2.6.3 Polarization

The wave radiated by an antenna consists of an electric field component and a magnetic field component. These two components are orthogonal and perpendicular to the direction of propagation of the wave; they vary at the frequency of the wave. By convention, the polarization of the wave is defined by the direction of the electric field. In general, the direction of the electric field is not fixed; i.e., during one period, the projection of the extremity of the vector representing the electric field onto a plane perpendicular to the direction of propagation of the wave describes an ellipse; the polarization is said to be elliptical (Figure 5.5).

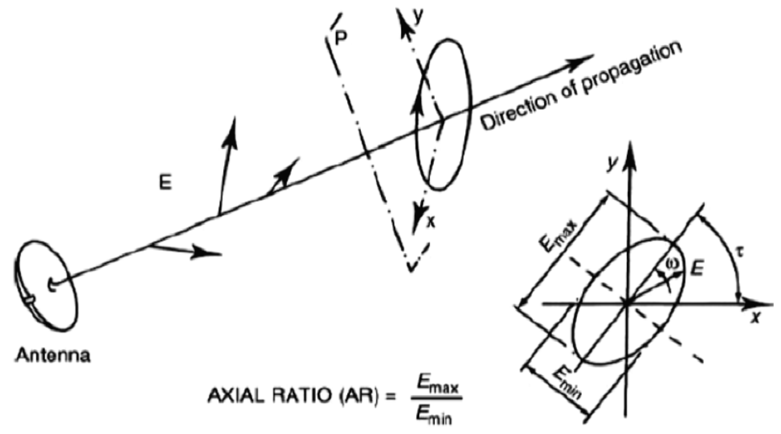
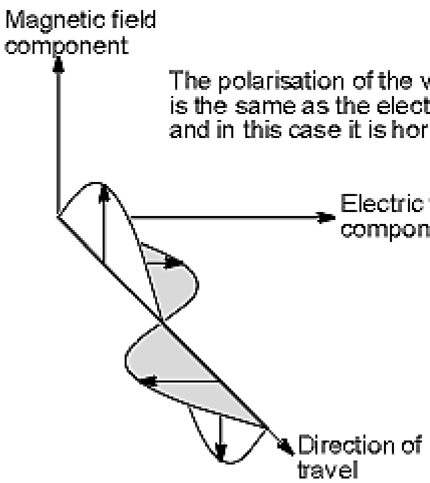


Figure 5.5 Characterisation of the polarisation of an electromagnetic wave.



Polarisation is characterised by the following parameters:

- direction of rotation (with respect to the direction of propagation): right-hand (clockwise) or left-hand (counter-clockwise).
- axial ratio (AR): AR = E_{max}/E_{min}, that is the ratio of the major and minor axes of the ellipse. When the ellipse is a circle (axial ratio = 1 = 0 dB), the polarisation is said to be circular. When the ellipse reduces to one axis (infinite axial ratio: the electric field maintains a fixed direction), the polarisation is said to be linear.
- inclination τ of the ellipse.

Two waves are in orthogonal polarisation if their electric fields describe identical ellipses in opposite directions. In particular, the following can be obtained:

1. two orthogonal circular polarisations described as right-hand circular and left-hand circular (the direction of rotation is for an observer looking in the direction of propagation).
2. two orthogonal linear polarisations described as horizontal and vertical (relative to a local reference).

An antenna designed to transmit or receive a wave of given polarisation can neither transmit nor receive in the orthogonal polarisation. This property enables two simultaneous links to be established at the same frequency between the same two locations; this is described as frequency re-use by orthogonal polarisation. To achieve this either, two polarised antennas must be provided at each end or, preferably, one antenna which operates with the two specified polarisations may be used. This situation is illustrated in Figure 5.6 which relates to the case of two orthogonal linear polarisations (but the illustration is equally valid for any two orthogonal polarisations). Let $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ be the amplitudes, assumed to be equal, of the electric field of the two waves transmitted simultaneously with linear polarisation, $\underline{\boldsymbol{a}_C}$ and $\underline{\boldsymbol{b}_C}$ the amplitudes received with the same polarisation and $\underline{\boldsymbol{a}_X}$ and $\underline{\boldsymbol{b}_X}$ the amplitudes received with orthogonal polarisations. The following are defined:

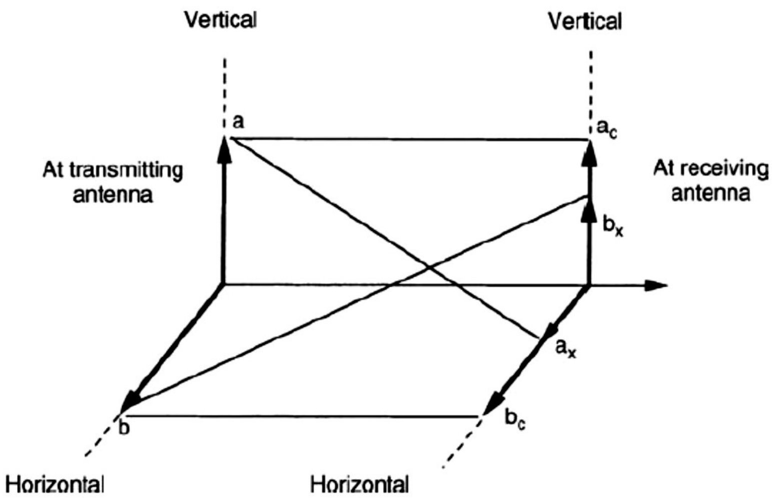


Figure 5.6 Amplitude of the transmitted and received electric field for the case of two orthogonal linear polarisations.

I. The cross-polarization isolation: ***XPI*** $= a_C/b_X$ ***or*** b_C/a_X, hence:

$$XPI\ (dB) = 20 \log(a_C/b_X) \text{ or } 20\ (b_C/a_X)\ (dB)$$

II. The cross-polarization discrimination (when a single polarisation is transmitted): ***XPD*** $= a_C/a_X$, hence:

$$XPD\ (dB) = 20 \log\ (a_C/a_X)\ (dB)$$

In practice, XPI and XPD are comparable and are often included in the term 'isolation'. For a quasi-circular polarisation characterised by its value of axial ratio AR, the cross-polarisation discrimination is given by:

$$XPD = 20 \log[(AR-1)/(AR-1)]\ (dB)$$

Conversely, the axial ratio AR can be expressed as a function of XPD by:

$AR = (10^{XPD/20} + 1)/(10^{XPD/20} - 1)$

The values and relative values of the components vary as a function of direction with respect to the antenna boresight. The antenna is thus characterised for a given polarisation by a radiation pattern for nominal polarisation (copolar) and a radiation pattern for orthogonal polarisation (cross-polar). Cross-polarisation discrimination is generally maximum on the antenna axis and degrades for directions other than that of maximum gain. antenna boresight. The antenna is thus characterised for a given polarisation by a radiation pattern for nominal polarisation (copolar) and a radiation pattern for orthogonal polarisation (cross-polar). Cross-polarisation discrimination is generally maximum on the antenna axis and degrades for directions other than that of maximum gain.

5.3 RADIATED POWER

5.3.1 Effective isotropic radiated power (EIRP)

The power radiated per unit solid angle by an isotropic antenna fed from a radio-frequency source of power P_T is given by:

$$P_T/4\pi \qquad \text{(W/steradian)}$$

In a direction where the value of transmission gain is G_T, any antenna radiates a power per unit solid angle equal to:

$$G_T P_T/4\pi \qquad \text{(W/steradian)}$$

The product $P_T G_T$ is called the 'effective isotropic radiated power' (EIRP). It is expressed in W.

5.3.2 Power flux density

A surface of area A situated at a distance R from the transmitting antenna subtends a solid angle A/R^2 at the transmitting antenna (see Figure 5.7). It receives a power equal to:

$$P_R = (P_T G_T/4\pi)(A/R^2) = \Phi A \quad \text{(W)} \qquad (5.10)$$

The $\Phi = P_T G_T/4\pi R^2$ magnitude is called the power flux density. It is expressed in W/m^2.

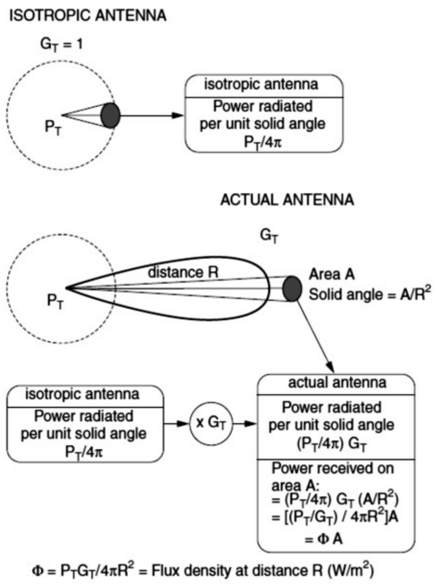


Figure 5.7 Power flux density.

5.4 RECEIVED SIGNAL POWER

5.4.1 Power captured by the receiving antenna and free space loss

As shown in Figure 5.8, a receiving antenna of effective aperture area A_{Reff} located at a distance R from the transmitting antenna receives power equal to:

$$P_R = \Phi A_{R\,eff} = (P_T G_T/4\pi R^2) A_{R\,eff} \quad \text{(W)} \qquad (5.11)$$

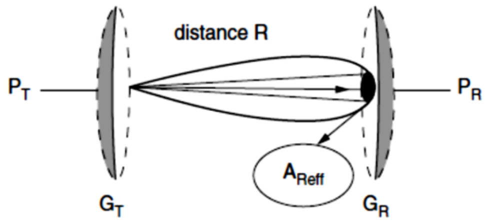


Figure 5.8 The power received by a receiving antenna.

The effective area of an antenna is expressed as a function of its receiving gain G_R according to equation (5.2):

$$A_{R\,eff} = G_R/(4\pi/\lambda^2) \quad (m^2) \tag{5.12}$$

Hence an expression for the received power:

$$\begin{aligned} P_R &= (P_T G_T/4\pi R^2)(\lambda^2/4\pi)G_R \\ &= (P_T G_T)(\lambda/4\pi R)^2 G_R \\ &= (P_T G_T)(1/L_{FS})G_R \quad (W) \end{aligned} \tag{5.13}$$

where $L_{FS} = (4\pi R/\lambda)^2$ free space loss and represents the ratio of the received and transmitted powers in a link between two isotropic antennas.

