Simple Stresses

Simple stresses are expressed as the ratio of the applied force divided by the resisting area or

$$\sigma = \textbf{Force / Area}.$$

It is the expression of force per unit area to structural members that are subjected to external forces and/or induced forces. Stress is the lead to accurately describe and predict the elastic deformation of a body.

Simple stress can be classified as normal stress, shear stress, and bearing stress. **Normal stress** develops when a force is applied perpendicular to the cross-sectional area of the material. If the force is going to pull the material, the stress is said to be **tensile stress** and **compressive stress** develops when the material is being compressed by two opposing forces. **Shear stress** is developed if the applied force is parallel to the resisting area. Example is the bolt that holds the tension rod in its anchor. Another condition of shearing is when we twist a bar along its longitudinal axis. This type of shearing is called torsion and covered in Chapter 3. Another type of simple stress is the **bearing stress**, it is the contact pressure between two bodies.

Suspension bridges are good example of structures that carry these stresses. The weight of the vehicle is carried by the bridge deck and passes the force to the stringers (vertical cables), which in turn, supported by the main suspension cables. The suspension cables then transferred the force into bridge towers.

Normal Stress

Stress

Stress is the expression of force applied to a unit area of surface. It is measured in psi (English unit) or in MPa (SI unit). Another unit of stress which is not commonly used is the dynes (cgs unit). Stress is the ratio of force over area.

stress = force / area

Simple Stresses

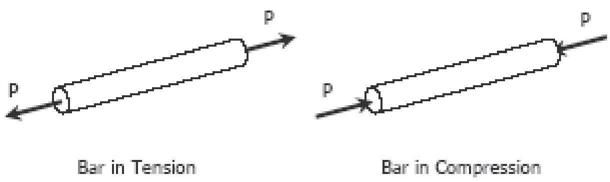
There are three types of simple stress namely; normal stress, shearing stress, and bearing stress.

Normal Stress

The resisting area is perpendicular to the applied force, thus normal. There are two types of normal stresses; tensile stress and compressive stress. Tensile stress applied to bar tends the bar to elongate while compressive stress tend to shorten the bar.

$$\sigma = \frac{P}{A}$$

where P is the applied normal load in Newton and A is the area in mm^2. The maximum stress in tension or compression occurs over a section normal to the load.



SOLVED PROBLEMS IN NORMAL STRESS

Problem 104

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.

Solution 104

$P = \sigma A$

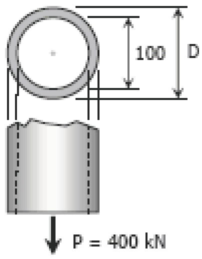
where:

$P = 400 \text{ kN} = 400\,000 \text{ N}$

$\sigma = 120 \text{ MPa}$

$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$

$= \frac{1}{4}\pi(D^2 - 10\,000)$



thus,

$400\,000 = 120[\frac{1}{4}\pi(D^2 - 10\,000)]$

$400\,000 = 30\pi D^2 - 300\,000\pi$

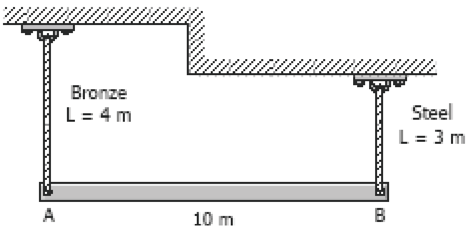
$D^2 = \dfrac{400\,000 + 300\,000\pi}{30\pi}$

$D = \mathbf{119.35 \text{ mm}}$

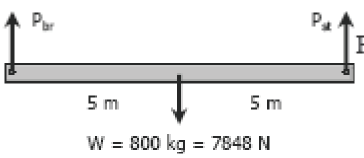
Problem 105

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in Fig. P-105. Calculate the smallest area of each cable if the stress is not to exceed 90 MPa in bronze and 120 MPa in steel.

