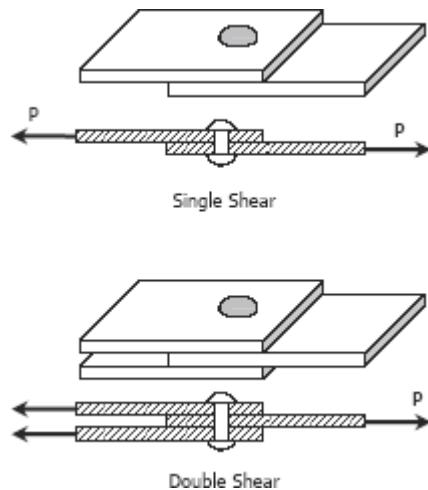


Shearing Stress

Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = \frac{V}{A}$$

where V is the resultant shearing force which passes through the centroid of the area A being sheared.

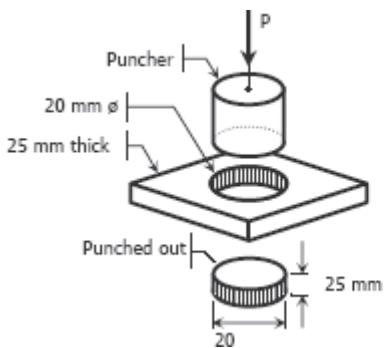


SOLVED PROBLEMS IN SHEARING STRESS

Problem 115

What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m².

Solution 115



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P .

$$\begin{aligned} V &= \tau A \\ P &= 350[\pi(20)(25)] \\ &= 549\,778.7 \text{ N} \\ &= 549.8 \text{ kN} \end{aligned}$$

Problem 116

As in Fig. 1-11c, a hole is to be punched out of a plate having a shearing strength of 40 ksi. The compressive stress in the punch is limited to 50 ksi. (a) Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. (b) If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.

Solution 116

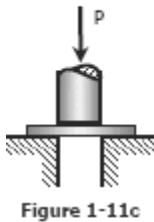


Figure 1-11c

(a) Maximum thickness of plate:

Based on puncher strength:

$$\begin{aligned}P &= \sigma A \\&= 50\left[\frac{1}{4}\pi(2.5^2)\right] \\&= 78.125\pi \text{ kips} \rightarrow \text{Equivalent shear force of the plate}\end{aligned}$$

Based on shear strength of plate:

$$\begin{aligned}V &= \tau A \quad \rightarrow V = P \\78.125\pi &= 40[\pi(2.5t)] \\t &= 0.781 \text{ inch}\end{aligned}$$

(b) Diameter of smallest hole:

Based on compression of puncher:

$$\begin{aligned}P &= \sigma A \\&= 50\left(\frac{1}{4}\pi d^2\right) \\&= 12.5\pi d^2 \quad \rightarrow \text{Equivalent shear force for plate}\end{aligned}$$

Based on shearing of plate:

$$V = \tau A \quad \rightarrow V = P$$

$$12.5\pi d^2 = 40[\pi d(0.25)]$$

$$d = 0.8 \text{ in}$$

Problem 117

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1-11b if $P = 400 \text{ kN}$. The shearing strength of the bolt is 300 MPa.

Solution 117

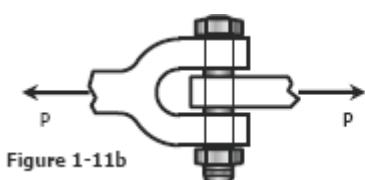


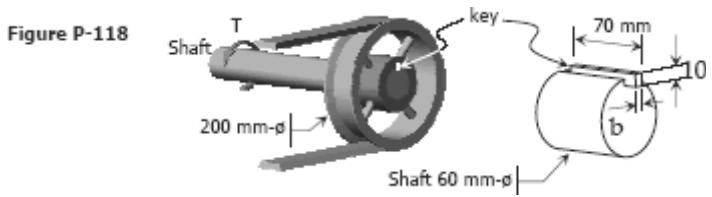
Figure 1-11b

The bolt is subject to double shear.

