ANTENNAS

Definition

An antenna is defined as a metallic device (as a rod or wire) for radiating or receiving radio waves, so we have a transmitting antenna and a receiving antenna. In other words the antenna is the transitional device between free-space and a transmission line (coaxial line or a waveguide) as shown in figure (1).

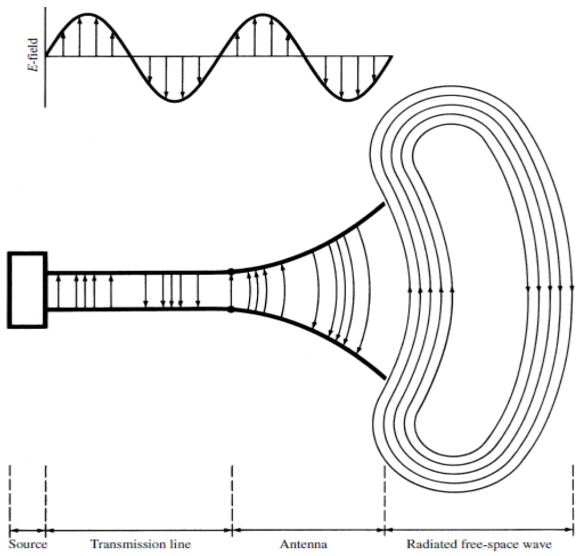


Figure (1): Antenna as a transition device.

Types of Antennas

1- Wire Antennas

Wire antennas are familiar to the layman because they are seen virtually everywhere on automobiles, buildings, ships, aircraft, spacecraft, and so on. There are various shapes of wire antennas such as a straight wire (dipole), loop, and helix which are shown

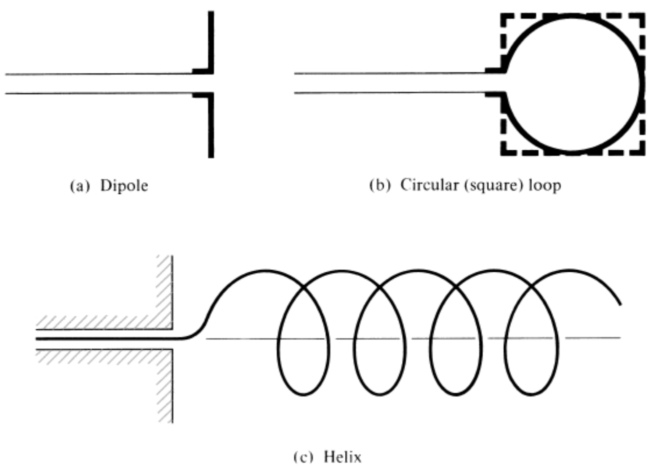


Figure 1.3 Wire antenna configurations.

2- Aperture Antennas

Antennas of this type are very useful for aircraft and spacecraft applications, because they can be very conveniently flush-mounted on the skin of the aircraft or spacecraft

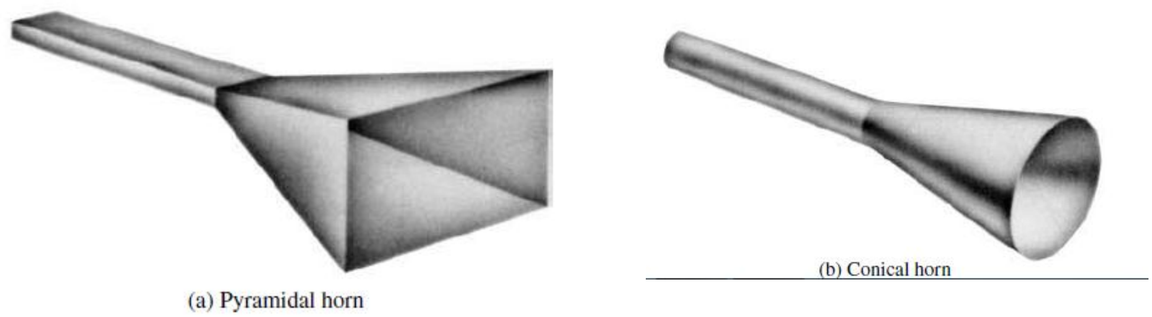


Figure: Aperture antenna configurations

3- Microstrip Antennas

These antennas consist of a metallic patch on a grounded substrate. The metallic patch can take many different configurations, as shown in Figure below:

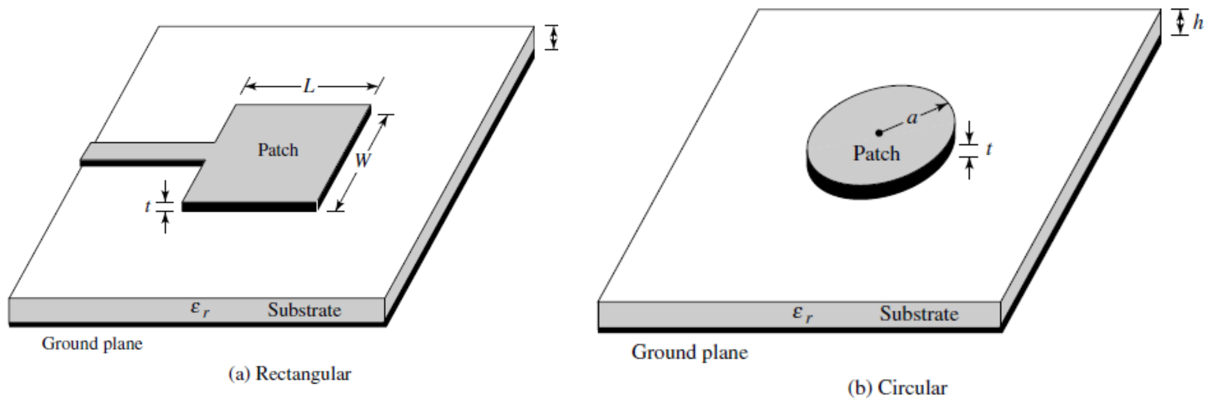
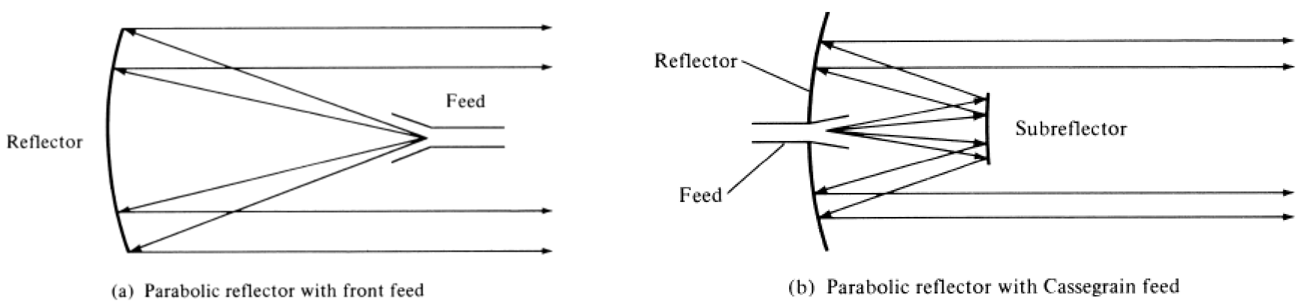


Figure: Rectangular and circular microstrip (patch) antennas.

