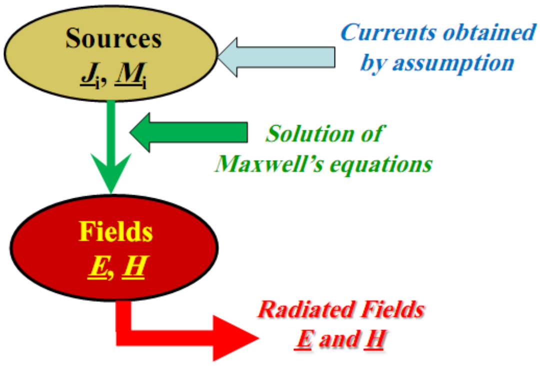
Lecture 8: Radiation Integrals and Auxiliary Potential Functions

As we know that when electric charges undergo acceleration or deceleration, electromagnetic radiation is produced. Hence it is the motion of charges (i.e., currents) that is the source of radiation. Some problems involving electric currents can be cast in equivalent forms involving magnetic currents (the use of magnetic currents is simply a mathematical tool, they have never been proven to exist). So,



Source of antenna radiation fields is

J= vector electric current density (A/m^2)

M= vector magnetic current density (V/m^2)

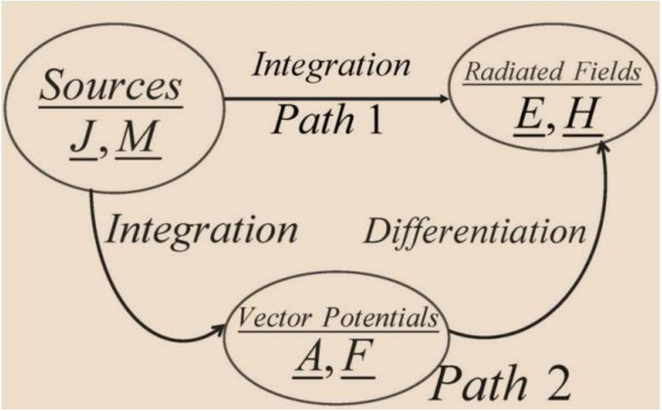
For mathematical simplicity, instead of calculating the radiated fields (E, H) directly from the sources (J, M) in a one-step procedure, a two-step procedure is commonly used.

Path-1: One Step Procedure

- Relates the E and H fields to J and M current sources by integral relations

Path-2: Two Step Procedure

- 1st Relates the A and F vector potential to the J and M current sources by integral relations
- 2nd Determined E and H fields by differentiating A and F



Block diagram for computing radiated fields from electric (J) and magnetic (M) current sources

Where (**A:** Magnetic vector potential **F:** Electric vector potential)

In order to account for both electric current and/or magnet tic current sources, the symmetric form of Maxwell's equations m must be utilized to determine the result ting radiation n fields. The s symmetric form of Maxwell's equations include additional radiation sources (electric charge density ρ and magnetic c charge density ρ_m).

Maxwell's equations (symmetric, time varying form)

Differential form

$$\nabla \times \mathscr{E} = -\mathscr{M}_i - \frac{\partial \mathscr{B}}{\partial t}$$

$$\nabla \times \mathscr{H} = \mathscr{J}_i + \mathscr{J}_c + \frac{\partial \mathscr{D}}{\partial t}$$

$$\nabla \cdot \mathscr{D} = q_{ev}$$

$$\nabla \cdot \mathscr{B} = q_{mv}$$

$$\nabla \cdot \mathscr{J}_{ic} = -\frac{\partial q_{ev}}{\partial t}$$

Integration form

$$\oint_C \mathscr{E} \cdot d\ell = -\iint_S \mathscr{M}_i \cdot ds - \frac{\partial}{\partial t}\iint_S \mathscr{B} \cdot ds$$

$$\oint_C \mathscr{H} \cdot d\ell = \iint_S \mathscr{J}_i \cdot ds + \iint_S \mathscr{J}_c \cdot ds + \frac{\partial}{\partial t}\iint_S \mathscr{D} \cdot ds$$

$$\oiint_S \mathscr{D} \cdot ds = \mathscr{Q}_e$$

$$\oiint_S \mathscr{B} \cdot ds = \mathscr{Q}_m$$

$$\oiint_S \mathscr{J}_{ic} \cdot ds = -\frac{\partial \mathscr{Q}_e}{\partial t}$$

Maxwell's equations (symmetric, time varying form)

