Advanced Digital Communication

Lecture 1 : Pulse code modulation (PCM)

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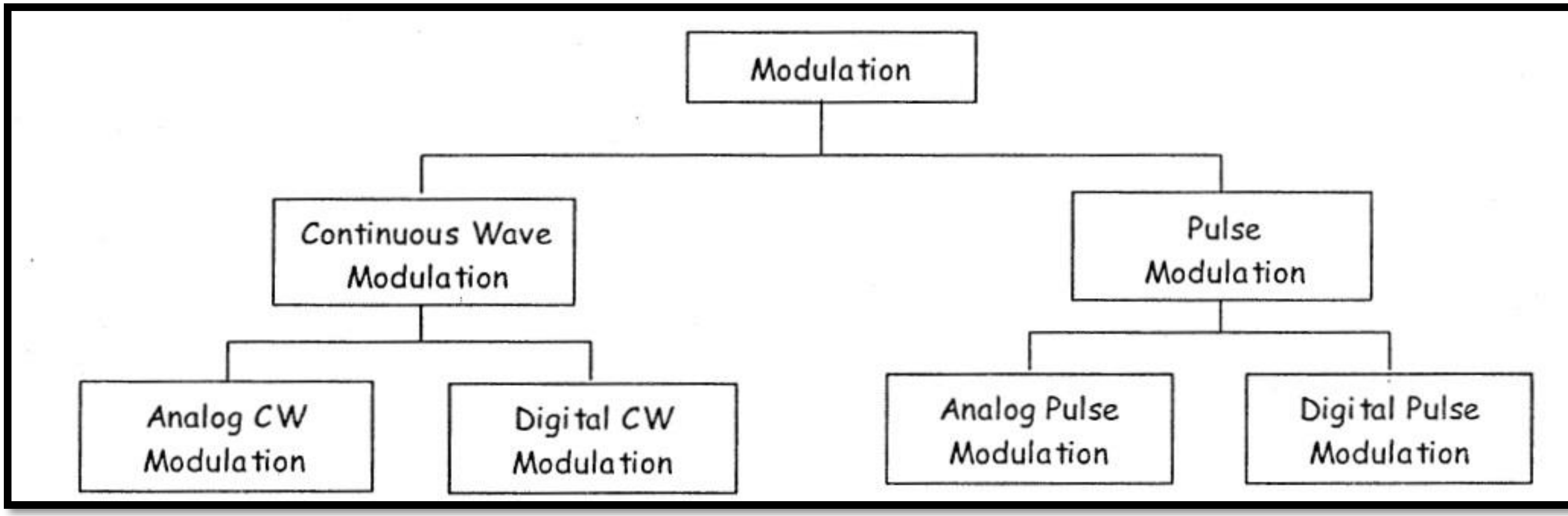
Modulation

- the process of varying one or more properties of a periodic waveform, called the *carrier signal*, with a modulating signal that typically contains information to be transmitted.

Need for Modulation

- Reduction in the height of antenna
- Avoids mixing of signals
- Increases the range of communication
- Multiplexing is possible

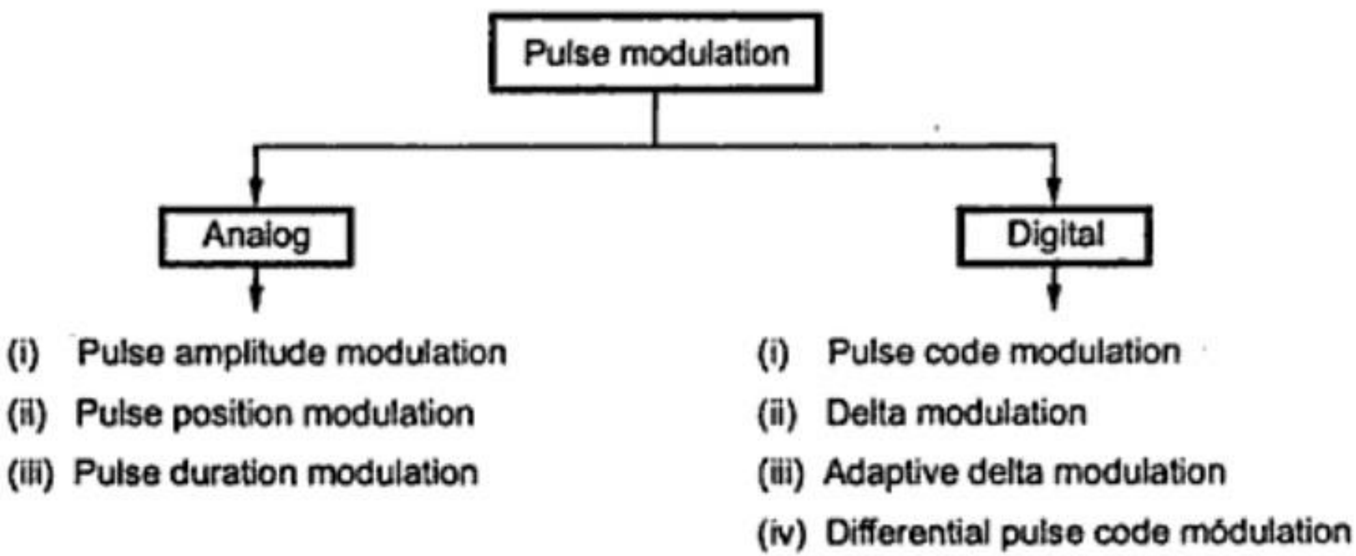
Types of Modulation



AM	ASK	PAM	PCM
FM	FSK	PWM	DM
PM	PSK	PPM	DPCM

Pulse modulation can be further classified as:

- Pulse analog modulation
- Pulse digital modulation



Pulse Code Modulation (PCM)

- Method used to digitally represent sampled analog signals(convert analog signal to digital)
- PCM consists of three steps to digitize an analog signal:
  1. Sampling
  2. Quantization
  3. Binary encoding

Advantages of Digital

- More reliable
- Flexible
- Compatibility with other digital systems
- Integrated networks
- Can be correct the error
- Only digitized information can be transported through a noisy channel without degradation

Advantages of PCM

- The PCM (pulse code modulation) convenient for long distance communication.
- It has a higher transmitter efficiency.
- It has a higher noise immunity.

