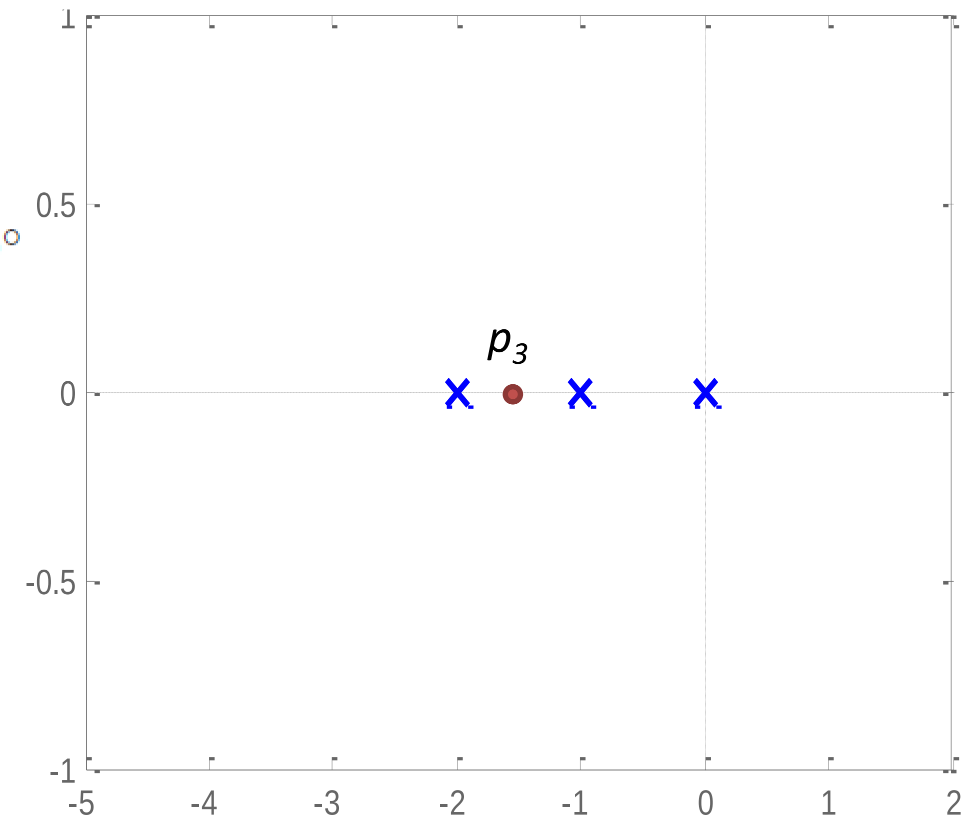
- Now, select a test point on the negative real axis between -1 and –2.
- Then

$$\underline{/s} = \underline{/s+1} = 180^\circ, \qquad \underline{/s+2} = 0^\circ$$

- Thus

$$-\underline{/s} - \underline{/s+1} - \underline{/s+2} = -360^\circ$$

- The angle condition is not satisfied. Therefore, the negative real axis between -1 and –2 is not a part of the root locus.