Differential form

$$\nabla \times \mathbf{E} = -\mathbf{M}_i - j\omega\mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_i + \mathbf{J}_c + j\omega\mathbf{D}$$

$$\nabla \cdot \mathbf{D} = q_{ev}$$

$$\nabla \cdot \mathbf{B} = q_{mv}$$

$$\nabla \cdot \mathbf{J}_{ic} = -j\omega q_{ev}$$

Integration form

$$\oint_C \mathbf{E} \cdot d\ell = -\iint_S \mathbf{M}_i \cdot ds - j\omega\iint_S \mathbf{B} \cdot ds$$

$$\oint_C \mathbf{H} \cdot d\ell = \iint_S \mathbf{J}_i \cdot ds + \iint_S \mathbf{J}_c \cdot ds + j\omega\iint_S \mathbf{D} \cdot ds$$

$$\oiint_S \mathbf{D} \cdot ds = Q_e$$

$$\oiint_S \mathbf{B} \cdot ds = Q_m$$

$$\oiint_S \mathbf{J}_{ic} \cdot ds = -j\omega Q_e$$

The use of potentials in n the solution n of radiation fields employs the concept of superposition of fields.

Electric current source (J, ρ)	⇒	Magnetic vector potential (A)	⇒	Radiation fields (E_A, H_A)
Magnetic current source (M, ρ_m)	⇒	Electric vector potential (F)	⇒	Radiation fields (E_F, H_F)

The total radiation fields (E, H) are e the sum of the fields due e to electric c currents (E_A, H_A) and the fields due to the e magnetic currents (E_F, H_F)

$$E = E_A + E_F$$

$$H = H_A + H_F$$

In the presence of magnetic sources only (j=0, M≠ 0), the Maxwell's equations become:

$$\nabla \times \boldsymbol{E}_A = -j\omega \boldsymbol{B}_A \qquad \text{(1a)}$$

$$\nabla \times \boldsymbol{H}_A = j\omega \boldsymbol{D}_A + \boldsymbol{J} \qquad \text{(1b)}$$

$$\nabla \cdot \boldsymbol{D}_A = \rho \qquad \text{(1c)}$$

$$\nabla \cdot \boldsymbol{B}_A = 0 \qquad \text{(1d)}$$

Similarly in the presence of electric sources only (j≠0, M= 0),

$$\nabla \times \boldsymbol{E}_F = -j\omega \boldsymbol{B}_F - \boldsymbol{M} \qquad \text{(2a)}$$

$$\nabla \times \boldsymbol{H}_F = j\omega \boldsymbol{D}_F \qquad \text{(2b)}$$

$$\nabla \cdot \boldsymbol{D}_F = 0 \qquad \text{(2c)}$$

$$\nabla \cdot \boldsymbol{B}_F = \rho_m \qquad \text{(2d)}$$

According g to the vector identity,

$$\nabla.(\nabla \times G) = 0 \quad (For\ any\ G)$$

Any vector with zero divergence (rotational or solenoidal field) can be expressed as curl of some other. So, from e equations (1d) and (2c), w we can write

$$B_A = \nabla \times A \qquad (3a)$$

$$D_F = -\nabla \times F \qquad (3b)$$

The flux density definitions in equations (3a) and d (3b) lead to the following g field definitions:

$$H_A = \frac{1}{\mu}\nabla \times A \qquad (4a)$$

$$E_F = -\frac{1}{\epsilon}\nabla \times F \qquad (4b)$$

Using (3a) in (1a) and (3b) in (2b), leads to

$$\nabla \times E_A = -j\omega(\nabla \times A) \qquad (5a)$$

$$\nabla \times H_F = -j\omega(\nabla \times F) \qquad (5b)$$

The above e two equations can be rewritten as:

$$\nabla \times [E_A + j\omega A] = 0 \qquad (6a)$$

$$\nabla \times [H_F + j\omega F] = 0 \qquad (6b)$$

Based on the vector identity

$$\nabla \times (\nabla g) = 0 \quad (for\ any\ scaler\ g)$$

The bracketed terms in equations (6a a) and (6b) represents non- -solenoidal (irrational fields) and may each be written as the gradient of some scalar.

$$E_A + j\omega A = -\nabla \emptyset_e \qquad (7a)$$

$$H_F + j\omega F = -\nabla \emptyset_m \qquad (7b)$$

where $\emptyset_e$ is the electric scalar potential and $\emptyset_m$ is the magnetic scalar potential.

