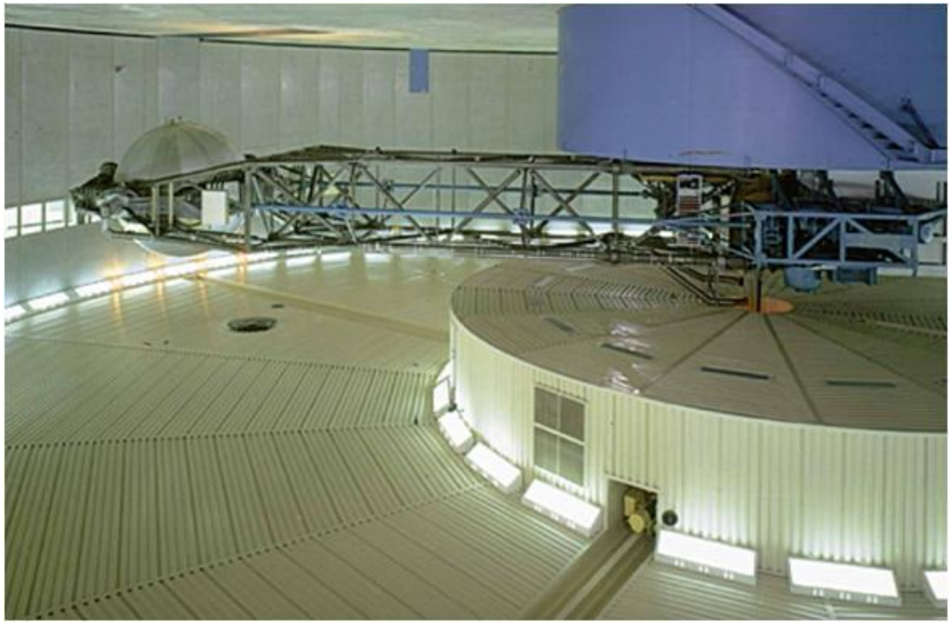
EQUATIONS OF MOTION: NORMAL AND TANGENTIAL COORDINATES

<u>Today's Objectives</u>:
Students will be able to:
1. Apply the equation of motion using normal and tangential coordinates.



<u>In-Class Activities</u>:
- Check Homework
- Reading Quiz
- Applications
- Equation of Motion using n-t Coordinates
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The "normal" component of the equation of motion is written as $\Sigma \boldsymbol{F_n} = m\boldsymbol{a_n}$, where $\Sigma \boldsymbol{F_n}$ is referred to as the _______.

   A) impulse
   B) centripetal force
   C) tangential force
   D) inertia force

2. The positive ***n*** direction of the normal and tangential coordinates is ____________.

   A) normal to the tangential component
   B) always directed toward the center of curvature
   C) normal to the bi-normal component
   D) All of the above.

APPLICATIONS

Race track turns are often banked to reduce the frictional forces required to keep the cars from sliding up to the outer rail at high speeds.

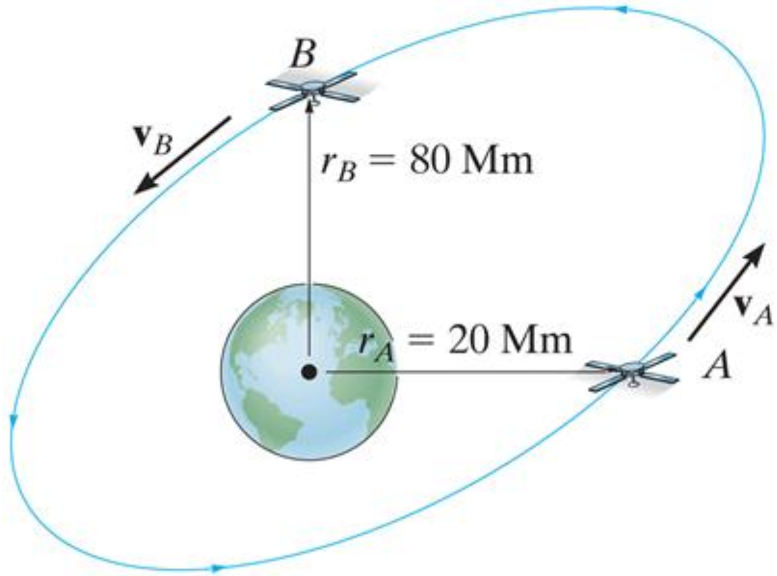
If the car's maximum velocity and a minimum coefficient of friction between the tires and track are specified, how can we determine the minimum banking angle (θ) required to prevent the car from sliding up the track?

