CHAPTER 1

SYSTEM OF LINEAR ALGEBRIC EQUATIONS

Consider the system:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = c_1 \qquad (1.a)$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = c_2 \qquad (1.b)$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = c_n \qquad (1.c)$$

$a's, c's$ are constant and n is the number of equations, in matrix form:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

Methods of Solution

(A) Direct Methods

(B) Iteration Methods

(A) Direct Methods:

1. *Gauss Elimination:* The matrix A is reduced to an *upper* and *lower* triangular matrix. the unknown are evaluated by backward substitution i.e.

$$[A] \Rightarrow \overbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}}^{[U]} \; or \; \Rightarrow \overbrace{\begin{bmatrix} 0 & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}}^{[L]}$$

To perform the above processes:

1. Eliminate x_1 from first equation through n^{th} equations. By multiplying Eq.(1.a) by a_{21}/a_{11} and subtract the resulting eq. from Eq.(1.b).

2. Procedure is repeated for the remaining Eq. such as Eq.(1.a) is multiplied by $\frac{a_{31}}{a_{11}}$ and the resulting Eq. is subtracted from third Eq. and so on till the upper triangular matrix is obtained.
3. Back substitution to obtain $x_1, x_2, \dots x_n$ as shown.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & c_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & 0 & a''_{33} & c''_3 \end{bmatrix} \xrightarrow{back\ subs.} \begin{aligned} x_3 &= c''_3/a''_{33} \\ x_2 &= (c'_2 - a'_{23}x_3)/a'_{22} \\ x_1 &= (c_1 - a_{12}x_2 - a_{13}x_3)/a_{11} \end{aligned}$$

Example: Use Gauss Elimination method to solve the following equations (carry out six significant figures during computation

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 && (a) \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 && (b) \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 && (c) \end{aligned}$$

Solution: Multiply (a) by $0.1/3$ and subtract from (b).
Multiply (a) by $0.3/3$ and subtract from (c).

That gives:

$$\left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0 & 7.00333 & -0.293333 & -19.5617 \\ 0 & -0.19000 & 10.0200 & 70.615 \end{array}\right] \begin{array}{c} a \\ b' \\ c' \end{array}$$

Multiply (b') by $-0.19000/7.00333$ and subtract from (c') gives:

$$\left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0 & 7.00333 & -0.293333 & -19.5617 \\ 0 & 0 & 10.0120 & 70.843 \end{array}\right] \begin{array}{c} a \\ b' \\ c'' \end{array}$$

Back substitution gives:

$x_3 = 7.00003$ $\quad$ $x_2 = -2.50000$ $\quad$ $x_1 = 3.00000$

2. *Gauss-Jordan Method:*

It is similar to G.E. method for solution system of eq. $Ax = b$ but in this method the matrix A reduced to diagonal matrix instead of triangular matrix.

Example: Use Gauss-Jordan method to solve the following equations

$$\begin{aligned} 4x_1 - 9x_2 + 2x_3 &= 5 && (a) \\ 2x_1 - 4x_2 + 6x_3 &= 3 && (b) \\ x_1 - x_2 + 3x_3 &= 4 && (c) \end{aligned}$$

Solution: Multiply (1) by $2/4$ and subtract from (2).

Multiply (1) by $1/4$ and subtract from (3).

$$\left[\begin{array}{ccc|c} 4 & -9 & 2 & 5 \\ 0 & 0.5 & 5 & 0.5 \\ 0 & 1.25 & 2.5 & 2.75 \end{array}\right] \quad \begin{array}{l} \ldots 1 \\ \ldots 2' \\ \ldots 3' \end{array}$$

Eliminate x_2 from $(3')$ and (1) by multiply $(2')$ by $1.25/0.5$ and subtract from $(3')$, multily $(2')$ by $-9/0.5$ and subtract from (1) gives:

$$\left[\begin{array}{ccc|c} 4 & 0 & 92 & 14 \\ 0 & 0.5 & 5 & 0.5 \\ 0 & 0 & -10 & 1.5 \end{array}\right] \quad \begin{array}{l} \ldots 1' \\ \ldots 2' \\ \ldots 3'' \end{array}$$

Eliminate x_3 from $(1')$ and $(2'')$

Multiply $(3'')$ by $92/-10$ and subtract from $(1')$.

Multiply $(3'')$ by $5/-10$ and subtract from $(2')$.

