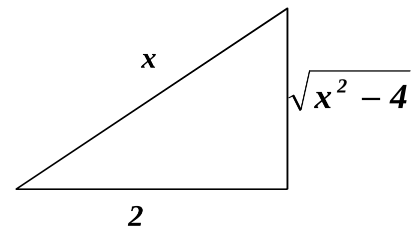
i) $x = a.Sin\theta \Rightarrow Sin\theta = \frac{x}{a} \Rightarrow Csc\theta = \frac{a}{x}$

$y = b\,tan\,\theta \Rightarrow tan\,\theta = \frac{y}{b} \Rightarrow Cot\theta = \frac{b}{y}$

Since $Csc^2\theta = Cot^2\theta + 1 \Rightarrow \frac{a^2}{x^2} = \frac{b^2}{y^2} + 1$



ii) $x = 2\,Sec\,\theta \Rightarrow Cos\,\theta = \frac{2}{x}$

$y = Cos\,2\theta \Rightarrow y = Cos^2\theta - Sin^2\theta$

$y = \frac{4}{x^2} - \frac{x^2 - 4}{x^2} \Rightarrow x^2 y = 8 - x^2$

EX-7- If $tan^2\theta - 2\,tan^2\beta = 1$, show that $2\,Cos^2\theta - Cos^2\beta = 0$.

Sol. –

$tan^2\,\theta - 2\,tan^2\,\beta = 1 \Rightarrow Sec^2\theta - 1 - 2(\,Sec^2\beta - 1\,) = 1$

$\Rightarrow Sec^2\theta - 2Sec^2\beta = 0 \Rightarrow \frac{1}{Cos^2\theta} - \frac{2}{Cos^2\beta} = 0$

$\Rightarrow 2Cos^2\theta - Cos^2\beta = 0$ Q.E.D.

EX-8- If $a\,Sin\theta = p - b\,Cos\theta$ and $b\,Sin\theta = q + a\,Cos\theta$ .Show that :
$a^2 + b^2 = p^2 + q^2$

Sol.-

$p = a.Sin\theta + b.Cos\theta$ and $q = b.Sin\theta - a.Cos\theta$

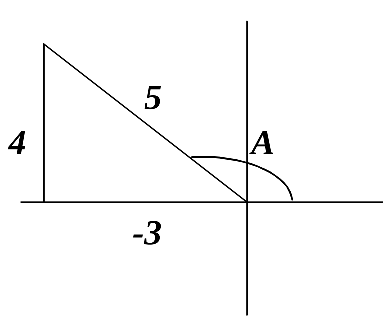
$p^2 + q^2 = (aSin\theta + bCos\theta)^2 + (bSin\theta - aCos\theta)^2$

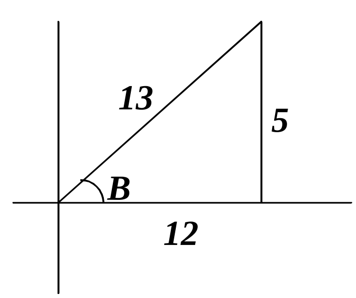
$= a^2(\,Sin^2\theta + Cos^2\theta\,) + b^2(Cos^2\theta + Sin^2\theta\,) = a^2 + b^2$

EX-9- If $Sin\,A = 4\,/\,5$ and $Cos\,B = 12\,/\,13$,where A is obtuse and B is acute . Find , without tables , the values of :

a) $Sin\,(\,A - B\,)$, b) $tan\,(\,A - B\,)$, c) $tan\,(\,A + B\,)$.