5.4.2 Example 1: Uplink received power

Consider the transmitting antenna of an earth station equipped with an antenna of diameter D=4m. This antenna is fed with a power P_T of 100 W, that is 20 dBW, at a frequency f_U = 14 GHz. It radiates this power towards a geostationary satellite situated at a distance of 40 000km from the station on the axis of the antenna. The beam of the satellite receiving antenna has a width $\theta_{3\,dB} = 2^\circ$.

It is assumed that the earth station is at the center of the region covered by the satellite antenna and consequently benefits from the maximum gain of this antenna. The efficiency of the satellite antenna is assumed to be η = 0.55 and that of the earth station to be η = 0.6.

—The power flux density at the satellite situated at earth station antenna boresight is calculated as:

$$\Phi_{max} = P_T G_{Tmax}/4\pi R^2 \quad (\mathrm{W/m^2})$$

The gain of the earth station antenna, from equation (5.3), is:

$$G_{Tmax} = \eta(\pi D/\lambda_U)^2 = \eta(\pi D f_U/c)^2$$
$$= 0.6(\pi \times 4 \times 14 \times 10^9/3 \times 10^8)^2 = 206\,340 = 53.1\ \mathrm{dBi}$$

The effective isotropic radiated power of the earth station (on the axis) is given by:

$$(\mathrm{EIRP_{max}})_{ES} = P_T G_{Tmax} = 53.1\ \mathrm{dBi} + 20\ \mathrm{dBW} = 73.1\ \mathrm{dBW}$$

The power flux density is given by:

$$\Phi_{max} = P_T G_{Tmax}/4\pi R^2 = 73.1\ \mathrm{dBW} - 10\log(4\pi(4 \times 10^7)^2)$$
$$= 73.1 - 163 = -89.9\ \mathrm{dBW/m^2}$$

—The power received (in dBW) by the satellite antenna is obtained using equation (5.13):

$$P_R = \mathrm{EIRP} - \text{attenuation of free space} + \text{gain of receiving antenna}$$

The attenuation of free space $L_{FS} = (4\pi R/\lambda_U)^2 = (4\pi R f_U/c)^2 = 207.4$ dB.
The gain of the satellite receiving antenna $G_R = G_{Rmax}$ is obtained using equation (5.3):

$$G_{Rmax} = \eta(\pi D/\lambda_U)^2$$

The value of D/λ_U is obtained using equation (5.7), hence $\theta_{3\,dB} = 70(\lambda_U/D)$, from which

$$D/\lambda_U = 70/\theta_{3\,dB} \text{ and } G_{Rmax} = \eta(70\pi/\theta_{3\,dB})^2 = 6650 = 38.2\ \mathrm{dBi}.$$

Notice that the antenna gain does not depend on frequency when the beamwidth, and hence the area covered by the satellite antenna, is imposed. In total:

$$P_R = 73.1 - 207.4 + 38.2 = -96.1\ \mathrm{dBW}, \quad \text{that is } 0.25\ \mathrm{nW} \text{ or } 250\ \mathrm{pW}$$

5.4.3 Example 2: Downlink received power

Consider the transmitting antenna of a geostationary satellite fed with a power P_T of 10 W, that is, 10dBW at a frequency f_D = 12 GHz, and radiating this power in a beam of width $\theta_{3\,dB} = 2^\circ$.. An earth station equipped with a 4m diameter antenna is located on the axis of the antenna at a distance of 40 000km from the satellite. The efficiency of the satellite antenna is assumed to be η = 0.55 and that of the earth station to be η = 0.6.

—The power flux density at the earth station situated at the satellite antenna boresight is calculated as:

$$\Phi_{max} = P_T G_{Tmax}/4\pi R^2 \quad (\mathrm{W/m^2})$$

The gain of the satellite antenna is the same in transmission as in reception since the beamwidths are made the same (notice that this requires two separate antennas on the satellite since the diameters cannot be the same and are in the ratio f_U/f_D = 14/12 = 1.17). Hence:

$$(\mathrm{EIRP}_{max})_{SL} = P_T G_{Tmax} = 38.2\,\mathrm{dBi} + 10\,\mathrm{dBW} = 48.2\,\mathrm{dBW}$$

The power flux density is:

$$\Phi_{max} = P_T G_{Tmax}/4\pi R^2 = 48.2\,\mathrm{dBW} - 10\log(4\pi(4\times 10^7)^2) = 48.2 - 163$$
$$= -114.8\,\mathrm{dBW/m^2}$$

— The power (in dBW) received by the antenna of the earth station is obtained using equation (5.13):

$$P_R = \mathrm{EIRP} - \text{attenuation of free space} + \text{gain of the receiving antenna}$$

The attenuation of free space is $L_{FS} = (4\pi R/\lambda_D)^2 = 206.1$ dB.
The gain $G_R = G_{Rmax}$ of the ground station receiving antenna is obtained using equation (5.3), hence:

$$G_{Rmax} = \eta(\pi D/\lambda_D)^2 = 0.6(\pi \times 4/0.025)^2 = 151\,597 = 51.8\,\mathrm{dB}$$

In total:

$$P_R = 48.2 - 206.1 + 51.8 = -106.1\,\mathrm{dBW}, \quad \text{that is } 25\,\mathrm{pW}$$

5.4.4 Additional losses

In practice, it is necessary to take account of additional losses due to various causes:

- attenuation of waves as they propagate through the atmosphere;
- losses in the transmitting and receiving equipment;
- depointing losses;
- polarization mismatch losses.

5.4.4.1 Attenuation in the atmosphere

The attenuation of waves in the atmosphere, denoted by L_A, is due to the presence of gaseous components in the troposphere, water (rain, clouds, snow and ice) and the ionosphere. The overall effect on the power of the received carrier can be taken into account by replacing L_{FS} in equation (5.13) by the path loss, L, where:

$$L = L_{FS} L_A \qquad (5.14)$$

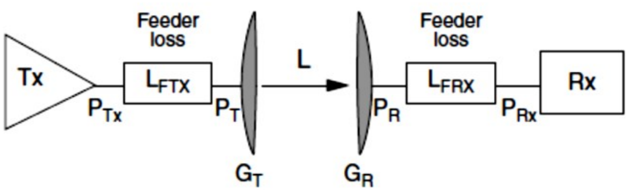


Figure 5.10 Losses in the terminal equipment.

5.4.4.2 Losses in the transmitting and receiving equipment

Figure 5.10 clarifies these losses:

- ✓ The feeder loss L_{FTX} between the transmitter and the antenna: to feed the antenna with a power P_T it is necessary to provide a power P_{TX} at the output of the transmission amplifier such that:

$$P_{TX} = P_T L_{FTX} \quad (W) \qquad (5.15)$$

Expressed as a function of the rated power of the transmission amplifier, the EIRP can be written:

$$\mathrm{EIRP} = P_T G_T = (P_{TX} G_T)/L_{FTX} \quad (W) \qquad (5.16)$$

- ✓ The feeder loss L_{FRX} between the antenna and the receiver: the signal power P_{RX} at the input of the receiver is equal to:

$$P_{RX} = P_R / L_{FRX} \quad (W) \qquad (5.17)$$

5.4.4.3 Depointing losses

Figure 5.11 shows the geometry of the link for the case of imperfect alignment of the transmitting and receiving antennas. The result is a fallout of antenna gain with respect to the maximum gain on transmission and on reception, called depointing loss. These depointing losses are a function of the misalignment of angles of transmission (θ_T) and reception (θ_R) and are evaluated using equation (5.5). Their value is given by:

$$L_T = 12(\theta_T/\theta_{3\,dB})^2 \quad (dB)$$
$$L_R = 12(\theta_R/\theta_{3\,dB})^2 \quad (dB) \qquad (5.18)$$

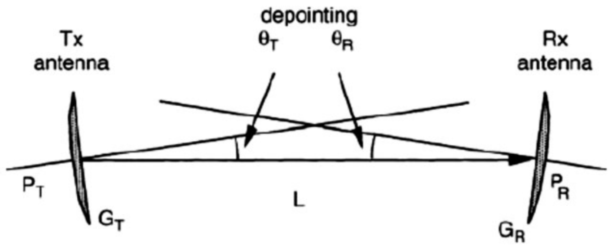


Figure 5.11 Geometry of the link.

5.4.4.4 Losses due to polarizations mismatch

It is also necessary to consider the polarizations mismatch loss L_{POL} observed when the receiving antenna is not oriented with the polarizations of the received wave. In a link with circular polarizations, the transmitted wave is circularly polarized only on the axis of the antenna and becomes elliptical off this axis. Propagation through the atmosphere can also change circular into elliptical polarizations. In a linearly polarized link, the wave can be subjected to a rotation of its plane of polarizations as it propagates through the atmosphere. Finally, with linear polarizations, the receiving antenna may not have its plane of polarizations aligned with that of the incident wave. If Ψ is the angle between the two planes, the polarizations mismatch loss L_{POL} (in dB) is equal to ***(-20 log cos Ψ)***. In the case where a circularly polarized antenna receives a linearly polarized wave, or a linearly polarized antenna receives a circularly polarized wave, L_{POL} has a value of 3 dB. Considering all sources of loss, the signal power at the receiver input is given by:

$$P_{RX} = (P_{TX}G_{Tmax}/L_T L_{FTX})(1/L_{FS}L_A)(G_{Rmax}/L_R L_{FRX} L_{POL}) \quad (W) \qquad (5.19)$$

5.5 NOISE POWER SPECTRAL DENSITY AT THE RECEIVER INPUT

5.5.1 The origins of noise

Noise consists of all unwanted contributions whose power adds to the wanted carrier power. It reduces the ability of the receiver to reproduce correctly the information content of the received wanted carrier.

The origins of noise are as follows:

- ❖ the noise emitted by natural sources of radiation located within the antenna reception area;
- ❖ the noise generated by components in the receiving equipment.

Carriers from transmitters other than those which it is wished to receive are also classed as noise. This noise is described as interference.

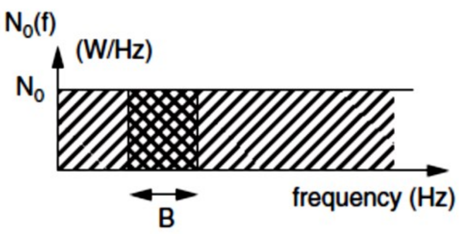


Figure 5.12 Spectral density of white noise.