APPLICATIONS (continued)

This picture shows a ride at the amusement park. The hydraulically-powered arms turn at a constant rate, which creates a centrifugal force on the riders.

We need to determine the smallest angular velocity of cars A and B such that the passengers do not lose contact with their seat. What parameters are needed for this calculation?

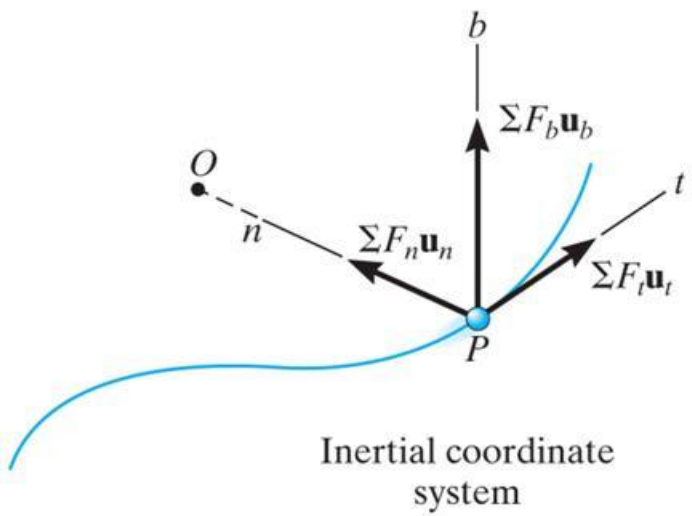
APPLICATIONS (continued)



Satellites are held in orbit around the earth by using the earth's gravitational pull as the centripetal force – the force acting to change the direction of the satellite's velocity.

Knowing the radius of orbit of the satellite, we need to determine the required speed of the satellite to maintain this orbit. What equation governs this situation?

NORMAL & TANGENTIAL COORDINATES (Section 13.5)

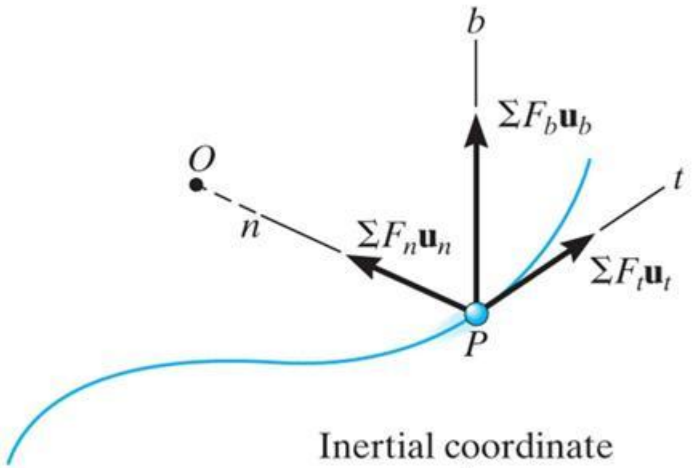


When a particle moves along a curved path, it may be more convenient to write the equation of motion in terms of normal and tangential coordinates.

The normal direction (n) *always* points toward the path's center of curvature. In a circle, the center of curvature is the center of the circle.

The tangential direction (t) is tangent to the path, usually set as positive in the direction of motion of the particle.

EQUATIONS OF MOTION



Inertial coordinate system

Since the equation of motion is a vector equation, $\sum \boldsymbol{F} = m\mathbf{a}$, it may be written in terms of the n & t coordinates as

$$\sum F_t \boldsymbol{u}_t + \sum F_n \boldsymbol{u}_n + \sum F_b \boldsymbol{u}_b = m\mathbf{a}_t + m\mathbf{a}_n$$

Here $\sum F_t$ & $\sum F_n$ are the sums of the force components acting in the t & n directions, respectively.