Sol. -





a) $Sin(A-B) = SinA.CosB - CosA.SinB$

$$= \frac{4}{5}.\frac{12}{13} + \frac{3}{5}.\frac{5}{13} = \frac{63}{65}$$

b) $tan(A-B) = \dfrac{tan A - tan B}{1 + tan A.tan B}$

$$= \frac{-\frac{4}{3} - \frac{5}{12}}{1 - \frac{4}{3}.\frac{5}{12}} = -\frac{63}{16}$$

c) $tan(A+B) = \dfrac{tan A + tan B}{1 - tan A.tan B}$

$$= \frac{-\frac{4}{3} + \frac{5}{12}}{1 + \frac{4}{3}.\frac{5}{12}} = \frac{33}{56}$$

<u>*EX-10*</u> – Prove the following identities:

a) $Sin(A+B) + Sin(A-B) = 2.SinA.CosB$

b) $tan A + tan B = \dfrac{Sin(A+B)}{CosA.CosB}$

c) $Sec(A+B) = \dfrac{SecA.SecB.CscA.CscB}{CscA.CscB - SecA.SecB}$

d) $\dfrac{Sin2\theta + Cos2\theta + 1}{Sin2\theta - Cos2\theta + 1} = Cot\theta$

Sol.-

a) L.H.S. $= Sin(A+B)+Sin(A-B)$

$$= SinA.CosB + CosA.SinB + SinA.CosB - CosA.SinB$$

$$= 2.SinA.CosB = R.H.S.$$

b) R.H.S. $= \dfrac{Sin(A+B)}{CosA.CosB} = \dfrac{SinA.CosB + CosA.SinB}{CosA.CosB}$

$$= tan\,A + tan\,B = L.H.S.$$

c) R.H.S $= \dfrac{SecA.SecB.CscA.CscB}{CscA.CscB - SecA.SecB} = \dfrac{\frac{1}{CosA}.\frac{1}{CosB}.\frac{1}{SinA}.\frac{1}{SinB}}{\frac{1}{SinA}.\frac{1}{SinB} - \frac{1}{CosA}.\frac{1}{CosB}}$

$$= \frac{1}{CosA.CosB - SinA.SinB} = \frac{1}{Cos(A+B)}$$

$$= Sec(A+B) = L.H.S.$$

d) L.H.S. $= \dfrac{Sin2\theta + Cos2\theta + 1}{Sin2\theta - Cos2\theta + 1} = \dfrac{2Sin\theta.Cos\theta + (Cos^2\theta - Sin^2\theta) + 1}{2Sin\theta.Cos\theta - (Cos^2\theta - Sin^2\theta) + 1}$

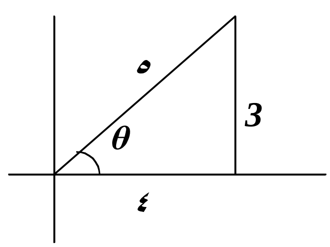
$$= \frac{2Sin\theta.Cos\theta + 2Cos^2\theta}{2Sin\theta.Cos\theta + 2Sin^2\theta} = \frac{Cos\theta}{Sin\theta} = Cot\theta = R.H.S.$$

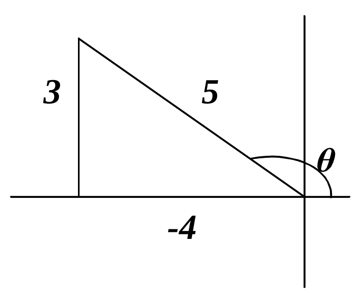
EX-11 **– Find , without using tables , the values of $Sin\ 2\theta$ and $Cos\ 2\theta$, when:**

a) $Sin\theta = 3/5$, **b)** $Cos\ \theta = 12/13$, **c)** $Sin\ \theta = -\sqrt{3}/2$.

Sol. –

a)

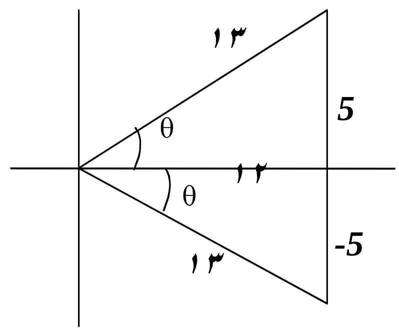




$$Sin2\theta = 2.Sin\theta.Cos\theta = 2.\frac{3}{5}.(\mp\frac{4}{5}) = \mp\frac{24}{25}$$

$$Cos2\theta = Cos^2\theta - Sin^2\theta = (\mp\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{7}{25}$$