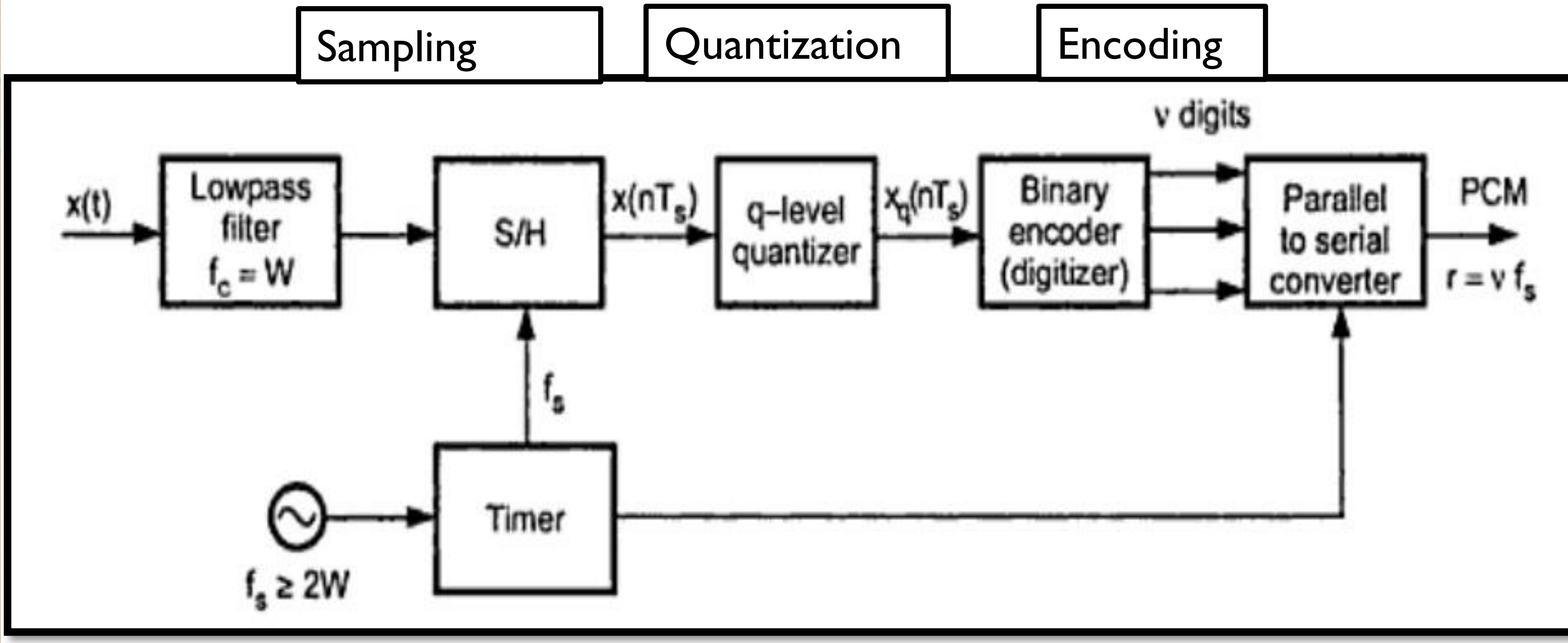
Disadvantages of PCM

- The PCM (pulse code modulation) requires large bandwidth as compared to analog system.
- Encoding, decoding and quantizing circuit of PCM is very complex.

Applications of PCM

- The PCM is used in the satellite transmission system.
- It is used in space communication.
- It is used in telephony.

Components of PCM encoder



Components of PCM encoder

- The signal x(t) is first passed the low pass filter with cutoff frequency (w)Hz,this filter blocks all frequnency commponent above (w)Hz.
- The sample and hold circuit then samples this signal at the rate (fs$\geq 2w$)
- Output of sample and hold x(nTs),q-level quantizer comparres input x(nTs) with fixed digital levels .thus out but of quantizer is a digital level cahhed $x_q(nTs)$.

Components of PCM encoder

- The quantized signale level $x_q(nTs)$ is given to binary encoder , the encoder convert input signal to (n) digits binary word .

- The binary digits (n) are convert to serial bit stream by parallel to serial convert .

Sampling

- A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed from its samples $x[n] = x(n\,T_s)$ if the samples are taken at a rate f_s which is greater than $2\,f_{max}$.

$$Fs \geq 2f_{max}$$

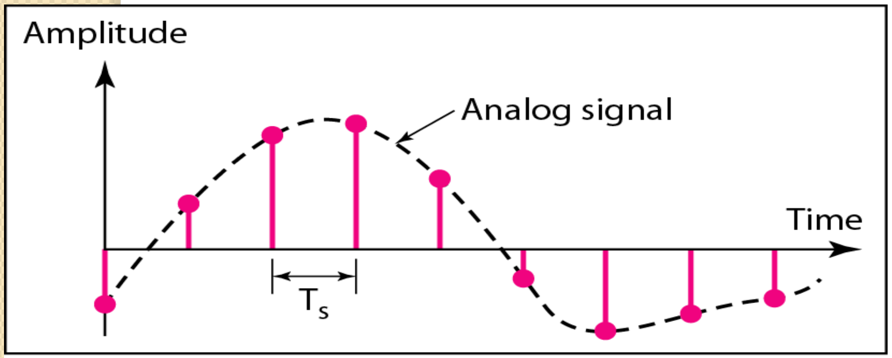
- Why CT signals are represent by samples ?