Figure P-105



Solution 105



By symmetry:

$P_{br} = P_{st} = \frac{1}{2}(7848)$

$= 3924 \text{ N}$

For bronze cable:

$P_{br} = \sigma_{br} A_{br}$

$3924 = 90 A_{br}$

$A_{br} = \mathbf{43.6 \text{ mm}^2}$

For steel cable:

$P_{st} = \sigma_{st} A_{st}$

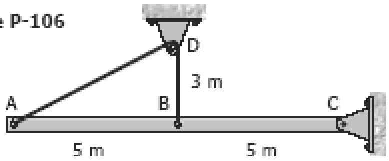
$3924 = 120 A_{st}$

$A_{st} = \mathbf{32.7 \text{ mm}^2}$

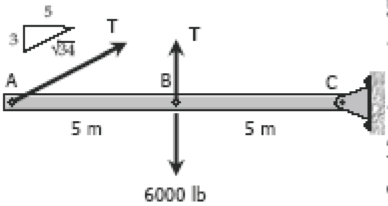
Problem 106

The homogeneous bar shown in Fig. P-106 is supported by a smooth pin at C and a cable that runs from A to B around the smooth peg at D. Find the stress in the cable if its diameter is 0.6 inch and the bar weighs 6000 lb.

Figure P-106



Solution 106



$\Sigma M_C = 0$

$5T + 10(\frac{3}{\sqrt{34}}T) = 5(6000)$

$T = 2957.13 \text{ lb}$

$T = \sigma A$

$2957.13 = \sigma[\frac{1}{4}\pi(0.6^2)]$

$\sigma = \mathbf{10\ 458.72\ psi}$

Problem 107

A rod is composed of an aluminum section rigidly attached between steel and bronze sections, as shown in Fig. P-107. Axial loads are applied at the positions indicated. If P = 3000 lb and the cross sectional area of the rod is 0.5 in^2, determine the stress in each section.

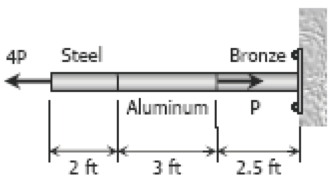
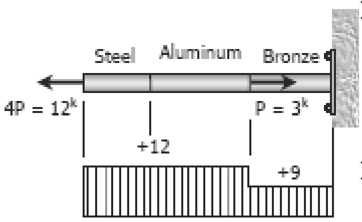


Figure P-107

Solution 107



For steel:

$\sigma_{st} A_{st} = P_{st}$

$\sigma_{st}(0.5) = 12$

$\sigma_{st} = \mathbf{24\ ksi}$

For aluminum:

$\sigma_{al} A_{al} = P_{al}$

$\sigma_{al}(0.5) = 12$

$\sigma_{al} = \mathbf{24\ ksi}$

For bronze:

$\sigma_{br} A_{br} = P_{br}$

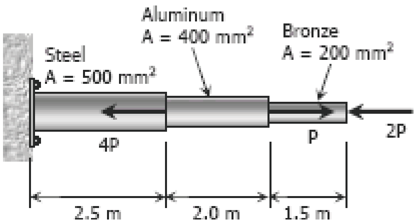
$\sigma_{br}(0.5) = 9$

$\sigma_{br} = \mathbf{18\ ksi}$

Problem 108

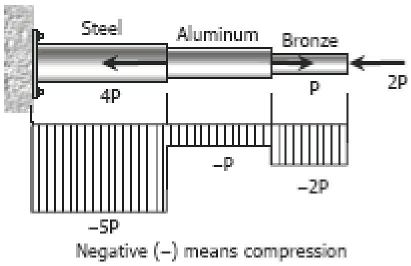
An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. P-108. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.

Figure P-108



Solution 108

For bronze:
$\sigma_{br} A_{br} = 2P$
$100(200) = 2P$
$P = 10\ 000$ N



For aluminum:
$\sigma_{al} A_{al} = P$
$90(400) = P$
$P = 36\ 000$ N
For Steel:
$\sigma_{st} A_{st} = 5P$
$P = 14\ 000$ N

For safe P, use $P = 10\ 000$ N = **10 kN**

Problem 109

Determine the largest weight W that can be supported by two wires shown in Fig. P-109. The stress in either wire is not to exceed 30 ksi. The cross-sectional areas of wires AB and AC are 0.4 in^2 and 0.5 in^2, respectively.