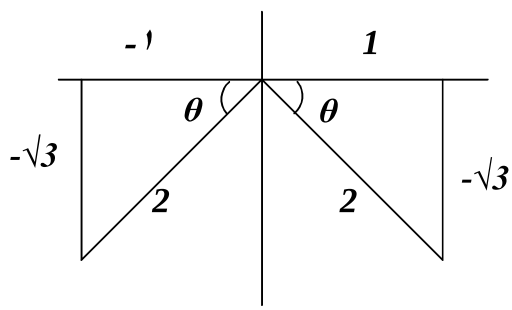
b)



$$Sin2\theta = 2.Sin\theta.Cos\theta = 2(\mp\frac{5}{13}).(\frac{12}{13}) = \mp\frac{120}{169}$$

$$Cos2\theta = Cos^2\theta - Sin^2\theta = (\frac{12}{13})^2 - (\mp\frac{5}{13})^2 = \frac{119}{169}$$

c)



$$Sin2\theta = 2Sin\theta.Cos\theta = 2(-\frac{\sqrt{3}}{2}).(\mp\frac{1}{2}) = \pm\frac{\sqrt{3}}{2}$$

$$Cos2\theta = Cos^2\theta - Sin^2\theta = (\mp\frac{1}{2})^2 - (-\frac{\sqrt{3}}{2})^2 = -\frac{1}{2}$$

EX-12- **Solve the following equations for values of θ from 0^o to 360^o inclusive:**

a) $Cos\ 2\theta + Cos\ \theta + 1 = 0$, **b)** $4\ tan\ \theta\ .\ tan\ 2\theta = 1$

Sol.-

$$a)\quad Cos2\theta + Cos\theta + 1 = 0 \Rightarrow 2Cos^2\theta - 1 + Cos\theta + 1 = 0$$

$$\Rightarrow Cos(2.Cos\theta + 1) = 0$$

$$either \quad Cos\theta = 0 \Rightarrow \theta = 90^o, 270^o$$

$$or \quad Cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^o, 240^o$$

$$\theta = \{90^o, 120^o, 240^o, 270^o\}$$

$$b)\quad 4.\tan\theta.\tan 2\theta = 1 \Rightarrow 4.\tan\theta.\frac{2\tan\theta}{1-\tan^2\theta} = 1$$

$$\Rightarrow 9\tan^2\theta = 1$$

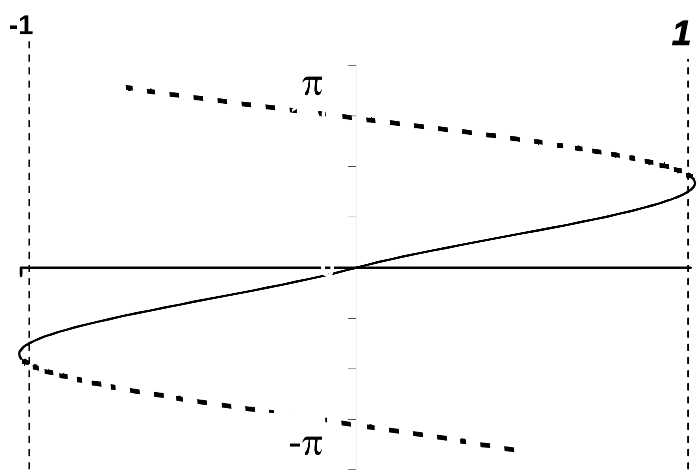
$$either \quad \tan\theta = \frac{1}{3} \Rightarrow \theta = 18.4^o, 198.4^o$$

$$or \quad \tan\theta = -\frac{1}{3} \Rightarrow \theta = 161.6^o, 341.6^o$$

$$\theta = \{18.4^o, 161.6^o, 198.4^o, 341.6^o\}$$

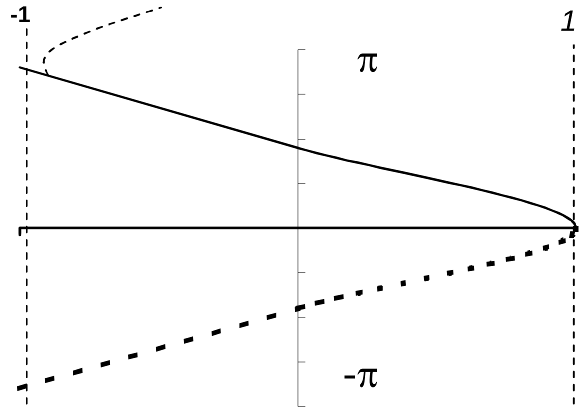
2-3- <u>The inverse trigonometric functions</u> : The inverse trigonometric functions arise in problems that require finding angles from side measurements in triangles :