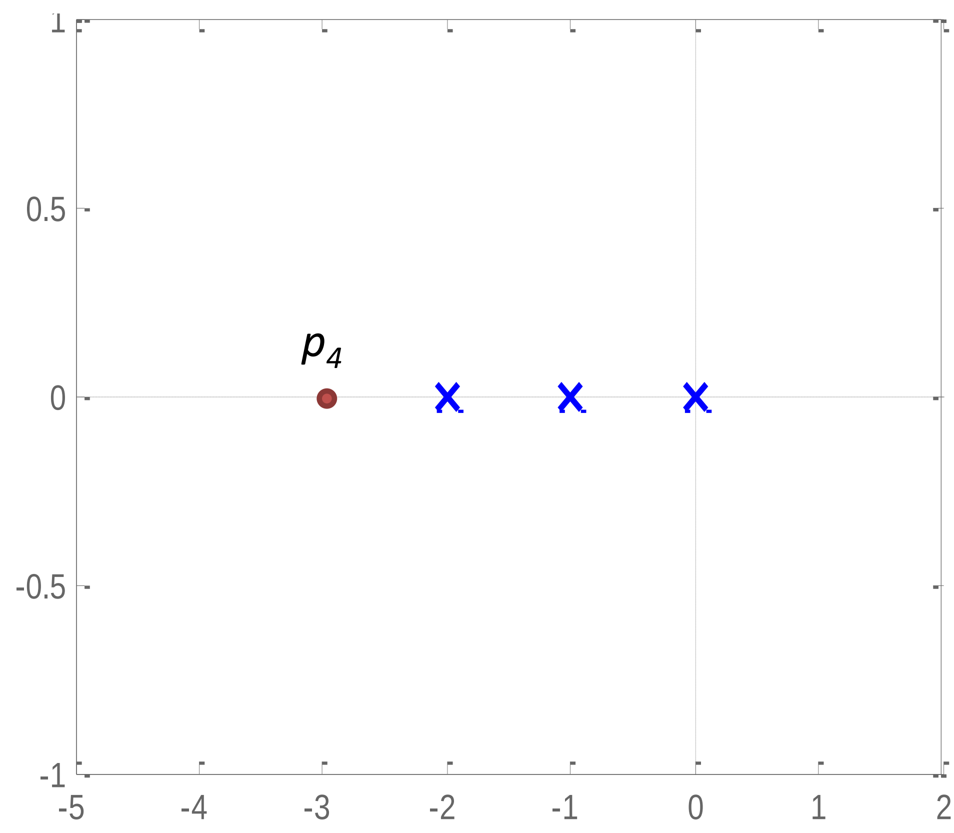


Construction of root loci

- Step-2: Determine the root loci on the real axis.

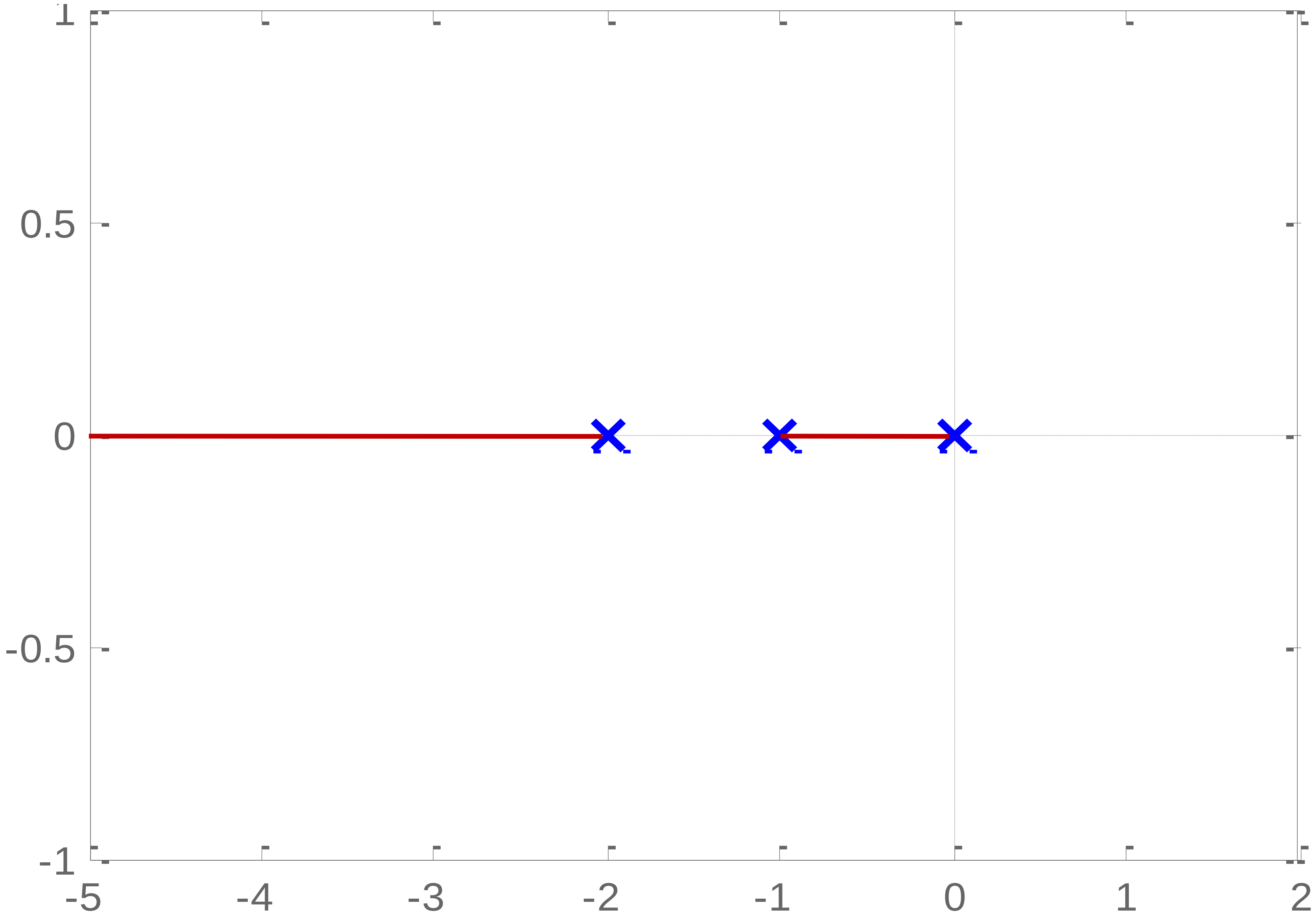
- Similarly, test point on the negative real axis between -2 and $-\infty$ satisfies the angle condition.

