LINEAR WIRE ANTENNAS

Linear wire antennas are some of the oldest, simplest, cheapest, and in many cases the most versatile for many applications.

1. **Infinitesimal Dipole (L ≤ λ /50)**
2. **Small Dipole (λ /50 < L ≤ λ /10)**
3. **Finite Length Dipole (L > λ /10)**
4. **Half-Wave Dipole (L= λ /2)**

1. ***Infinitesimal Dipole***

An infinitesimal linear dipole is positioned symmetrically at the origin along the *z*-axis, as shown in Figure (9). In addition to being very small *($l << \lambda$)*, is very thin *($a << \lambda$)*. The current distribution is assumed to be constant:

$$I(z) = I_0 \boldsymbol{a_z}$$

$$\vec{A} = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I(z) \frac{e^{-j\beta r}}{r} dl \quad , \quad dl = dz \vec{A} = \frac{\mu I_0}{4\pi r} e^{-j\beta r} \boldsymbol{a_z} \int_{-l/2}^{l/2} dz = \frac{\mu I_0 l}{4\pi r} e^{-j\beta r} \boldsymbol{a_z}$$

$$A_z = \frac{\mu I_0 l}{4\pi r} e^{-j\beta r} \qquad A_x = A_y = 0$$

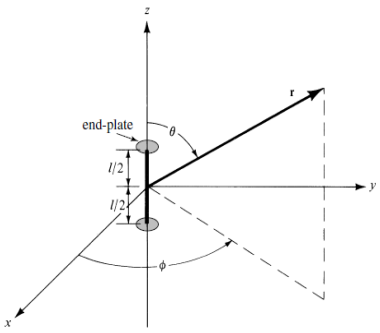


Figure (9)

The transformation between rectangular and spherical coordinates is given, in matrix form, by:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r = A_z \cos\theta = \frac{\mu I_0 l}{4\pi r} \cos\theta e^{-j\beta r}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l}{4\pi r} \sin\theta e^{-j\beta r}$$

$$A_\phi = 0$$

Since no variation in ϕ direction is assumed, so that $\frac{\partial}{\partial \phi} = 0$

The magnetic field *"**H**"* can be obtained by:

$$\text{curl A} = \nabla \times A = \mu H \rightarrow H = \frac{1}{\mu}(\nabla \times A)$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}\left(-r\frac{\mu I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu I_0 l}{4\pi r}\cos\theta\, e^{-j\beta r}\right)\right]$$

$$H_\phi = \frac{1}{\mu r}\left[j\beta\frac{\mu I_0 l}{4\pi}\sin\theta\, e^{-j\beta r} + \frac{\mu I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right] = j\beta\frac{I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}\right)e^{-j\beta r}$$

$$H_\phi = j\beta\frac{I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}\right)e^{-j\beta r}$$

$$H_r = H_\theta = 0$$

The electric field ***E*** can be found using:

$$E = \frac{1}{j\omega\epsilon}(\nabla \times H)$$

$$E_r = \eta\frac{I_0 l}{2\pi r^2}\cos\theta\left(1+\frac{1}{j\beta r}\right)e^{-j\beta r}$$

$$E_\theta = j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}-\frac{1}{(\beta r)^2}\right)e^{-j\beta r}$$

$$E_\phi = 0$$

In far-field region βr >>1, the electric and magnetic fields are approximated as:

$$E_\theta \cong j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$H_\phi \cong j\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$E_r = E_\phi = H_r = H_\theta = 0$$

The average power density can be found using:

$$W_{av} = W_r\boldsymbol{a_r} = \frac{1}{2}Re\left[E_\theta \times H_\phi^*\right] = \frac{1}{2}\left(j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right) * \left(-j\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{j\beta r}\right)\boldsymbol{a_r}$$

$$W_{av} = W_r\boldsymbol{a_r} = \eta\frac{\beta^2 I_0^2 l^2}{32\pi^2 r^2}sin^2\theta\,\boldsymbol{a_r} = \eta\frac{\left(\frac{2\pi}{\lambda}\right)^2 I_0^2 l^2}{32\pi^2 r^2}sin^2\theta\,\boldsymbol{a_r}$$

$$W_{av} = \frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2\frac{sin^2\theta}{r^2}\,\boldsymbol{a_r}$$

The radiation power is:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_r\boldsymbol{a_r}.r^2 sin\theta d\theta d\phi\,\boldsymbol{a_r} = \int_0^{2\pi}\int_0^{\pi}\frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2\frac{sin^2\theta}{r^2}.r^2 sin\theta d\theta d\phi$$

$$P_{rad} = \frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2 2\pi\int_0^{\pi} sin^3\theta d\theta = \frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2 2\pi\frac{4}{3} = \frac{I_0^2}{2}\eta\frac{2\pi}{3}\left(\frac{l}{\lambda}\right)^2$$

$$P_{rad} = \eta\frac{\pi I_0^2}{3}\left(\frac{l}{\lambda}\right)^2$$

$$P_{rad} = \frac{I_0^2}{2} R_{rad}$$

And the radiation resistance is:

$$R_{rad} = \eta \frac{2\pi}{3} \left(\frac{l}{\lambda}\right)^2 = 120\pi \frac{2\pi}{3} \left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \quad (\Omega)$$

$$R_{in} = \frac{R_{rad}}{\sin^2\left(\frac{\beta l}{2}\right)}$$

The radiation resistance of an infinitesimal dipole is very small, so a very large mismatch occurs when connected to practical transmission lines have characteristic impedances of 50 or 75 Ω, and the radiation efficiency will be very small.

