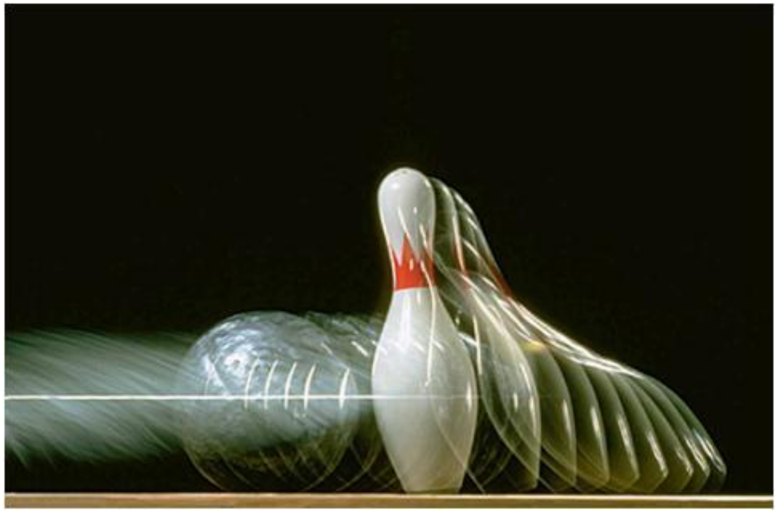
CONSERVATION OF MOMENTUM

Today's Objectives:

Students will be able to:

1. Understand necessary conditions for conservation of linear and angular momentum.
2. Use conservation of linear/ angular momentum for solving problems.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Conservation of Linear and Angular Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. If there are no external impulses acting on a body ________________.

   A) only linear momentum is conserved

   B) only angular momentum is conserved

   C) both linear momentum and angular momentum are conserved

   D) neither linear momentum nor angular momentum are conserved

2. If a rigid body rotates about a fixed axis passing through its center of mass, the body's linear momentum is ____________.

   A) constant

   B) zero

   C) $m\,v_G$

   D) $I_G\,\omega$

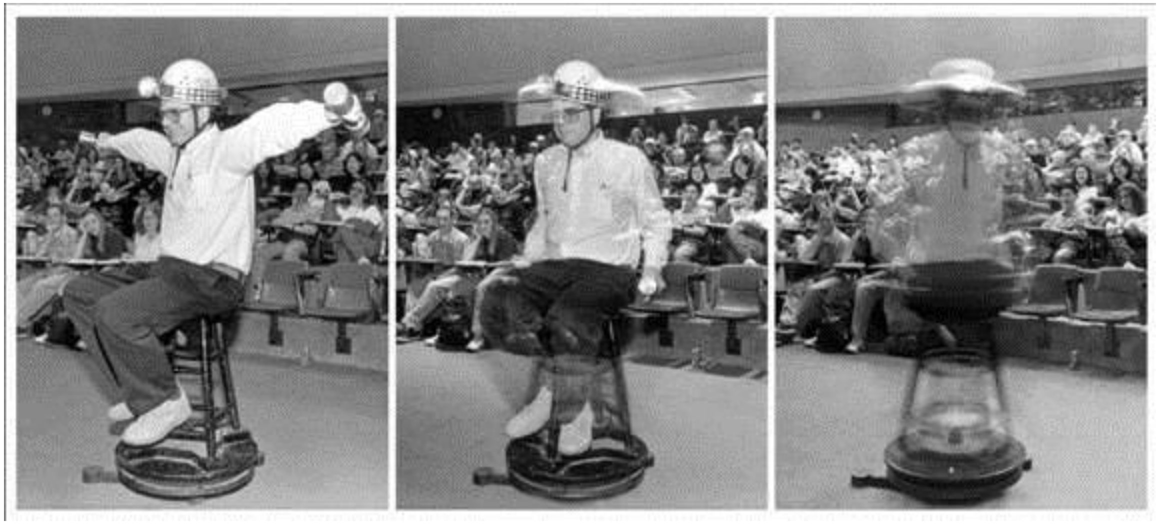
APPLICATIONS

A skater spends a lot of time either spinning on the ice or rotating through the air. To spin fast, or for a long time, the skater must develop a large amount of angular momentum.

If the skater's angular momentum is constant, can the skater vary her rotational speed? How?

The skater spins faster when the arms are drawn in and slower when the arms are extended. Why?

APPLICATIONS (continued)



Conservation of angular momentum allows cats to land on their feet and divers to flip, twist, spiral and turn. It also helps teachers make their heads spin!

Conservation of angular momentum makes water circle the drain faster as it gets closer to the drain.