4- Reflector Antennas

A very common antenna form for such an application is a parabolic reflector shown in Figures below:



Fundamentals Parameters of Antennas

1. Radiation Pattern

An antenna radiation pattern or antenna pattern is defined as ***a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates***. In most cases, the radiation pattern is determined in the far field region and is represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity, phase or polarization.

1.1 Radiation Pattern Lobes

Various parts of a radiation pattern are referred to as lobes, which may be subclassified into major or **main**, **minor**, **side**, and **back** lobes as shown in figure(2).

- A major lobe (also called main beam) is defined as "***the radiation lobe containing the direction of maximum radiation***." In some antennas, there may exist more than one major lobe.
- A minor lobe is ***any lobe except a major lobe***.
- A side lobe is ***adjacent to the main lobe and occupies the hemisphere in the direction of the mainbeam.***
- A back lobe is ***a radiation lobe whose axis makes an angle of approximately 180^o with respect to the major (main) lobe***.

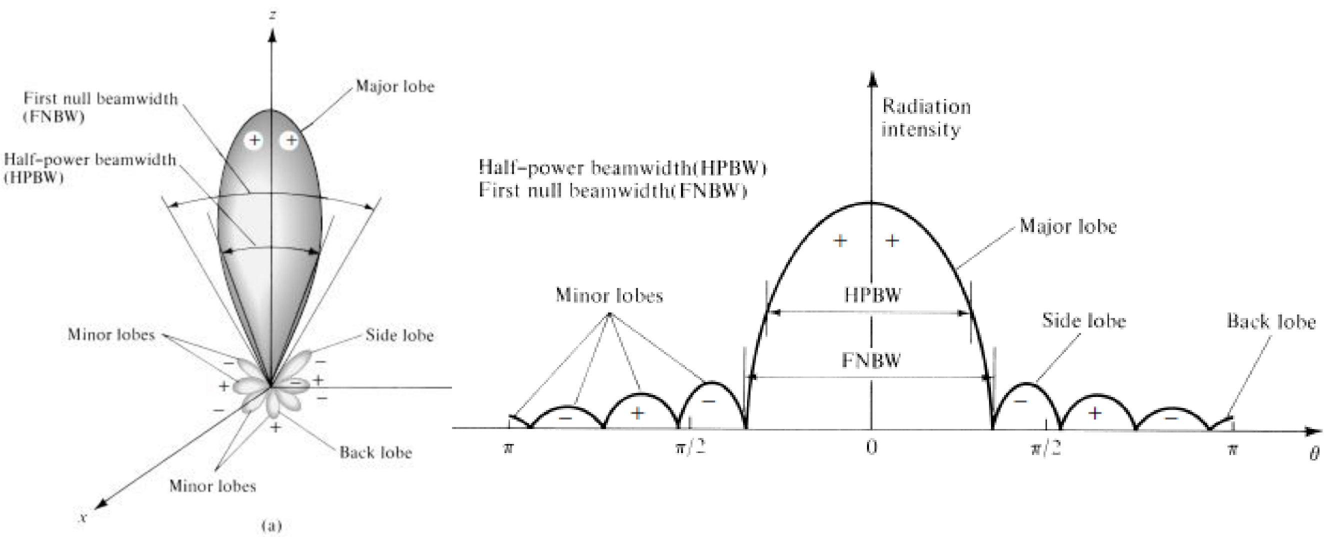
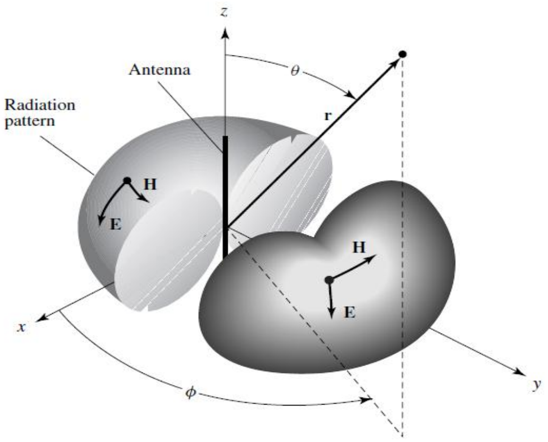


Figure (2) Radiation lobes and beamwidths of an antenna pattern

1.2 Isotropic, Directional, and Omni-directional Patterns

- An isotropic radiator is defined as ***a hypothetical lossless antenna having equal radiation in all directions.***
- A directional antenna is ***one having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others***.
- An omni-directional, is defined as one ***having an essentially nondirectional pattern in a given plane and a directional pattern in any orthogonal plane***. An omnidirectional pattern is a special type of a directional pattern.

It is seen that the pattern in figure(3) is ***nondirectional*** in the ***azimuth plane*** [f(φ), θ=π/2] and directional in the ***elevation plane*** [g(θ), φ = constant].



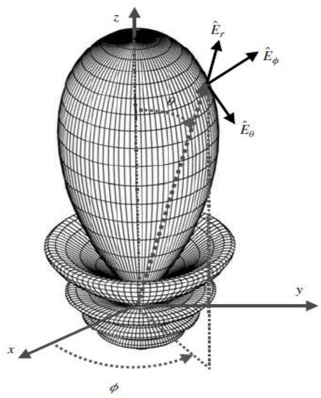
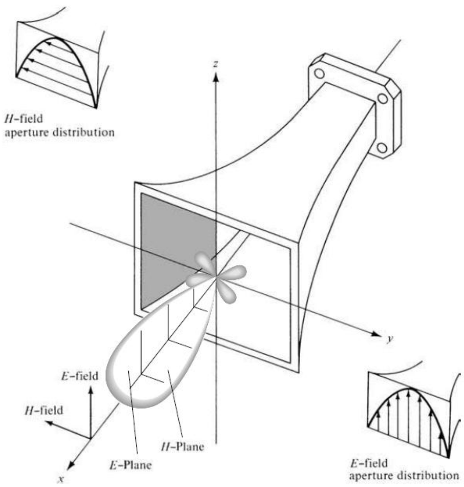


Figure (3): Omni-directional & directional antenna pattern.

1.3 Principal Patterns

For a linearly polarized antenna, performance is often described in terms of its principal E- and H-plane patterns. **The E-plane** is defined as ***the plane containing the electric field vector*** and the direction of maximum radiation, and **the H-plane** as the plane ***containing the magnetic-field vector*** and the direction of maximum radiation. An illustration is shown in figure (4), for this example, the x-z plane (elevation plane; φ = 0) is the principal E-plane and the x-y plane (azimuth plane; θ = π/2) is the principal H-plane.



1.4 Field Regions

The space surrounding an antenna is usually subdivided into three regions as shown in figure (5):

- Reactive near-field: is defined as that portion of the near-field region immediately surrounding the antenna, R < $0.62\sqrt{D^3/\lambda}$ where λ is the wavelength and D is the largest dimension of the antenna.
- Radiating near-field (Fresnel) region: is defined as that region of the field of an antenna between the reactive near-field region and the far-field region. The inner boundary is taken to be the distance R < $0.62\sqrt{D^3/\lambda}$ and the outer boundary the distance R < $2D^2/\lambda$.
- Far-field (Fraunhofer) region: is defined as that region of the field of an antenna where the angular ***field distribution is independent of the distance from the antenna***. The far-field region is commonly taken to exist at distances R> $2D^2/\lambda$ from the antenna.