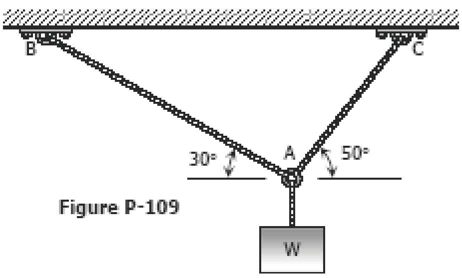


Figure P-109

Solution 109

For wire AB:

By sine law (from the force polygon):

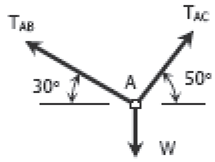
$$\frac{T_{AB}}{\sin 40°} = \frac{W}{\sin 80°}$$

$T_{AB} = 0.6527W$

$\sigma_{AB}A_{AB} = 0.6527W$

$30(0.4) = 0.6527W$

$W = 18.4$ kips



FBD of knot A

For wire AC:

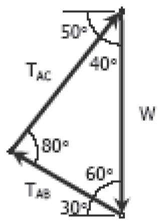
$$\frac{T_{AC}}{\sin 60°} = \frac{W}{\sin 80°}$$

$T_{AC} = 0.8794W$

$T_{AC} = \sigma_{AC}A_{AC}$

$0.8794W = 30(0.5)$

$W = 17.1$ kips



Force polygon of forces on knot A

Safe load W = **17.1 kips**

Problem 110

A 12-inches square steel bearing plate lies between an 8-inches diameter wooden post and a concrete footing as shown in Fig. P-110. Determine the maximum value of the load P if the stress in wood is limited to 1800 psi and that in concrete to 650 psi.

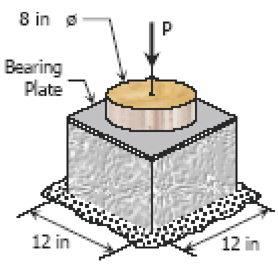


Figure P-110

Solution 110

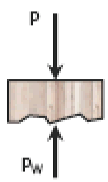
For wood:

$P_w = \sigma_w A_w$

$\quad = 1800[\frac{1}{4}\pi(8^2)]$

$\quad = 90\ 477.9$ lb

FBD of Wood



From FBD of Wood:

$P = P_w = 90\ 477.9$ lb

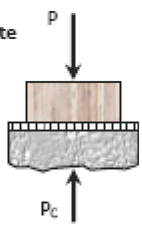
For concrete:

$P_c = \sigma_c A_c$

$\quad = 650(12^2)$

$\quad = 93\ 600$ lb

FBD of Concrete



From FBD of Concrete:

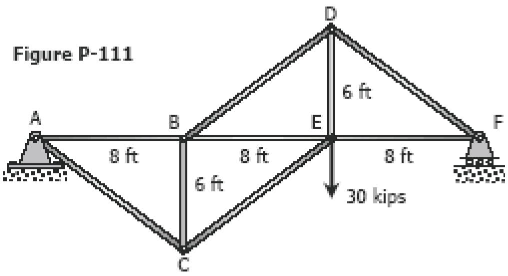
$P = P_c = 93\ 600$ lb

Safe load P = **90 478 lb**

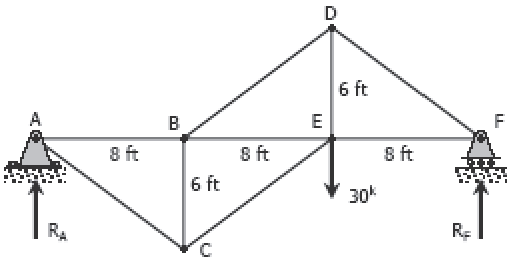
Problem 111

For the truss shown in Fig. P-111, calculate the stresses in members CE, DE, and DF. The crosssectional area of each member is 1.8 in^2. Indicate tension (T) or compression (C).