5.5.2 Noise characterization

Harmful noise power is that which occurs in the bandwidth **B** of the wanted modulated carrier. A popular noise model is that of white noise, for which the power spectral density N_0(W/Hz) is constant in the frequency band involved (Figure 5.12). The equivalent noise power N(W) captured by a receiver with equivalent noise bandwidth B_N, usually matched to B ($B=B_N$), is given by:

$$N = N_0 B_N \quad (W) \qquad (5.20)$$

- **white noise is a random signal having equal intensity at different frequencies, giving it a constant power spectral density**

5.5.2.1 Noise temperature of a noise source

The noise temperature of a two-port noise source delivering an available noise power spectral density N_0 is given by:

$$T = N_0/k \text{ (K)} \quad (5.21)$$

where k is Boltzmann's constant = 1.379 x 10^{-23} = -228.6 dBW/Hz K, T represents the thermodynamic temperature of a resistance which delivers the same available noise power as the source under consideration (Figure 5.13). Available noise power is the power delivered by the source to a device which is impedance matched to the source.

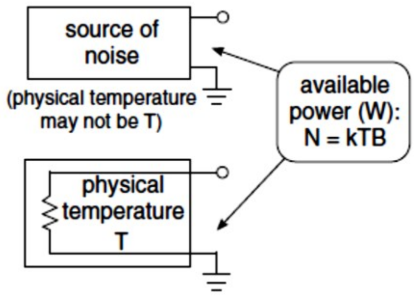


Figure 5.13 Definition of the noise temperature of a noise source.

5.5.2.2 Effective input noise temperature

The effective input noise temperature T_e of a four-port element is the thermodynamic temperature of a resistance which, placed at the input of the element assumed to be noise-free, establishes the same available noise power at the output of the element as the actual element without the noise source at the input (Figure 5.14). T_e is thus a measure of the noise generated by the internal components of the four-port element. The noise figure of this four-port element is the ratio of the total available noise power at the output of the element to the component of this power engendered by a source at the input of the element with a noise temperature equal to the reference temperature $T_0 = 290$ K. Assume that the element has a power gain G, a bandwidth B and is driven by a source of noise temperature T_0; the total power at the output is $Gk(T_e + T_0)B$. The component of this power originating from the source is GkT_0B: The noise figure is thus:

$$F = [Gk(T_e + T_0)B]/[GkT_0B] = (T_e + T_0)/T_0 = 1 + T_e/T_0 \quad (5.22)$$

The noise figure is usually quoted in decibels (dB), according to:

$$F(dB) = 10 \log F$$

Figure 5.15 displays the relationship between noise temperature and noise figure (dB).

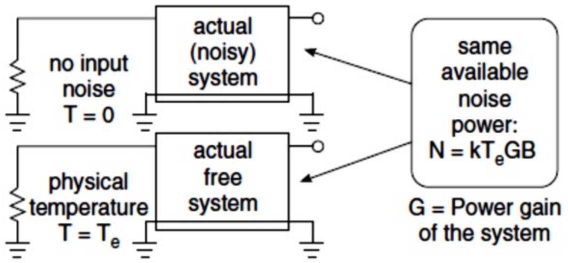


Figure 5.14 Effective input noise temperature of a four-port element.

5.5.2.3 Effective input noise temperature of an attenuator

An attenuator is a four-port element containing only passive components (which can be classed as resistances) all at temperature T_{ATT} which is generally the ambient temperature. If L_{ATT} is the attenuation caused by the attenuator, the effective input noise temperature of the attenuator is:

$$T_{eATT} = (L_{ATT} - 1)T_{ATT} \quad \text{(K)} \qquad (5.23)$$

If $T_{ATT} = T_0$, the noise figure of the attenuator from a comparison of equations (5.22) and (5.23), is:

$$F_{ATT} = L_{ATT}$$

5.5.2.4 Effective input noise temperature of cascaded elements

Consider a chain of N four-port elements in cascade, each element j having a power gain $G_j = (j = 1, 2, \ldots, N)$ and an effective input noise temperature T_{ej}.

The overall effective input noise temperature is:

$$T_e = T_{e1} + T_{e2}/G_1 + T_{e3}/G_1G_2 + \cdots + T_{iN}/G_1G_2, \ldots, G_{N-1} \quad \text{(K)} \qquad (5.24)$$

The noise figure is obtained from equation (5.22):

$$F = F_1 + (F_2 - 1)/G_1 + (F_3 - 1)/G_1G_2 + \cdots + (F_N - 1)/G_1G_2, \ldots, G_{N-1} \qquad (5.25)$$

5.5.2.5 Effective input noise temperature of a receiver

Figure 5.16 shows the arrangement of the receiver. By using equation (5.24), the effective input noise temperature T_{eRX} of the receiver can be expressed as:

$$T_{eRX} = T_{LNA} + T_{MX}/G_{LNA} + T_{IF}/G_{LNA}G_{MX} \quad \text{(K)} \qquad (5.26)$$

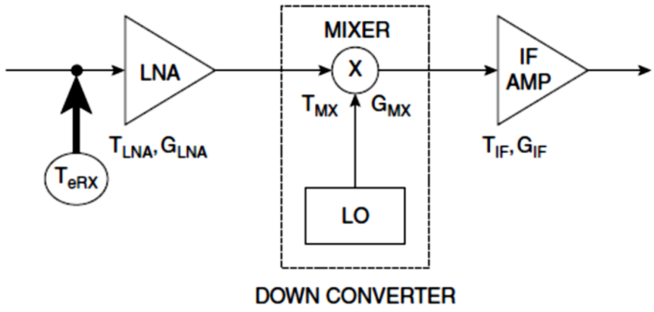


Figure 5.16 The organisation of a receiver.

Example:

Low noise amplifier (LNA): $T_{LNA} = 150\,K, G_{LNA} = 50\,dB$

Mixer: $T_{MX} = 850\,K, G_{MX} = -10\,dB\ (L_{MX} = 10\,dB)$

IF amplifier: $T_{IF} = 400\,K, G_{IF} = 30\,dB$

Hence:

$$\begin{aligned} T_{eRX} &= 150 + 850/10^5 + 400/10^5 10^{-1} \\ &= 150\,K \end{aligned}$$

5.5.3 Noise temperature of an antenna

An antenna picks up noise from radiating bodies within the radiation pattern of the antenna. The noise output from the antenna is a function of the direction in which it is pointing, its radiation pattern and the state of the surrounding environment. The antenna is assumed to be a noise source characterized by a noise temperature called the noise temperature of the antenna T_A(K). Let $T_b(\theta, \varphi)$ be the brightness temperature of a radiating body located in a direction (θ, φ), where the gain of the antenna has a value G(θ, φ). The noise temperature of the antenna is obtained by integrating the contributions of all the radiating bodies within the radiation pattern of the antenna.

The noise temperature of the antenna is thus:

$$T_A = (1/4\pi)\iint T_b(\theta, \varphi)G(\theta, \varphi)\sin\theta\, d\theta\, d\varphi \quad (K) \qquad (5.27)$$

There are two cases to be considered:

- a satellite antenna (the uplink);
- an earth station antenna (the downlink).

5.5.3.1 Noise temperature of a satellite antenna (uplink)

The noise captured by the antenna is noise from the earth and from outer space. The beamwidth of a satellite antenna is equal to or less than the angle of view of the earth from the satellite, that is 17.5^0 for a geostationary satellite. Under these conditions, the major contribution is that from the earth. For a beamwidth θ_{3dB} of 17.5^0, the antenna noise temperature depends on the frequency and the orbital position of the satellite. For a smaller width (a spot beam), the temperature depends on the frequency and the area covered.

5.5.3.2 Noise temperature of an earth station antenna (the downlink)

The noise captured by the antenna consists of noise from the sky and noise due to radiation from the earth. Figure 5.19 shows the situation.

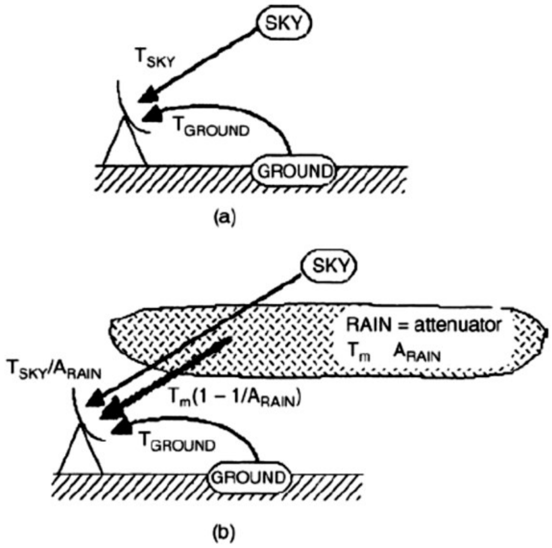


Figure 5.19 Contributions to the noise temperature of an earth station: (a) 'clear sky' conditions and (b) conditions of rain.

5.5.3.2.1 'Clear sky' conditions

At frequencies greater than 2 GHz, the greatest contribution is that of the non-ionised region of the atmosphere which, being an absorbent medium, is a noise source. In the absence of meteorological formations (conditions described as 'clear sky'), the antenna noise temperature contains contributions due to the sky and the surrounding ground (Figure 5.19a). The sky noise contribution is determined from equation (5.27), where $T_b(\theta, \varphi)$ is the brightness temperature of the sky in the direction(θ, φ). In practice, only that part of the sky in the direction of the antenna boresight contributes to the integral as the gain has a high value only in that direction. As a consequence, the noise contribution of the clear sky T_{SKY} can be assimilated with the brightness temperature for the angle of elevation of the antenna.