  - A CT signal cannot be processed in the digital processor or computer

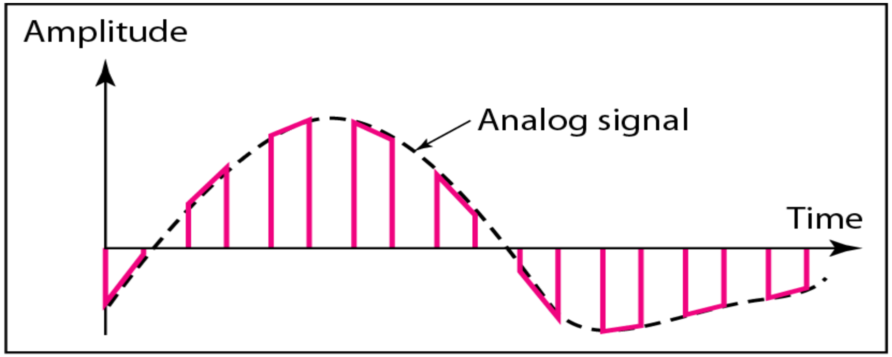
  - TO enable digital transmission of CT signals

There are three sampling methods:

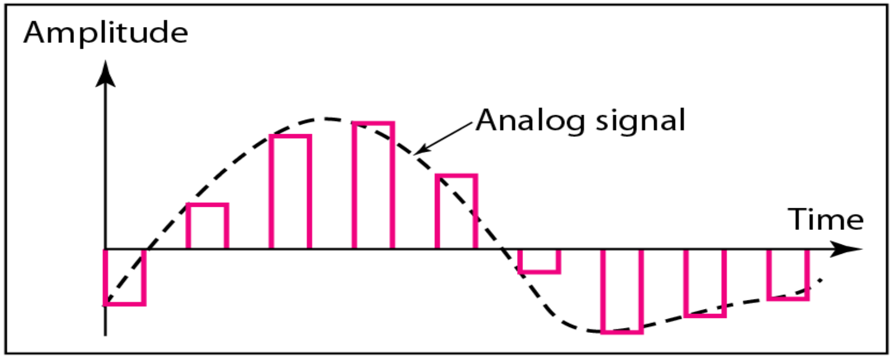
- Ideal - an impulse at each sampling instant
- Natural - a pulse of short width with varying amplitude
- Flattop - sample and hold, like natural but with single amplitude value



a. Ideal sampling



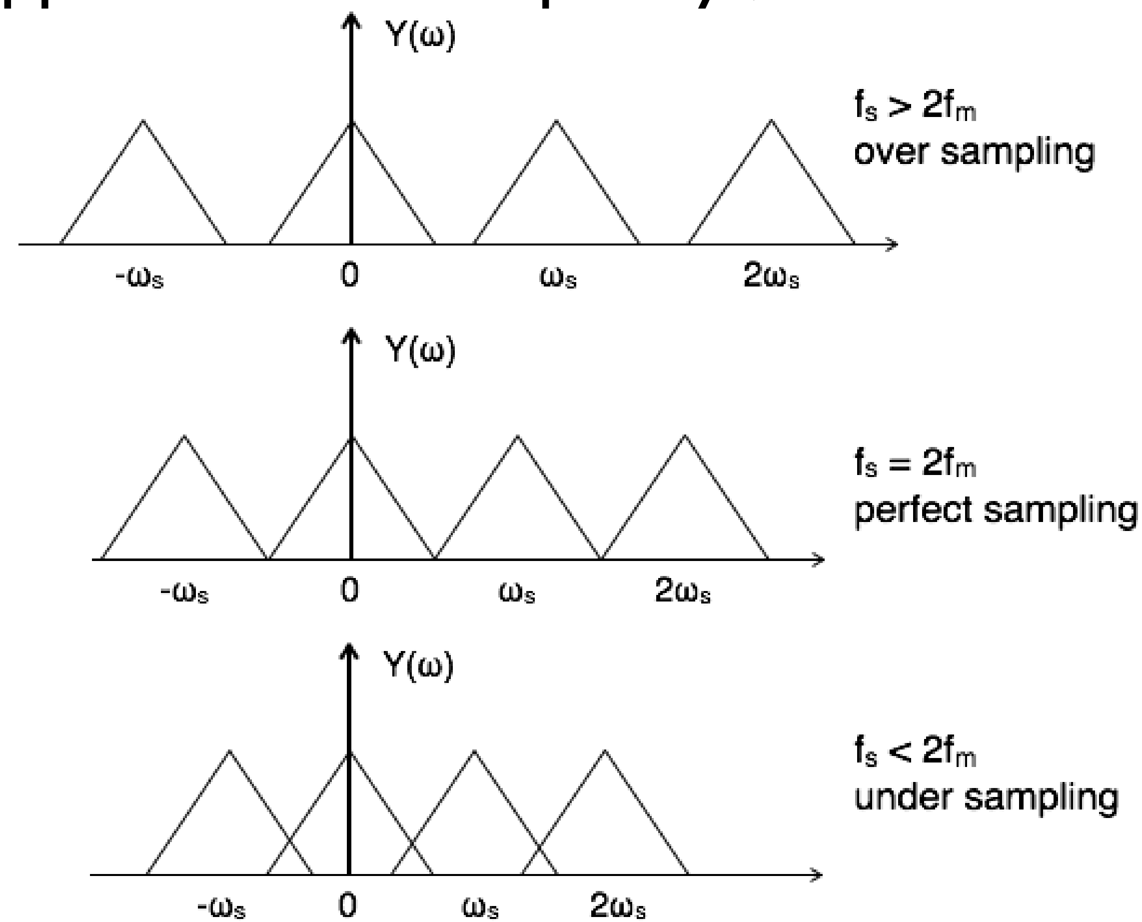
b. Natural sampling



c. Flat-top sampling

Undersampling (Aliasing)

- When the high frequency interferes with low frequency and appears as low frequency ,that mean Fs <2fmax



Effect of aliasing

- Since high and low frequencies interfere with each other ,distortion is generated
- The data is lost and it cannot recovered

Different ways to avoid aliasing

-Sampling rate fs >= 2fmax
-Strictly band limit the signal to fmax

Nyquist Rate and Nyquist interval

- Nyquist Rate : when the sampling rate becomes exactly equal to (2 fmax)
- Nyquist interval: it is the time interval between any two adjacent sample when sampling rate is Nyquist rate

$$\text{Nyquist Rate} = 2f_{max}$$

$$\text{Nyquist interval} = \frac{1}{2fmax}$$

Example : find the Nyquist Rate and Nyquist interval for the following signals

1) $m1(t) = \frac{1}{2\pi}\cos(4000\ \pi t)\cos(100\ \pi t)$

$$m_1(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$$

$$= \frac{1}{2\pi}\left\{\frac{1}{2}\left[\cos(4000\pi t - 1000\pi t) + \cos(4000\pi t + 1000\pi t)\right]\right\}$$

$$= \frac{1}{4\pi}\left[\cos 3000\pi t + \cos 5000\pi t\right]$$

$$= \frac{1}{4\pi}\left[\cos 2\pi f_1 t + \cos 2\pi f_2 t\right]$$

F1=1500Hz ;   f2=2500Hz

the high frequency = 2500Hz

Nyquist Rate = 2fmax

=2*2500

= 5000 Hz

Nyquist interval $= \frac{1}{2fmax}$

$= \frac{1}{5000}$

= 0.2 msec

Quantization

Quantizing operation approximates the analog values by using a finite number of levels (q) .