Figure P-111



Solution 111



Free Body Diagram of Truss

From the *FBD* of the truss:

$\Sigma M_A = 0$

$24R_B = 16(30)$

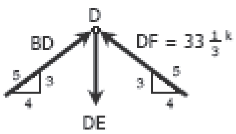
$R_F = 20^k$

At joint *F*:

$\Sigma F_V = 0$

$\frac{3}{5} DF = 20$

$DF = 33\frac{1}{3}{}^k\ (C)$



Joint D

At joint *D*: (by symmetry)

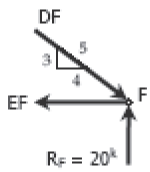
$BD = DF = 33\frac{1}{3}{}^k\ (C)$

$\Sigma F_V = 0$

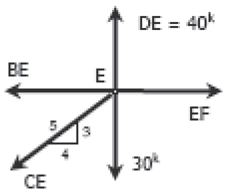
$DE = \frac{3}{5} BD + \frac{3}{5} DF$

$= \frac{3}{5}(33\frac{1}{3}) + \frac{3}{5}(33\frac{1}{3})$

$= 40^k\ (T)$



Joint F



Joint E

At joint *E*:

$\Sigma F_V = 0$

$\frac{3}{5} CE + 30 = 40$

$CE = 16\frac{2}{3}{}^k\ (T)$

Stresses:

Stress = Force/Area

$\sigma_{CE} = \dfrac{16\frac{2}{3}}{1.8} = \mathbf{9.26\ ksi}\ (T)$

$\sigma_{DE} = \dfrac{40}{1.8} = \underline{\mathbf{22.22}}\ \mathbf{ksi}\ (T)$

$\sigma_{DF} = \dfrac{33\frac{1}{3}}{1.8} = \mathbf{18.52\ ksi}\ (C)$

Problem 112

Determine the crosssectional areas of members AG, BC, and CE for the truss shown in Fig. P-112 above. The stresses are not to exceed 20 ksi in tension and 14 ksi in compression. A reduced stress in compression is specified to reduce the danger of buckling.

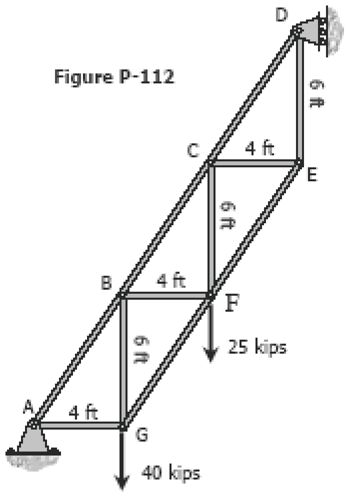
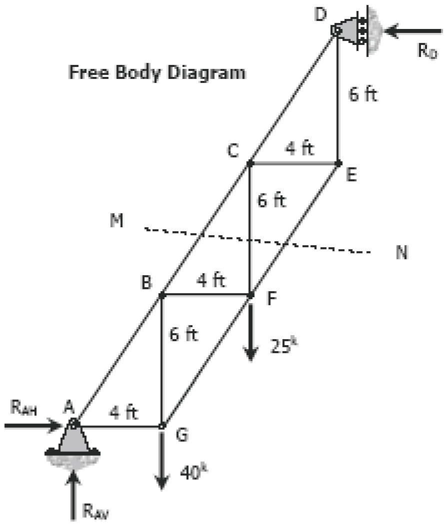


Figure P-112

Solution 112



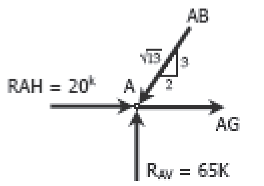
Free Body Diagram

$\Sigma F_V = 0$
$R_{AV} = 40 + 25$
$= 65^k$

$\Sigma M_A = 0$
$18R_D = 8(25) + 4(40)$
$R_D = 20^k$

$\Sigma F_H = 0$
$R_{AH} = R_D = 20^k$

Check:
$\Sigma M_D = 0$
$12R_{AV} = 18(R_{AH}) + 4(25) + 8(40)$
$12(65) = 18(20) + 4(25) + 8(40)$
$780 \text{ ft·kip} = 780 \text{ ft·kip}$ (OK!)