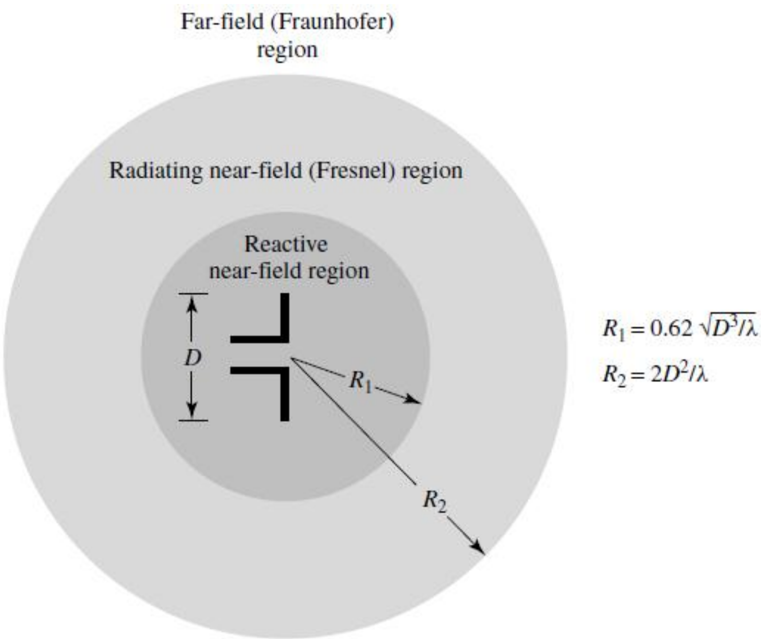


Figure (5) field regions of an antenna.

2. Radiation Power Density (W)

Power and energy are associated with electromagnetic fields. The quantity used to describe the power associated with an electromagnetic wave is the instantaneous **Poynting vector** defined as:

$W = E \times H$

W = instantaneous Poynting vector (W/m^2)

E = instantaneous electric-field intensity (V/m)

H = instantaneous magnetic-field intensity (A/m)

The average power density can be written as:

$$W_{av} = W_r \boldsymbol{a_r} = \frac{1}{2} Re[E \text{ x } H^*] = \frac{1}{2} Re\left[E_\theta \text{ x } H_\phi^*\right] = -\frac{1}{2} Re\left[E_\phi \text{ x } H_\theta^*\right] = \frac{E^2}{2\eta} = \frac{\eta H^2}{2}$$

The average power radiated by an antenna (radiated power) can be written as:

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} W_{av} \,.\, ds = \int_0^{2\pi} \int_0^{\pi} W_r \boldsymbol{a_r} \,.\, r^2 sin\theta d\theta d\phi \, \boldsymbol{a_r} \ , ds = r^2 sin\theta d\theta d\phi \, a_r$$

Example 1: The radial component of the radiated power density of an antenna is given by**:** $W_{av} = A_o \frac{sin\theta}{r^2} \, a_r \quad W/m^2$, where A_0 is the peak value of the power density, Determine the total radiated power?

Solution:

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} W_{av} . ds$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} A_o \frac{sin\theta}{r^2} \, a_r \ . r^2 sin\theta d\theta d\phi \, a_r = \pi^2 A_o \ \ (W)$$

The total power radiated of Isotropic Antenna is given by:

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} w_o \,.\, ds = \int_0^{2\pi} \int_0^{\pi} w_o \ . r^2 sin\theta d\theta d\phi \, a_r = 4\pi r^2 w_o \ \ (W)$$

and the power density by

$$w_o = \frac{P_{rad}}{4\pi r^2} \, a_r$$

3. *Radiation Intensity* (U)

Radiation intensity in a given direction is defined as “the power radiated from an antenna per unit solid angle.” In mathematical form it is expressed as:

$$U = r^2 W_{rad}$$

"U" is the radiation intensity (W/unit solid angle)

w_{rad}: radiation density (W/m2)

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U.d\Omega = \int_0^{2\pi} \int_0^{\pi} U.\ sin\theta d\theta d\phi$$

(dΩ) =Unit solid angle = sinθdθdϕ

$$P_{rad} = I_{rms}^2 R_{rad} = \frac{I_0^2}{2} R_{rad} \qquad R_{rad} = radiation\ resistance$$

The radiation intensity of an isotropic source as:

$$U_o = \frac{P_{rad}}{4\pi}$$

Example: The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$\boldsymbol{a}$) $\boldsymbol{U = B_0 cos^2(\theta)}$ $\boldsymbol{b}$) $\boldsymbol{U = B_0 cos^3(\theta)}$

For each, find the maximum power density (in watts/square meter) at a distance of 1m?

4. Beamwidth

In an antenna pattern, there are a number of beamwidths. One of the most widely used beamwidths is the Half-Power Beamwidth **(HPBW)**, which is defined as ***the angle between the two directions in which the radiation intensity is one-half value of the maximum beam***. Another important beamwidth is ***the angular separation between the first nulls of the pattern***, and it is referred to as the First-Null Beamwidth (**FNBW**), as shown in figure (6).

θ_m : is the angle of the max beam.

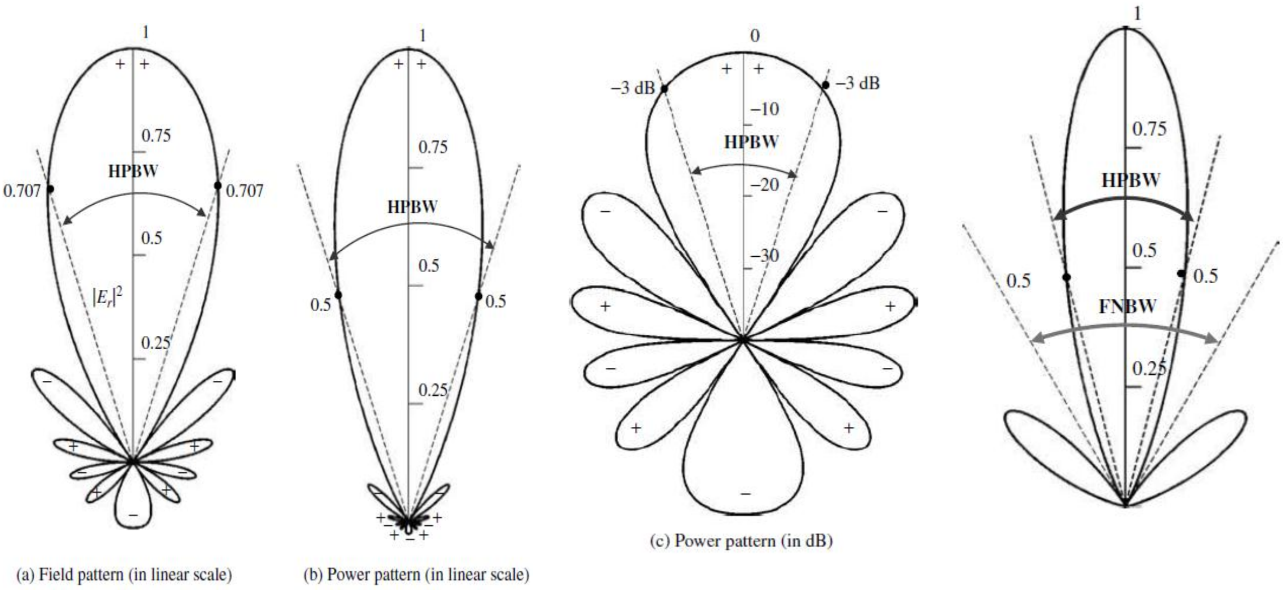


Figure (6)

$$HPBW = 2|\theta_m - \theta_h|$$
$$FNBW = 2|\theta_m - \theta_n|$$

To find θ_m:

[Normalized U(θ)] $_{\theta = \theta m}$=1

OR [normalized E(θ)] $_{\theta = \theta m}$=1

To find θ_h:

[Normalized U(θ)]$_{\theta=\theta h}$=0.5

OR [normalized E(θ)]$_{\theta= \theta h}$=0.707

To find θ_n:

[Normalized U(θ)]$_{\theta=\theta n}$=0

OR [normalized E(θ)]$_{\theta= \theta n}$=0

Example 2

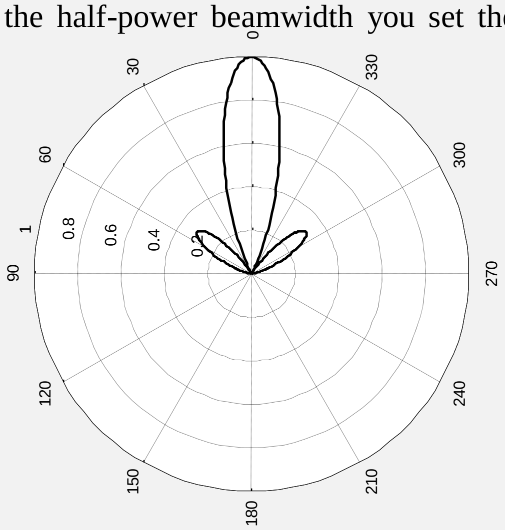
The normalized radiation intensity of an antenna is defined by **U(θ) = cos²(θ)cos²(3θ)** , $(0 \le \theta \le 90^o, 0^o \le \varphi \le 360^o)$. Find the:

a. HPBW (in radians and degrees)

b. FNBW (in radians and degrees)

Solution:

a. Since U(θ) represents the power pattern, to find the half-power beamwidth you set the function equal to half of its maximum,



$U(\theta)|_{\theta=\theta h}= \cos^2(\theta)\ \cos^2(3\theta)|_{\theta=\theta h}= 0.5$

$\cos(\theta_h)\ \cos(3\theta_h) = 0.707$

$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$

By using trial and error we get:

$\theta_h \approx 0.25$ (radians) = 14.325^o

Since the function U(θ) is symmetric about the maximum at $\theta_m = 0$, then:

$HPBW = 2(\theta_m - \theta_h) \approx 0.50\ (radians) = 28.65^\circ$

b. To find the first-null beamwidth (FNBW), set the U(θ) equal to zero :

$U(\theta)|_{\ \theta=\theta h} = \cos^2(\theta)\ \cos^2(3\theta)|_{\theta=\theta n} = 0$

$\cos\theta_n = 0 \rightarrow \theta_n = \pi/2\ (radians) = 90^o$

OR $\cos 3\theta_n = 0 \rightarrow \theta_n = \pi/6\ (radians) = 30^o$

$FNBW = 2\theta_n = \pi/3\ (radians) = 60^o$

H.W: Find (HPBW) and (FNBW), in radians and degrees, for the following normalized radiation intensities, then draw the radiation pattern:

(a) U(θ) = cos(2θ) (b) U(θ) = cos²(2θ)

(c) U(θ) = cos(2θ) cos(3θ) (d) U(θ) = cos²(2θ) cos²(3θ)

5. Directivity D

Directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. In mathematical form, it can be written as:

$$D = \frac{U}{U_o} = \frac{4\pi U}{P_{rad}} \qquad \text{(dimensionless)}$$

The maximum directivity is expressed as:

$$D_{max} = D_o = \frac{U_{max}}{U_o} = \frac{4\pi U_{max}}{P_{rad}} \qquad \text{(dimensionless)}$$

The general expression for the directivity and maximum directivity is:

$$D(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{\int_0^{2\pi}\int_0^{\pi} U(\theta, \phi)\,\sin\theta d\theta d\phi}$$

$$D_0 = 4\pi \frac{U(\theta, \phi)|_{max}}{\int_0^{2\pi}\int_0^{\pi} U(\theta, \phi)\,\sin\theta d\theta d\phi} = \frac{4\pi}{\Omega_A}$$

Where Ω_A is the beam solid angle, and it is given by**:**

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} U_n(\theta, \phi)\,\sin\theta d\theta d\phi$$

Example 3

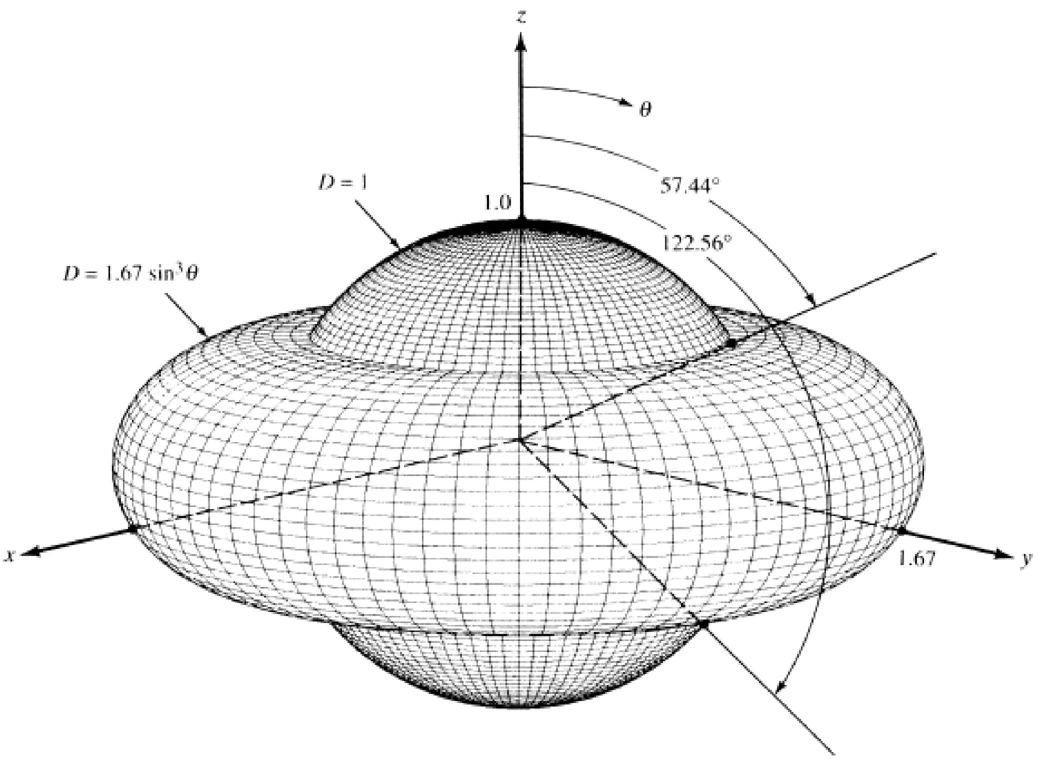
The radiation intensity of the major lobe of many antennas can be adequately represented by: $\boldsymbol{U = B_0 \cos\theta}$. The radiation intensity exists only in the upper hemisphere $(0 \le \theta \le \pi/2, 0 \le \varphi \le 2\pi)$, Find the maximum directivity D?