From the above two equations, we can have

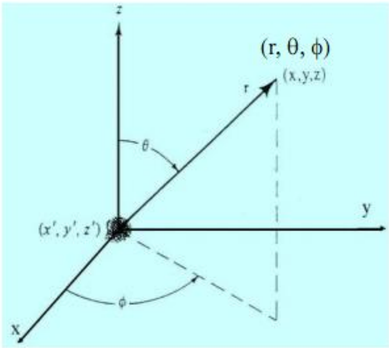
$$E_A = -j\omega A - \nabla \emptyset_e \qquad (8a)$$

$$H_F = -j\omega F - \nabla \emptyset_m \qquad (8b)$$

So, now the fields (E_A, H_A) due to electric sources and (E_F, H_F) due to magnetic sources were derived through equations (4a, 8a) and (4b, 8b).

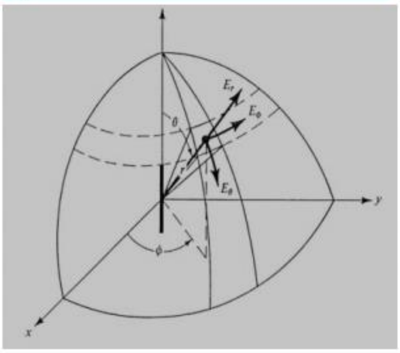
Electric current source (J) at origin

$$\underline{A} = \frac{\mu}{4\pi} \iiint_V \underline{J} \frac{e^{-jkr}}{r} dv' \quad \text{(@ origin)}$$



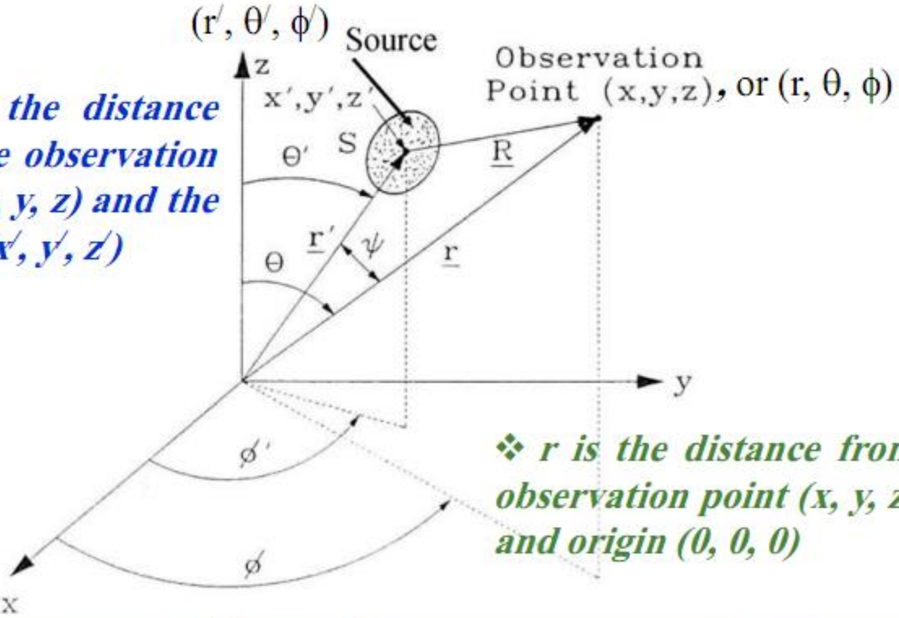
Electric field orientation

$\underline{E}_r, \underline{E}_\theta, \underline{E}_\phi$



Electric J and magnetic M sources not at origin

- ***R is the distance from the observation point (x, y, z) and the source (x′, y′, z′)***