This vector equation will be satisfied provided the individual components on each side of the equation are equal, resulting in the two scalar equations: $\sum F_t = ma_t$ and $\sum F_n = ma_n$.

Since there is no motion in the binormal (b) direction, we can also write $\sum F_b = 0$.

NORMAL AND TANGENTIAL ACCELERATION

The tangential acceleration, $a_t = dv/dt$, represents the time rate of change in the magnitude of the velocity. Depending on the direction of $\sum F_t$, the particle's speed will either be increasing or decreasing.

The normal acceleration, $a_n = v^2/\rho$, represents the time rate of change in the direction of the velocity vector. Remember, a_n always acts toward the path's center of curvature. Thus, $\sum F_n$ will always be directed toward the center of the path.

Recall, if the path of motion is defined as $y = f(x)$, the radius of curvature at any point can be obtained from

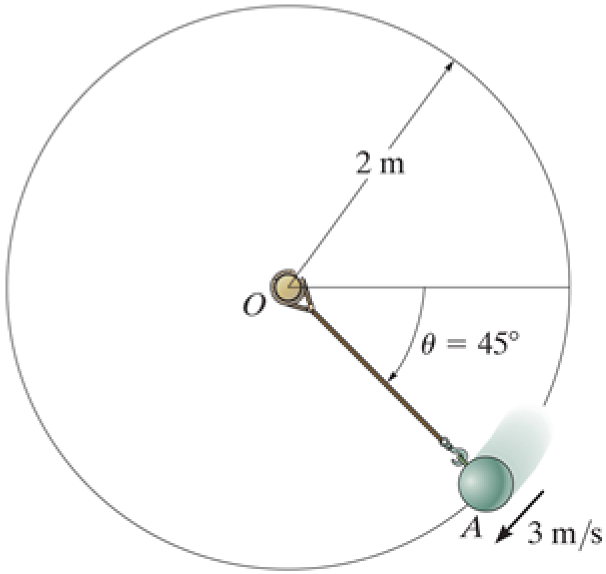
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

SOLVING PROBLEMS WITH n-t COORDINATES

- Use n-t coordinates when a particle is moving along a known, curved path.
- Establish the n-t coordinate system on the particle.
- Draw free-body and kinetic diagrams of the particle. The normal acceleration (a_n) always acts "inward" (the positive n-direction). The tangential acceleration (a_t) may act in either the positive or negative t direction.
- Apply the equations of motion in scalar form and solve.
- It may be necessary to employ the kinematic relations:

$$a_t = dv/dt = v\, dv/ds \qquad a_n = v^2/\rho$$

EXAMPLE



Given: The 10-kg ball has a velocity of 3 m/s when it is at A, along the vertical path.

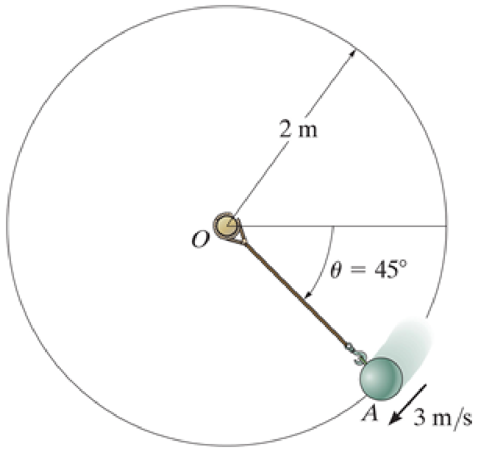
Find: The tension in the cord and the increase in the speed of the ball.

Plan:
1) Since the problem involves a curved path and requires finding the force perpendicular to the path, use n-t coordinates. Draw the ball's free-body and kinetic diagrams.
2) Apply the equation of motion in the n-t directions.