CONSERVATION OF LINEAR MOMENTUM
(Section 19.3)

Recall that the linear impulse and momentum relationship is

$$\boldsymbol{L}_1 + \cancelto{0}{\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt} = \boldsymbol{L}_2 \quad \text{or} \quad (m\,\boldsymbol{v}_G)_1 + \cancelto{0}{\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt} = (m\,\boldsymbol{v}_G)_2$$

If the sum of all the linear impulses acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, the linear momentum for a rigid body (or system) is constant, or conserved. So $\boldsymbol{L}_1 = \boldsymbol{L}_2$.

This equation is referred to as the conservation of linear momentum. The conservation of linear momentum equation can be used if the linear impulses are small or non-impulsive.

CONSERVATION OF ANGULAR MOMENTUM

Similarly, if the sum of all the angular impulses due to external forces acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, angular momentum is conserved

$$(\boldsymbol{H}_G)_1 + \sum \cancelto{0}{\int_{t_1}^{t_2} \boldsymbol{M}_G \, dt} = (\boldsymbol{H}_G)_2 \quad \text{or} \quad I_G \boldsymbol{\omega}_1 + \sum \cancelto{0}{\int_{t_1}^{t_2} \boldsymbol{M}_G \, dt} = I_G \boldsymbol{\omega}_2$$

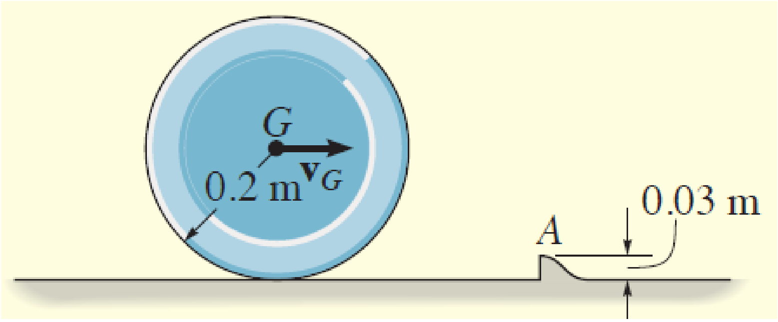
The resulting equation is referred to as the conservation of angular momentum or $(\boldsymbol{H}_G)_1 = (\boldsymbol{H}_G)_2$.

If the initial condition of the rigid body (or system) is known, conservation of momentum is often used to determine the final linear or angular velocity of a body just after an event occurs.

PROCEDURE FOR ANALYSIS

- Establish the x, y, z inertial frame of reference and draw FBDs.
- Write the conservation of linear momentum equation.
- Write the conservation of angular momentum equation about a fixed point or at the mass center G.
- Solve the conservation of linear or angular momentum equations in the appropriate directions.
- If the motion is complicated, use of kinematic equations that relate the velocity of the mass center, G, and the angular velocity, ω, may be necessary.

EXAMPLE



Given: A 10 kg wheel ($I_G = 0.156$ kg·m^2) rolls without slipping and does not rebound.

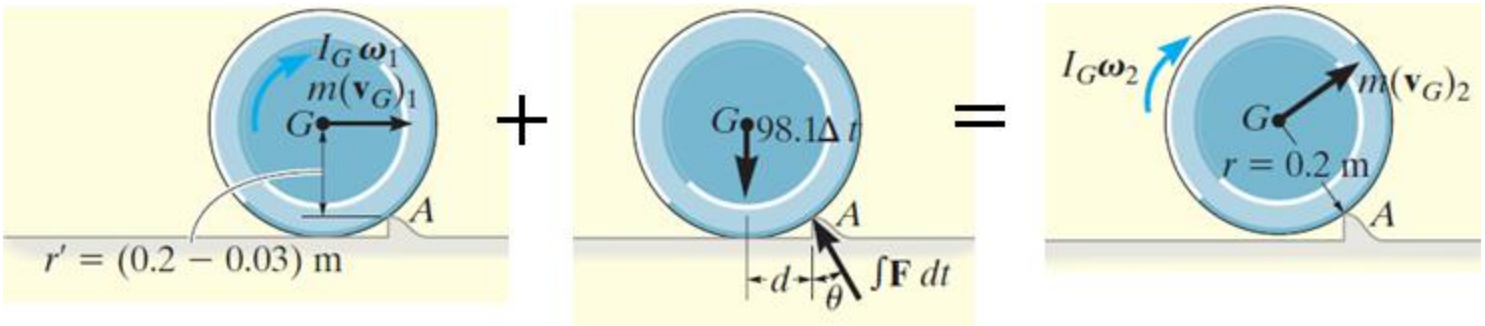
Find: The minimum velocity, v_G, the wheel must have to just roll over the obstruction at A.