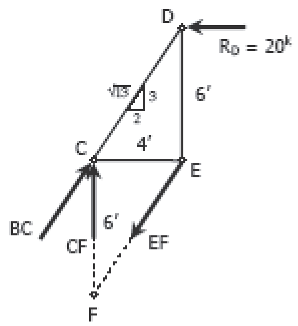
Joint A

For member AG:
At joint A:
$\Sigma F_V = 0$
$\frac{3}{\sqrt{13}} AB = 65$

$AB = \dfrac{65\sqrt{13}}{3}$
$= 78.12^k$

$\Sigma F_H = 0$
$AG + 20 = \frac{2}{\sqrt{13}} AB$
$AG = \frac{2}{\sqrt{13}}(78.12) - 20$
$= 20.33^k$ Tension

$AG = \sigma_{tension} A_{AG}$
$20.33 = 20\, A_{AG}$
A_{AG} = **1.17 in²**

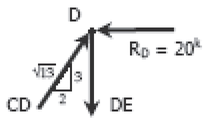


Section through MN

For member BC:
At section through MN
$\Sigma M_F = 0$
$6(\frac{2}{\sqrt{13}} BC) = 12(20)$
$BC = 20\sqrt{13}$
$= 72.11^k$ Compression

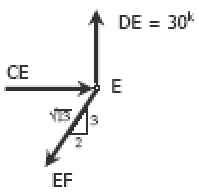
$BC = \sigma_{compression} A_{BC}$
$72.11 = 14\, A_{BC}$
A_{BC} = **5.15 in²**

For member CE:
At joint D:
$\Sigma F_H = 0$
$\frac{2}{\sqrt{13}} CD = 20$
$CD = 10\sqrt{13}$
$= 36.06^k$



Joint D

$\Sigma F_V = 0$
$DE = \frac{3}{\sqrt{13}} CD$
$= \frac{3}{\sqrt{13}}(36.06)$
$= 30^k$



Joint E

At joint E:
$\Sigma F_V = 0$
$\frac{3}{\sqrt{13}} EF = 30$
$EF = 10\sqrt{13} = 36.06^k$

$\Sigma F_H = 0$
$CE = \frac{2}{\sqrt{13}} EF$
$= \frac{2}{\sqrt{13}}(36.06)$
$= 20^k$ Compression

$CF = \sigma_{compression} A_{CE}$
$20 = 14\, A_{CE}$
A_{CE} = **1.43 in²**

Problem 113

Find the stresses in members BC, BD, and CF for the truss shown in Fig. P-113. Indicate the tension or compression. The cross sectional area of each member is 1600 mm^2.

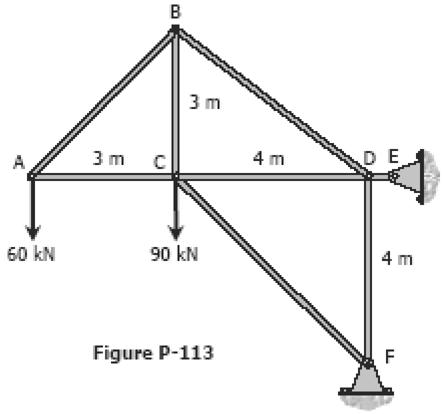
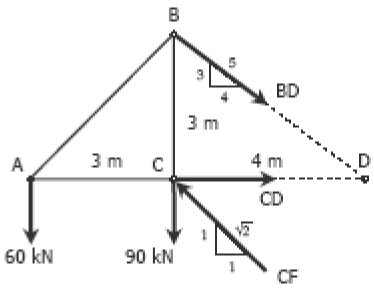
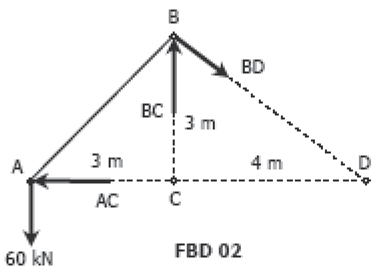


