Alfurat Al-Awsat Technical University
Technical College / Al-Najaf
Department : Building \& Construction Technology Engineering
Subject: Theory of Structures
Class: Third year
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## Statically determinate beams

Shear force and bending moments in beams
Lecture (5+6) Concept of S.F. and B.M.

Consider this beam :


Suppose that the beam is cut at section $x-x$ as shown .Separate it into two segments. For equilibrium a S.F. $=\mathrm{V}$ and a moment $=\mathrm{M}$ must act on each side of the section (or each segment). Take one segment as a free body (for example the left -side). To find V ,

$\mathrm{V}+6+3 \times 2-\mathrm{R}_{1}=0$ $\qquad$ $. V=R_{1}-12$

To find $M$, use $\Sigma M=0$ at the section
$M=R_{1} \times 2-6 \times 5-3 \times 2 \times 2 / 2$
If the right-side segment is taken, then :
$V=-R_{2}+3 \times 10$
$M=R_{2} \times 10-3 \times 10 \times 10 / 2$
Here :
$V=S . F$. (Cutting at the section)
$\mathrm{M}=\mathrm{B} . \mathrm{M}$.(Bending at the section)

From the equations:
S.F.=sum of all lateral forces on one side of the section.
B.M. =sum of all moments of all forces on one side of the section. The moments are taken about the cut section.

Example:
Find $V$ and $M$ at $x-x$ section of the beam shown,


Solution:
First find the external reactions,
$R_{1}=[4 \times 10 \times 10 / 2-10 \times 3] / 10=17 t$ Up

$R_{2}$ is not needed if left -side segment is used
$V=17-4 \times 4=1 t$
$M=17 \times 4-4 \times 4 \times 4 / 2=36$ t.m (Sagging)

Sign conventions:
v



For S.F. :Shear force is positive (+ve) if its direction is down on left side segment or up on right side segment. Or Shear force is positive (+ve) if $\sum F y$ on left side is up or $\Sigma F y$ is down on right side .


For B.M. : The B.M. at a section will cause curvature .There is compression on the inside and tension on the outside of the curvature. Usually +B.M. is drawn on the compression faces.


## Relation between SHEAR FORCE, BEDING MOMENT AND LOADING ( w )

Take a small piece (or segment) of length dx . Here V \& M act on one side (at section x ), then $\mathrm{V}+\mathrm{dV}$ and $\mathrm{M}+\mathrm{dM}$ will act at section ( $\mathrm{x}+\mathrm{dx}$ ).

By vertical equilibrium, V+dV-V +w dx=0
Then,
$d V / d x=-w$


Take moments at right end,
$M+V . d x-(M+d M)-w . d x . d x / 2=0$

$$
\frac{d M}{d x}=V-\frac{w \cdot d x}{2} \ldots \ldots \ldots .\left(\text { Neglecting } \frac{w \cdot d x}{2}\right)
$$

Then, $\quad \frac{d M}{d x}=V$

And

$$
\frac{d^{2} M}{d x^{2}}=\frac{d V}{d x}=-w
$$

Example: Draw S.F.D \& B.M. for this beam.


Solution:
$R_{1}=[4 x 5 x 0.5 / 6]=10 / 6 t \quad U p$
$R_{2}=[4 \times 5 \times 5.5 / 6]=110 / 6 t \quad \mathrm{Up}$
Check: $R_{1}+R_{2}=$ ? $4 \times 5=20 t$
$10 / 6+110 / 6=120 / 6=20 \mathrm{t} . . . . . . . . . . . . .$. O.K.