- Therefore, the negative real axis between -2 and $-\infty$ is part of the root locus.



Construction of root loci

- Step-2: Determine the root loci on the real axis.



Construction of root loci

- Step-3: Determine the *asymptotes* of the root loci.

$$Angle\ of\ \ asymptotes = \psi = \frac{\pm 180^\circ(2k+1)}{n-m}$$

- where
- n-----> number of poles
- m-----> number of zeros
- For this Transfer Function $G(s)H(s) = \dfrac{K}{s(s+1)(s+2)}$

$$\psi = \frac{\pm 180^\circ(2k+1)}{3-0}$$

Construction of root loci

- Step-3: Determine the *asymptotes* of the root loci.

$$\begin{aligned} \psi &= \pm 60^\circ & \text{when } k = 0 \\ &= \pm 180^\circ & \text{when } k = 1 \\ &= \pm 300^\circ & \text{when } k = 2 \\ &= \pm 420^\circ & \text{when } k = 3 \end{aligned}$$

- Since the angle repeats itself as k is varied, the distinct angles for the asymptotes are determined as 60°, –60°, -180°and 180°.
- Thus, there are three asymptotes having angles 60°, –60°, 180°.

Construction of root loci

- Step-3: Determine the *asymptotes* of the root loci.

- Before we can draw these asymptotes in the complex plane, we must find the point where they intersect the real axis.

- Point of intersection of asymptotes on real axis (or centroid of asymptotes) can be find as out

$$\sigma = \frac{\sum poles - \sum zeros}{n - m}$$

Construction of root loci

- Step-3: Determine the *asymptotes* of the root loci.