Radiation from the ground in the vicinity of the earth station is captured by the side lobes of the antenna radiation pattern and partly by the main lobe when the elevation angle is small. The contribution of each lobe is determined by $T_i = G_i(\Omega_i/4\pi)T_G$, where G_i is the mean gain of the lobe of solid angle G_i and T_G the brightness temperature of the ground. The sum of these contributions yields the value T_{GROUND}. The following can be taken as a first approximation:

$-T_G = 290\,\text{K}$ for lateral lobes whose elevation angle E is less than $-10°$
$-T_G = 150\,\text{K}$ for $-10° < E < 0°$
$-T_G = 50\,\text{K}$ for $0° < E < 10°$
$-T_G = 10\,\text{K}$ for $10° < E < 90°$

The antenna noise temperature is thus given by:

$$T_A = T_{SKY} + T_{GROUND} \quad (\text{K}) \tag{5.28}$$

To this noise may be added that of individual sources which are located in the vicinity of the antenna boresight. For a radio source of apparent angular diameter α and noise temperature T_n at the frequency considered and measured at ground level after attenuation by the atmosphere, the additional noise temperature ΔT_A for an antenna of beamwidth θ_{3dB} is given by:

$$\begin{array}{lll} \Delta T_A = T_n(\alpha/\theta_{3\,dB})^2 & \text{if} \quad \theta_{3\,dB} > \alpha & (\text{K}) \\ \Delta T_A = T_n & \text{if} \quad \theta_{3\,dB} < \alpha & (\text{K}) \end{array} \tag{5.29}$$

For earth stations pointing towards a geostationary satellite, only the sun and the moon need to be considered.

5.5.3.2.2 Conditions of rain

The antenna noise temperature increases during the presence of meteorological formations, such as clouds and rain (Figure 5.19b), which constitute an absorbent, and consequently emissive, medium. Using equation (5.23), the antenna noise temperature becomes:

$$T_A = T_{SKY}/A_{RAIN} + T_m(1 - 1/A_{RAIN}) + T_{GROUND} \quad (\text{K}) \tag{5.30}$$

where A_{RAIN} is the attenuation and T_m the mean thermodynamic temperature of the formations in question. For T_m, a value of 275K can be assumed.

In conclusion, the antenna noise temperature T_A, is a function of:

- the frequency,
- the elevation angle,
- the atmospheric conditions (clear sky or rain).

Consequently, the figure of merit of an earth station must be specified for particular conditions of frequency, elevation angle and atmospheric conditions.

5.5.4 System noise temperature

Consider the receiving equipment shown in Figure 5.22. This consists of an antenna connected to a receiver. The connection (feeder) is a lossy one and is at a thermodynamic temperature T_F (which is close to T_0 = 290 K). It introduces an attenuation L_{FRX}, which corresponds to a gain $G_{FRX} = 1/L_{FRX}$ and is less than 1 ($L_{FRX} \leq 1$). The effective input noise temperature T_e of the receiver is T_{eRX}. The noise temperature may be determined at two points as follows:

- ✓ at the antenna output, before the feeder losses, temperature T_1;
- ✓ at the receiver input, after the losses, temperature T_2.

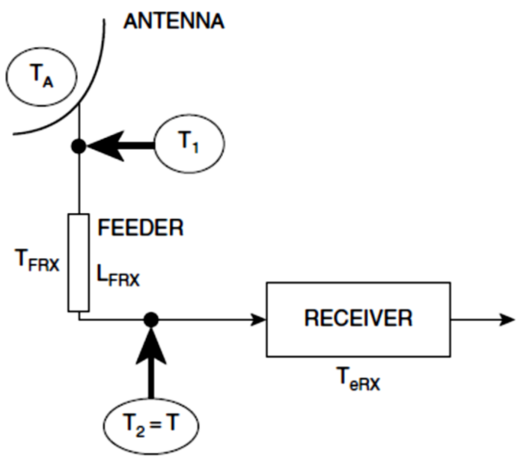


Figure 5.22 A receiving system: T is the system noise temperature at the receiver input.

The noise temperature T_1 at the antenna output is the sum of the noise temperature of the antenna T_A and the noise temperature of the subsystem consisting of the feeder and the receiver in cascade. The noise temperature of the feeder is given by equation (5.23). From equation (5.24) the noise temperature of the subsystem is $(L_{FRX}-1)T_F + T_{eRX}/G_{FRX}$. Adding the contribution of the antenna, considered as a noise source, this becomes:

$$T_1 = T_A + (L_{FRX}-1)T_F + T_{eRX}/G_{FRX} \quad \text{(K)} \qquad (5.31)$$

Now consider the receiver input. This noise must be attenuated by a factor L_{FRX}. Replacing G_{FRX} by $1/L_{FRX}$, one obtains the noise temperature T_2 at the receiver input: This noise temperature T_2, which takes account of the noise generated by the antenna and the feeder together with the receiver noise, is called the ***system noise temperature*** T at the receiver input.

$$T_2 = T_1/L_{FRX} = T_A/L_{FRX} + T_F(1-1/L_{FRX}) + T_{eRX} \quad \text{(K)} \qquad (5.32)$$

5.5.5 System noise temperature: Example

Consider the receiving system of Figure 5.22 with the following values:

- ✓ Antenna noise temperature: T_A = 50 K;

- ✓ Thermodynamic temperature of the feeder: $T_F = 290$ K;
- ✓ Effective input noise temperature of the receiver: $T_{eRX} = 50$ K.

The system noise temperature at the receiver input will be calculated for two cases:

1. no feeder loss between the antenna and the receiver and
2. feeder loss $L_{FRX} = 1$ dB. Using equation (5.31);

$T = T_A/L_{FRX} + T_F(1 - 1/L_{FRX}) + T_{eRX}$:

For case (1), $T = 50 + 50 = 100$ K

For case (2), $T = 50/10^{0.1} + 290(1 - 1/10^{0.1}) + 50 = 39.7 + 59.6 + 50 = 149.3$ K or around 150 K

5.6 INDIVIDUAL LINK PERFORMANCE

The link performance is evaluated as the ratio of the received carrier power, C, to the noise power spectral density, N_0, and is quoted as the C/N_0 ratio, expressed in hertz. One can evaluate the link performance using other ratios besides C/N_0; for instance:

- ❖ ***C/T*** represents the carrier power over the system noise temperature; expressed in units of watts per Kelvin (W/K), it is given by $C/T = (C/N_0)k$, where k is the Boltzmann constant.
- ❖ ***C/N*** represents the carrier power over the noise power; dimensionless, it is given by $C/N = (C/N_0)(1/B_N)$, where B_N is the receiver noise bandwidth.

5.6.1 Carrier power to noise power spectral density ratio at receiver input

The power received at the receiver input, as given by equation (5.19), is that of the carrier. Hence

$$C = P_{RX}$$

The noise power spectral density at the same point is $N_0 = kT$, where T is given by equation (5.32).Hence:

$$C/N_0 = [P_{TX}G_{Tmax}/L_T L_{FTX})(1/L_{FS}L_A)(G_{Rmax}/L_R L_{FRX} L_{POL})] / [T_A/L_{FRX} + T_F(1 - 1/L_{FRX}) + T_{eRX}](1/k) \quad (\text{Hz}) \tag{5.34}$$

This expression can be interpreted as follows:

$$C/N_0 = (\text{transmitter EIRP})(1/\text{path loss}) \times (\text{composite receiving gain/noise temperature}) \times (1/k) \quad (\text{Hz}) \tag{5.35}$$

C/N_0 can also be expressed as a function of the power flux density φ:

$$C/N_0 = \Phi(\lambda^2/4\pi)(\text{composite receiving gain/noise temperature})(1/k) \quad (\text{Hz}) \tag{5.36}$$

where $\varphi = (\text{transmitter EIRP})/(4\pi R^2) \quad (\text{W/m}^2)$

Finally, it can be verified that evaluation of C/N_0 is independent of the point chosen in the receiving chain as long as the carrier power and the noise power spectral density are calculated at the same point.

Equation (5.35) for C/N_0 introduces three factors:

- *EIRP*, which characterizes the transmitting equipment:
- *1/L*, which characterizes the transmission medium;
- the composite receiving gain / noise temperature, which characterizes the receiving equipment; it is called the ***figure of merit***, or ***G/T***, of the receiving equipment.

By examining equation (5.34) it can be seen that the figure of merit ***G/T*** of the receiving equipment is a function of the antenna noise temperature T_A and the effective input noise temperature T_{eRX} of the receiver. These magnitudes will now be quantified. In conclusion, equation (5.34) boils down to:

$$C/N_0 = (\mathrm{EIRP})(1/L)(G/T)(1/k) \quad (\mathrm{Hz}) \qquad (5.37)$$

5.6.2 Clear sky uplink performance

Figure 5.23 shows the geometry of the uplink. It is assumed that the transmitting earth station is on the edge of the 3 dB coverage of the satellite receiving antenna. The data are as follows:

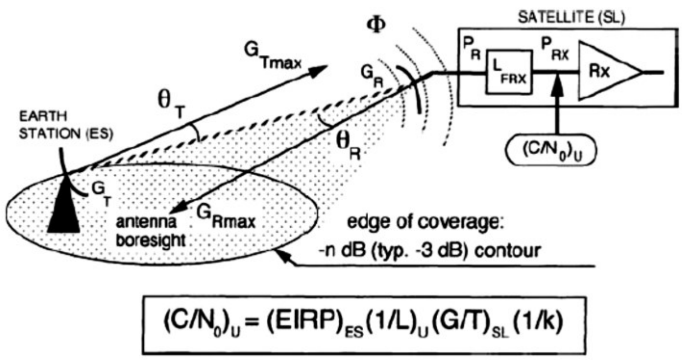


Figure 5.23 The geometry of an uplink.