Plan: Since no slipping or rebounding occurs, the wheel pivots about point A. The force at A is much greater than the weight, and since the time of impact is very short, the weight can be considered non-impulsive. The reaction force at A is a problem as we don't know either its direction or magnitude. This force can be eliminated by applying the conservation of angular momentum equation about A.

EXAMPLE (continued)

Solution:

Impulse-momentum diagram:



Conservation of angular momentum:

$$(H_A)_1 = (H_A)_2$$

$$r' \, m \, (v_G)_1 + I_G \, \omega_1 = r \, m \, (v_G)_2 + I_G \, \omega_2$$

$$(0.2 - 0.03) \, 10 \, (v_G)_1 + 0.156 \, \omega_1 = 0.2(10) \, (v_G)_2 + 0.156 \, \omega_2$$

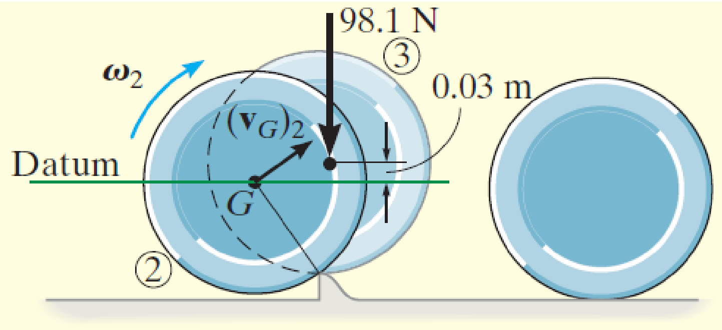
Kinematics: Since there is no slip, $\omega = v_G/r = 5 \, v_G$.

Substituting and solving the momentum equation yields

$$(v_G)_2 = 0.892 \, (v_G)_1$$

EXAMPLE (continued)

To complete the solution, conservation of energy can be used. Since it cannot be used for the impact (why?), it is applied just after the impact. In order to roll over



the bump, the wheel must go to position 3 from 2. When $(v_G)_2$ is a minimum, $(v_G)_3$ is zero. Why?

Energy conservation equation : $T_2 + V_2 = T_3 + V_3$

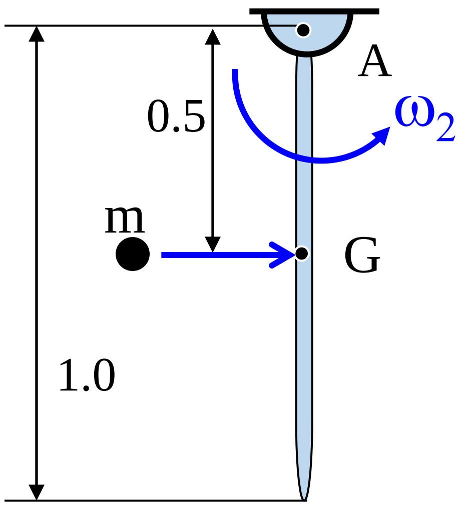
$$\{½\,(10)\,(v_G)_2^2 + ½\,(0.156)\,\omega_2^2\} + 0 = 0 + 98.1\,(0.03)$$

Substituting $\omega_2 = 5\,(v_G)_2$ and $(v_G)_2 = 0.892\,(v_G)_1$ and solving yields

$(v_G)_1 =$ <u>0.729 m/s</u>

CONCEPT QUIZ

1. A slender rod (mass = M) is at rest. If a bullet (mass = m) is fired with a velocity of v_b, the angular momentum of the bullet about A just before impact is ___________.