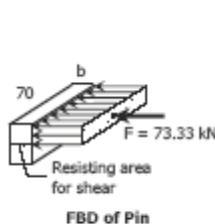
$$\begin{aligned}V &= \tau A \\400(1000) &= 300[2\left(\frac{1}{4}\pi d^2\right)] \\d &= 29.13 \text{ mm}\end{aligned}$$

Problem 118

A 200-mm-diameter pulley is prevented from rotating relative to 60-mm-diameter shaft by a 70-mm-long key, as shown in Fig. P-118. If a torque $T = 2.2 \text{ kN}\cdot\text{m}$ is applied to the shaft, determine the width b if the allowable shearing stress in the key is 60 MPa.



Solution 118



$$T = 0.03F$$

$$2.2 = 0.03F$$

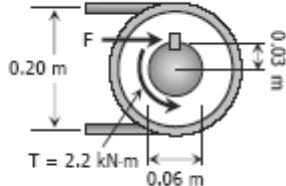
$$F = 73.33 \text{ kN}$$

$$V = \tau A$$

$$\text{Where: } V = F = 73.33 \text{ kN} \\ A = 70b; \tau = 60 \text{ MPa}$$

$$73.33(1000) = 60(70b)$$

$$b = 17.46 \text{ mm}$$



Problem 119

Compute the shearing stress in the pin at B for the member supported as shown in Fig. P-119. The pin diameter is 20 mm.

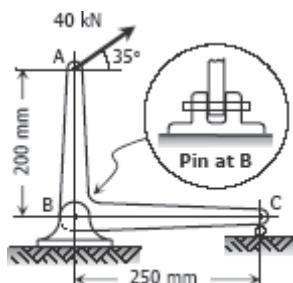


Figure P-119

Solution 119

From the FBD:

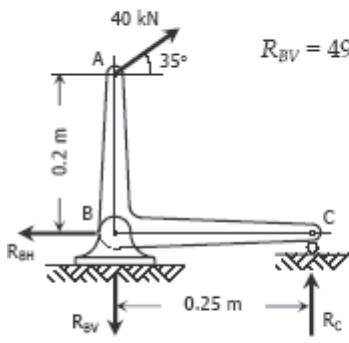
$$\sum M_C = 0$$

$$0.25R_{BV} = 0.25(40 \sin 35^\circ) + 0.2(40 \cos 35^\circ)$$

$$R_{BV} = 49.156 \text{ kN}$$

$$\sum F_H = 0$$

$$R_{BH} = 40 \cos 35^\circ = 32.766 \text{ kN}$$



Free Body Diagram

$$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$$

$$= \sqrt{32.766^2 + 49.156^2}$$

$$= 59.076 \text{ kN} \rightarrow \text{shear force of pin at B}$$

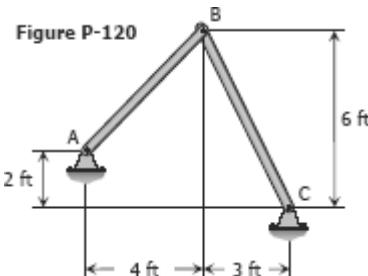
$$V_B = \tau_B A \rightarrow \text{double shear}$$

$$59.076 (1000) = \tau_B [2(\frac{1}{4}\pi(20^2))]$$

$$\tau_B = 94.02 \text{ MPa}$$

Problem 120

The members of the structure in Fig. P-120 weigh 200 lb/ft. Determine the smallest diameter pin that can be used at A if the shearing stress is limited to 5000 psi. Assume single shear.

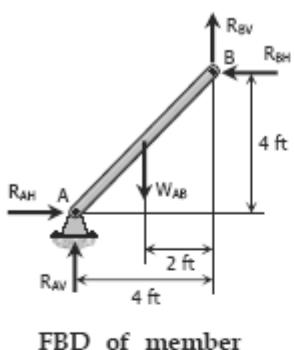


Solution 120

For member AB:

$$\begin{aligned} \text{Length, } L_{AB} &= \sqrt{4^2 + 4^2} \\ &= 5.66 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Weight, } W_{AB} &= 5.66(200) \\ &= 1132 \text{ lb} \end{aligned}$$