$$q = 2^n$$

q= number of levels

n=number of bit

Two of quantization

- Uniform quantizer (linear quantizer)
- Non uniform type Quantizer or compande

Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (\max - \min)/L$$

Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Quantization Zones

- Assume we have a voltage signal with amplitutes V_{min}=-20V and V_{max}=+20V.
- We want to use L=8 quantization levels.
- Zone width $\Delta = (20 - -20)/8 = 5$
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

Assigning Codes to Zones

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$n_b = \log_2 q$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

Q)It is impossible to reconstruct exact original signal x(t)

- Because of permanent quantization error introduced during quantization at the transmitter

- Reduced the quantization error

- Increasing the binary levels (increase number of bits n). but increasing bits (n) increase the signaling rate (r) ,increase bandwidth

Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples -> higher bit rate

Quantization Error and SN_QR

- Signals with lower amplitude values will suffer more from quantization error as the error range: $\Delta/2$, is fixed for all signal levels.
- Non linear quantization is used to alleviate this problem. Goal is to keep SN_QR fixed for all sample values.
- Two approaches:
  - The quantization levels follow a logarithmic curve. Smaller Δ's at lower amplitudes and larger Δ's at higher amplitudes.
  - Companding: The sample values are compressed at the sender into logarithmic zones, and then expanded at the receiver. The zones are fixed in height.

Maximum signal to quantization noise ratio for linear quantization

$$\frac{S}{N} = \frac{3p}{x^3} . 2^{2n}$$

If we assume that input x(t) is normalized

Xmax=1;

Then

$$\frac{S}{N} = 3.2^{2n}.p$$

If the destination noise power (p) is normalized

$$p \leq 1$$

$$\frac{S}{N} \leq 3.2^{2n}$$

Signal to noise ratio in decibels

$$\left(\frac{S}{N}\right)dB \leq (4.8 + 6n)dB$$

Signal to quantization noise ratio

- For normalized values of power (p) and amplitude of input x(t)

$$\left(\frac{S}{N}\right)dB \leq (4.8 + 6n)dB$$

- for sinusiodal signal

$$\left(\frac{S}{N}\right)dB = (1.8 + 6n)dB$$

Transmission Bandwidth in PCM

- Let the quantizer level (q) and (n) number bit for each level .

$$q = 2^n$$

- Each sample is convert to (n bit) number of bits per sample , we know number of samples per second = fs
- Number of bits per sample = (number of bits per sample) *(samples per second)

$$= n * fs$$

$$r = n*fs$$

- Signaling rate in PCM :

$$r = n * fs$$

- Bandwidth needed for PCM transmission

$$B_T \geq \frac{1}{2} r$$

$$B_T \geq \frac{1}{2} n * fs$$

$$B_T \geq \frac{1}{2} n * \mathrm{fmax}$$

Example : A television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM . The number levels is 512 .calculate

1)code word length 2) transmission bandwidth

3) final bit rate 4) signal to quantization noise ratio

- **1)code word length**

$$q = 2^n$$

$$512 = 2^n$$

$$log512 = nlog2$$

n=9bits ;

The code word length is 9 bits

2) transmission bandwidth

$$B_T \geq n * fmax$$

$$\geq 9 * 4.2 * 10^6$$

$$B_T \geq 37.8\ MHz$$

3) final bit rate

$$r = n * fs$$

$$fs \geq 2 * fmax$$

$$\geq 2 * 4.2$$

$$fs \geq 8.4\ MHz$$

$$\therefore r = 9 * 8.4 * 10^6$$

$$r = 75.6 MHz$$

4) signal to quantization noise ratio

$$\left(\frac{S}{N}\right)dB \leq (4.8 + 6n)dB$$
$$\leq 4.8 + 6 * 9$$
$$\leq 58.8\ dB$$

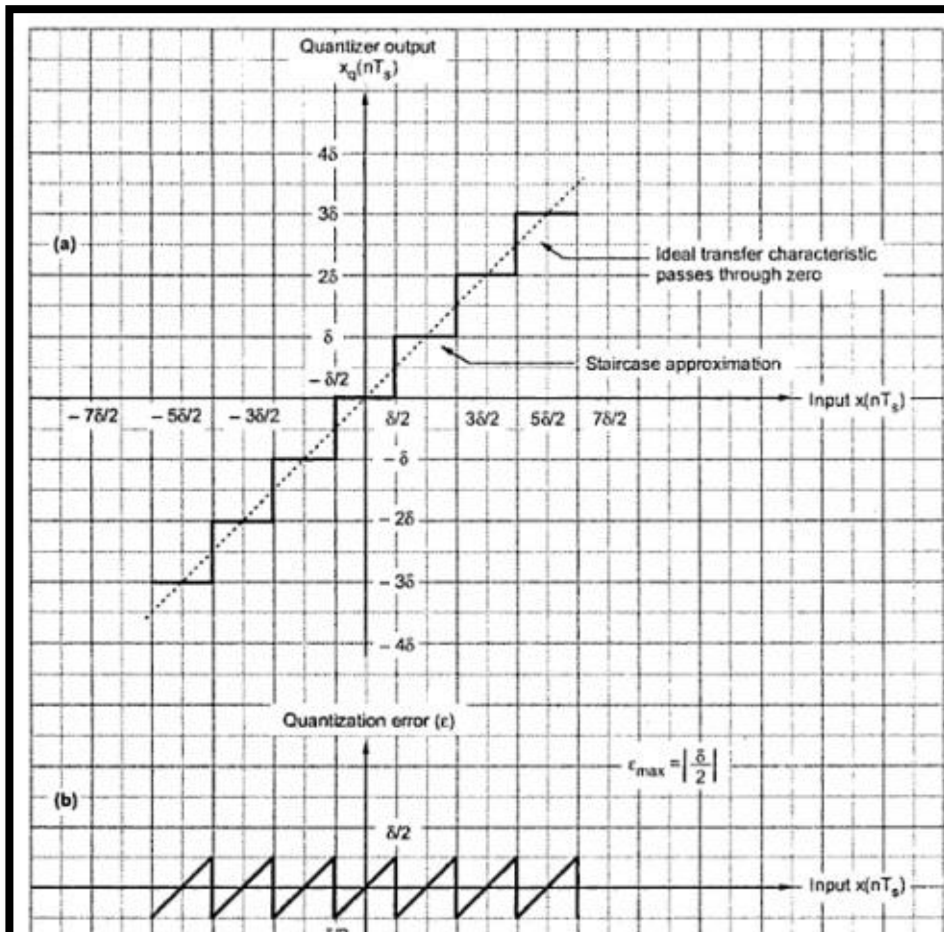
Uniform quantization (linear quantization)

