Alfurat Al-Awsat Technical University
Technical College / Al-Najaf
Department : Building & Construction Technology Engineering
Subject: Theory of Structures
Class: Third year
Lecturer : Professor Dr. Hakim Alkurayshi

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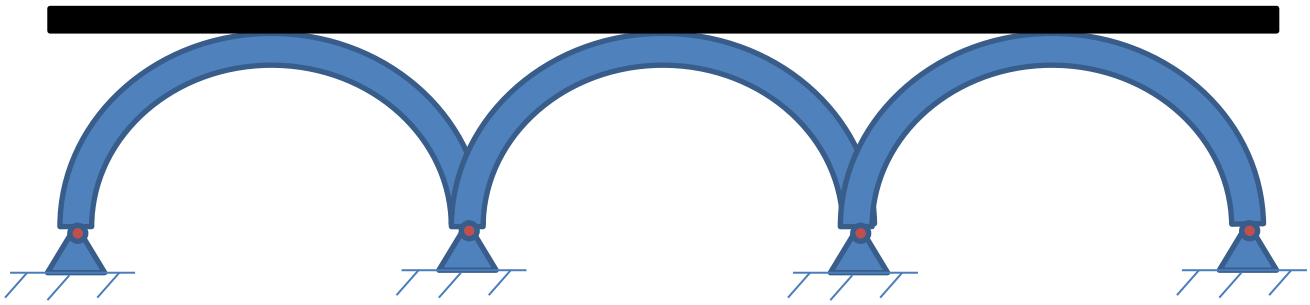
Statically determinate arches

Shear force and bending moments in beams

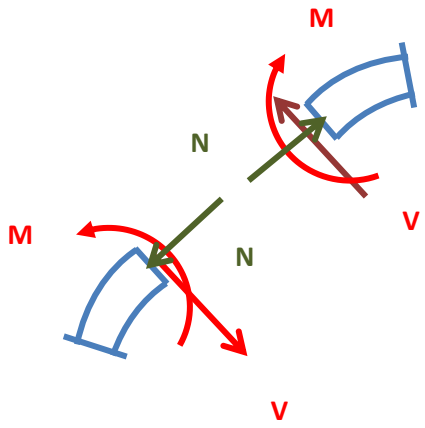
Lecture (7) Concept of S.F. and B.M.

An arch is a curved beam for large spans.

Beam resting on the arch (Bridges)



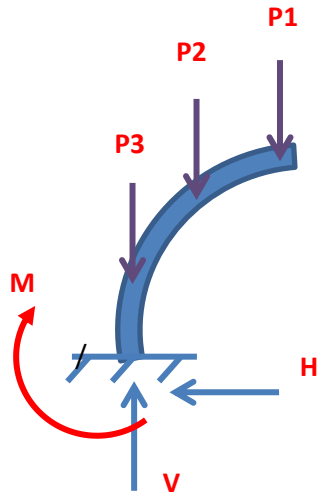
At any x-section of the arch, there are axial force, shearing force and bending moment.



For the whole arch or any portion there are three equations of equilibrium:

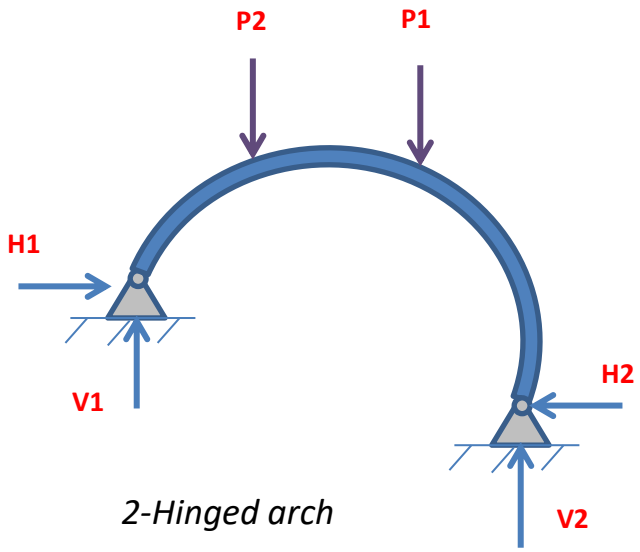
ΣFx=0 , ΣFy=0 & ΣM=0 ……Or any alternatives.