Solution:

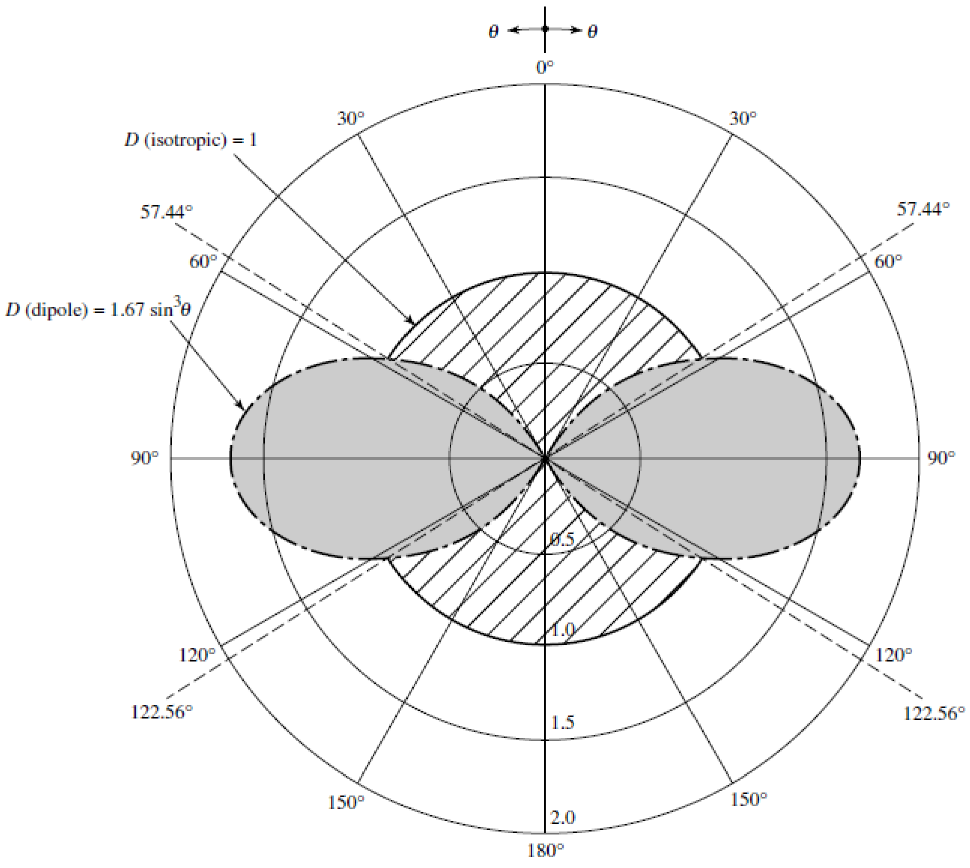
$$\Omega_A = \int_0^{2\pi}\int_0^{\pi/2} \cos\theta\,\sin\theta d\theta d\phi = 2\pi * \frac{1}{2}\int_0^{\pi/2} \sin(2\theta)\,d\theta = \pi$$

$$D_o = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi} = 4 = 6.02 \text{ dB}$$

Two- and three-dimensional directivity patterns of Isotropic and λ /2 dipole.

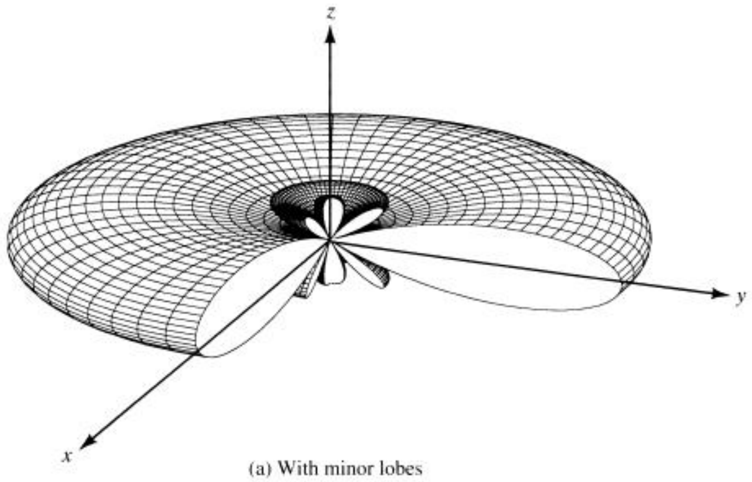


(b) Three-dimensional

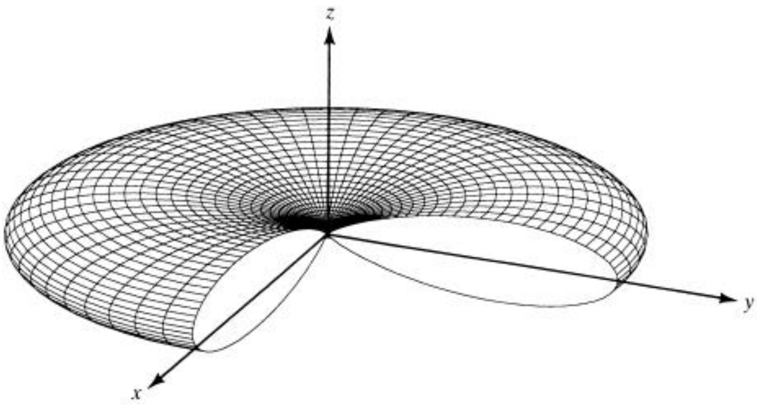


(a) Two-dimensional

Omnidirectional patterns **with** and **without** minor lobes

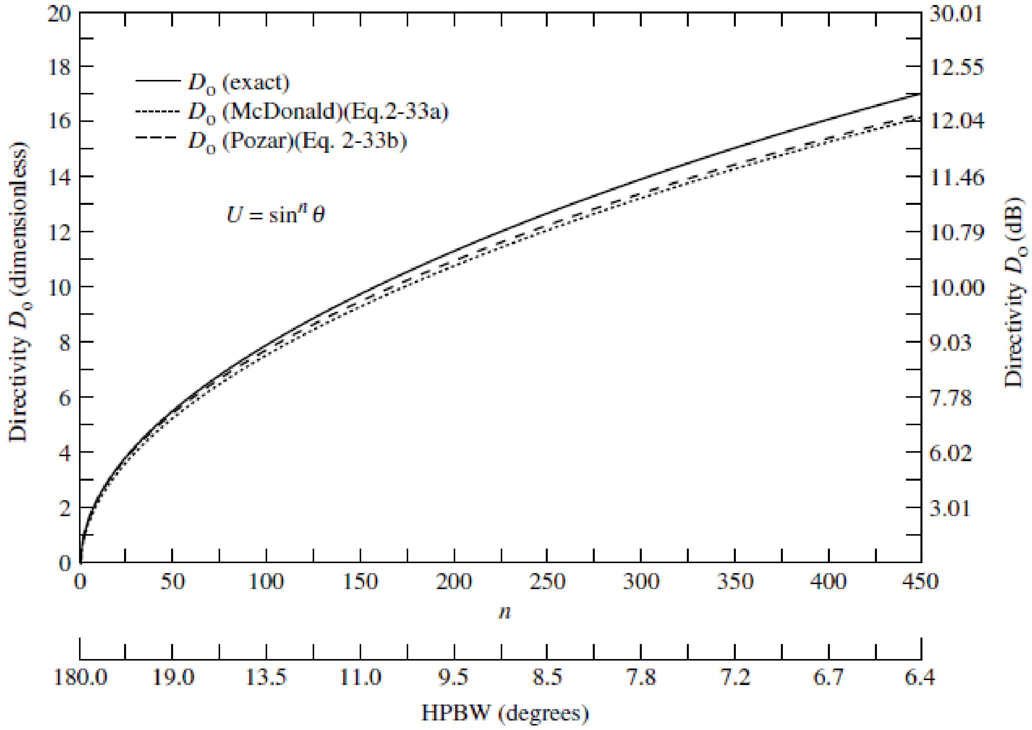


(a) With minor lobes



(b) Without minor lobes

Directivity for Omnidirectional $U = \sin^n \theta$ power patterns



6. Antenna Efficiency

Associated with an antenna are a number of efficiencies and can be defined using Figure 8. The total antenna efficiency e_0 is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due, referring to Figure 8(b), to