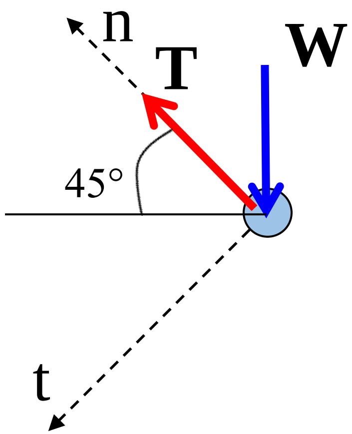
EXAMPLE (continued)

Solution:

1) The n-t coordinate system can be established on the ball at Point A, thus at an angle of 45°. Draw the free-body and kinetic diagrams of the ball.



Free-body diagram



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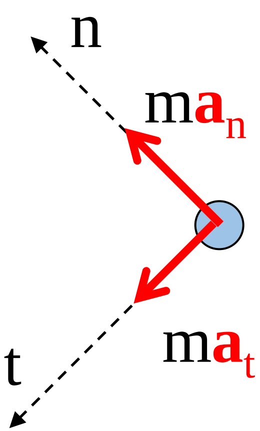
Kinetic diagram



EXAMPLE (continued)

2) Apply the equations of motion in the n-t directions.

(a) $\sum F_n = ma_n \Rightarrow T - W \sin 45° = m\, a_n$

Using $a_n = v^2/\rho = 3^2/2$, W = 10(9.81) N, and m = 10 kg

$\Rightarrow \quad T - 98.1 \sin 45° = (10)\,(3^2/2)$

$\Rightarrow \quad T = \underline{114\ \text{N}}$

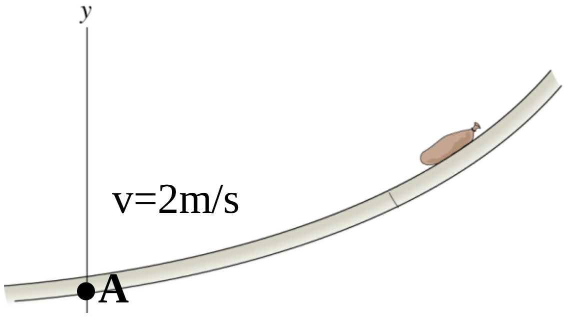
(b) $\sum F_t = ma_t \Rightarrow W \cos 45° = ma_t$

$\Rightarrow \quad 98.1 \cos 45° = 10\, a_t$

$\Rightarrow \quad a_t = (dv/dt) = \underline{6.94\ \text{m/s}^2}$

CONCEPT QUIZ

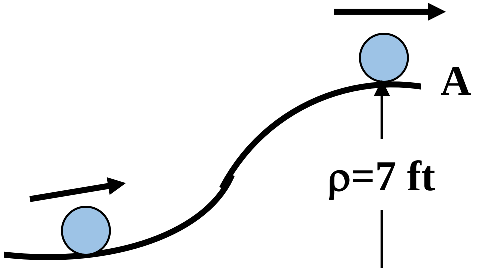
1. A 10 kg sack slides down a smooth surface. If the normal force at the flat spot on the surface, A, is 98.1 N (↑) , the radius of curvature is ____.



A) 0.2 m B) 0.4 m

C) 1.0 m D) None of the above.

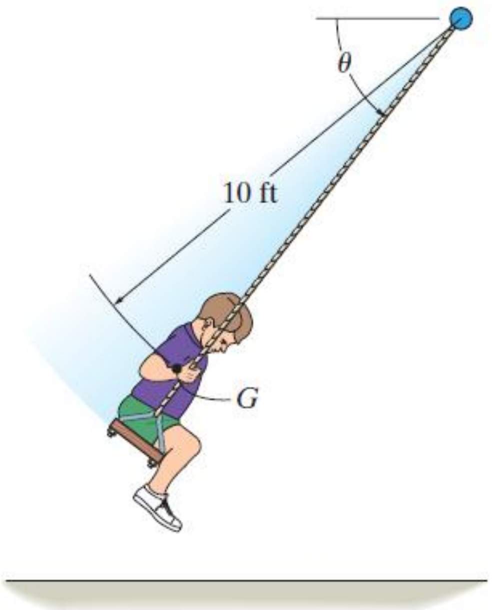
2. A 20 lb block is moving along a smooth surface. If the normal force on the surface at A is 10 lb, the velocity is ________.