FBD of member AB

$$\sum M_A = 0$$

$$\begin{aligned} 4R_{BH} + 4R_{BV} &= 2W_{AB} \\ 4R_{BH} + 4R_{BV} &= 2(1132) \\ R_{BH} + R_{BV} &= 566 \quad \rightarrow (1) \end{aligned}$$

For member BC:

$$\begin{aligned} \text{Length, } L_{BC} &= \sqrt{3^2 + 6^2} \\ &= 6.71 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Weight, } W_{BC} &= 6.71(200) \\ &= 1342 \text{ lb} \end{aligned}$$

$$\sum M_C = 0$$

$$\begin{aligned} 6R_{BH} &= 1.5W_{BC} + 3R_{BV} \\ 6R_{BH} - 3R_{BV} &= 1.5(1342) \\ 2R_{BH} - R_{BV} &= 671 \quad \rightarrow (2) \end{aligned}$$

Add equations (1) and (2)

$$R_{BH} + R_{BV} = 566 \quad \rightarrow (1)$$

$$2R_{BH} - R_{BV} = 671 \quad \rightarrow (2)$$

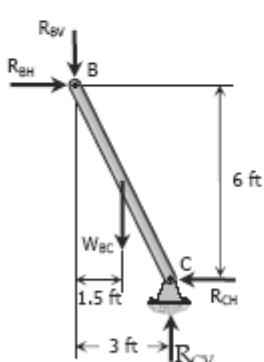
$$\frac{3R_{BH}}{3} = 1237$$

$$R_{BH} = 412.33 \text{ lb}$$

From equation (1):

$$412.33 + R_{BV} = 566$$

$$R_{BV} = 153.67 \text{ lb}$$



FBD of member BC

From the FBD of member AB

$$\sum F_H = 0$$

$$R_{AH} = R_{BH} = 412.33 \text{ lb}$$

$$\sum F_V = 0$$

$$R_{AV} + R_{BV} = W_{AB}$$

$$R_{AV} + 153.67 = 1132$$

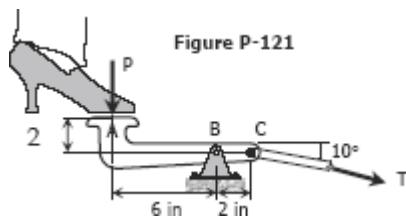
$$R_{AV} = 978.33 \text{ lb}$$

$$\begin{aligned}
 R_A &= \sqrt{R_{AH}^2 + R_{AV}^2} \\
 &= \sqrt{412.33^2 + 978.33^2} \\
 &= 1061.67 \text{ lb} \rightarrow \text{shear force of pin at A}
 \end{aligned}$$

$$\begin{aligned}
 V &= \tau A \\
 1061.67 &= 5000(\frac{1}{4}\pi d^2) \\
 d &= 0.520 \text{ in}
 \end{aligned}$$

Problem 121

Referring to Fig. P-121, compute the maximum force P that can be applied by the machine operator, if the shearing stress in the pin at B and the axial stress in the control rod at C are limited to 4000 psi and 5000 psi, respectively. The diameters are 0.25 inch for the pin, and 0.5 inch for the control rod. Assume single shear for the pin at B.

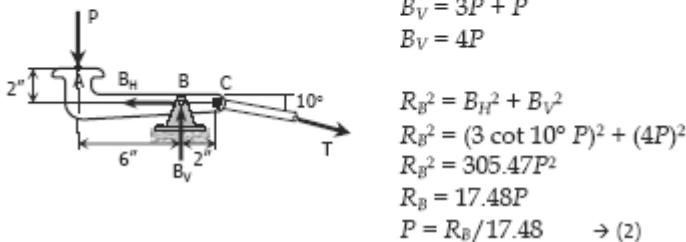


Solution 121

$$[\sum M_B = 0] \quad 6P = 2T \sin 10^\circ \rightarrow (1)$$

$$\begin{aligned}
 [\sum F_H = 0] \quad B_H &= T \cos 10^\circ \rightarrow \text{from (1), } T = 3P/\sin 10^\circ \\
 B_H &= (3P/\sin 10^\circ) \cos 10^\circ \\
 B_H &= 3 \cot 10^\circ P
 \end{aligned}$$

$$\begin{aligned}
 [\sum F_V = 0] \quad B_V &= T \sin 10^\circ + P \rightarrow \text{from (1), } T \sin 10^\circ = 3P \\
 B_V &= 3P + P \\
 B_V &= 4P
 \end{aligned}$$



