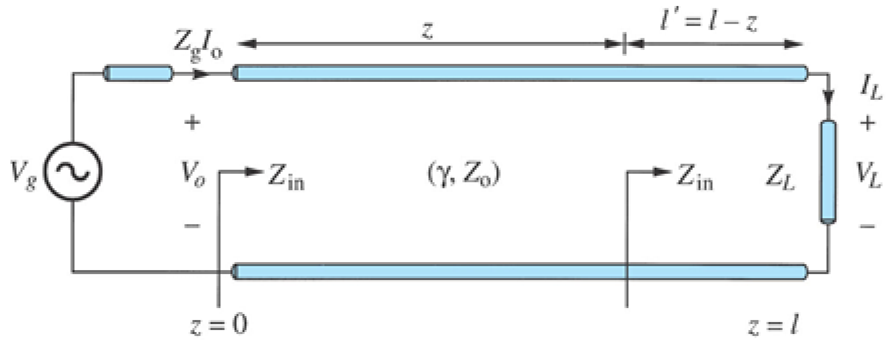
FINITE LENGTH LINE TERMINATED IN A GENERAL Z_L



Reflections occur when $Z_L \neq Z_0$.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V^+ + V^-$$

$$I(z) = I_0^+ e^{-\gamma z} - I_0^- e^{+\gamma z} = I^+ - I^-$$

Reflection Coefficient:

$$\Gamma = \frac{\text{Reflected Voltage (or Current) at pt. } z}{\text{Forward Voltage (or Current) at pt. } z} = \frac{V^-}{V^+} = \frac{I^-}{I^+}$$

The load reflection Coefficient

$$\Gamma_L = \frac{V^-(z=l)}{V^+(z=l)} = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

Defining $z = l - z'$, we express *V* and *I* as:

$$V = V_0^+ e^{-\gamma(l-z')} + V_0^- e^{\gamma(l-z')}$$

$$I = \frac{V_0^+}{Z_0} e^{-\gamma(l-z')} - \frac{V_0^-}{Z_0} e^{\gamma(l-z')}$$ or,

$$V = V_0^+ e^{-\gamma l}\left[e^{\gamma z'} + \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-\gamma z'}\right]$$

$$I = \frac{V_0^+}{I_0} e^{-\gamma l}\left[e^{\gamma z'} - \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-\gamma z'}\right]$$

So,

$$V = V_0^+ e^{-\gamma l}\left[e^{\gamma z'} + \Gamma_L e^{-\gamma z'}\right]$$

$$I = \frac{V_0^+}{I_0} e^{-\gamma l}\left[e^{\gamma z'} - \Gamma_L e^{-\gamma z'}\right]$$

Since,

$$V^+ = V_0^+ e^{-\gamma z} = V_0^+ e^{-\gamma l} e^{\gamma z'}$$

$$V^- = V_0^- e^{\gamma z} = V_0^- e^{\gamma l} e^{-\gamma z'}$$

$$\Gamma = \frac{V^-}{V^+} = \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-2\gamma z'} = \Gamma_L e^{-2\gamma z'}$$

So the reflection coefficient at any point on the transmission line:

$$\Gamma = \Gamma_L e^{-2\gamma z'} = \Gamma_L e^{-2\alpha z'} e^{-j2\beta z'}$$

Or,

$$\Gamma = \Gamma_L e^{-2\gamma z'} = \Gamma_L e^{-2\alpha z'} \angle -2\beta z'$$

If,

$$\Gamma = |\Gamma| \angle \phi \qquad \Gamma_L = |\Gamma_L| \angle \phi_L$$

Then

$$\Gamma = |\Gamma| \angle \phi = |\Gamma_L| e^{-2\alpha z'} \angle \phi_L - 2\beta z'$$

$$|\Gamma| = |\Gamma_L| e^{-2\alpha z'}$$

$$\phi = \phi_L - 2\beta z'$$

As a special point, take $z' = l$

$$\Gamma_{in} = \Gamma_L e^{-2\gamma l} \angle \phi_L - 2\beta l$$

Γ_L CAN BE CALCULATED FROM Z_L AND Z_0

If we put $z'=0$, $V=V_L$ and $I=I_L$ so,

$$V_L = V_0^+ e^{-\gamma l}\left(1+\Gamma_L\right)$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l}\left(1-\Gamma_L\right)$$

But,

$$Z_L = \frac{V_L}{I_L} = Z_o \frac{1+\Gamma_L}{1-\Gamma_L}$$

Eliminating Γ_L gives,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$$

Where,

$\bar{Z}_L = \frac{Z_L}{Z_0}$, normalized load impedance.

Defining, $Y_L = \frac{1}{Z_L}, \quad Y_0 = \frac{1}{Z_0} \quad \bar{Y}_L = \frac{Y_L}{Y_0}$

$$\Gamma_L = \frac{Y_o - Y_L}{Y_0 + Y_L} = \frac{1-\bar{Y}_L}{1+\bar{Y}_L} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$$

If the impedance at any point on the line is defined as:

$$Z = \frac{V}{I}$$

The above equations may be generalized as:

$$Z = Z_o \frac{1+\Gamma}{1-\Gamma} \qquad Y = Y_o \frac{\Gamma - 1}{\Gamma + 1}$$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{Y_0 - Y}{Y_0 + Y}$$

STANDING WAVE PATTEREN

Consider a lossless line, then $\gamma = j\beta,\ \alpha = 0.$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z}\left(1 + \frac{V_0^+}{V_0^-} e^{j2\beta z}\right)$$