A) 7.6 ft/s B) 9.6 ft/s

C) 10.6 ft/s D) 12.6 ft/s

GROUP PROBLEM SOLVING I



Given: The boy has a weight of 60 lb. At the instant θ = 60, the boy's center of mass *G* experiences a speed v = 15 ft/s.

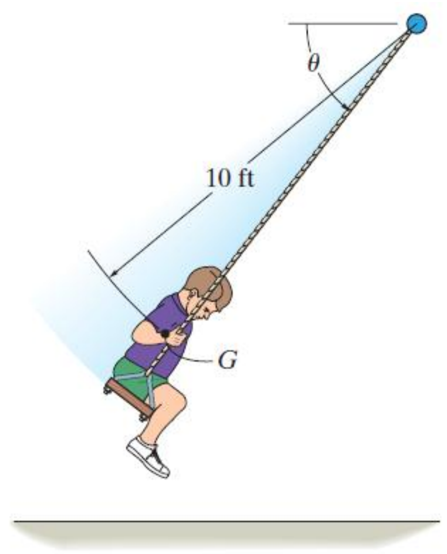
Find: The tension in each of the two supporting cords of the swing and the rate of increase in his speed at this instant.

Plan:
1) Use n-t coordinates and treat the boy as a particle. Draw the free-body and kinetic diagrams.
2) Apply the equation of motion in the n-t directions.

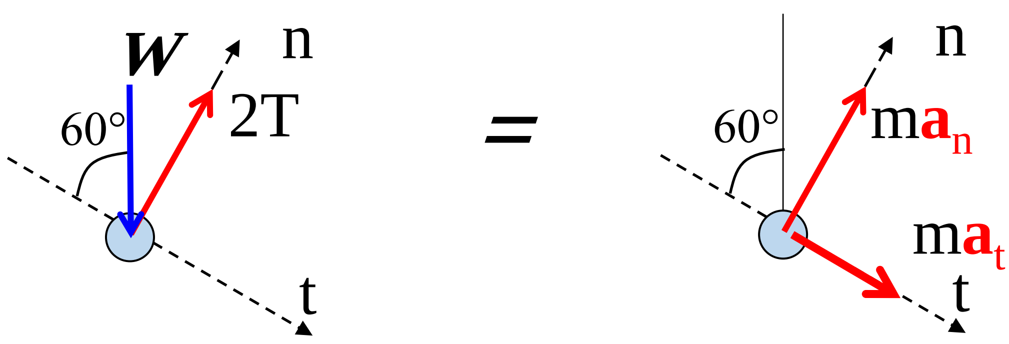
GROUP PROBLEM SOLVING I (continued)

Solution:

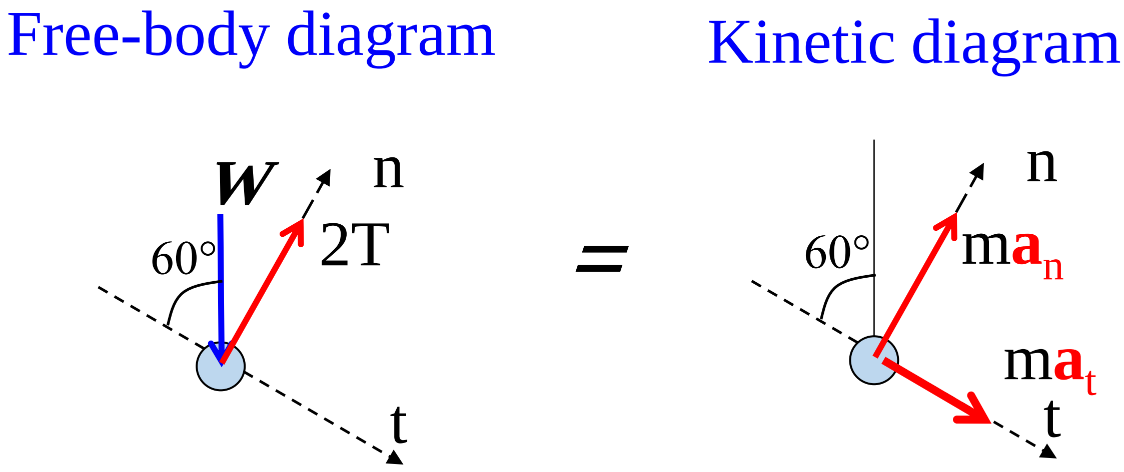
1) The n-t coordinate system can be established on the boy at angle 60°. Approximating the boy as a particle, the free-body and kinetic diagrams can be drawn**:**