- For $G(s)H(s)=\dfrac{K}{s(s+1)(s+2)}$

$$\sigma=\frac{(0-1-2)-0}{3-0}$$

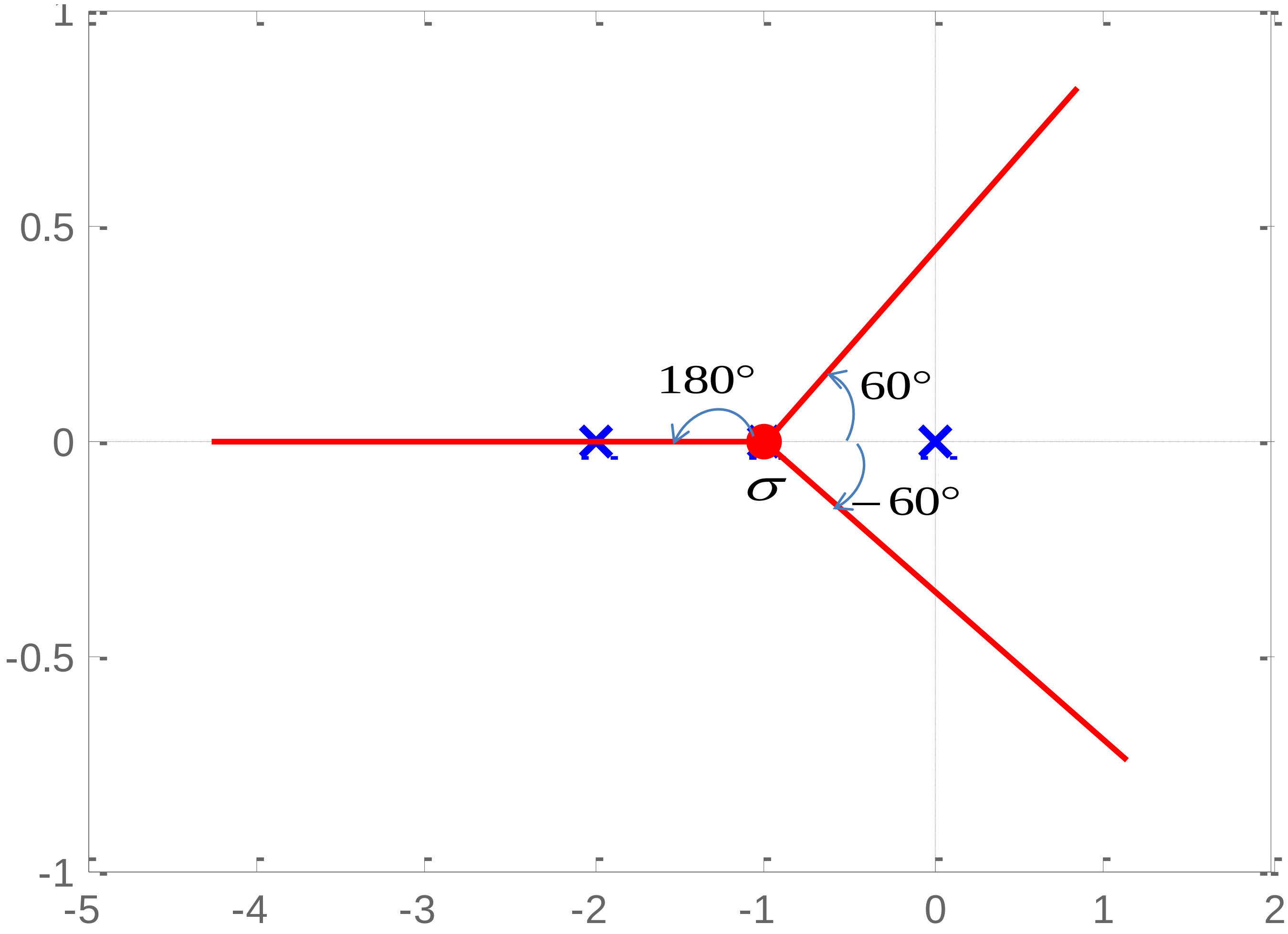
$$\sigma=\frac{-3}{3}=-1$$

Construction of root loci

- Step-3: Determine the *asymptotes* of the root loci.

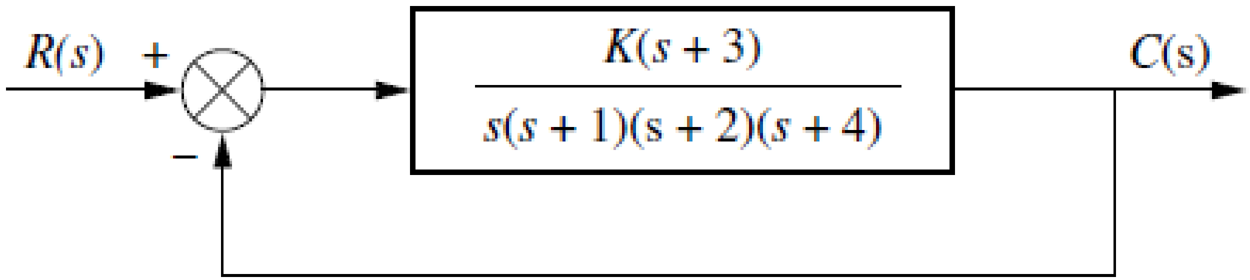
$$\psi = 60°, -60°, 180°$$

$$\sigma = -1$$



Home Work

- Consider following unity feedback system.