That gives:

$$\left[\begin{array}{ccc|c} 4 & 0 & 0 & 27.8 \\ 0 & 0.5 & 0 & 1.25 \\ 0 & 0 & -10 & 1.5 \end{array}\right]$$

Then

$x_3 = 27.8/4 = 6.95$ $\qquad$ $x_2 = 2.5$ $\qquad$ $x_1 = -0.15$

3. ***Matrix Inversion By Gauss Method:***

This method start with

a) $\left[\begin{array}{c|c} A & I \end{array}\right] \xrightarrow{\text{Gauss Elim.}} \left[\begin{array}{c|c} U \ or \ L & New\ matrix \end{array}\right]$

b) Back substitution

c) $\left[\; U \ or \ L \;\right] x_1 = \left[\begin{array}{c} 1^{st}\ column\ of \\ New\ matrix \end{array}\right] \Rightarrow First\ column\ of\ A^{-1}$

$\left[\; U \ or \ L \;\right] x_2 = \left[\begin{array}{c} 2^{nd}\ column\ of \\ New\ matrix \end{array}\right] \Rightarrow Second\ column\ of\ A^{-1}$

Example: Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 8 & -2 \\ -6 & 49 & -10 \\ -4 & 34 & -5 \end{bmatrix}$$

Solution:

$$\left[\begin{array}{ccc|ccc} -1 & 8 & -2 & 1 & 0 & 0 \\ -6 & 49 & -10 & 0 & 1 & 0 \\ -4 & 34 & -5 & 0 & 0 & 1 \end{array}\right] \overset{G.E.}{\Longrightarrow} \left[\begin{array}{ccc|ccc} -1 & 8 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -6 & 1 & 0 \\ 0 & 0 & -1 & 8 & -2 & 1 \end{array}\right]$$

$$\begin{bmatrix} -1 & 8 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} x_1 = \begin{bmatrix} 1 \\ -6 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 95 \\ 10 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 8 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} x_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -28 \\ -3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 8 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 18 \\ 2 \\ -1 \end{bmatrix}$$

Then

$$A^{-1} = \begin{bmatrix} 95 & -28 & 18 \\ 10 & -3 & 2 \\ -8 & 2 & -1 \end{bmatrix}$$

4. ***Matrix Inversion By Gauss-Jordan Method:***

In this method the matrix A is reduced to an identify matrix i.e.

$$\left[\begin{array}{c|c} A & I \end{array}\right] \Rightarrow \left[\begin{array}{c|c} I & A^{-1} \end{array}\right]$$

Example: Find the inverse of the matrix

$$\begin{bmatrix} 4 & -9 & 2 \\ 2 & -4 & 6 \\ 1 & -1 & 3 \end{bmatrix}$$

Solution:

$$\left[\begin{array}{c|c} A & I \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc|ccc} 4 & -9 & 2 & 1 & 0 & 0 \\ 2 & -4 & 6 & 0 & 1 & 0 \\ 1 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 4 & -9 & 2 & 1 & 0 & 0 \\ 0 & 0.5 & 5 & -0.5 & 1 & 0 \\ 0 & 1.25 & 2.5 & -0.25 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 92 & -8 & 18 & 0 \\ 0 & 0.5 & 5 & -0.5 & 1 & 0 \\ 0 & 0 & -10 & 1 & -2.5 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 1.2 & -5 & 9.2 \\ 0 & 0.5 & 0 & 0 & -0.25 & 0.5 \\ 0 & 0 & -10 & 1 & -2.5 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.3 & -1.25 & 2.3 \\ 0 & 1 & 0 & 0 & -0.5 & 1 \\ 0 & 0 & 1 & -0.1 & 0.25 & -0.1 \end{array}\right] \Leftrightarrow \left[\begin{array}{c|c} I & A^{-1} \end{array}\right]$$

Or $$A^{-1} = \begin{bmatrix} 0.3 & -1.25 & 2.3 \\ 0 & -0.5 & 1 \\ -0.1 & 0.25 & -0.1 \end{bmatrix}$$

5. ***Choleski's Decomposition Process***

A square matrix A is expressed as the product of LU i.e.

$$[A] = [L][U]$$

To find $[L]$ and $[U]$, the above matrices can be represented by:

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}$$

From which;

$$U_{11} = a_{11} \qquad U_{12} = a_{12}, \qquad U_{13} = a_{13}$$

$$L_{21} = \frac{a_{21}}{U_{11}} = \frac{a_{21}}{a_{11}}, \qquad L_{31} = \frac{a_{31}}{U_{11}} = \frac{a_{31}}{a_{11}}, \qquad U_{22} = a_{22} - L_{21}U_{12}$$

$$U_{23} = a_{23} - L_{21}U_{13} \qquad L_{32} = \frac{(a_{32} - L_{31}U_{12})}{U_{22}} \qquad U_{33} = a_{33} - L_{31}U_{13} - L_{32}U_{23}$$