- Frequency: $f_U = 14$ GHz
- For the earth station (ES):

  Transmitting amplifier power: $P_{TX} = 100$W

  Loss between amplifier and antenna: $L_{FTX} = 0.5$ dB

  Antenna diameter: D = 4m

  Antenna efficiency: $\eta = 0.6$

Maximum pointing error: $\theta_T = 0.1^0$

- ☒ Earth station–satellite distance: R = 40 000 km
- ☒ Atmospheric attenuation: $L_A = 0.3$ dB (typical value for attenuation by atmospheric gases at this frequency for an elevation angle of 10^0).
- ☒ For the satellite (SL):

Receiving beam half power angular width: $\theta_{3\ dB}=2^0$

Antenna efficiency: $\eta = 0.55$

Receiver noise figure: F = 3 dB

Loss between antenna and receiver: $L_{FRX} = 1$ dB

Thermodynamic temperature of the connection: $T_F = 290$ K

Antenna noise temperature: $T_A = 290$ K

To calculate the EIRP of the earth station:

$$(\text{EIRP})_{ES} = (P_{TX}G_{Tmax}/L_T L_{FTX}) \quad (\text{W}) \qquad (5.38)$$

with:

$$P_{TX} = 100\ \text{W} = 20\ \text{dBW}$$
$$G_{Tmax} = \eta(\pi D/\lambda_U)^2 = \eta(\pi D f_U/c)^2 = 0.6\ [\pi \times 4 \times (14 \times 10^9)/(3 \times 10^8)]^2 = 206\,340$$
$$= 53.1\ \text{dBi}$$
$$L_T(\text{dB}) = 12(\theta_T/\theta_{3\,dB})^2 = 12(\theta_T D f_U/70c)^2 = 0.9\ \text{dB}$$
$$L_{FTX} = 0.5\ \text{dB}$$

Hence:

$$(\text{EIRP})_{ES} = 20\ \text{dBW} + 53.1\ \text{dB} - 0.9\ \text{dB} - 0.5\ \text{dB} = 71.7\ \text{dBW}$$

To calculate the attenuation on the upward path (U):

$$L_U = L_{FS}L_A \qquad (5.39)$$

With:

$$L_{FS} = (4\pi R/\lambda_U)^2 = (4\pi R f_U/c)^2 = 5.5 \times 10^{20} = 207.4\ \text{dB}$$
$$L_A = 0.3\ \text{dB}$$

Hence:

$$L_U = 207.4\ \text{dB} + 0.3\ \text{dB} = 207.7\ \text{dB}$$

To calculate the figure of merit G/T of the satellite (SL):

$$(G/T)_{SL} = (G_{Rmax}/L_R L_{FRX} L_{POL})/[T_A/L_{FRX} + T_F(1-1/L_{FRX}) + T_{eRX}] \quad (K^{-1}) \qquad (5.40)$$

with:

$$G_{R\max} = \eta(\pi D/\lambda_U)^2 = \eta(\pi.70/\theta_{3\,dB})^2 = 0.55(\pi.70/2)^2 = 6650 = 38.2\,dBi$$
$$L_R = 12(\theta_R/\theta_{3\,dB})^2$$

As the earth station is on the edge of the 3 dB coverage area, $\theta_R = \theta_{3\,dB}/2$ and $L_R = 3\,dB$.

$$\text{Assume } L_{POL} = 0\,dB$$
$$L_{FRX} = 1\,dB$$
$$\text{Given} \quad T_A = 290\,K$$
$$T_F = 290\,K$$
$$T_{eRX} = (F-1)T_0 = (10^{0.3}-1)290 = 290\,K$$

Hence:

$$(G/T)_{SL} = 38.2-3-1-10\log[290/10^{0.1} + 290(1-1/10^{0.1}) + 290]$$
$$= 6.6\,dBK^{-1}$$

Notice that when the thermodynamic temperature of the feeder between the antenna and the satellite receiver is close to the antenna noise temperature, which is the case in practice, the uplink system noise temperature at the receiver input is $T_U \approx T_F + T_{eRX} \approx 290+T_{eRX}$. It is, therefore, needlessly costly to install a receiver with a very low noise figure on board a satellite.

To calculate the ratio C/N_0 for the uplink:

$$(C/N_0)_U = (EIRP)_{ES}(1/L_U)(G/T)_{SL}(1/k) \quad (Hz) \qquad (5.41)$$

Hence:

$$(C/N_0)_U = 71.7\,dBW - 207.7\,dB + 6.6\,dBK^{-1} + 228.6\,dBW/HzK = 99.2\,dBHz$$

5.6.3 Clear sky downlink performance

Figure 5.25 shows the geometry of the downlink. It is assumed that the receiving earth station is located on the edge of the 3 dB coverage area of the satellite receiving antenna. The data are as follows:

- ☒ Frequency: $f_D = 12$ GHz
- ☒ For the satellite (SL):
  Transmitting amplifier power: $P_{TX} = 10W$
  Loss between amplifier and antenna: $L_{FTX} = 1$ dB
  Transmitting beam half power angular width: $\theta_{3\,dB} = 2^0$
  Antenna efficiency: $\eta = 0.55$
- ☒ Earth station–satellite distance: R = 40 000 km
- ☒ Atmospheric attenuation: $L_A = 0.3$ dB (typical attenuation by atmospheric gases at this frequency for an elevation angle of 10^0)

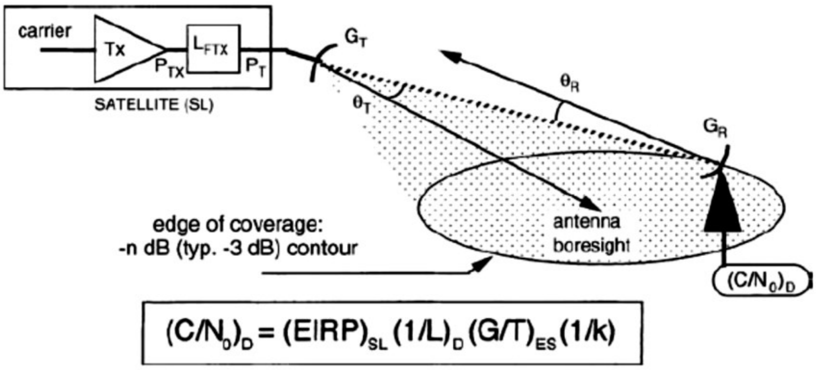


Figure 5.25 The geometry of a downlink.

☒ For the earth station (ES):

Receiver noise figure: F = 1 dB

Loss between antenna and receiver: L_{FRX} = 0:5 dB

Thermodynamic temperature of the feeder: T_F = 290 K

Antenna diameter: D = 4m

Antenna efficiency: Z = 0.6

Maximum pointing error: $\theta_R = 0.1^0$

Ground noise temperature: T_{GROUND} = 45 K

To calculate the EIRP of the satellite:

$$(\mathrm{EIRP})_{SL} = P_{TX}G_{Tmax}/L_T L_{FTX} \quad (\mathrm{W}) \tag{5.42}$$

with:

$$\begin{aligned} P_{TX} &= 10\,\mathrm{W} = 10\,\mathrm{dBW} \\ G_{Tmax} &= \eta(\pi D/\lambda_D)^2 = \eta(\pi 70/\theta_{3\,\mathrm{dB}})^2 = 0.55(\pi 70/2)^2 = 6650 = 38.2\,\mathrm{dBi} \\ L_T(\mathrm{dB}) &= 3\,\mathrm{dB}\ (\text{earth station on edge of coverage}) \\ L_{FTX} &= 1\,\mathrm{dB} \end{aligned}$$

Hence:

$$(\mathrm{EIRP})_{SL} = 10\,\mathrm{dBW} + 38.2\,\mathrm{dBi} - 3\,\mathrm{dB} - 1\,\mathrm{dB} = 44.2\,\mathrm{dBW}$$

To calculate the attenuation on the downlink (D):

$$L_D = L_{FS}L_A \tag{5.43}$$

with:

$$\begin{aligned} L_{FS} &= (4\pi R/\lambda_D)^2 = (4\pi R f_D/c)^2 = 4.04 \times 10^{20} = 206.1\,\mathrm{dB} \\ L_A &= 0.3\,\mathrm{dB} \end{aligned}$$

Hence:

$$L_D = 206.1\,\mathrm{dB} + 0.3\,\mathrm{dB} = 206.4\,\mathrm{dB}$$

To calculate the figure of merit G/T of the earth station in the satellite direction:

$$(G/T)_{ES} = (G_{Rmax}/L_R L_{FRX} L_{POL})/T_D \quad (K^{-1})$$

T_D is the downlink system noise temperature at the receiver input given by:

$$T_D = T_A/L_{FRX} + T_F(1-1/L_{FRX}) + T_{eRX}$$

and:

$$\begin{aligned}
G_{Rmax} &= \eta(\pi D/\lambda_D)^2 = \eta(\pi D f_D/c)^2 = 0.6(\pi \times 4 \times 12 \times 10^9/3 \times 10^8)^2 \\
&= 151\,597 = 51.8\ \text{dBi} \\
L_R(\text{dB}) &= 12(\theta_R/\theta_{3\,\text{dB}})^2 = 12(\theta_R D f_D/70c)^2 = 0.6\ \text{dB} \\
L_{FRX} &= 0.5\ \text{dB} \\
L_{POL} &= 0\ \text{dB}
\end{aligned}$$

$T_A = T_{SKY} + T_{GROUND}$ with $T_{SKY} = 20\,\text{K}$ (see Figure 5.20 for $f = 12\,\text{GHz}$ and $E = 10°$) and $T_{GROUND} = 45\,\text{K}$, from which $T_A = 65\,\text{K}$

$$\begin{aligned}
T_F &= 290\ \text{K} \\
T_{eRX} &= (F-1)T_0 = (10^{0.1}-1)290 = 75\ \text{K}
\end{aligned}$$

Hence:

$$T_D = 65/10^{0.05} + 290(1-1/10^{0.05}) + 75 = 164.5\ \text{K}$$

then:

$$\begin{aligned}
(G/T)_{ES} &= 51.8-0.6-0.5-10\log[65/10^{0.05} + 290(1-1/10^{0.05}) + 75] \\
&= 28.5\ \text{dBK}^{-1}
\end{aligned}$$

To calculate the ratio C/N_0 for the downlink:

$$(C/N_0)_D = (\text{EIRP})_{SL}(1/L_D)(G/T)_{ES}(1/k) \quad (\text{Hz}) \tag{5.44}$$

Hence:

$$\begin{aligned}
(C/N_0)_D &= 44.2\ \text{dBW} - 206.4\ \text{dB} + 28.5\ \text{dBK}^{-1} + 228.6\ \text{dBW/HzK} \\
&= 94.9\ \text{dBHz}
\end{aligned}$$

5.7.4 Link performance under rain conditions

5.7.4.1 Uplink performance

In the presence of rain, propagation attenuation is greater due to the attenuation A_{RAIN} caused by rain in the atmosphere. This is in addition to the attenuation due to gases in the atmosphere (0.3 dB).

A typical value of attenuation due to rain for an earth station situated in a temperate climate (for example, in Europe) can be considered to be A_{RAIN} = 10 dB. Such an attenuation would not be exceeded, at a frequency of 14 GHz, for more than 0.01% of an average year. This gives ***L_A = 0.3 dB+10 dB = 10.3 dB***.