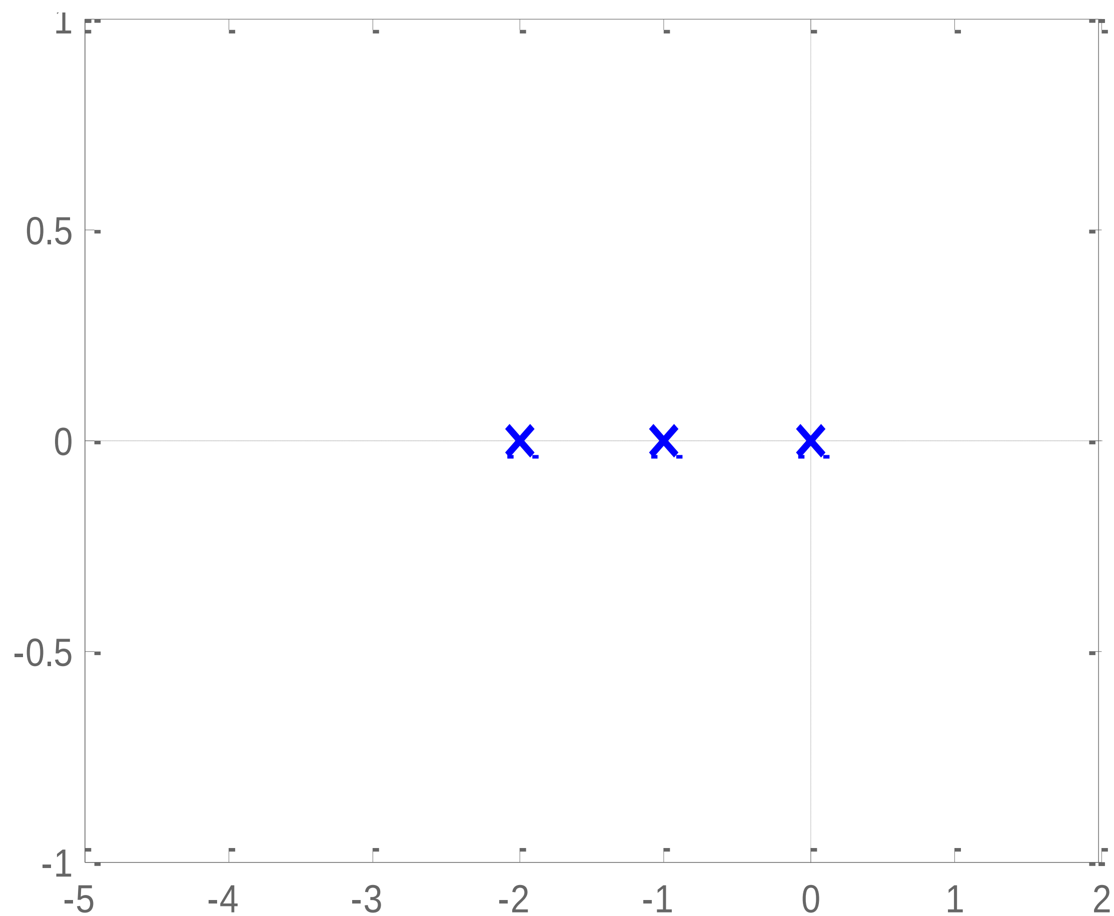
- Determine
  - Root loci on real axis
  - Angle of asymptotes
  - Centroid of asymptotes

Construction of root loci

- Step-4: Determine the *breakaway point*.

  - The breakaway point corresponds to a point in the s plane where multiple roots of the characteristic equation occur.

  - It is the point from which the root locus branches leaves real axis and enter in complex plane.