1. ***Reflections*** because of the mismatch between the transmission line and the antenna
2. I^2R losses (***conduction and dielectric***)

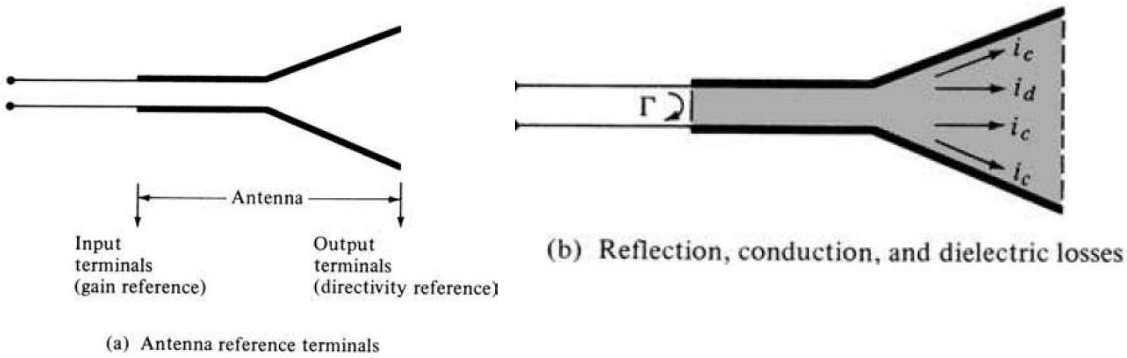


Figure 8 Reference terminals and losses of an antenna.

In general, the overall efficiency can be written as

$$e_o = e_r e_c e_d$$

e_0 = total efficiency (dimensionless)
e_r = reflection(mismatch) efficiency = $(1 - |\Gamma|^2)$ (dimensionless)
e_c = conduction efficiency (dimensionless)
e_d = dielectric efficiency (dimensionless)
Γ = voltage reflection coefficient at the input terminals of the antenna

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

where Z_{in} = antenna input impedance,
Z_0 = characteristic impedance of the transmission line

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 + |\Gamma|}$$

VSWR: voltage standing wave ratio

Usually e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write as

$$e_o = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2)$$

where e_{cd} = e_c e_d = ***antenna radiation efficiency***, which is used to relate the gain and directivity

7. Gain

Gain of an antenna is defined as "***the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically***"

$$G = \frac{\text{radiation intensity}}{\text{total input (accepted) power}/4\pi} = 4\pi \frac{U(\theta, \phi)}{Pin}$$

According to the IEEE Standards, "***gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses)***."

$$P_{rad} = e_{cd} P_{in}$$

which is related to the directivity by

$$G = e_{cd} \left[4\pi \frac{U(\theta, \phi)}{P_{rad}} \right] = e_{cd} D$$

We can introduce an ***absolute gain*** G_{abs} that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line), and it can be written as

$$G_{abs} = e_r e_{cd} D = e_{cd}(1 - |\Gamma|^2) D$$

Example:

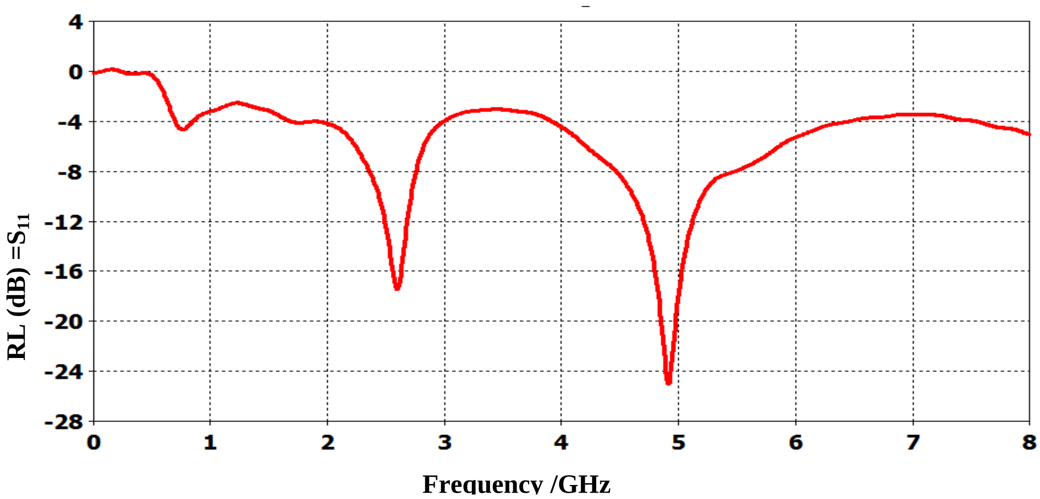
A lossless antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms, assuming that the pattern of the antenna is given by $\boldsymbol{U = 5 \sin^3 \theta}$ **.** Find the maximum absolute gain of this antenna?

8. Return Loss (RL) (S_{11})

The return loss is defined as ***the ratio of amplitude of reflected wave to the amplitude of incident wave***. The return loss value describes the reduction in the amplitude of reflected wave compared to the forward energy. This return loss also can be used to determine the matching condition is achieved or not. The frequency can operate when the return loss value less than 10 dB. The equation for return loss is:

$$\text{Return loss(RL)} = S_{11} = 20\log|\Gamma|$$

When the transmitter and the antenna are perfectly matched **(Γ=0, RL=-∞),** this means that ***no power is reflected***. On the other hand, when (**Γ=1, RL=0dB**), ***all input power is reflected***.



Return Loss of monopole Sierpinski antenna.

9. Bandwidth

The bandwidth can be define as: ***the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency***.

For ***broadband antennas***, the bandwidth is usually expressed as ***the ratio of the upper-to-lower frequencies*** of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.

For ***narrowband antennas***, the bandwidth is expressed as a percentage of the frequency difference (***upper minus lower***) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency difference of acceptable operation is 5% of the center frequency of the bandwidth

10. Polarization

Polarization is defined as ***the orientation of electric field***. ***In addition, polarization is describing the time varying direction and relative magnitude of the electric field vector***.

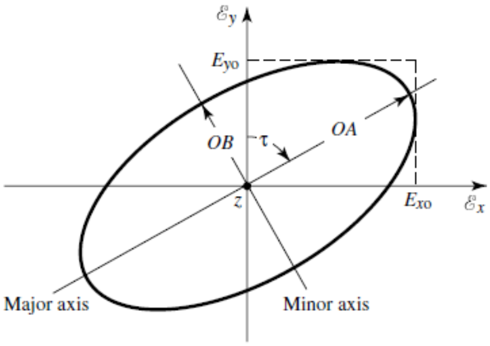
Polarization may be classified as **linear**, **circular** or **elliptical**.

a- *If the vector that describes the electric field at a point in space is always directed along a line the field is said to be* ***linearly polarized****.*

b- *The figure that the electric field traces is an ellipse is said to be* ***elliptically polarized****.*

Linear and circular polarizations are special cases of elliptical and they can be obtained when the ellipse becomes a straight line or a circle respectively. The ratio of the major axis to the minor axis is referred to as the **axial ratio** (AR), and it is equal to

$$AR = \frac{major\ axis}{minor\ axis} = \frac{OA}{OB} \qquad 1 \le AR \le \infty$$



(b) Polarization ellipse

In general the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave as shown in figure below. This is commonly stated as polarization mismatch the amount of power extracted by the antenna from the incoming signal will not be maximum because of the ***polarization loss***.