At each interior hinge , the B.M. =0 . Therefore , the arch is statically determinate if **number of external reactions = 3+ Number of interior hinges.** See the following examples:



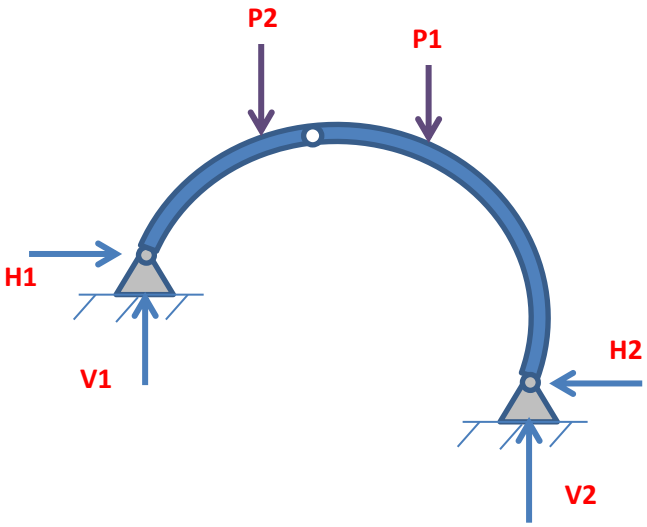
Cantilevering arch

3 unknown reactions and no interior hinge ,so it is statically determinate.



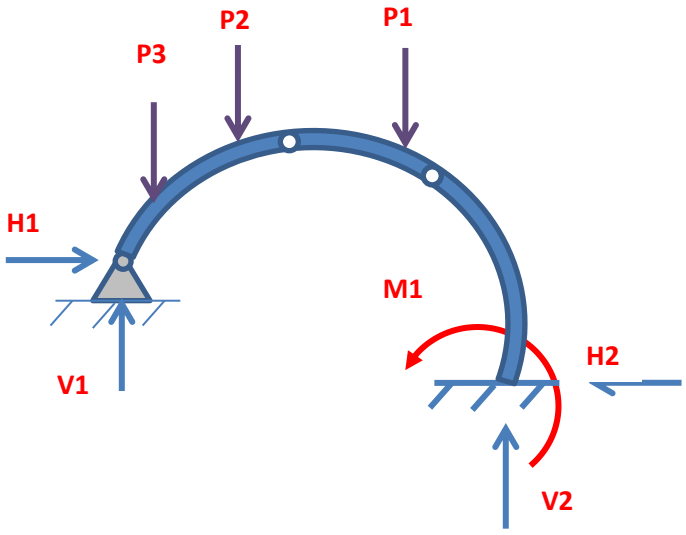
2-Hinged arch

4 unknown reactions and no interior hinge ,so it is statically indeterminate.



3-Hinged arch

4 unknown reactions and one interior hinge ,3+1 =4 ,so it is statically determinate.



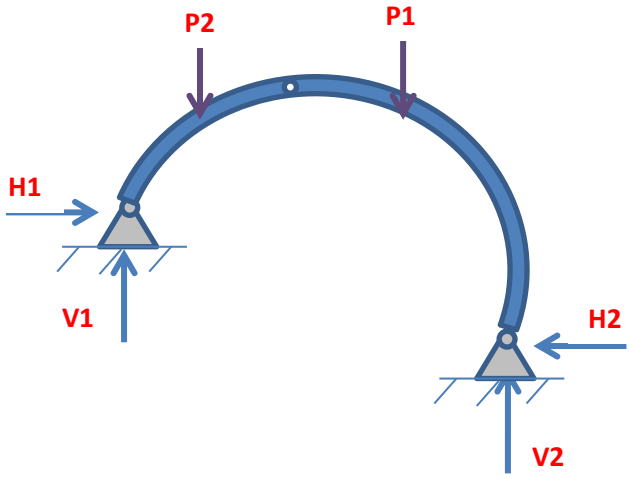
3-Hinged arch

5 unknown reactions and two interior hinge ,3+2 =5,so it is statically determinate.

Analysis of Three hinged arch

This is statically determinate . It has 3-hinges (interior and exterior). The external reactions must be found first in order to calculate the axial force and the bending moment at any x-section .There are three methods to find the external reactions .

1ST Method (Using bending moment =0 at each hinge)



To find V1 and H1 . B.M.=0 at interior hinge H ……(1)

ΣM=0 , Or (B.M.=0) at hinged end (2) ……(2)

Two equations will solve two unknowns (H1 and V1) .