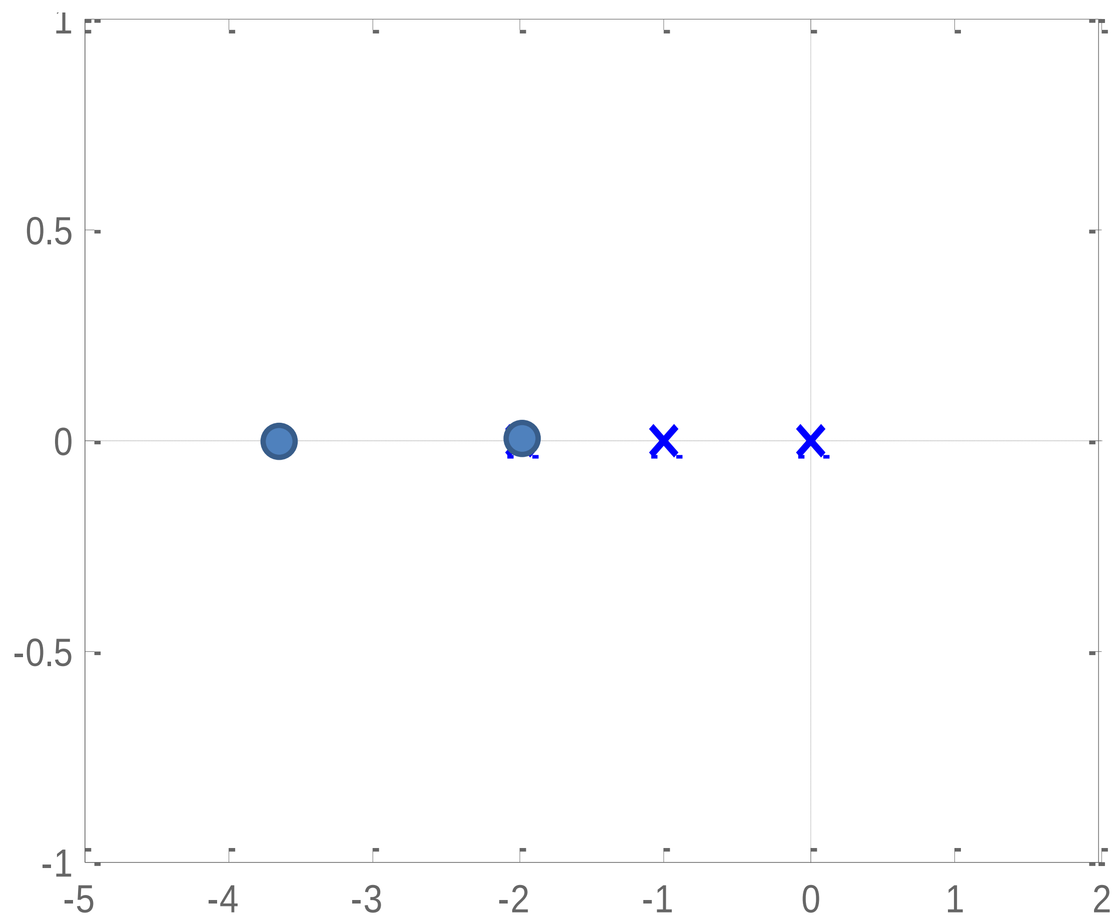


Construction of root loci

- Step-4: Determine the *break-in point*.

  - The break-in point corresponds to a point in the s plane where multiple roots of the characteristic equation occur.

  - It is the point where the root locus branches arrives at real axis.



Construction of root loci

- Step-4: Determine the *breakaway point* or *break-in point*.

  - The breakaway or break-in points can be determined from the roots of

$$\frac{dK}{ds} = 0$$

  - It should be noted that not all the solutions of dK/ds=0 correspond to actual breakaway points.

  - If a point at which dK/ds=0 is on a root locus, it is an actual breakaway or break-in point.

  - Stated differently, if at a point at which dK/ds=0 the value of K takes a real positive value, then that point is an actual breakaway or break-in point.

Construction of root loci

- Step-4: Determine the *breakaway point* or *break-in point*.

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

- The characteristic equation of the system is

$$1+G(s)H(s) = 1+\frac{K}{s(s+1)(s+2)} = 0$$

$$\frac{K}{s(s+1)(s+2)} = -1$$

$$K = -\left[s(s+1)(s+2)\right]$$

- The breakaway point can now be determined as

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s(s+1)(s+2)\right]$$

Construction of root loci

- Step-4: Determine the *breakaway point* or *break-in point*.

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s(s+1)(s+2)\right]$$

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s^3 + 3s^2 + 2s\right]$$

$$\frac{dK}{ds} = -3s^2 - 6s - 2$$

- Set *dK/ds=0* in order to determine breakaway point.

$$-3s^2 - 6s - 2 = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.4226$$

$$= -1.5774$$

Construction of root loci

- Step-4: Determine the *breakaway point* or *break-in point*.

$$s = -0.4226$$
$$= -1.5774$$

- Since the breakaway point must lie on a root locus between 0 and –1, it is clear that *s=–0.4226* corresponds to the actual breakaway point.

- Point s=–1.5774 is not on the root locus. Hence, this point is not an actual breakaway or break-in point.