Hence:

$$L_U = 207.4\,\text{dB} + 10.3\,\text{dB} = 217.7\,\text{dB}$$

Referring to the example of Section 5.6.2, the uplink performance under rain conditions becomes:

$$(C/N_0)_U = 71.7\,\text{dBW} - 217.7\,\text{dB} + 6.6\,\text{dBK}^{-1} + 228.6\,\text{dBW/Hz K} = 89.2\,\text{dBHz}$$

The ratio $(C/N_0)_U$ for the uplink would be greater than the value calculated in this way for 99.99% of an average year.

5.7.4.2 Downlink performance

Referring now to the example of Section 5.6.3, A_{RAIN} = 7 dB is taken as the typical value of attenuation due to rain for an earth station situated in a temperate climate (for example, in Europe) which will not be exceeded, at a frequency of 12 GHz, for more than 0.01% of an average year; this gives $L_A = 0.3\,dB + 7\,dB = 7.3\,dB$. Hence, $L_D = 206.1 + 7.3\,dB = 213.4\,dB$. The antenna noise temperature is given by:

$$T_A = T_{SKY}/A_{RAIN} + T_m(1 - 1/A_{RAIN}) + T_{GROUND} \quad (\text{K}) \tag{5.49}$$

Taking

$$\begin{aligned} T_m &= 275\,\text{K} \\ T_A &= 20/10^{0.7} + 275(1 - 1/10^{0.7}) + 45 = 269\,\text{K} \\ T_D &= 269/10^{0.05} + 290(1 - 1/10^{0.05}) + 75 = 346\,\text{K} \end{aligned}$$

Hence

$$\begin{aligned} (G/T)_{ES} &= 51.8 - 0.6 - 0.5 - 10\log[269/10^{0.05} + 290(1 - 1/10^{0.05}) + 75] \\ &= 25.3\,\text{dBK}^{-1} \end{aligned}$$

To calculate the ratio C/N_0 for the downlink:

$$(C/N_0)_D = (\text{EIRP})_{SL}(1/L_D)(G/T)_{ES}(1/k) \quad (\text{Hz})$$

Hence:

$$(C/N_0)_D = 44.2\,\text{dBW} - 213.4\,\text{dB} + 25.3\,\text{dBK}^{-1} + 228.6\,\text{dBW/HzK} = 84.7\,\text{dBHz}$$

The ratio $(C/N_0)_D$ for the downlink would be greater than the value calculated in this way for 99.99% of an average year.

5.9 OVERALL LINK PERFORMANCE WITH TRANSPARENT SATELLITE

Section 5.6 presents the individual link performance in terms of C/N_0. This section discusses the expression for the overall station-to-station link performance; that is, the link involving one uplink and one downlink via a transparent satellite (no on-board demodulation and re-modulation). Up to now, noise on the uplink and on the downlink has been considered to be thermal noise only. In practice, one has to account for interference noise originating from other carriers in the considered frequency bands and intermodulation noise resulting from multicarrier operation of non-linear amplifiers. The overall link performance is discussed (Section 5.9.2) without intermodulation or interference, then expressions are introduced considering interference and finally intermodulation.

The following notation is used:

- **$(C\backslash N_0)_U$** is the uplink carrier power-to-noise power spectral density ratio (Hz) at the satellite receiver input, considering no other noise contribution than the uplink system thermal noise temperature T_U.
- **$(C/N_0)_D$** is the downlink carrier power-to-noise power spectral density ratio (Hz) at the earth station receiver input, considering no other noise contribution than the downlink system thermal noise temperature T_D.
- **$(C/N_0)_I$** is the carrier power-to-interference noise power spectral density ratio (Hz) at the input of the considered receiver.
- **$(C/N_0)_{IM}$** is the carrier power-to-intermodulation noise power spectral density ratio (Hz) at the output of the considered non-linear amplifier.
- $(C/N_0)_T$ is the overall carrier power-to-noise power spectral density ratio (Hz) at the earth station receiver input.

5.9.1 Characteristics of the satellite channel

Figure 5.33 shows a transparent payload, where carriers are power amplified and frequency down-converted. Due to technology power limitations, the overall bandwidth is split into several sub-bands, the carriers in each sub-band being amplified by a dedicated power amplifier. The amplifying chain associated with each sub-band is called a ***satellite channel***, ***or transponder***. The satellite channel amplifies one or several carriers. Here is some more notation:

- **C_U** is the considered carrier power at the satellite receiver input; at saturation, it is denoted **$(C_U)_{sat}$**
- **P_{in}** is the power at the input to the satellite channel amplifier (i = input, n = number of carriers in the channel).
- **P_{on}** is the power at the output of the satellite channel amplifier (o = output, n = number of carriers in the channel)
- **$n = 1$** corresponds to a single-carrier operation of the satellite channel.
- **$(P_{i1})_{sat}$** is the power at the input to the satellite channel amplifier at saturation in single-carrier operation;
- **$(P_{o1})_{sat}$** is the power at the output of the satellite channel amplifier at saturation in single-carrier operation.

Saturation refers to the operation of the amplifier at maximum output power in single-carrier operation. The satellite operator provides characteristic values of a satellite channel in terms of flux density at saturation, $\mathbf{\Phi_{sat}}$, and EIRP at saturation, $\mathbf{EIRP_{sat}}$.

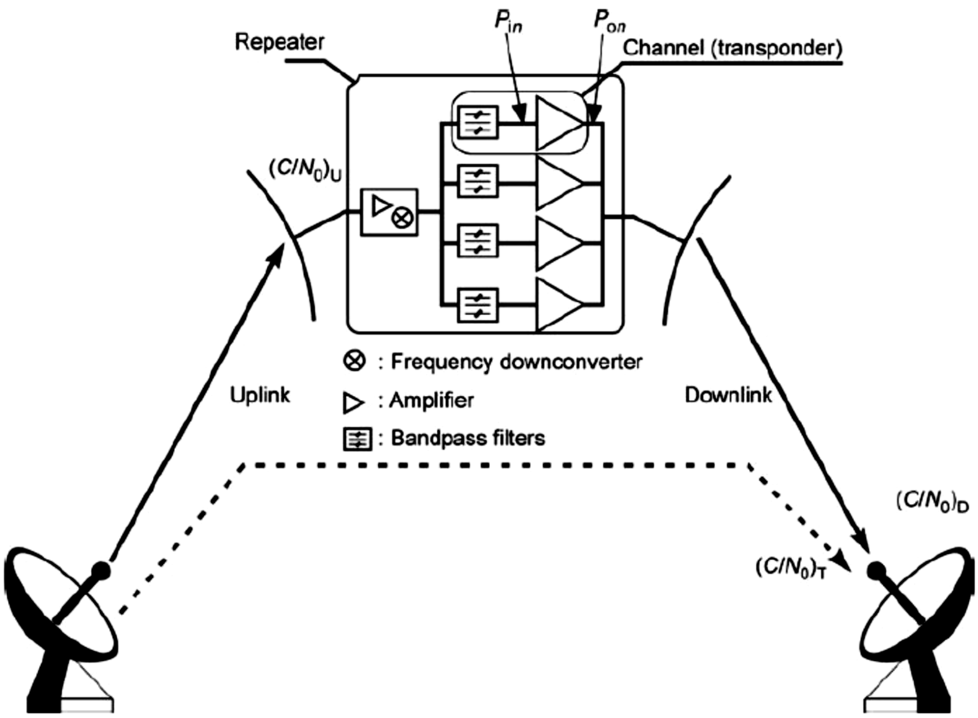


Figure 5.33 Overall station-to-station link for a transparent satellite.

5.9.1.1 Satellite power flux density at saturation

Its nominal value to drive the satellite channel amplifier at saturation is given by:

$$\Phi_{\text{sat, nom}} = \frac{(P_{i1})_{\text{sat}}}{G_{\text{FE}}} \frac{L_{\text{FRX}}}{G_{\text{Rmax}}} \frac{4\pi}{\lambda_U^2} \quad (\text{W/m}^2) \qquad (5.58)$$

where G_{FE} is the front end gain from the input to the satellite receiver to the input to the satellite channel amplifier; L_{FRX} is the loss from the output of the satellite receive antenna to the input of the satellite receiver; and G_{Rmax} is the satellite receive antenna maximum gain. The formula assumes that the transmit earth station is located at the center of the satellite receive coverage (satellite antenna boresight).

In practice, the flux to be provided from a given earth station to drive the satellite channel amplifier to saturation depends on the location of the transmit earth station within the satellite coverage and the polarization mismatch of the satellite receiving antenna with respect to the uplink carrier polarization. Assuming that the receive satellite antenna gain in the direction of the transmit earth station experiences a gain fallout L_R with respect to the maximum gain, and a polarization mismatch loss L_{POL}, the actual flux density to be provided by the transmit earth station is larger than or equal to:

$$\Phi_{sat} = \Phi_{sat,\ nom} L_R L_{POL} = \frac{(P_{i1})_{sat}}{G_{FE}} \frac{L_{FRX}}{G_{Rmax}} \frac{4\pi}{\lambda_U^2} L_R L_{POL} \quad (W/m^2)$$

5.9.1.2 Satellite EIRP at saturation:

$EIRP_{sat;\ max}$, relates to the satellite channel amplifier output power at saturation, $(P_{o1})_{sat}$, as follows:

$$EIRP_{sat,\ max} = \frac{(P_{o1})_{sat}}{L_{FTX}} G_{Tmax} \quad (W) \qquad (5.59)$$

where L_{FTX} is the loss from the output of the power amplifier to the transmit antenna, and G_{Tmax} is the satellite transmit antenna maximum gain (at boresight).

In practice, the satellite $EIRP_{sat}$ which conditions the available carrier power at a given earth station receiver input is reduced by the transmit satellite antenna gain fallout L_T (the gain fallout is defined in the direction of the receiving earth station, with respect to the maximum gain) when the earth station is not located at the center of transmit coverage (satellite antenna boresight):

$$EIRP_{sat} = \frac{EIRP_{sat,max}}{L_T} = \frac{(P_{01})_{sat}}{L_{FTX}} \frac{G_{Tmax}}{L_T} = \frac{(P_{01})_{sat}}{L_{FTX}} G_T \quad (W) \qquad (5.60)$$

5.9.1.3 Satellite repeater gain

The satellite repeater gain, G_{SR}, is the power gain from the satellite receiver input to the satellite channel amplifier output. At saturation, it is called $G_{SR\ sat}$.

$$G_{SR} = G_{FE} G_{CA} \qquad (5.61)$$

where G_{FE} is the front end gain (from satellite receiver input to satellite channel amplifier input) and G_{CA} is the satellite channel amplifier gain.