A) $0.5\ m\ v_b$

B) $m\ v_b$

C) $0.5\ m\ v_b^2$

D) zero

2. For the rod in question 1, the angular momentum about A of the rod and bullet just after impact will be ___________.

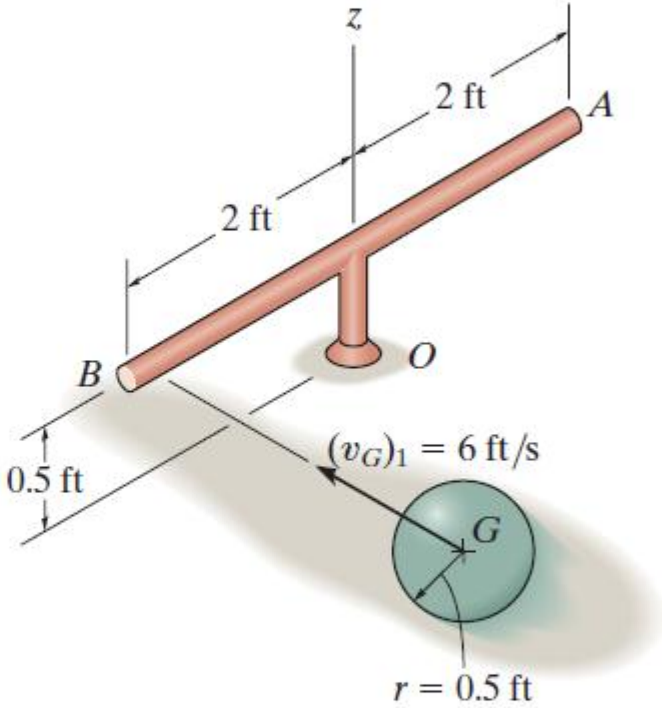
A) $m\ v_b + M(0.5)\omega_2$

B) $m(0.5)^2\omega_2 + M(0.5)^2\omega_2$

C) $m(0.5)^2\omega_2 + M(0.5)^2\omega_2 + (1/12)\ M\ \omega_2$

D) zero

GROUP PROBLEM SOLVING



Given: The mass center, G, of the 3-lb ball has a velocity of $(v_G)_1 = 6$ ft/s when it strikes the end of the smooth 5-lb slender bar, which is at rest.

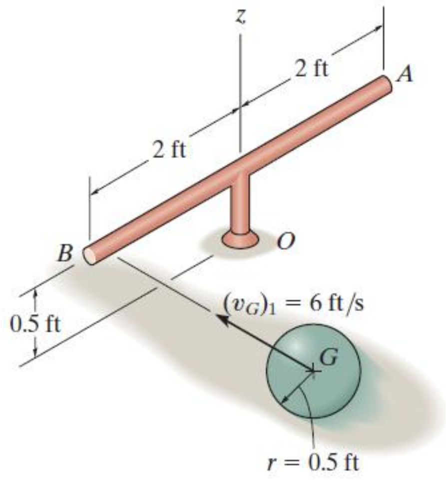
Find: The angular velocity of the bar about the *z* axis just after impact if $e = 0.8$.

Plan: The force due to impact is internal to the system (the slender bar and ball), so the impulses sum to zero. Thus, angular momentum is conserved and the conservation of angular momentum can be used to find the angular velocity.

GROUP PROBLEM SOLVING (continued)

Solution:

To use conservation of angular momentum, the mass moment of inertia of the slender bar about its z-axis must be found.



$$I_z = (1/12)\,(5/32.2)\,(4^2) = 0.2070 \text{ slug}\cdot\text{ft}^2$$

Kinematics: $\omega_2 = (v_{Bar})_2 / r = (v_{Bar})_2 / 2$

Apply the conservation of angular momentum equation:

$$(H_z)_1 = (H_z)_2$$

$$\{m_{ball}\,(v_G)_1\}\,2 = (I_z)\,\omega_2 + \{m_{ball}\,(v_G)_2\}\,(2)$$

$$(3/32.2)\,6\,(2) = 0.2070\,\{(v_{Bar})_2/2\} + (3/32.2)\,(v_G)_2\,(2)$$

$$\Rightarrow 1.118 = 0.1035\,(v_{Bar})_2 + 0.1863\,(v_G)_2 \qquad (1)$$

GROUP PROBLEM SOLVING (continued)

Apply the coefficient of restitution for the impact:

$$e = [(v_{Bar})_2 - (v_G)_2] / [(v_G)_1 - (v_{Bar})_1]$$

$$0.8 = [(v_{Bar})_2 - (v_G)_2] / [6 - 0]$$

$$\Rightarrow (v_{Bar})_2 - (v_G)_2 = 4.8 \qquad (2)$$

Solving Eqs. (1) and (2) yields

$(v_G)_2 = 2.143$ ft/s

$(v_{Bar})_2 = 6.943$ ft/s

The angular velocity of the slender rod is given by

$\omega_2 = (v_{Bar})_2 / 2 =$ 3.47 rad/s

ATTENTION QUIZ

1. Using conservation of linear and angular momentum requires that _______________.

   A) all linear impulses sum to zero

   B) all angular impulses sum to zero

   C) both linear and angular impulses sum to zero

   D) None of the above

2. The angular momentum of a body about a point A that is the fixed axis of rotation but not the mass center (G) is _______________.

   A) $I_A \omega$

   B) $I_G \omega$

   C) $r_G (m\, v_G) + I_G\, \omega$

   D) Both A & C

End of the Lecture

Let Learning Continue