$PLF = |COS\psi_P|^2$ (Dimensionless)

Where: ψ_P is the angle between the two unit vectors

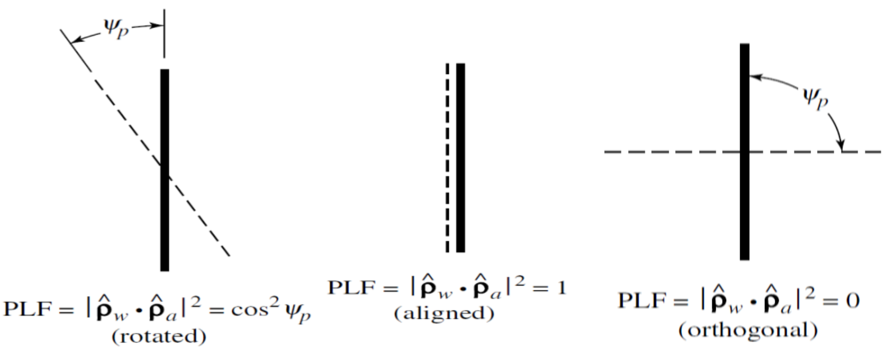
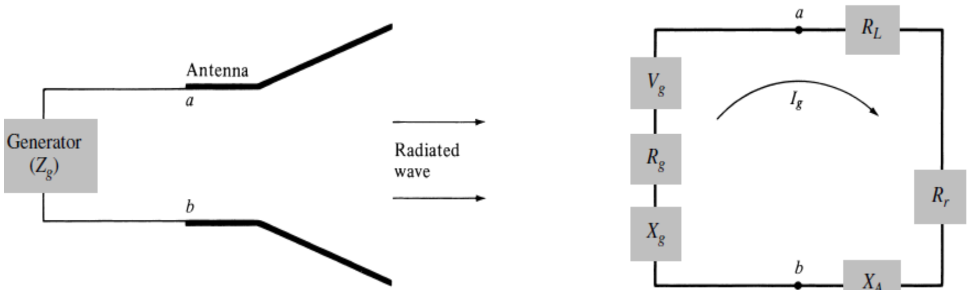


Figure: PLF for transmitting and receiving of linear wire antennas.

11. Input Impedance

Input impedance is defined as the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals "a" and "b" as shown in figure (7a).



(a) Antenna in transmitting mode

(b) Thevenin equivalent

Figure (7)

$$Z_A = R_A + jX_A$$

In general the resistive part consists of two components:

$$R_A = R_{rad} + R_{loss}$$

If we assume that the antenna is attached to a generator with internal impedance:

$$Z_g = R_g + jX_g$$

The current developed within the loop which is given by:

$$I_g = \frac{V_g}{Z_g + Z_A} = \frac{V_g}{(R_{rad} + R_{loss} + R_g) + j(X_A + X_G)}$$

The power delivered to the antenna for radiation is given by:

$$P_{rad} = \frac{1}{2}|I_g|^2 R_{rad}$$

and the loss power (as a heat) is :

$$P_{loss} = \frac{1}{2}|I_g|^2 R_{loss}$$

The remaining power is dissipated as heat on the internal resistance of the generator (R_g), and it is given by:

$$P_g = \frac{1}{2}|I_g|^2 R_g$$

The power supplied by the generator during conjugate matching is:

$$P_s = \frac{1}{2} V_g I_g^*$$

The ***maximum power*** delivered to the antenna occurs when we have conjugate matching; that is when:

$R_{rad} + R_{loss} = R_g$ and $X_A = -X_g$

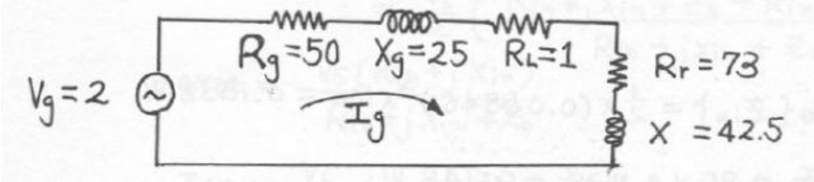
$$I_g = \frac{V_g}{(R_{rad} + R_{loss} + R_{rad} + R_{loss})} = \frac{V_g}{2(R_{rad} + R_{loss})}$$

Example:

A λ **/2** dipole, with a total loss resistance of 1 ohm, is connected to a generator whose internal impedance is 50 + *j*25 ohms. Assuming that the peak voltage of the generator is 2 V and the impedance of the dipole, excluding the loss resistance, is 73 + *j*42.5 ohms, find the power

(a) Supplied by the source (real) (b) Radiated by the antenna (c) Dissipated by the antenna

Solution:



$$I_g = \frac{V_g}{Z_g + Z_A} = \frac{V_g}{(R_{rad} + R_{loss} + R_g) + j(X_A + X_g)} = \frac{2}{(73 + 1 + 50) + j(42.5 + 25)}$$

$$= 14.166 \times 10^{-3} \lfloor -28.56$$

$$a)\quad P_s = \frac{1}{2} V_g I_g^* = \frac{1}{2}(2)(\ 14.166)$$

$$b)\quad P_{rad} = \frac{1}{2}\left|I_g\right|^2 R_{rad} = \frac{1}{2}(14.166 \times 10^{-3})^2(73) = 7.84\, mW$$

$$c)\quad P_g = \frac{1}{2}\left|I_g\right|^2 R_g = \frac{1}{2}(14.166 \times 10^{-3})^2(50) = 5.016\, mW$$

$$P_{Loss} = \frac{1}{2}\left|I_g\right|^2 R_{Loss} = \frac{1}{2}(14.166 \times 10^{-3})^2(1) = 0.1003\, mW$$

12. Antenna Radiation Efficiency

The ***conduction-dielectric efficiency*** e_{cd} is defined as the ratio of the power delivered to the radiation resistance R_r to the power delivered to R_r and R_L.

the radiation efficiency can be written as:

$$\text{Radiation Efficiency } (e_{cd}) = \frac{P_{rad}}{P_{in}} = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{R_{rad}}{R_{rad} + R_{loss}} \quad ,$$

$$R_{loss} = R_{hf} = \frac{L}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}} \qquad for\ uniform\ current\ distribution$$

$$R_{loss} = \frac{1}{2} R_{hf} = \frac{L}{4\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}} \qquad for\ nonuniform\ current\ distribution$$

L = length of the dipole, b = radius of the wire, ω = angular frequency,

μ_0 = permeability of free-space ($4\pi * 10^{-7}$), and σ = conductivity of the metal.