And generally;

$U_{1j} = a_{1j}$

$L_{i1} = a_{i1}/U_{11}$

$U_{ij} = a_{ij} - \sum_{k=1}^{j-1} L_{1k}U_{kj}$ $\qquad 1 \le i \le j$

$L_{ij} = \left(a_{ij} - \sum_{k=1}^{j-1} L_{1k}U_{kj}\right)/U_{jj}$ $\qquad i \ge j > 1$

Example: Express the following matrix in LU form

$$A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 5 \\ 2 & -3 & -4 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 5 \\ 2 & -3 & -4 \end{bmatrix} \xrightarrow{resulting\ matrix} \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 1/2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -2 \\ 0 & -3/4 & 11/2 \\ 0 & 0 & -36 \end{bmatrix}$$

5.1. Application of Choleski's Decomposition to Solution of Simultaneous Linear Equations

If the matrix A is decomposes into LU then any equation such as $[A][x] = [B]$, where A is a square matrix $(n \times n)$, can be written in the form.

$$[L][U][x] = [B]$$

Then the equations are solved as follows:

1. $[L][Y] = [B]$
2. $[U][x] = [Y]$

The second Eq. is written in the form;

$$L_{11}Y_1 = B_1$$
$$L_{21}Y_1 + L_{22}Y_2 = B_2$$
$$L_{31}Y_1 + L_{32}Y_2 + L_{33}Y_3 = B_3$$

Which give the values of Y by forward substitution then the first Eq. can be written as

$$U_{11}x_1 + U_{12}x_2 + \cdots + U_{1n}x_n = Y_1$$
$$U_{22}x_2 + U_{23}x_3 + \cdots + U_{2n}x_n = Y_2$$
$$\vdots$$
$$U_{nn}x_n = Y_n$$

Which give the value of x by backward substitution.

Example: Solve the following set

$$2x_1 + x_3 = 4$$
$$-3x_1 + 4x_2 - 2x_3 = -3$$
$$x_1 + 7x_2 - 5x_3 = 6$$

Solution:

The matrix form;

$$\begin{bmatrix} 2 & 0 & 1 \\ -3 & 4 & -2 \\ 1 & 7 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix}$$

$[L][U][x] = [B]$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ -1.5 & 1 & 0 \\ 0.5 & 1.75 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & -0.5 \\ 0 & 0 & -4.625 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix}$$

$[L][Y] = [B]$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1.5 & 1 & 0 \\ 0.5 & 1.75 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix} \overset{gives}{\Longrightarrow} Y_1 = 4, \qquad Y_2 = 3, \qquad Y_3 = -1.25$$

$[U][x] = [Y]$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & -0.5 \\ 0 & 0 & -4.625 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -1.25 \end{bmatrix}$$

$$\therefore \quad x_1 = \frac{69}{37}, \qquad x_2 = \frac{29}{37}, \qquad x_3 = \frac{10}{37}$$

5.1. Matrix Inversion by Choleski's Decomposition

Example: Find the inverse of the matrix

$$A = \begin{bmatrix} 0.7 & -5.4 & 1.0 \\ 3.5 & 2.2 & 0.8 \\ 1.0 & -1.5 & 4.3 \end{bmatrix}$$

$\big[A\big]\big[x\big] = \big[B\big]$ Where $\big[B\big]$ is the identity matrix.

The resulting LU matrices are

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 1.0 & 0.213 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & -5.4 & 1.0 \\ 0 & 29.2 & -4.2 \\ 0 & 0 & 3.75 \end{bmatrix}$$

$\big[L\big]\big[Y\big] = \big[B\big]$ Will be

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 1.0 & 0.213 & 1 \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This gives the values of $\big[Y\big] = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -0.38 & -0.21 & 1 \end{bmatrix}$

And $\big[U\big]\big[x\big] = \big[Y\big]$ becomes

$$\begin{bmatrix} 0.7 & -5.4 & 1.0 \\ 0 & 29.2 & -4.2 \\ 0 & 0 & 3.75 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -0.38 & -0.21 & 1 \end{bmatrix}$$

Which gives complete values of $\begin{bmatrix} x \end{bmatrix}$

$$\begin{bmatrix} 0.11 & 0.32 & -0.08 \\ -0.19 & 0.03 & -0.04 \\ -0.1 & -0.06 & 0.27 \end{bmatrix} \Rightarrow \begin{bmatrix} A \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 1.003 & 0.002 & -0.02 \\ -0.113 & 1.138 & 0.024 \\ 0.035 & 0.017 & 1.021 \end{bmatrix}$$

(B)<u>Iterative Methods:</u>

1.<u>Jacobi Iteration Method:</u> Also called simulation displacement.