$$y = Sinx \quad \Leftrightarrow \quad x = Sin^{-1}y$$

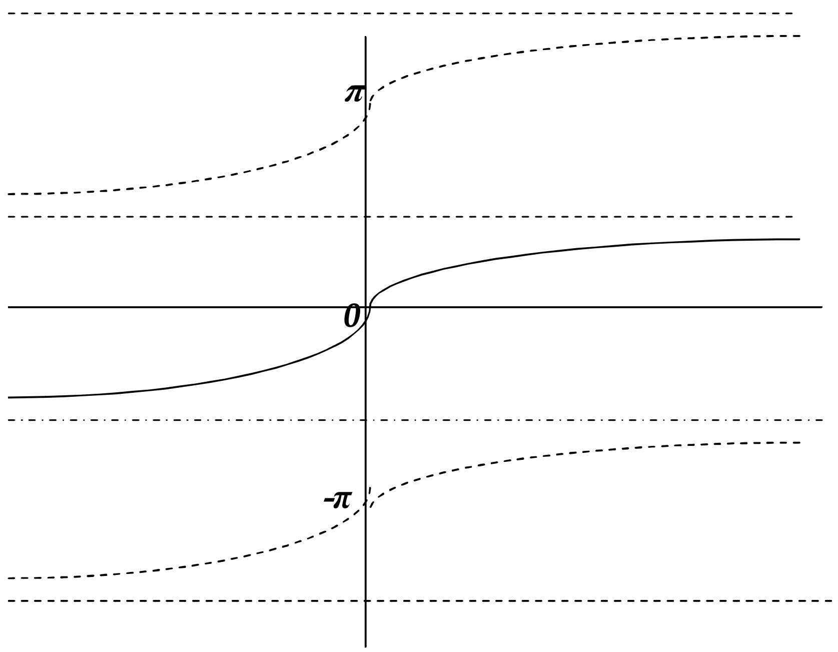


$$y = Sin^{-1}x \quad D_x : -1 \le x \le 1$$

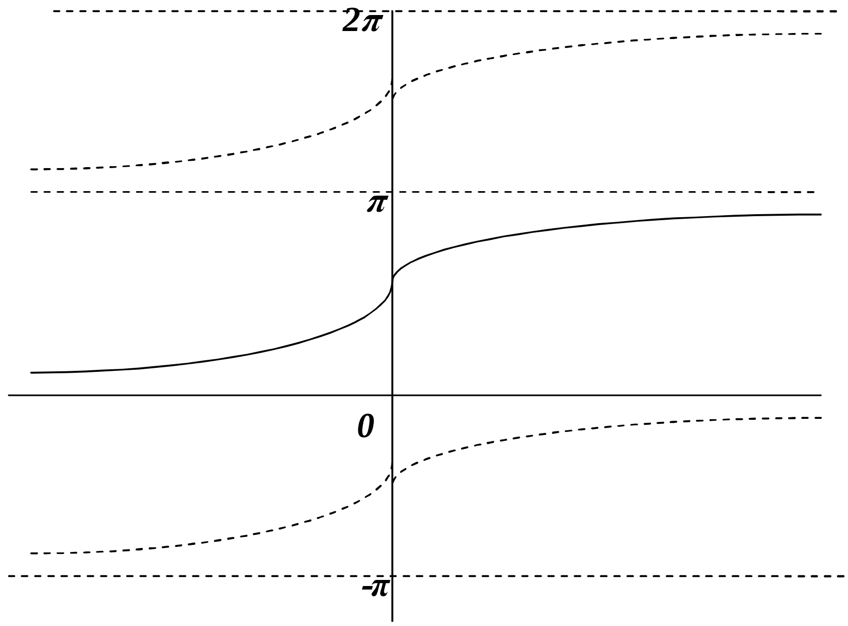
$$R_y : -90 \le y \le 90$$



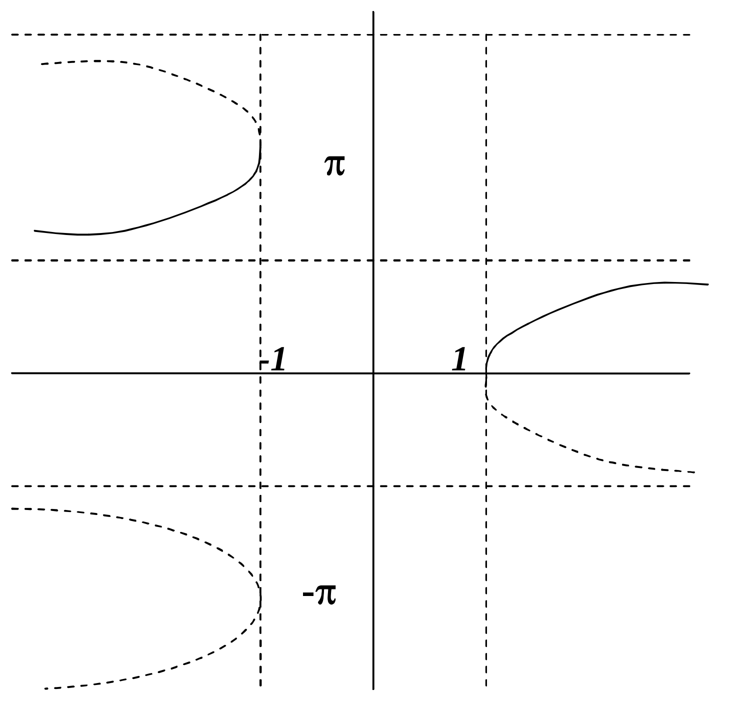
$$y = Cos^{-1}x \quad D_x : -1 \le x \le 1$$
$$R_y : 0 \le y \le 180$$



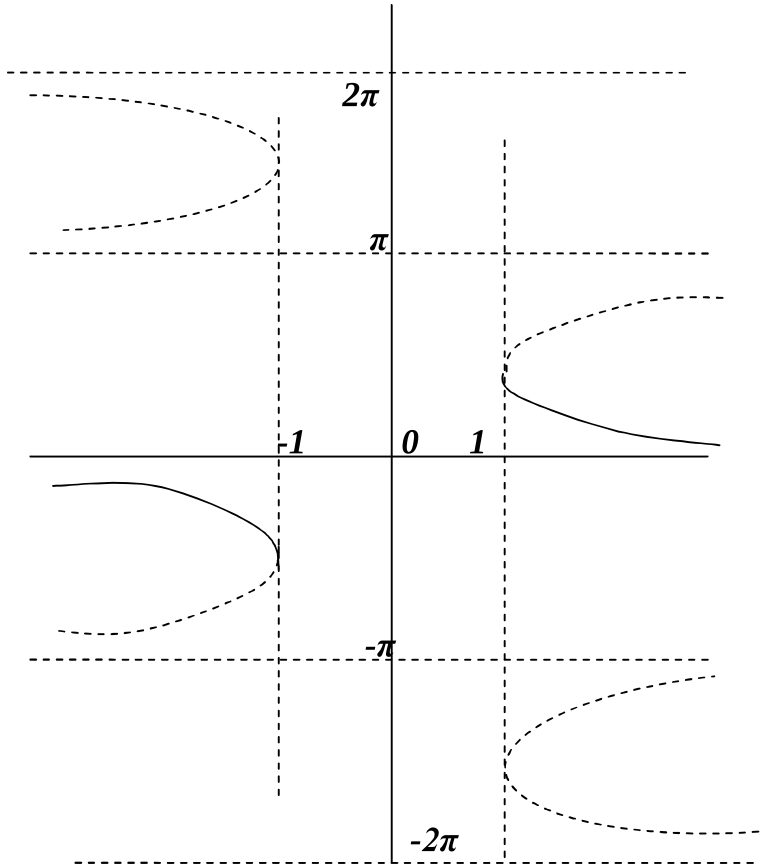
$$y = tan^{-1}\ x \quad D_x : \forall x$$
$$R_y : -90 \le y \le 90$$



$$y = Cot^{-1} x \quad D_x : \forall x$$
$$R_y : 0 \le y \le \pi$$



$$y = Sec^{-1} x \quad D_x : \forall |x| \ge 1$$
$$R_y : 0 \le y \le \pi , y \ne \frac{\pi}{2}$$



$$y = Csc^{-1}x \quad D_x : \forall |x| \geq 1$$

$$R_y : -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$$

The following are some properties of the inverse trigonometric functions :