5.9.1.4 Input and Output Back-Off

In practice, the satellite channel power amplifier is not always operated at saturation, and it is convenient to determine the operating point Q of the satellite channel amplifier determined by the input power $(P_{in})_Q$ and the output power $(P_{on})_Q$. It is convenient to normalize these quantities with respect to $(P_{i1})_{sat}$ and $(P_{o1})_{sat}$ respectively. This defines the input back-off (IBO) and the output back-off (OBO):

$$\mathrm{IBO} = (P_{in})_Q/(P_{i1})_{sat} \quad (5.62)$$

$$\mathrm{OBO} = (P_{on})_Q/(P_{o1})_{sat} \quad (5.63)$$

From now on, the operating power value is denoted without the Q subscript.

5.9.1.5 Carrier power at satellite receiver input

The carrier power required at the satellite receiver input to drive the satellite channel amplifier to operate at the considered operating point Q is given by:

$$C_U = \frac{(P_{in})_Q}{G_{FE}} = \frac{P_{in}}{G_{FE}} = \mathrm{IBO}\frac{(P_{in})_{sat}}{G_{FE}} \quad (\mathrm{W}) \qquad (5.64)$$

The carrier power can also be expressed as a function of the satellite channel amplifier output power:

$$C_U = \mathrm{IBO}\frac{P_{on}}{G_{FE}G_{CA}} = \mathrm{IBO}\frac{(P_{01})_{sat}}{G_{FE}(G_{CA})_{sat}} \quad (\mathrm{W}) \qquad (5.65)$$

where $(G_{CA})_{sat}$ is the satellite amplifier gain *at saturation*. Finally, C_U can be expressed as:

$$C_U = \mathrm{IBO}(C_U)_{sat} \quad (\mathrm{W}) \qquad (5.66)$$

where

$$(C_U)_{sat} = \frac{(P_{i1})_{sat}}{G_{FE}} = \frac{(P_{o1})_{sat}}{G_{FE}(G_{CA})_{sat}}$$

is the carrier power required at the satellite receiver input to drive the satellite channel amplifier at saturation. $(C_U)_{sat}$ can also be expressed as a function of Φ_{sat}:

$$(C_U)_{sat} = \Phi_{sat}\frac{G_{Rmax}}{L_{FRX}}\frac{\lambda_U^2}{4\pi}\frac{1}{L_R L_{POL}} \quad (\mathrm{W}) \qquad (5.67)$$

or

$$(C_U)_{sat} = \Phi_{sat,\ nom}\frac{G_{Rmax}}{L_{FRX}}\frac{\lambda_U^2}{4\pi}$$

Note that the input back-off IBO can also be expressed as the ratio of the power flux density Φ required to operate the satellite channel amplifier at the considered operating point to the satellite power flux density at saturation:

$$\mathrm{IBO} = \frac{C_U}{(C_U)_{sat}} = \frac{\Phi}{\Phi_{sat}}$$

5.9.2 Expression for $(C/N_0)_T$

5.9.2.1 Expression for $(C/N_0)_T$ without interference from other systems or intermodulation:

The power of the carrier received at the input of the earth station receiver is C_D. The noise at the input of the earth station receiver corresponds to the sum of the following:

— the downlink system noise considered in isolation ($T_D = T_2$, given by equation (5.32)) which defines the ratio C/N_0 for the downlink $(C/N_0)_D$ and can be calculated as in the example of Section 5.6.3 with $(N_0)_D = kT_D$;
— the uplink noise retransmitted by the satellite.

Hence:

$$(N_0)_T = (N_0)_D + G(N_0)_U \quad (\mathrm{W/Hz}) \tag{5.68}$$

where $G = G_{SR}G_TG_R/L_{FTX}L_DL_{FRX}$ is the total power gain between the satellite receiver input and the earth station receiver input. G takes into account the satellite repeater gain G_{SR} from the input to the satellite receiver to the output of the satellite channel amplifier; the gain G_T/L_{FTX} of the satellite transmit antenna including the gain fallout and the loss L_{FTX} from the output of the power amplifier to the transmit antenna; the downlink path loss L_D and the receiving station composite gain G_R/L_{FRX}. This gives

$$\begin{aligned}(C/N_0)_T^{-1} &= (N_0)_T/C_D \\ &= [(N_0)_D + G(N_0)_U]/C_D = (N_0)_D/C_D + (N_0)_U/G^{-1}C_D \quad (\mathrm{Hz}^{-1})\end{aligned} \tag{5.69}$$

In the above expression, the term $G^{-1}C_D$ represents the carrier power at the satellite receiver input. Hence $(N_0)_U/G^{-1}C_D = (C/N_0)_U^{-1}$. Finally:

$$(C/N_0)_T^{-1} = (C/N_0)_U^{-1} + (C/N_0)_D^{-1} \quad (\mathrm{Hz}^{-1}) \tag{5.70}$$

In this expression:

$$\begin{aligned}(C/N_0)_U &= (P_{i1})/(N_0)_U = \mathrm{IBO}(P_{i1})_{sat}/(N_0)_U \\ &= \mathrm{IBO}(P_{o1})_{sat}/G_{SR\,sat}(N_0)_U \\ &= \mathrm{IBO}(C/N_0)_{U\,sat} \quad (\mathrm{Hz}) \\ (C/N_0)_D &= \mathrm{OBO}(\mathrm{EIRP}_{sat})_{SL}(1/L_D)(G/T)_{ES}(1/k) \\ &= \mathrm{OBO}(C/N_0)_{D\,sat} \quad (\mathrm{Hz})\end{aligned}$$

$(C/N_0)_{U\,sat}$ and $(C/N_0)_{D\,sat}$ are the values of C/N_0 for the uplink and downlink when the satellite channel operates at saturation. L_D represents the attenuation on the downlink and is given by equation (5.14) and $(G/T)_{ES}$, the figure of merit of the earth station in the satellite direction.

5.9.2.2 Expression for $(C/N_0)_T$ taking account of interference:

Interference is the unwanted power contribution of other carriers in the frequency band occupied by the wanted carrier.

Four types of interference between systems can be distinguished:

- ☒ a satellite interfering with a terrestrial station;
- ☒ a terrestrial station interfering with a satellite;
- ☒ an earth station interfering with a terrestrial station;

- A terrestrial station interfering with an earth station.

The carriers emitted by other systems are superimposed on the wanted carrier of the station to- station link at two levels:

- at the input of the satellite repeater on the uplink;
- at the input of the earth station receiver on the downlink

The effect of interference is similar to an increase of the thermal noise on the link affected by interference. It is allowed for in the equations in the form of an increase of the spectral density:

$$N_0 = (N_0)_{\text{without interference}} + (N_0)_I \quad (\text{W/Hz}) \qquad (5.71)$$

where $(N_0)_I$ represents the increase of the noise power spectral density due to interference. A ratio $(C/N_0)_I$ which expresses the signal power in relation to the spectral density of the interference can be associated with $(N_0)_I$; these are $(C/N_0)_{I,U}$ for the uplink and $(C/N_0)_{I,D}$ for the downlink. This leads to modification of equation (5.70), replacing $(C/N_0)_U$ and $(C/N_0)_D$ by the following expressions:

$$(C/N_0)_U^{-1} = [(C/N_0)_U^{-1}]_{\text{without interference}} + (C/N_0)_{I,U}^{-1} \quad (\text{Hz}^{-1})$$

$$(C/N_0)_D^{-1} = [(C/N_0)_D^{-1}]_{\text{without interference}} + (C/N_0)_{I,D}^{-1} \quad (\text{Hz}^{-1}) \qquad (5.72)$$

The total expression becomes:

$$(C/N_0)_T^{-1} = (C/N_0)_U^{-1} + (C/N_0)_D^{-1} + (C/N_0)_I^{-1} \quad (\text{Hz}^{-1}) \qquad (5.73)$$

where $(C/N_0)_U$ and $(C/N_0)_D$ are the values appearing in equation (5.70) and:

$$(C/N_0)_I^{-1} = (C/N_0)_{I,U}^{-1} + (C/N_0)_{I,D}^{-1} \quad (\text{Hz}^{-1}) \qquad (5.74)$$

5.9.2.3 Expression for$(C/N_0)_T$ taking account of intermodulation and interference

The ratio of the carrier power to the intermodulation noise spectral density is$(C/N_0)_{IM}$. Equation (5.74) for the carrier power-to-noise power spectral density ratio for the overall station-to-station link $(C/N_0)_T$ is modified as follows:

$$(C/N_0)_T^{-1} = (C/N_0)_U^{-1} + (C/N_0)_D^{-1} + (C/N_0)_I^{-1} + (C/N_0)_{IM}^{-1} \quad (\text{Hz}^{-1}) \qquad (5.75)$$

with:

$$(C/N_0)_{IM}^{-1} = (C/N_0)_{IM,U}^{-1} + (C/N_0)_{IM,D}^{-1}$$

— input back-off *per carrier*:

- IBO_1 = single carrier input power/single carrier input power at saturation $= P_{i1}/(P_{i1})_{sat}$
  or, in dB:
- $IBO_1\ (\text{dB}) = 10 \log \{P_{i1}/(P_{i1})_{sat}\}$

In this case, the expressions for the ratios $(C/N_0)_U$, $(C/N_0)_D$ and $(C/N_0)_{IM}$ are to be used with values of input and output back-off IBO and OBO for operation of the amplifier in multicarrier mode with carriers of *equal power*. The output power of the amplifier is shared among the carriers, the thermal noise and the intermodulation noise to which the interference noise for the channel is added.