Consider the system;

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

Which are arranged for solution in the form;

$$x_1 = \frac{1}{a_{11}}\langle b_1 - a_{12}x_2 - a_{13}x_3 - \cdots - a_{1n}x_n\rangle$$
$$x_2 = \frac{1}{a_{22}}\langle b_2 - a_{21}x_1 - a_{23}x_3 - \cdots - a_{2n}x_n\rangle$$
$$\vdots$$
$$x_n = \frac{1}{a_{nn}}\langle b_n - a_{n1}x_1 - a_{n2}x_2 - \cdots - a_{n(n-1)}x_{n-1}\rangle$$

For initial guesses put all $(x's)$ zero

Noted as $x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \ldots, x_n^{(1)}$ and substitution them into the right side of the above equation, a new set $x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, \ldots, x_n^{(2)}$ can be calculated.

<u>Example:</u> Solve the following set

$$3x_1 + x_2 + x_3 = 10$$
$$x_1 + 5x_2 + 2x_3 = 21$$
$$x_1 + 2x_2 + 5x_3 = 30$$

<u>Solution:</u>

$$x_1^n = \frac{1}{3}\left(10 - x_2^{(n-1)} - x_3^{(n-1)}\right)$$
$$x_2^n = \frac{1}{5}\left(21 - x_1^{(n-1)} - 2x_3^{(n-1)}\right)$$
$$x_3^n = \frac{1}{5}\left(30 - x_1^{(n-1)} - 2x_2^{(n-1)}\right)$$
$$x_1^{(1)} = \frac{b_1}{a_{11}} = \frac{10}{3}$$

$$x_2^{(1)} = \frac{b_2}{a_{22}} = \frac{21}{5}$$

$$x_3^{(1)} = \frac{b_3}{a_{33}} = \frac{30}{5}$$

$$x_1^{(2)} = \frac{1}{3}\left(10 - \frac{21}{5} - \frac{30}{5}\right) = -0.067$$

$$x_2^{(2)} = \frac{1}{5}\left(21 - \frac{10}{3} - 2 * \frac{30}{5}\right) = 1.133$$

$$x_3^{(2)} = \frac{1}{5}\left(30 - \frac{10}{3} - 2 * \frac{21}{5}\right) = 3.653$$

$\vdots$

Continue till

$x_1^{(17)} = 1.001,\ \ x_2^{(17)} = 2.001,\ \ x_3^{(17)} = 5.001$

And $\ x_1^{(18)} = 0.999,\ \ x_2^{(18)} = 2.000,\ \ x_3^{(18)} = 5.000$

2. <u>Gauss-Seidel Iteration Method:</u> To compare with the previous example, rearrange the equation

$$x_1^n = \frac{1}{3}\left(10 - x_2^{(n-1)} - x_3^{(n-1)}\right)$$

$$x_2^n = \frac{1}{5}\left(21 - x_1^{(n)} - 2x_3^{(n-1)}\right)$$

$$x_3^n = \frac{1}{5}\left(30 - x_1^{(n)} - 2x_2^{(n)}\right)$$

$$x_1^{(1)} = \frac{b_1}{a_{11}} = \frac{10}{3}$$

$$x_2^{(1)} = \frac{b_2}{a_{22}} = \frac{21}{5}$$

$$x_3^{(1)} = \frac{b_3}{a_{33}} = \frac{30}{5}$$

Substitution in the arranged equation

$$x_1^{(2)} = \frac{1}{3}\left(10 - \frac{21}{5} - \frac{30}{5}\right) = -0.067$$

$$x_2^{(2)} = \frac{1}{5}\left(21 + 0.067 - 2 * \frac{30}{5}\right) = 1.813$$

$$x_3^{(2)} = \frac{1}{5}(30 + 0.067 - 2 * 1.813) = 5.288$$

And so on till

$x_1^{(6)} = 1.001,\ \ x_2^{(6)} = 2.000,\ \ x_3^{(6)} = 5.000$

And $\ x_1^{(7)} = 1.000,\ \ x_2^{(7)} = 2.000,\ \ x_3^{(7)} = 5.000$

H.W: Solve the set using;

1. Jacobi Iteration method
2. Gauss-Seidel Iteration method

$$10.27x_1 - 1.23x_2 + 0.67x_3 = 4.27$$
$$2.39x_1 - 12.62x_2 + 1.13x_3 = 1.26$$
$$1.79x_1 + 3.61x_2 + 15.11x_3 = 12.71$$