- In uniform quantization , the quantization step or difference between two quantization level remains constant over the complete amplitude range
- Types of linear quantization

1) Midtread quantizer
2) Midriser quantizer
3) Biased quantizer

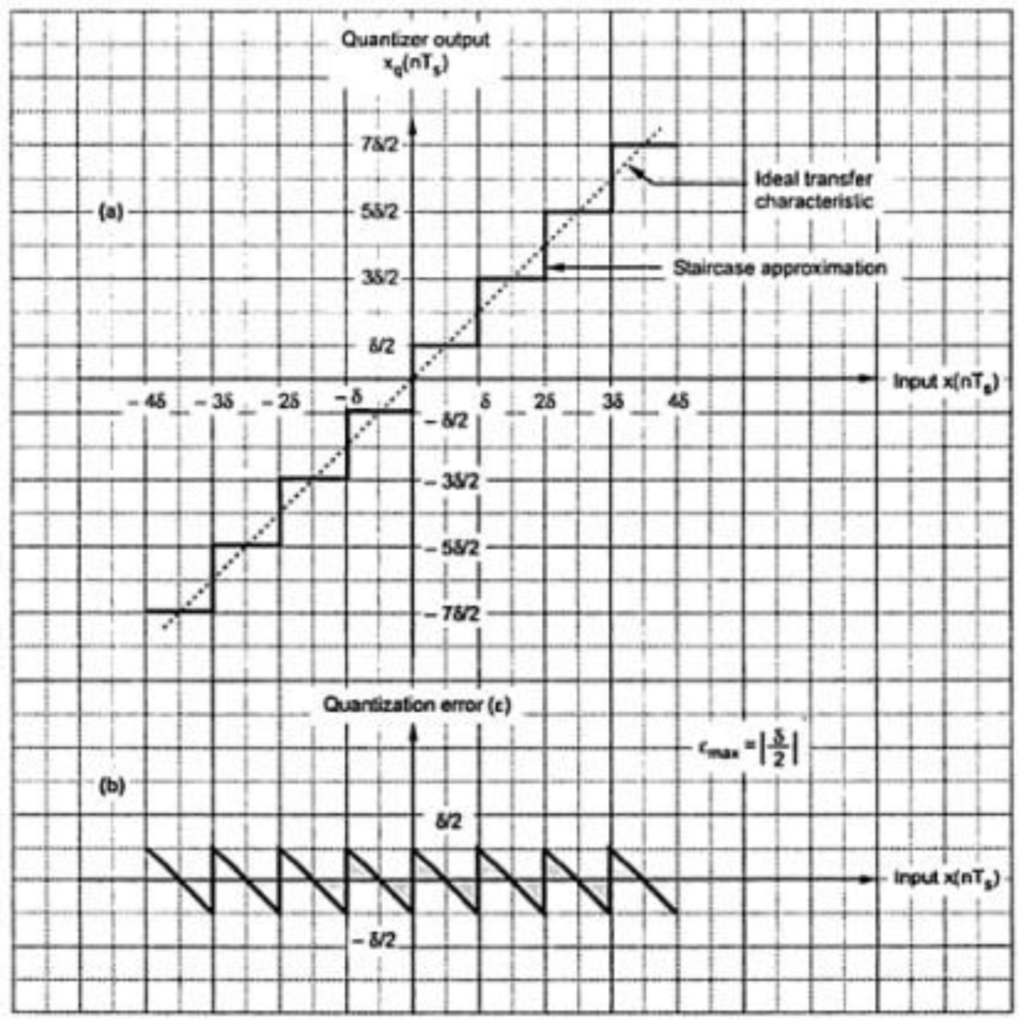
Midtread quantizer

- It called midtread quantization because quantizer output is zero when x(nTs) is zero .