Figure P-113

Solution 113



FBD 01



FBD 02

For member BD: (See FBD 01)

$\Sigma M_C = 0$

$3(\frac{4}{5}BD) = 3(60)$

$BD = 75$ kN Tension

$BD = \sigma_{BD} A$

$75\,(1000) = \sigma_{BD}\,(1600)$

$\sigma_{BD} = 46.875$ MPa (Tension)

For member CF: (See FBD 01)

$\Sigma M_D = 0$

$4(\frac{1}{\sqrt{2}}CF) = 4(90) + 7(60)$

$CF = 195\sqrt{2}$

$\quad = 275.77$ kN Compression

$CF = \sigma_{CF} A$

$275.77\,(1000) = \sigma_{CF}\,(1600)$

$\sigma_{CF} = 172.357$ MPa (Compression)

For member BC: (See FBD 02)

$\Sigma M_D = 0$

$4BC = 7(60)$

$BC = 105$ kN Compression

$BC = \sigma_{BC} A$

$105\,(1000) = \sigma_{BC}\,(1600)$

$\sigma_{BC} = 65.625$ **MPa (Compression)**

Problem 114

The homogeneous bar ABCD shown in Fig. P-114 is supported by a cable that runs from A to B around the smooth peg at E, a vertical cable at C, and a smooth inclined surface at D. Determine the mass of the heaviest bar that can be supported if the stress in each cable is limited to 100 MPa. The area of the cable AB is 250 mm^2 and that of the cable at C is 300 mm^2.

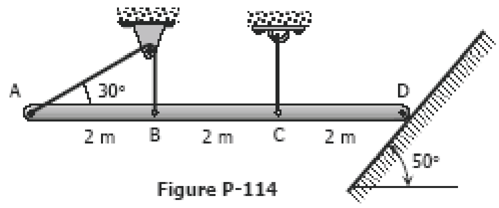
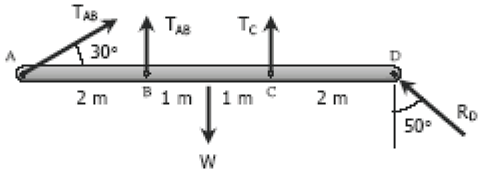


Figure P-114

Solution 114



$\Sigma F_H = 0$
$T_{AB} \cos 30° = R_D \sin 50°$
$R_D = 1.1305T_{AB}$

$\Sigma F_V = 0$
$T_{AB} \sin 30° + T_{AB} + T_C + R_D \cos 50° = W$
$T_{AB} \sin 30° + T_{AB} + T_C + (1.1305T_{AB}) \cos 50° = W$
$2.2267T_{AB} + T_C = W$
$T_C = W - 2.2267T_{AB}$

$\Sigma M_D = 0$
$6(T_{AB} \sin 30°) + 4T_{AB} + 2T_C = 3W$
$7T_{AB} + 2(W - 2.2267T_{AB}) = 3W$
$2.5466T_{AB} = W$
$T_{AB} = 0.3927W$

$T_C = W - 2.2267T_{AB}$
$= W - 2.2267(0.3927W)$
$= 0.1256W$

Based on cable AB:
$T_{AB} = \sigma_{AB} A_{AB}$
$0.3927W = 100(250)$
$W = 63\ 661.83$ N

Based on cable at C:
$T_2 = \sigma_C A_C$
$0.1256W = 100(300)$
$W = 238\ 853.50$ N

Safe weight $W = 63\ 669.92$ N
$W = mg$
$63\ 669.92 = m\,(9.81)$
$m = 6\ 490$ kg
$= \mathbf{6.49\ Mg}$