Example 5

A resonant half-wavelength dipole is made out of copper *(σ = 5.7 × 10^7 S/m)* wire. Determine the radiation efficiency of the dipole antenna at f = 100 MHz if the radius of the wire b=3×$10^{-4}\lambda$, and the radiation resistance of the $\lambda/2$ dipole is 73Ω

Solution:

$\lambda = c/f = 3 \times 10^8/10^8 = 3\ m$

$L = \lambda/2 = 1.5\ m$

For a $\lambda/2$ dipole with a sinusoidal current distribution:

$R_{loss} = 0.5R_{hf}$

$$R_{loss} = 0.5\frac{1.5}{2\pi * 3\ \times\ 10^{-4} * 3}\sqrt{\frac{\pi(10^8)(4\pi\ \times\ 10^{-7})}{5.7\ \times\ 10^7}} = 0.349\Omega$$

$$e_{cd} = \frac{R_{rad}}{R_{rad} + R_{loss}} = \frac{73}{73 + 0.349} = 0.9952\ =\ 99.52\%$$

13. Effective Aperture (Area)

Effective aperture is defined as “the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction.

The relationship between directivity and effective aperture is illustrated by the following formulas:

$$Ae = \frac{\lambda^2}{4\pi}D \qquad and \qquad Ae_{max} = \frac{\lambda^2}{4\pi}D_0$$

Example 4

Find the maximum directivity and the maximum effective aperture of the antenna whose radiation intensity is that of example1and λ=3m. Write an expression for the directivity as a function of the directional angles θ and φ.

Solution: The radiation intensity is given by:

$U = r^2W_{rad} = A_0 \sin\theta$

The maximum radiation is directed along $\theta_m = \pi/2$, thus:

$U_{max} = A_0$

$P_{rad} = \pi^2A_0$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4}{\pi} = 1.27 \quad ,A_{em} = \frac{3^2}{4\pi} * 1.27 = 0.9\ m^2$$

$D = D_0 \sin\theta = 1.27 \sin\theta$

Example 6

The radiation intensity of an antenna is given by: $\boldsymbol{U = BI_0^2 \sin(\pi \sin\theta)}$ Find:

- Radiation power and Radiation resistance
- Maximum directivity
- Maximum effective aperture
- HPBW and FNBW
- Sketch the power pattern

Solution :

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} BI_0^2 \sin(\pi\sin\theta)\, sin\theta d\theta d\phi = 2\pi BI_0^2 \int_0^{\pi} \sin(\pi\sin\theta)\, sin\theta d\theta$$

This integral can be solved numerically (Trapezoidal or Simpson's rule)

θi	0	18	36	54	72	90	108	126	144	162	180
F(θi)	0	0.255	0.565	0.456	0.145	0	0.145	0.456	0.565	0.255	0

$$\int_0^{\pi} \sin(\pi\sin\theta)\, sin\theta d\theta = \frac{\pi}{18}\left[\left(f(0)+f(\theta n)\right) + 2\sum f(\theta \mathrm{i})\right]$$

$P_{rad} = 2\pi * 0.9BI_0^2 = 1.8\pi BI_0^2 \quad (W)$

$R_{rad} = 4\pi * 1.574B = 6.52\pi B \quad (\Omega)$

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi BI_0^2}{2\pi * 1.63BI_0^2} = 1.227$$

$$Ae_{max} = \frac{\lambda^2}{4\pi} D_{max} = \frac{1.227}{4\pi}\lambda^2 = 0.1\lambda^2 \quad (m^2)$$

$$U(\theta)|_{\theta=\theta_m} = \sin(\pi\sin\theta) = 1 \to \pi\sin\theta_m = \frac{\pi}{2} \to \sin\theta_m = \frac{1}{2} \to \quad \theta_m = 30^o\ ,150$$

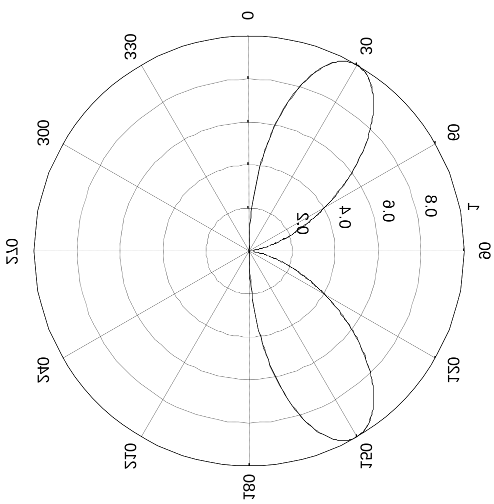
$$U(\theta)|_{\theta=\theta_h} = \sin(\pi\sin\theta) = 0.5 \to \begin{cases} \pi\sin\theta_h = \frac{\pi}{6} \to \sin\theta_h = \frac{1}{6} \to \theta_h = 9.6^o \\ \pi\sin\theta_h = \frac{5\pi}{6} \to \sin\theta_h = \frac{5}{6} \to \theta_h = 56.4^o \end{cases}$$

$HPBW = 56.4^o - 9.6^o = 46.8^o$

$U(\theta)|_{\theta=\theta_n} = \sin(\pi\sin\theta) = 0$

$$\to \begin{cases} \pi\sin\theta_n = 0 \to \sin\theta_n = 0 \to \theta_n = 0\ , 180^o \\ \pi\sin\theta_n = \pi \to \sin\theta_n = 1 \to \theta_n = 90^o \end{cases}$$

$FNBW = 90^o - 0^o = 90^o$

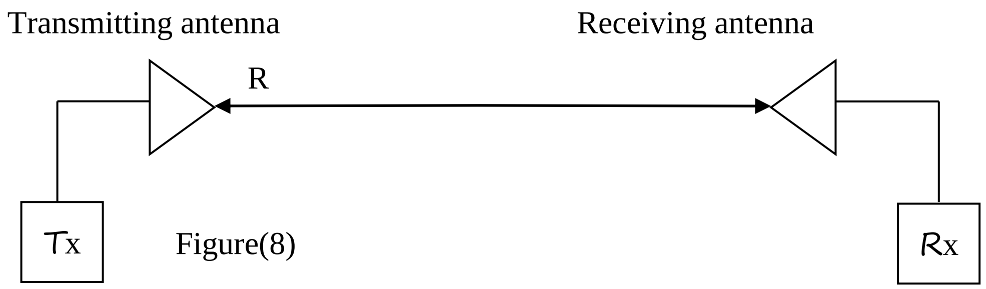


H.W: Repeat Example 5 with

$U = BI_0^2 \cos\left(\frac{\pi}{2} cos\,\theta\right)$

14. Friis Transmission Equation

The Friis Transmission Equation relates the ***power received*** to the ***power transmitted*** between two antennas separated by a distance R > $2D^2/\lambda$, where D is the largest dimension of either antenna. Referring to figure (8), let us assume that the transmitting antenna is initially isotropic.



Figure(8)

The amount of the received power P_r can be written as:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Where: P_r : is the received power, G_r : receiving antenna gain, G_t : transmitting antenna gain, W_t : transmitting power density.

If mismatch and polarization loss factors are also included then:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\, PLF$$

Example 7

In a microwave link, two identical antennas of gain =4dB operating at 2.4 GHz. If the transmitter power is 1W, find the received power if the range =3 m?

Solution

$$G_t = G_r = 4dB = 10^{0.4}$$

$$\lambda = \frac{3 * 10^8}{2.4 * 10^9} = 0.125\, m$$

$$P_r = 1 * 10^{0.4} * 10^{0.4} \left(\frac{0.125}{4\pi * 3}\right)^2 = 0.02\, W$$

$$P_r(dB) = 20 \log 0.02 = -33.9\, dB$$