But,

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

So,

$$\frac{V_0^-}{V_0^+} = \Gamma_L e^{-2\gamma l}$$ and

$$V(z) = V_0^+ e^{-j\beta z}\left(1 + \Gamma_L e^{j2\beta(z-l)}\right)$$

With $\Gamma_L = |\Gamma| e^{j\phi_L}$

$$V(z) = V_0^+ e^{-j\beta z}\left(1 + |\Gamma_L| e^{j(2\beta z - 2\beta l + \phi_L)}\right)$$

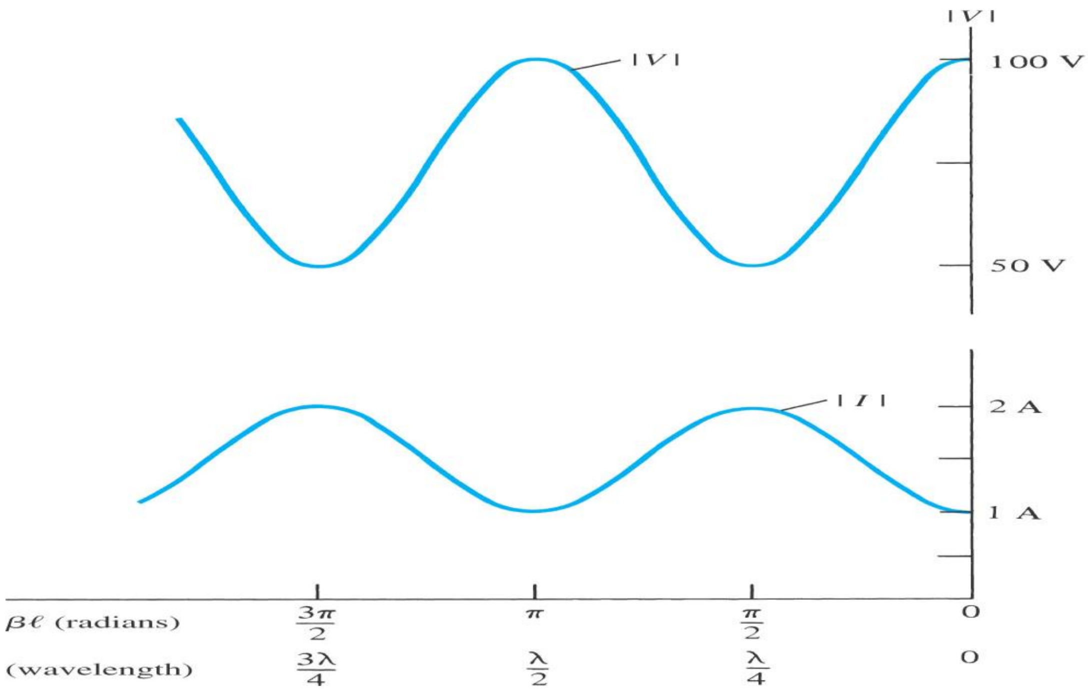
The Standing-Wave Pattern (Voltage):

$$|V(z)| = |V_0^+| \left|\left(1 + |\Gamma_L| e^{j(2\beta z - 2\beta l + \phi_L)}\right)\right|$$

$$|V(z)| = |V_0^+| \left|\left(1 + |\Gamma_L| e^{-j(2\beta(l-z)) - \phi_L)}\right)\right|$$

$$\left|V(z)\right| = \left|V_0^+\right| \left| \left(1 + \left|\Gamma_L\right| e^{-j(2\beta z' - \phi_L)}\right) \right|$$

$$\left|V(z)\right| = \left|V_0^+\right| \left(1 + \left|\Gamma_L\right|^2 + 2\left|\Gamma_L\right| \cos\left(2\beta z' - \phi_L\right)\right)^{1/2}$$



Maxima of Voltage:

$$2\beta z'_{\max,m} - \phi_L = 2\pi m, \quad m = 0, 1, \ldots$$

Minima of Voltage:

$$2\beta z'_{\min,n} - \phi_L = (2n-1)\pi, \quad n = 1, 2, \ldots$$

Current Standing Wave Pattern:

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z} \left(1 - \frac{V_0^+}{V_0^-} e^{2j\beta z} \right)$$

$$|I(z)| = \frac{|V_0^+|}{|Z_0|} \left(1 + |\Gamma_L|^2 - 2|\Gamma_L| \cos\left(2\beta z' - + \phi_L \right) \right)^{1/2}$$

Current maxima occur at voltage minima and current minima exist at voltage maxima.

Standing -Wave Ratio (SWR)

$$|V(z)|_{\max} = V_0^+ \left(1 + |\Gamma_L| \right) \qquad |V(z)|_{\min} = V_0^+ \left(1 - |\Gamma_L| \right)$$

$$SWR = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$SWR \geq 1$$

If $\Gamma = 0,\ SWR = 1$ (no reflections exits)

If $\Gamma = 1,\ SWR = \infty$

$$|\Gamma_L| = \frac{SWR - 1}{SWR + 1}$$

i) $Z_L = 0\ (short\ circuit) \rightarrow \Gamma_L = -1 \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$

ii) $Z_L = \infty\ (open\ circuit) \rightarrow \Gamma_L = +1 \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$

iii) $Z_L = Z_0\ (matched\ circuit) \rightarrow \Gamma_L = 0 \rightarrow SWR = 1$

iv) $Z_L = jX_L\ (reactive\ load) \rightarrow \Gamma_L = e^{j\theta_L} \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$