Denoting by P_{in} and P_{on} respectively the input and output and power of one carrier among the n amplified ones, input and output back-off are defined as follows:

— output back-off *per carrier*:
  – OBO_1 = single carrier output power/single carrier output power at saturation $= P_{o1}/(P_{o1})_{sat}$
  or, in dB:
  – $IBO_1\ (dB) = 10\log\{P_{o1}/(P_{o1})_{sat}\}$
— total input back-off:
  – IBO_t = sum of all input carrier power/single carrier input power at saturation $= \Sigma P_{in}/(P_{i1})_{sat}$
  or, in dB:
  – $IBO_t\ (dB) = 10\log\{\Sigma P_{in}/(P_{i1})_{sat}\}$
— total output back-off:
  – OBO_t = sum of all output carrier power/single carrier output power at saturation $= \Sigma P_{on}/(P_{o1})_{sat}$
  or, in dB:
  – $OBO_t\ (dB) = 10\log\{\Sigma P_{on}/(P_{o1})_{sat}\}$

With n equally powered carriers:

— $IBO_1 = IBO_t/n$ or, in dB, $IBO_1\ (dB) = IBO_t\ (dB) - 10\log n$
— $OBO_1 = OBO_t/n$ or, in dB, $OBO_1\ (dB) = OBO_t\ (dB) - 10\log n$

5.9.3 Overall link performance for a transparent satellite without interference or intermodulation

It is required to establish a satellite link between two earth stations (Figure 5.33), assumed to be located at the center of the satellite antenna's coverage. The data are as follows:

- Uplink frequency: f_U = 14 GHz.
- Downlink frequency: f_D =12 GHz.
- Downlink path loss: L_D = 206 dB.
- For the satellite (SL):
  - Power flux density required to saturate the satellite channel amplifier:

$$(\Phi_{sat,\ nom})_{SL} = -90\ \mathrm{dBW/m^2}$$

- Satellite receiving antenna gain at boresight: G_{Rmax} = 30 dBi
- Satellite figure of merit at boresight: $(G/T)_{SL}$= 3.4 dBK^{-1}
- Satellite channel amplifier characteristic (single carrier operation) modeled by:

$$OBO(dB) = IBO(dB) + 6 - 6\exp[IBO(dB)/6]$$

– Satellite effective isotropic radiated power at saturation in the direction of the considered receiving earth station (i.e. at boresight of the satellite transmitting antenna)

$$(\mathrm{EIRP_{sat}})_{SL} = 50\,\mathrm{dBW}$$

– Satellite transmitting antenna gain at boresight: $G_{\mathrm{Tmax}} = 40$ dBi

The following losses are considered:

— Satellite reception and transmission feeder losses: $L_{\mathrm{FRX}} = L_{\mathrm{FTX}} = 0\,\mathrm{dB}$
— Satellite antenna polarisation mismatch loss $L_{\mathrm{POL}} = 0\,\mathrm{dB}$
— Satellite antenna depointing losses: $L_{\mathrm{R}} = L_{\mathrm{T}} = 0$ dB (earth stations at boresight)
— For the earth station (ES): Figure of merit of earth station in satellite direction $(G/T)_{\mathrm{ES}} = 25\,\mathrm{dBK^{-1}}$

It is assumed that there is no interference.

5.9.3.1 Satellite repeater gain at saturation $G_{SR\ sat}$

$$G_{\mathrm{SR\,sat}} = (P_o^1)_{\mathrm{sat}}/(C_{\mathrm{U}})_{\mathrm{sat}}$$

where $(C_{\mathrm{U}})_{\mathrm{sat}}$ is the carrier power required at the satellite receiver input to drive the satellite channel amplifier at saturation. From equation (5.60):

$$(P_{o1})_{\mathrm{sat}} = (\mathrm{EIRP_{sat}})_{\mathrm{SL}} L_T L_{\mathrm{FTX}}/G_{\mathrm{Tmax}} \quad (\mathrm{W})$$

Hence:

$$(P_{o1})_{\mathrm{sat}} = 50\ \mathrm{dBW} - 40\ \mathrm{dBi} = 10\,\mathrm{dBW} = 10\,\mathrm{W}$$

From equation (5.67)

$$(C_{\mathrm{U}})_{\mathrm{sat}} = (\Phi_{\mathrm{sat}})_{\mathrm{SL}} G_{\mathrm{Rmax}}/L_{\mathrm{FRX}} L_{\mathrm{R}} L_{\mathrm{POL}}(4\pi/\lambda_{\mathrm{U}}^2) \quad (\mathrm{W})$$

hence:

$$\begin{aligned}(C_{\mathrm{U}})_{\mathrm{sat}} &= -90\,\mathrm{dBW/m^2} + 30\,\mathrm{dBi} - 44.4\,\mathrm{dBm^2} = -104.4\,\mathrm{dBW} = 36\,\mathrm{pW} \\ G_{\mathrm{SR\ sat}} &= (P_{o1})_{\mathrm{sat}}/(C_{\mathrm{U}})_{\mathrm{sat}} = 10\,\mathrm{dBW} - (-104.4\ \mathrm{dBW}) = 114.4\,\mathrm{dB}\end{aligned}$$

5.9.3.2 Calculation of C/N_0 for the up- and downlinks and the overall link when the repeater operates at saturation

$$(C/N_0)_{\mathrm{U\ sat}} = (C_U)_{\mathrm{sat}}/kT_{\mathrm{U}} = (C_U)_{\mathrm{sat}}(G/T)_{\mathrm{SL}}/(kG_{\mathrm{Rmax}}/L_R L_{FRX} L_{POL})$$

$$(C/N_0)_{\mathrm{U\ sat}} = -104.4 + 3.4 - (-228.6) - 30 = 97.6\,\mathrm{dBHz}$$

$$(C/N_0)_{\mathrm{D\ sat}} = (\mathrm{EIRP_{sat}})_{\mathrm{SL}}(1/L_{\mathrm{D}})(G/T)_{\mathrm{ES}}(1/k) \quad (\mathrm{Hz})$$

$$(C/N_0)_{\mathrm{D\ sat}} = 50 - 206 + 25 - (-228.6) = 97.6\,\mathrm{dBHz}$$

$$(C/N_0)_{\mathrm{T\ sat}}^{-1} = (C/N_0)_{\mathrm{U\ sat}}^{-1} + (C/N_0)_{\mathrm{D\ sat}}^{-1} \quad (\mathrm{Hz^{-1}})$$

$$(C/N_0)_{\mathrm{T\ sat}} = 94.6\,\mathrm{dBHz}$$

5.9.3.3 Calculation of the input and output back-off to achieve $(C/N_0)_T$ =80 dBHz and the corresponding values of $(C/N_0)_U$ and $(C/N_0)_D$

One must have:

$$(C/N_0)_U^{-1} + (C/N_0)_D^{-1} = 10^{-8}\,\text{Hz}^{-1}$$

Hence:

$$\text{IBO}^{-1}(C/N_0)_{U\,\text{sat}}^{-1} + \text{OBO}^{-1}(C/N_0)_{D\,\text{sat}}^{-1} = 10^{-8}\,\text{Hz}^{-1}$$

This gives:

$$10^{-\text{IBO(dB)}/10} + 10^{-\text{OBO(dB)}/10} = 10^{1.76}$$

with:

$$\text{OBO(dB)} = \text{IBO(dB)} + 6 - 6\exp(\text{IBO(dB)}/6)$$

Numerical solution gives:

$$\begin{aligned}\text{IBO} &= -16.4\,\text{dB}\\ \text{OBO} &= -10.8\,\text{dB}\end{aligned}$$

Hence:

$$\begin{aligned}(C/N_0)_U &= \text{IBO}(C/N_0)_{U\,\text{sat}} = -16.4\,\text{dB} + 97.6\,\text{dBHz} = 81.2\,\text{dBHz}\\ (C/N_0)_D &= \text{OBO}(C/N_0)_{D\,\text{sat}} = -10.8\,\text{dB} + 97.6\,\text{dBHz} = 86.8\,\text{dBHz}\end{aligned}$$

5.9.3.4 Value of $(C/N_0)_T$ under rain conditions causing an attenuation of 6 dB on the uplink

The attenuation of 6 dB on the uplink reduces the input back-off by 6 dB. The new value of IBO becomes:

$$\text{IBO(dB)} = -16.4\,\text{dB} - 6\,\text{dB} = -22.4\,\text{dB}$$

The new value of output back-off corresponding to this is:

$$\text{OBO(dB)} = \text{IBO(dB)} + 6 - 6\exp(\text{IBO(dB)}/6) = -16.5\,\text{dB}$$

Hence:

$$\begin{aligned}(C/N_0)_U &= \text{IBO}(C/N_0)_{U\,\text{sat}} = -22.4\,\text{dB} + 97.6\,\text{dBHz} = 75.2\,\text{dBHz}\\ (C/N_0)_D &= \text{OBO}(C/N_0)_{D\,\text{sat}} = -16.5\,\text{dB} + 97.6\,\text{dBHz} = 81.1\,\text{dBHz}\end{aligned}$$

and, from equation (5.70)

$$(C/N_0)_T = 74.2\,\text{dBHz}$$

To regain the required value $(C/N_0)_T = 80$ dBHz, it is necessary to increase the $(\text{EIRP})_{ES}$ of the transmitting earth station by 6 dB.

5.9.3.5 Value of $(C/N_0)_T$ under rain conditions causing an attenuation of 6 dB on the downlink with a reduction of 2 dB in the figure of merit of the earth station due to the increase of antenna noise temperature

The value of $(C/N_0)_D$ reduces by 8 dB, hence: $(C/N_0)_D = 86.8\,\text{dBHz} - 8\,\text{dB} = 78.8$ dBHz. From which: $(C/N_0)_T = 76.8$ dBHz.

To regain the required value $(C/N_0)_T = 80$ dBHz, it is necessary to increase the $(\text{EIRP})_{ES}$ of the transmitting earth station in such a way that the value of IBO satisfies the equation:

$$\text{IBO}^{-1}(C/N_0)^{-1}_{\text{U sat}} + \text{OBO}^{-1}(C/N_0)^{-1}_{\text{D sat}} = 10^{-8}\ \text{Hz}^{-1}$$

in which:

$$(C/N_0)_{\text{U sat}} = 97.6\,\text{dBHz}$$
$$(C/N_0)_{\text{D sat}} = 97.6\,\text{dBHz} - 8\,\text{dB} = 89.6\,\text{dBHz}$$

This gives:

$$\text{IBO} = -13\,\text{dB}$$
$$\text{OBO} = -7.7\,\text{dB}$$

It is necessary to increase the $(\text{EIRP})_{ES}$ of the earth station transmission by $-13\,\text{dB} - (-16.4\,\text{dB}) = 3.4\,\text{dB}$.

Hence:

$$(C/N_0)_U = \text{IBO}(C/N_0)_{\text{U sat}} = -13\,\text{dB} + 97.6\,\text{dBHz} = 84.6\,\text{dBHz}$$
$$(C/N_0)_D = \text{OBO}(C/N_0)_{\text{D sat}} = -7.7\,\text{dB} + 89.6\,\text{dBHz} = 81.9\,\text{dBHz}$$