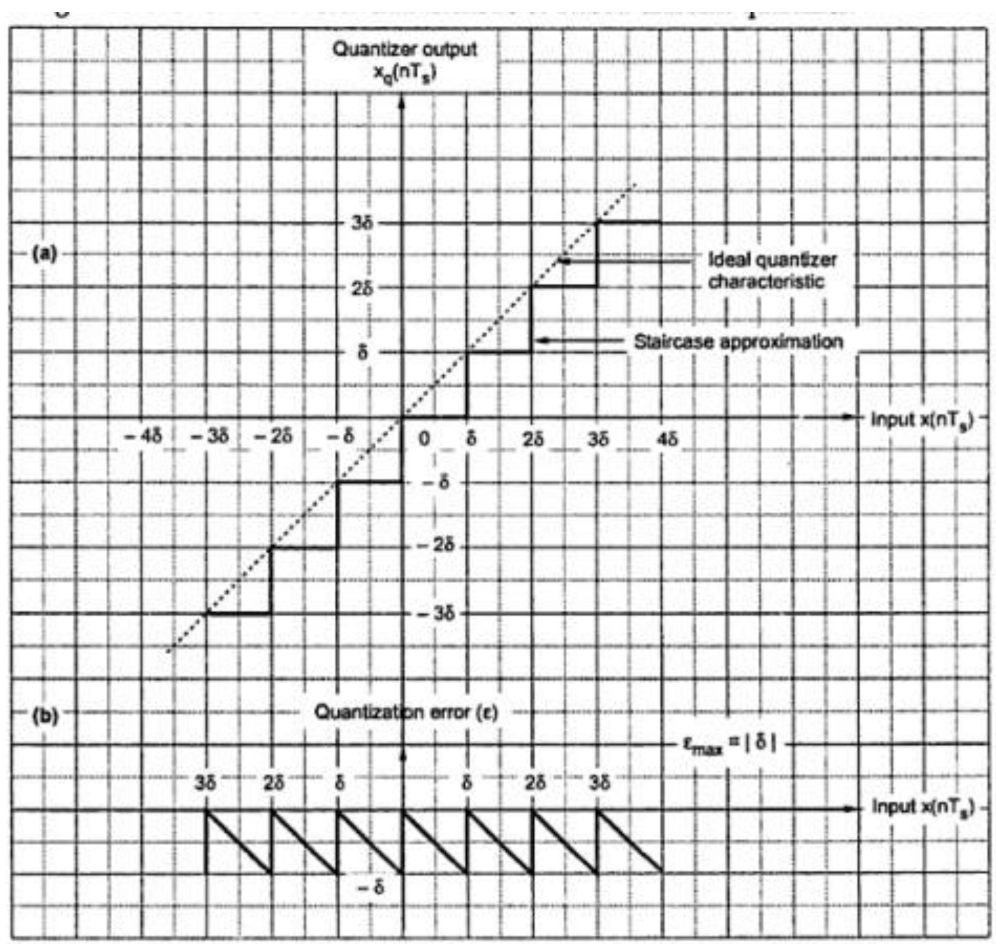
Midriser quantizer

- When an input is between 0 and δ ,the output is $\frac{\delta}{2}$.



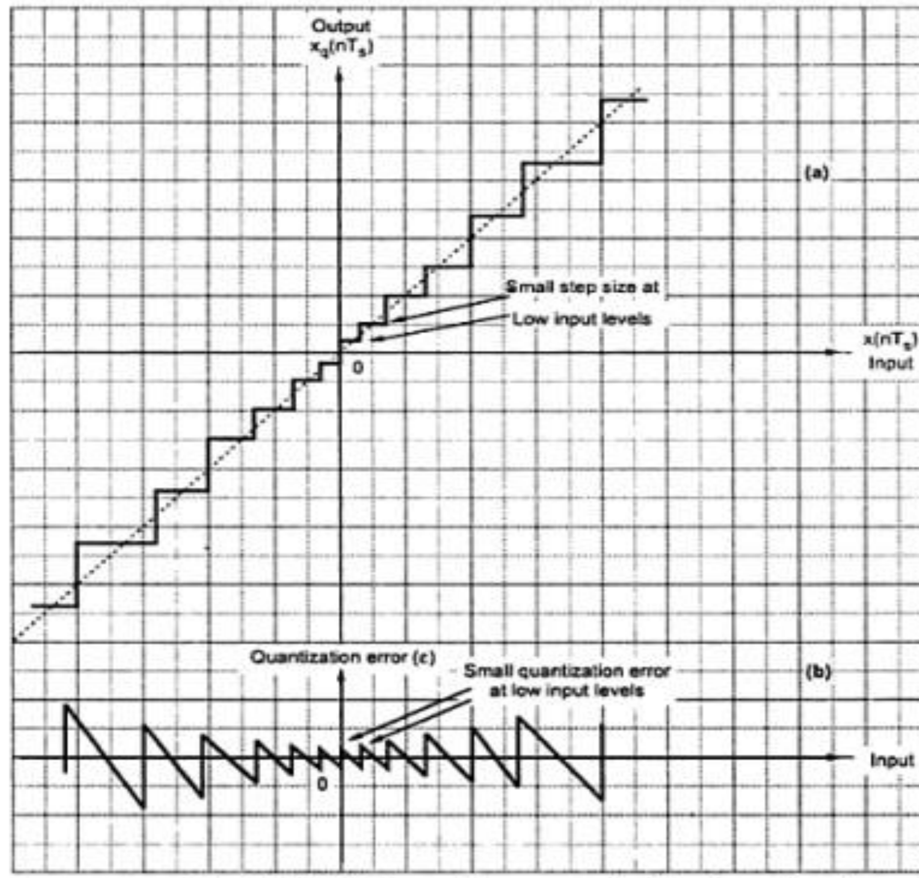
Biased quantizer

- The midriser and midtread quantizers are rounding quantizer ,but biased quantizer is truncation quantizer . When input is between 0 and δ , the output is zero

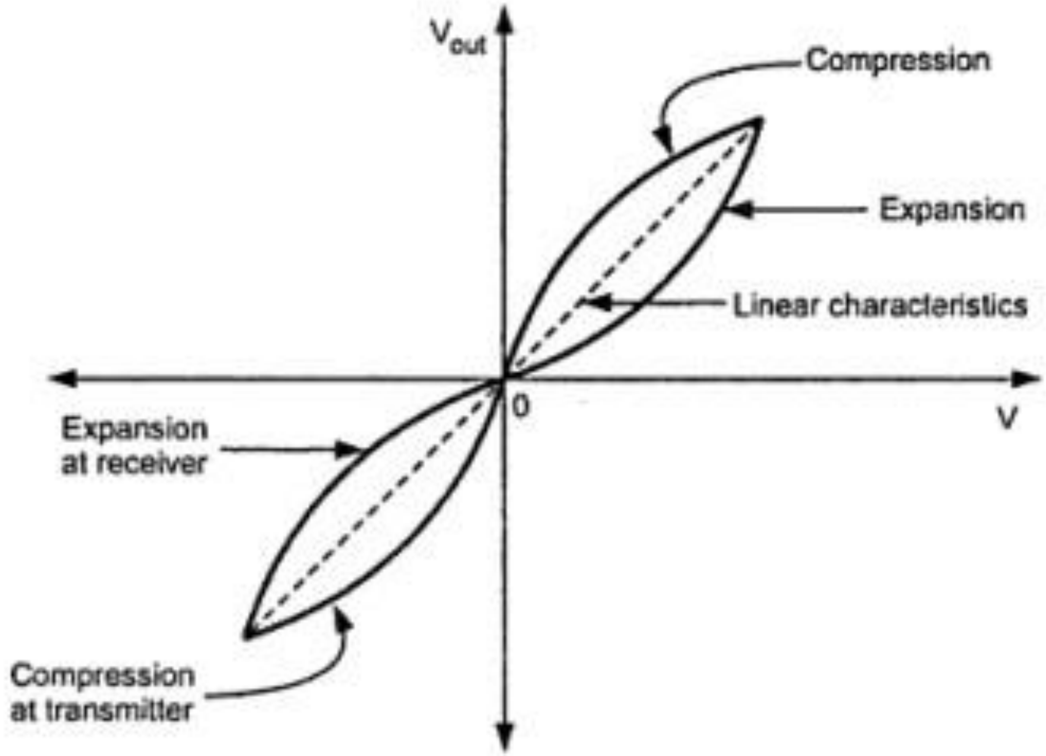


Nonuniform quantization

- In the Nonuniform quantization , the step size is no fixed . It varies according to certain law or as per input signal amplitude .