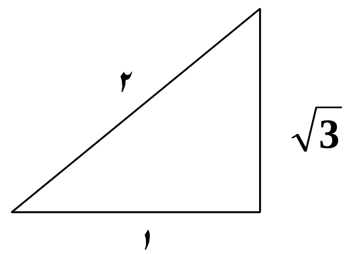
1. $Sin^{-1}(-x) = -Sin^{-1}x$
2. $Cos^{-1}(-x) = \pi - Cos^{-1}x$
3. $Sin^{-1}x + Cos^{-1}x = \frac{\pi}{2}$
4. $tan^{-1}(-x) = -tan^{-1}x$
5. $Cot^{-1}x = \frac{\pi}{2} - tan^{-1}x$
6. $Sec^{-1}x = Cos^{-1}\frac{1}{x}$
7. $Csc^{-1}x = Sin^{-1}\frac{1}{x}$
8. $Sec^{-1}(-x) = \pi - Sec^{-1}x$

and noted that $(Sinx)^{-1} = \frac{1}{Sinx} = Cscx \neq Sin^{-1}x$

EX-13- **Given that** $\alpha = Sin^{-1}\dfrac{\sqrt{3}}{2}$ **, find :**

$$Csc\alpha \;\; ,Cos\alpha \;\; ,Sec\alpha \;\; ,tan\,\alpha \;\; ,and \;\; ,Cot\alpha$$

Sol.-



$$\alpha = Sin^{-1}\frac{\sqrt{3}}{2} \Rightarrow Sin\alpha = \frac{\sqrt{3}}{2} = \frac{x}{y} \Rightarrow r = \sqrt{4-3} = 1$$

$$Csc\alpha = \frac{2}{\sqrt{3}} \;\; ,Cos\alpha = \frac{1}{2} \;\; ,Sec\alpha = 2 \;\; ,tan\,\alpha = \sqrt{3} \;\; ,Cot\alpha = \frac{1}{\sqrt{3}}$$

EX-14 **– Evaluate the following expressions :**

$$a)\;\; Sec(Cos^{-1}\frac{1}{2}) \quad b)\;\; Sin^{-1}1 - Sin^{-1}(-1) \quad c)\;\; Cos^{-1}(-Sin\frac{\pi}{6})$$

Sol.-

$$a)\;\; Sec(Cos^{-1}\frac{1}{2}) = Sec\frac{\pi}{3} = 2$$

$$b)\;\; Sin^{-1}1 - Sin^{-1}(-1) = \frac{\pi}{2} - (-\frac{\pi}{2}) = \pi$$

$$c)\;\; Cos^{-1}(-Sin\frac{\pi}{6}) = Cos^{-1}(-\frac{1}{2}) = \frac{2}{3}\pi$$

EX-15- **Prove that :**

$$a)\;\; Sec^{-1}x = Cos^{-1}\frac{1}{x} \qquad b)\;\; Sin^{-1}(-x) = -Sin^{-1}x$$

Sol.

$$a) \qquad Let \;\; y = Sec^{-1}x \Rightarrow x = Secy \Rightarrow x = \frac{1}{Cosy}$$

$$\Rightarrow y = Cos^{-1}\frac{1}{x} \Rightarrow Sec^{-1}x = Cos^{-1}\frac{1}{x}$$

$$b)\quad Let \;\; y = -Sin^{-1}x \Rightarrow x = Sin(-y) \Rightarrow x = -Siny$$

$$\Rightarrow y = Sin^{-1}(-x) \Rightarrow Sin^{-1}(-x) = -Sin^{-1}x$$