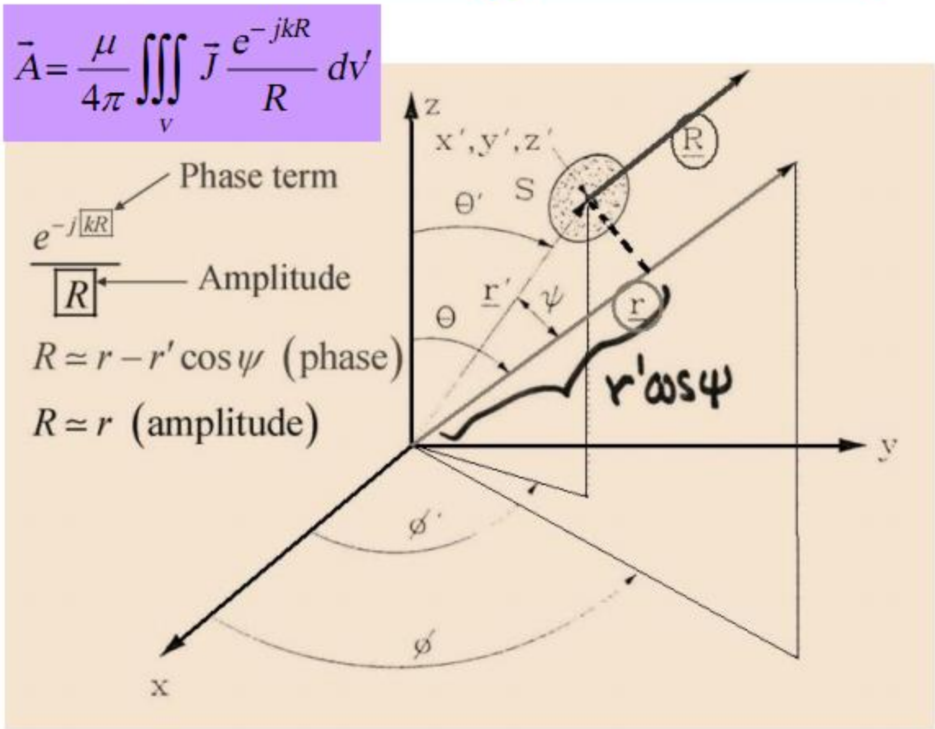


- ***r is the distance from observation point (x, y, z) and origin (0, 0, 0)***

$$\underline{A} = \frac{\mu}{4\pi}\iiint_V \underline{J}\,\frac{e^{-jkR}}{R}\,dv'$$

$$\underline{F} = \frac{\varepsilon}{4\pi}\iiint_V \underline{M}\,\frac{e^{-jkR}}{R}\,dv'$$

Far-Field Approximations



1) Magnetic vector potential $\underline{A}$ for eclectic current source $\underline{J}$

$$(\underline{J} \neq 0,\ \underline{M} = 0)$$

$$1.\quad \underline{A} = \frac{\mu}{4\pi} \iiint_V \underline{J} \frac{e^{-jkR}}{R} dv'$$

$$2.\quad \underline{H}_A = +\frac{1}{\mu} \nabla \times \underline{A}$$

$$3.\quad \underline{E}_A = -j\omega \underline{A} - j\frac{1}{\omega\mu\varepsilon} \nabla(\nabla \cdot \underline{A})$$

or

$$3a.\quad \nabla \times \underline{H}_A = \overset{0}{\cancel{\underline{J}}} + j\omega\varepsilon \underline{E}_A \Rightarrow \boxed{\underline{E}_A = \frac{\nabla \times \underline{H}_A}{j\omega\varepsilon}}$$

2) Electric vector potential $\underline{F}$ for magnetic current source $\underline{M}$

$$(\underline{M} \neq 0,\ \underline{J} = 0)$$

$$1.\ \underline{F} = \frac{\varepsilon}{4\pi} \iiint_V \underline{M} \frac{e^{-jkR}}{R} dv'$$

$$2.\ \underline{E}_F = -\frac{1}{\varepsilon} \nabla \times \underline{F}$$

$$3.\ \underline{H}_F = -j\omega \underline{F} - j\frac{1}{\omega\mu\varepsilon} \nabla(\nabla \cdot \underline{F})$$

or

$$3a.\ \nabla \times \underline{E}_F = -\overset{0}{\cancel{\underline{M}}} + j\omega\mu \underline{H}_F \Rightarrow \boxed{\underline{H}_F = -\frac{\nabla \times \underline{E}_F}{j\omega\mu}}$$

Summary

1) ***Specify <u>J</u> and <u>M</u> (electric and magnetic current density sources)***
2) ***Find <u>A</u> due to <u>J</u> using:*** $$\vec{A} = \frac{\mu}{4\pi} \iiint_V \vec{J} \frac{e^{-jkR}}{R} dv'$$
3) ***Find <u>F</u> due to <u>M</u> using:*** $$\vec{F} = \frac{\varepsilon}{4\pi} \iiint_V \vec{M} \frac{e^{-jkR}}{R} dv'$$
4) ***Find $\underline{E}_A$ and $\underline{H}_A$ due to the magnetic potential vector <u>A</u>***
5) ***Find $\underline{E}_F$ and $\underline{H}_F$ due to the electric potential vector <u>F</u>***
6) ***<u>Total fields are</u>:***

- $E_t = \underline{E}_A + \underline{E}_F$
- $H_t = \underline{H}_A + \underline{H}_F$