The ***radiation intensity*** which is given by:

$$U = r^2 W_{av} = r^2 * \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2 \frac{sin^2\theta}{r^2}$$

$$U = \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2 sin^2\theta \quad , \qquad U_{max} = \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2$$

The maximum directivity and maximum effective aperture are:

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = 4\pi \frac{\frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2}{\frac{I_0^2}{2} \eta \frac{2\pi}{3} \left(\frac{l}{\lambda}\right)^2}$$

$$D_o = \frac{3}{2}$$

$$Aem = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} * \frac{3}{2}$$

$$Aem = 0.119\lambda^2 \quad (m^2)$$

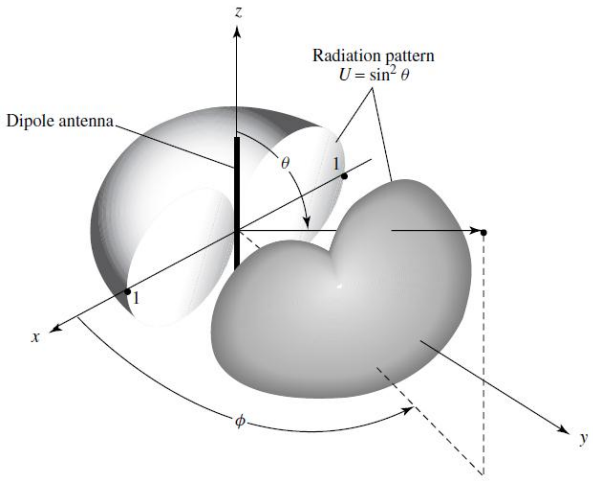
To find the HPBW : $\theta_m = 90^o$

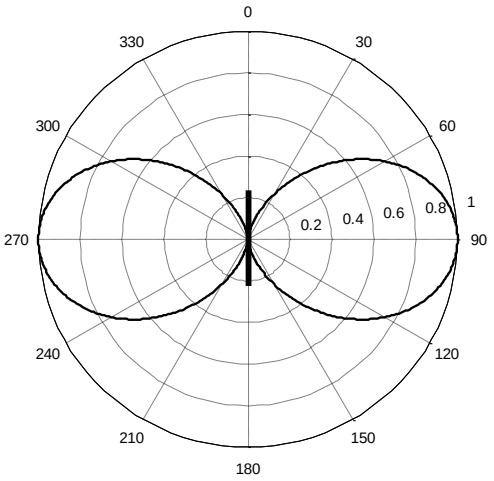
$U_n(\theta)|_{\theta=\theta_h} = 0.5 \rightarrow sin^2\theta_h = 0.5 \rightarrow sin\theta_h = 0.707 \rightarrow \theta_h = 45^o$

$HPBW = 2|\theta_m - \theta_h| = 2|90 - 45|$

$$HPBW = 90^o$$

Figure(10) shows the two and three dimensional radiation pattern of infinitesimal dipole.





Figure(10)

Example:

Find the radiation resistance of an infinitesimal dipole whose overall length is a) $l = \lambda/50$. b) $l = \lambda/75$

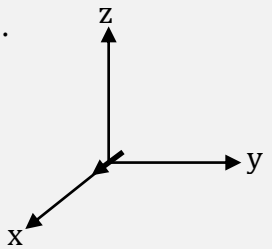
Solution:

$$a)\ R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda/50}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{50}\right)^2 = 0.316\ \Omega$$

$$b)\ R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda/75}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{75}\right)^2 = 0.14\ \Omega$$

Example 8

Derive an expression for the electric fields, magnetic fields, and radiation intensity in far region if an infinitesimal dipole is placed symmetrically along the x-axis.



solution

In general:

$$E \cong j\eta\frac{\beta I_0 l}{4\pi r}\sin\gamma\ e^{-j\beta r} \quad sin\gamma = \sqrt{1-cos^2\gamma}$$

$$cos\,\gamma = \vec{\boldsymbol{a}_x} \cdot \vec{\boldsymbol{a}_r} = \vec{\boldsymbol{a}_x} \cdot \left(\vec{\boldsymbol{a}_x}\sin\theta\cos\phi + \vec{\boldsymbol{a}_y}\sin\theta\sin\phi + \vec{\boldsymbol{a}_z}\cos\theta\right)$$

$$cos\,\gamma = \sin\theta\cos\phi$$

$$E \cong j\eta\frac{\beta I_0 l}{4\pi r}\sqrt{1-(\sin\theta\cos\phi)^2}\,e^{-j\beta r}$$

$$and \quad H \cong j\frac{\beta I_0 l}{4\pi r}\sqrt{1-(\sin\theta\cos\phi)^2}e^{-j\beta r}$$

$$U = W_{av}r^2 = \frac{1}{2}Re[E \times H^*].r^2 = \eta\frac{\beta^2 I_0^2 l^2}{32\pi^2}[1-(\sin\theta\cos\phi)^2]$$