Free-body diagram Kinetic diagram



GROUP PROBLEM SOLVING I (continued)



2) Apply the equations of motion in the n-t directions.

$$\sum F_n = ma_n \Rightarrow 2T - W \sin 60° = ma_n$$

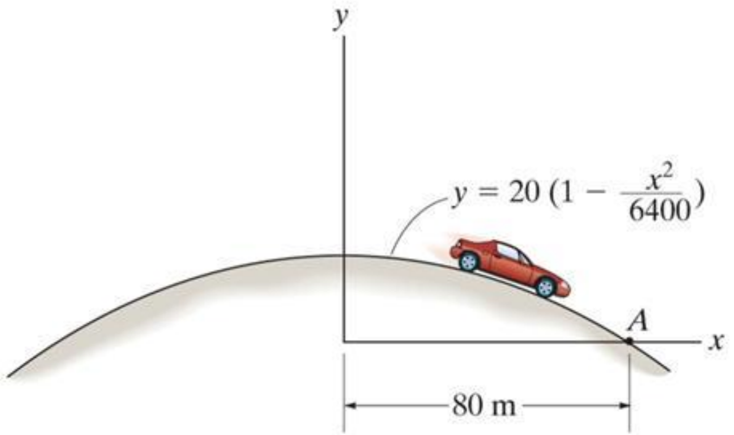
Using $a_n = v^2/\rho = 15^2/10$, W = 60 lb,

we get: T = 46.9 lb

$$\sum F_t = ma_t \Rightarrow 60 \cos 60° = (60 / 32.2)\, a_t$$

$a_t = \dot{v} = $ 16.1 ft/s^2

GROUP PROBLEM SOLVING II



Given: A 800 kg car is traveling over a hill with the shape of a parabola. When the car is at point A, its v = 9 m/s and a = 3 m/s^2. (Neglect the size of the car.)

Find: The resultant normal force and resultant frictional force exerted on the road at point A by the car.

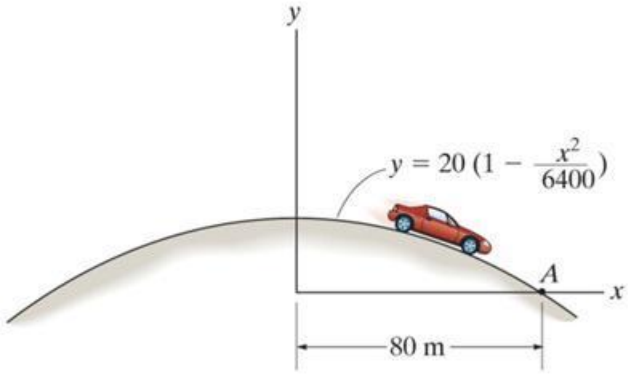
Plan:

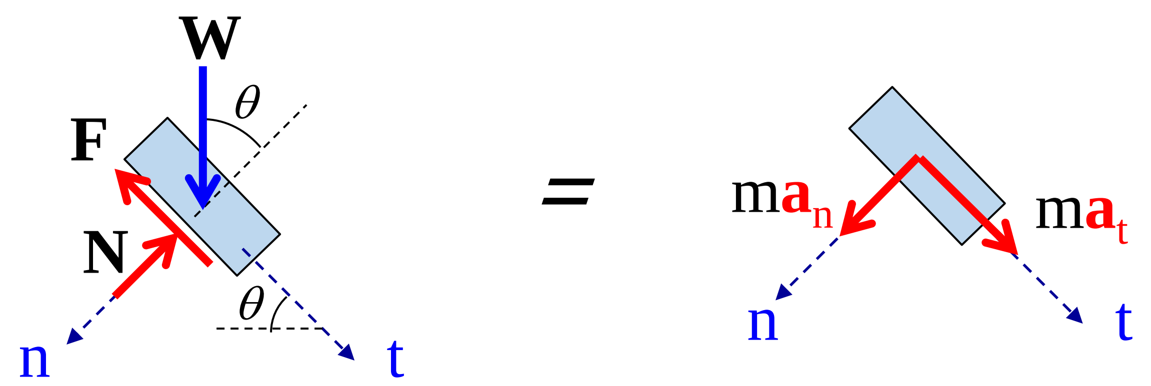
1) Treat the car as a particle. Draw its free-body and kinetic diagrams.
2) Apply the equations of motion in the n-t directions.
3) Use calculus to determine the slope and radius of curvature of the path at point A.

