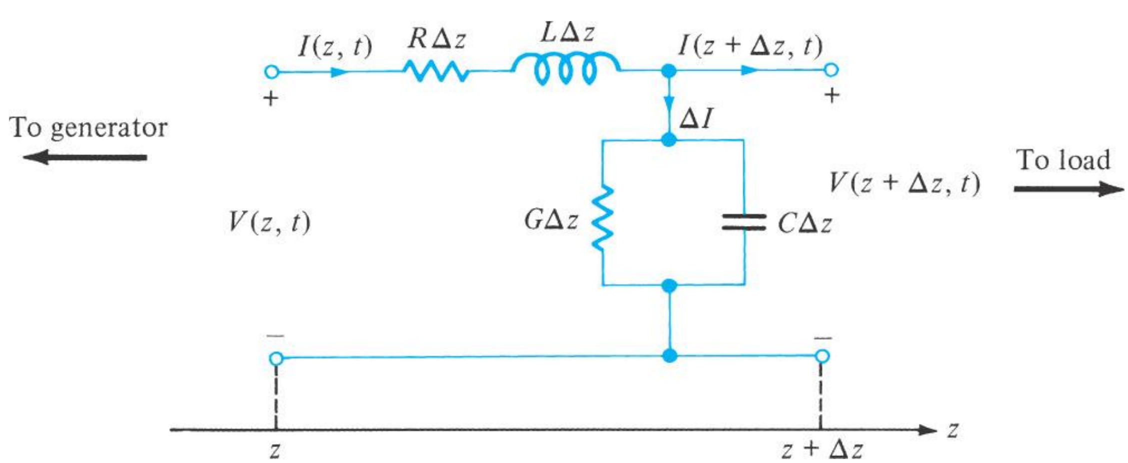
# TRANSMISSION LINE THEORY

(TEM Line)

A uniform transmission line is defined as the one whose dimensions and electrical properties are identical at all planes transverse to the direction of propagation.

## Circuit Representation of TL's

A uniform TL may be modeled by the following circuit representation:



***R*****:** Series resistance per unit length of line (for both conductors (ohm/m)).

***L*****:** Series inductance per unit length of line (Henry/m).

***G*****:** Shunt conductance per unit length of line (mho/m).

**C**: Shunt capacitance per unit length of line (Farad/m).

The line is pictured as a cascade of identical sections, each of $\Delta z$ long.

Since $\Delta z$ can always be chosen small compared to the operating wavelength, an individual section of line may be analyzed using ordinary ac circuit theory. In the following analysis, we let $\Delta z \to 0$, so the results are valid at all frequencies (hence for any physical time variation).

Applying the Kirchhoff's voltage law to the line section gives:

$$v(z,t) = (R\Delta z)\,i(z,t) + (L\,\Delta z)\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t)$$

Rearranging yields:

$$\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -R\;i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Letting $\Delta z \to 0$, we get,

$$\frac{\partial v(z,t)}{\partial z} = -R\;i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Now applying Kirchhoff's current law to the line section gives:

$$i(z,t) = \left(G\Delta z\right)v\left(z+\Delta z,t\right) + \left(C\Delta z\right)\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z,t)$$

Rearranging yields:

$$\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = -G\;v(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

Letting $\Delta z \to 0$

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Then the time domain TL or telegrapher equations are:

$$\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

The solution of these equations, together with the electrical properties of the generator and load, allow us to determine the instantaneous voltage and current at any time *t* and any place *z* along the uniform TL.

**Lossless Line**: For the case of perfect conductors (R=0) and insulators (G=0), the telegrapher equations reduce to the following form:

$$\frac{\partial v(z,t)}{\partial z} = -L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -C\frac{\partial v(z,t)}{\partial t}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial v(z,t)}{\partial z}\right) = -L\frac{\partial}{\partial z}\left(\frac{\partial i(z,t)}{\partial t}\right)$$

$$-L\frac{\partial}{\partial t}\left(\frac{\partial i(z,t)}{\partial z}\right) = -L\frac{\partial}{\partial t}\left(-C\frac{\partial v}{\partial t}\right)$$

Or,

$$\frac{\partial^2 v(z,t)}{\partial z^2} - LC\frac{\partial^2 i(z,t)}{\partial t^2} = 0$$

$$\frac{\partial^2 i(z,t)}{\partial z} - LC\frac{\partial^2 i(z,t)}{\partial t^2} = 0$$

**Wave equation's for voltage and current on a lossless TL.**

Although real lines are never lossless, lolessness approximation for practical TL's is very usefull.

## TRANSMISSION LINES WITH SINUSOIDAL EXCITATION

We will only consider the sinusoidal steady-state solutions.

**Transmission-Line Equations:**

Under sinusoidal steady state conditions, the TL equations take the form:

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$
$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

Where $V(z)$ and $I(z)$ are <u>voltage and current phasors.</u>

The real sinusoidal voltage and current waveforms are obtained from:

$$v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right]$$
$$i(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right]$$

**Wave Propagation on a TL**

The second order differential equations for $V(z)$ and $I(z)$ are:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$
$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

where $\gamma = \alpha + j\beta = \left[(R + j\omega L)(G + j\omega C)\right]^{1/2}$

$\gamma =$ complex propagation constant.

$\alpha =$ attenuation constant (Np/m).

$\beta =$ phase constant (rad/m).