Compression and Expansion curves



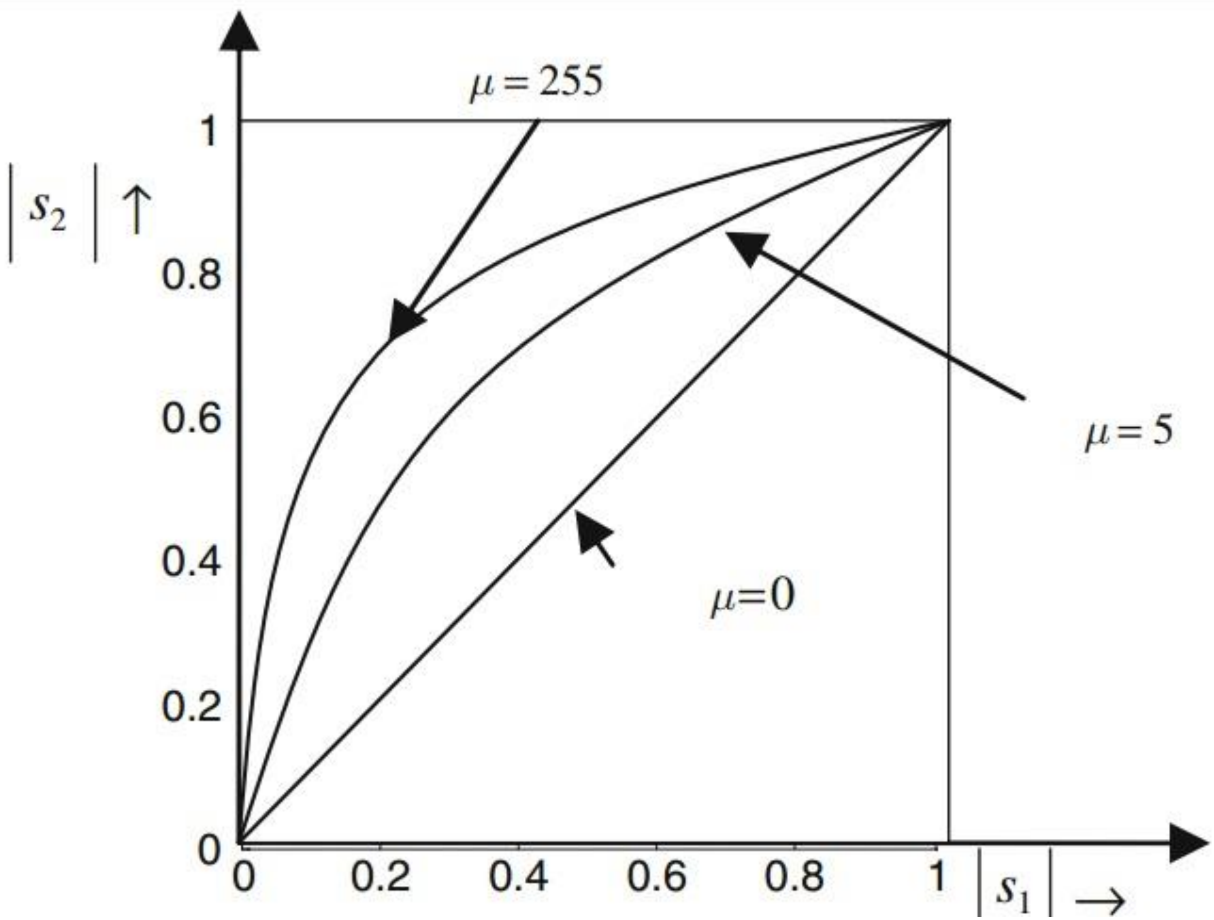
μ-Law

- The compression algorithm following μ-law is given by

$$|s2| = \frac{\log(1 + \mu|s1|)}{\log(1 + \mu)}$$

- $s1$ and $s2$ are normalized input and output voltage/current respectively
- μ is a non-negative parameter which determine the degree of compression
- For $\mu = 0$,
  we obtain uniform quantization