Based on tension of rod (equation 1):

$$P = \frac{1}{3} T \sin 10^\circ$$

$$P = \frac{1}{3} [5000 \times \frac{1}{4}\pi(0.5)^2] \sin 10^\circ$$

$$P = 56.83 \text{ lb}$$

Based on shear of rivet (equation 2):

$$P = 4000 \times \frac{1}{4}\pi(0.25)^2 / 17.48$$

$$P = 11.23 \text{ lb}$$

Safe load $P = 11.23 \text{ lb}$

Problem 122

Two blocks of wood, width w and thickness t , are glued together along the joint inclined at the angle θ as shown in Fig. P-122. Using the free-body diagram concept in Fig. 1-4a, show that the shearing stress on the glued joint is $\tau = P \sin 2\theta / 2A$, where A is the cross-sectional area.

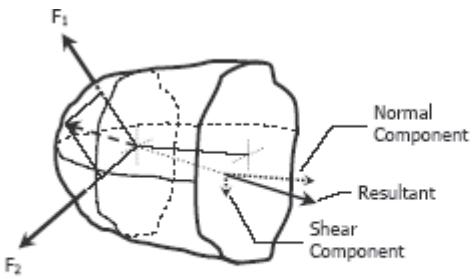


Figure 1-4a Normal and shear components of the resultant on arbitrary section.

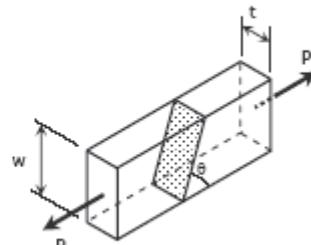
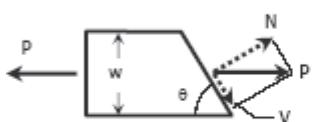


Figure P-122

Solution 122



$$\begin{aligned} \text{Shear area, } A_{\text{shear}} &= t(w \csc \theta) \\ &= tw \csc \theta \\ &= A \csc \theta \end{aligned}$$

$$\text{Shear force, } V = P \cos \theta$$

$$\begin{aligned} V &= \tau A_{\text{shear}} \\ P \cos \theta &= \tau (A \csc \theta) \\ \tau &= P \sin \theta \cos \theta / A \\ &= P (2 \sin \theta \cos \theta) / 2A \\ &= P \sin 2\theta / 2A \quad (\text{ok!}) \end{aligned}$$

Problem 123

A rectangular piece of wood, 50 mm by 100 mm in cross section, is used as a compression block shown in Fig. P-123. Determine the axial force P that can be safely applied to the block if the compressive stress in wood is limited to 20 MN/m^2 and the shearing stress parallel to the grain is limited to 5 MN/m^2 . The grain makes an angle of 20° with the horizontal, as shown. (Hint: Use the results in Problem 122.)

Solution 123

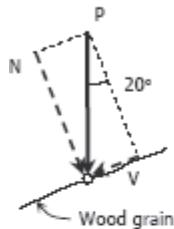
Based on maximum compressive stress:

Normal force:

$$N = P \cos 20^\circ$$

Normal area:

$$A_N = 50 (100 \sec 20^\circ) \\ = 5320.89 \text{ mm}^2$$



$$N = \sigma A_N$$

$$P \cos 20^\circ = 20 (5320.89)$$

$$P = 113\,247 \text{ N}$$

$$= 133.25 \text{ kN}$$

Based on maximum shearing stress:

Shear force:

$$V = P \sin 20^\circ$$

Shear area:

$$A_V = A_N$$

$$= 5320.89 \text{ mm}^2$$

$$V = \tau A_V$$

$$P \sin 20^\circ = 5 (5320.89)$$

$$P = 77\,786 \text{ N}$$

$$= 77.79 \text{ kN}$$

For safe compressive force, use $P = 77.79 \text{ kN}$