GROUP PROBLEM SOLVING II (continued)

Solution:

1) The n-t coordinate system can be established on the car at point A. Treat the car as a particle and draw the free-body and kinetic diagrams:





W = mg = weight of car

N = resultant normal force on road

F = resultant friction force on road

GROUP PROBLEM SOLVING II (continued)

2) Apply the equations of motion in the n-t directions:

$$\sum F_n = ma_n \;\Rightarrow\; W \cos\theta - N = ma_n$$

Using $W = mg$ and $a_n = v^2/\rho = (9)^2/\rho$

$$\Rightarrow\; (800)(9.81)\cos\theta - N = (800)\,(81/\rho)$$

$$\Rightarrow\; N = 7848\cos\theta - 64800\,/\,\rho \qquad \text{Eq. (1)}$$

$$\sum F_t = ma_t \Rightarrow\; W \sin\theta - F = ma_t$$

Using $W = mg$ and $a_t = 3\ \text{m/s}^2$ (given)

$$\Rightarrow\; (800)(9.81)\sin\theta - F = (800)\,(3)$$

$$\Rightarrow\; F = 7848\sin\theta - 2400 \qquad \text{Eq. (2)}$$

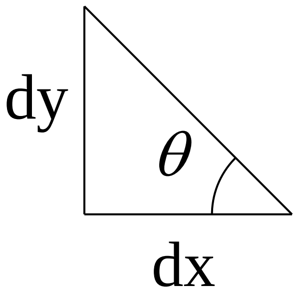
GROUP PROBLEM SOLVING II (continued)

3) Determine ρ by differentiating y = f(x) at x = 80 m:

$$y = 20(1 - x^2/6400) \Rightarrow dy/dx = (-40)\,x / 6400$$
$$\Rightarrow d^2y/dx^2 = (-40) / 6400$$

$$\rho\Big|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{3/2}}{|0.00625|} = 223.6\text{ m}$$

Determine θ from the slope of the curve at A:



$$\tan\,\theta = dy/dx\Big|_{x = 80\text{ m}}$$

$$\theta = \left|\tan^{-1}(dy/dx)\right| = \left|\tan^{-1}(-0.5)\right| = 26.6°$$

GROUP PROBLEM SOLVING II (continued)

From Eq. (1): $N = 7848 \cos \theta - 64800 / \rho$

$= 7848 \cos (26.6°) - 64800 / 223.6 = \underline{6728\ \text{N}}$

From Eq. (2): $F = 7848 \sin \theta - 2400$

$= 7848 \sin (26.6°) - 2400 = \underline{1114\ \text{N}}$

ATTENTION QUIZ

1. The tangential acceleration of an object

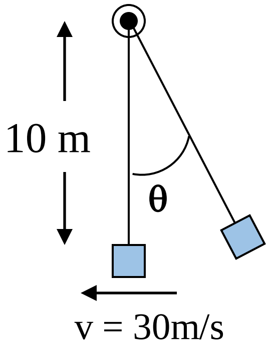
   A) represents the rate of change of the velocity vector's direction.

   B) represents the rate of change in the magnitude of the velocity.

   C) is a function of the radius of curvature.

   D) Both B and C.

2. The block has a mass of 20 kg and a speed of v = 30 m/s at the instant it is at its lowest point. Determine the tension in the cord at this instant.



   A) 1596 N B) 1796 N

   C) 1996 N D) 2196 N

End of the Lecture

Let Learning Continue