Compression by μ-law



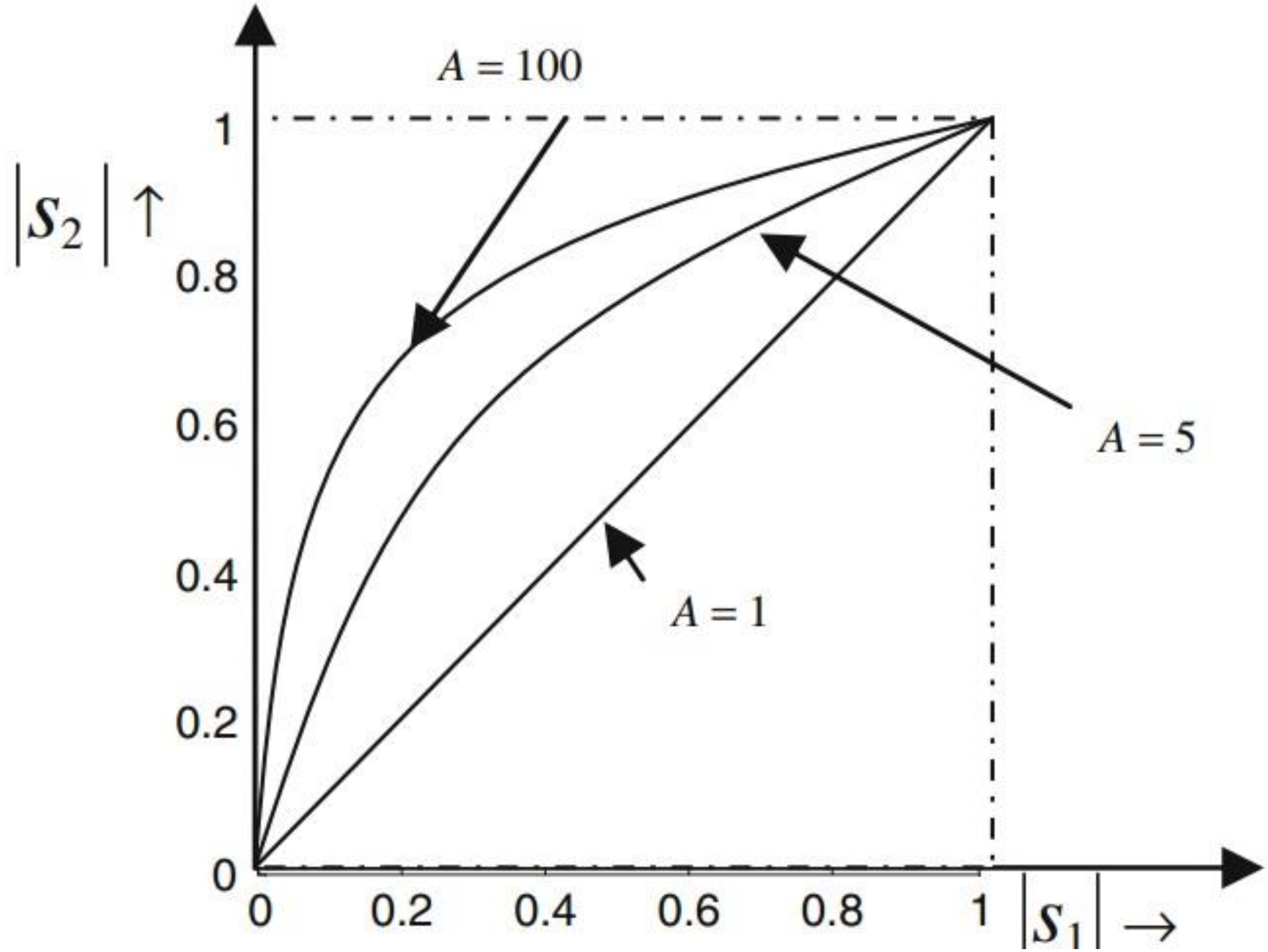
A-Law

- Another well known compression algorithm following A-law is given by

$$|s2| = \frac{A|s1|}{1 + \log A} \qquad ; 0 \le |s1| \le \frac{1}{A}$$

$$|s2| = \frac{1 + \log(A|s1|)}{1 + \log A} \qquad ; \frac{1}{A} \le |s1| \le 1$$

Compression by A-law



Encoding

- The output of the quantizer is one of *q* possible signal levels.
  - If we want to use a binary transmission system, then we need to map each quantized sample into an ***n*** bit binary word.
- ***Encoding*** is the process of representing each quantized sample by an n bit code word.
  - The mapping is one-to-one so there is no distortion introduced by encoding.
  - Some mappings are better than others.
    - A ***Gray code*** gives the best end-to-end performance.
    - The weakness of Gray codes is poor performance when the sign bit (MSB) is received in error.

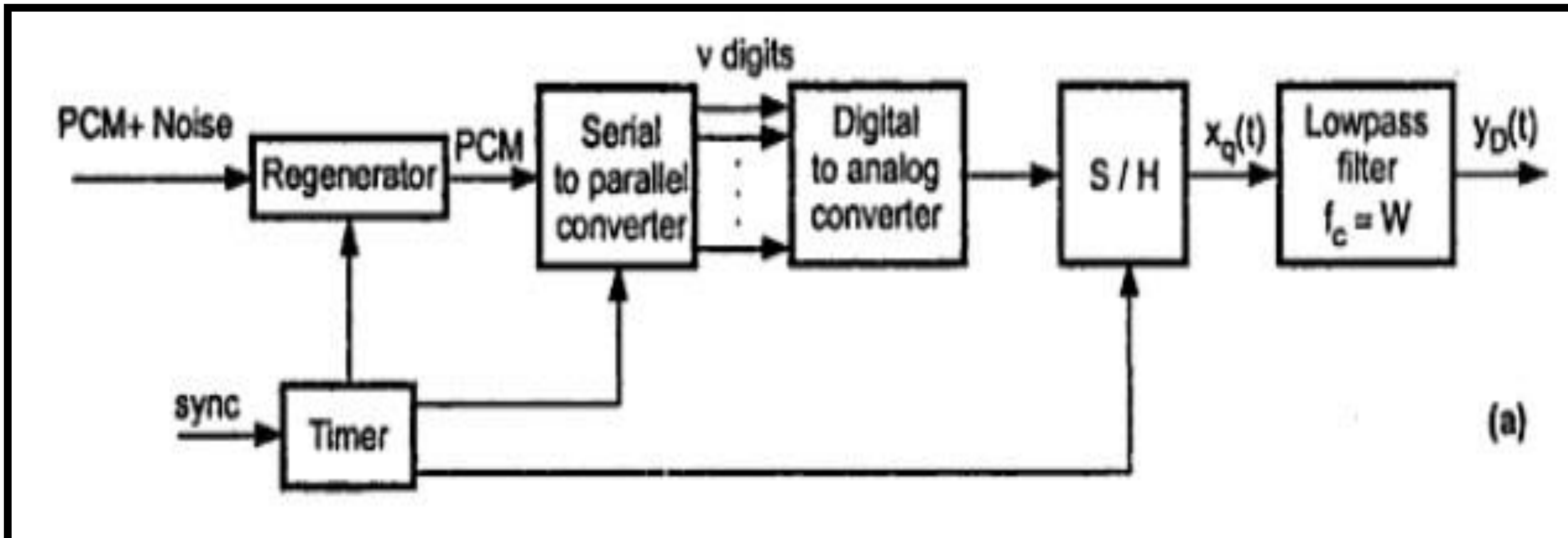
Gray Codes

- With ***gray codes*** adjacent samples differ only in one bit position.
- Example (3 bit quantization):

X_Q	Natural coding	Gray Coding
+7	111	110
+5	110	111
+3	101	101
+1	100	100
-1	011	000
-3	010	001
-5	001	011
-7	000	010

- With this gray code, a single bit error will result in an amplitude error of only 2.
  - Unless the MSB is in error.

PCM Receiver



PCM Receiver

- The regenerator at the star of PCM receiver reshapes the pulses and remove the noise .
- This signal is convert to parallel digital words for each sample .
- The digital word is convert to analog value xq(t) along with sample and hold .
- The out put of S/H is passed through low pass reconstruction filter to get y(t).

Reconstruction signal