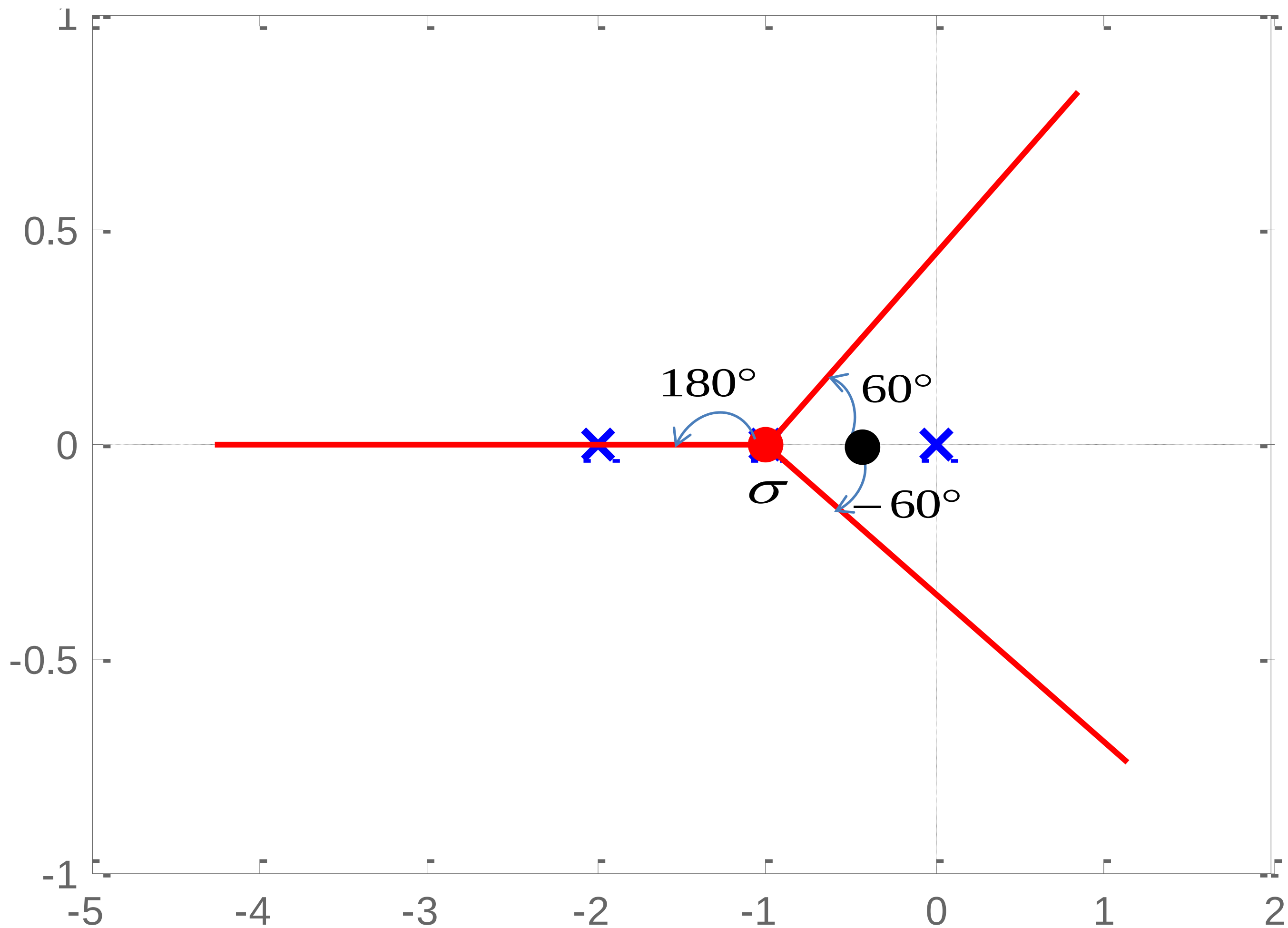
- In fact, evaluation of the values of K corresponding to s=–0.4226 and s=–1.5774 yields

$$K = 0.3849, \qquad \text{for } s = -0.4226$$
$$K = -0.3849, \qquad \text{for } s = -1.5774$$

Construction of root loci

- Step-4: Determine the *breakaway point*.

$$s = -0.4226$$



Construction of root loci

- Step-4: Determine the *breakaway point*.

$$s = -0.4226$$