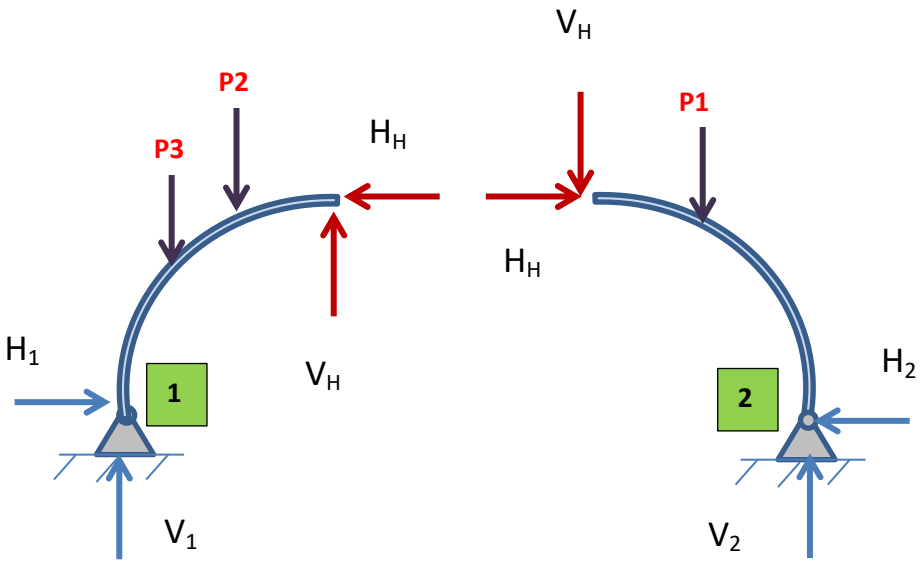
Use Σ Fx =0 to find H2,

Use Σ Fy =0 to find V2,

Check by ΣM =0 at hinged end (1).

2ND Method (Separating into segments at the interior hinges)

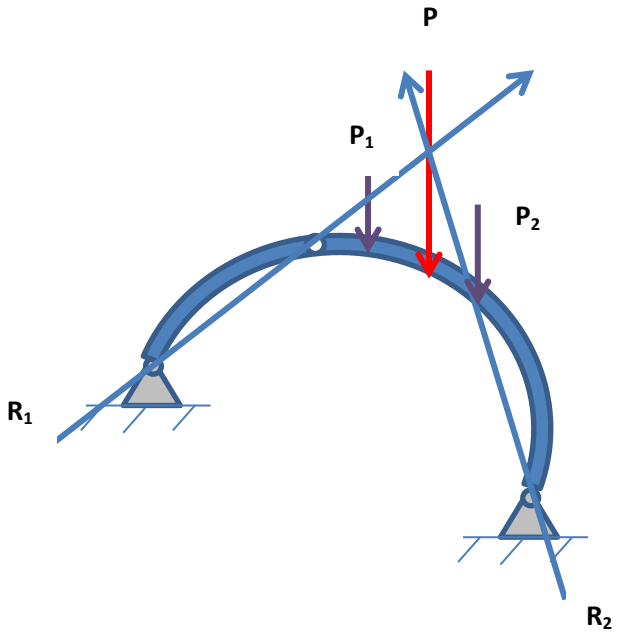
Let V_H and H_H be the internal reactions at the interior hinge . There are 6 unknowns and there are 3x2=6 equations .



First find V_H and H_H by using ΣM =0 at hinged end (1) for left side . ΣM =0 at hinged end (2)for right side. When V_H and H_H are found, then use Σ Fx =0 and Σ Fy =0 for each segment. Check by taking the whole arch and use ΣM =0 about any point.

3RD Method (Graphical)

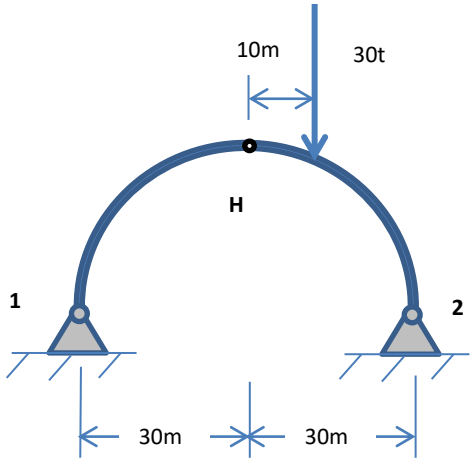
Here the loads must act on only one segment. The whole arch will be under three concurrent forces.

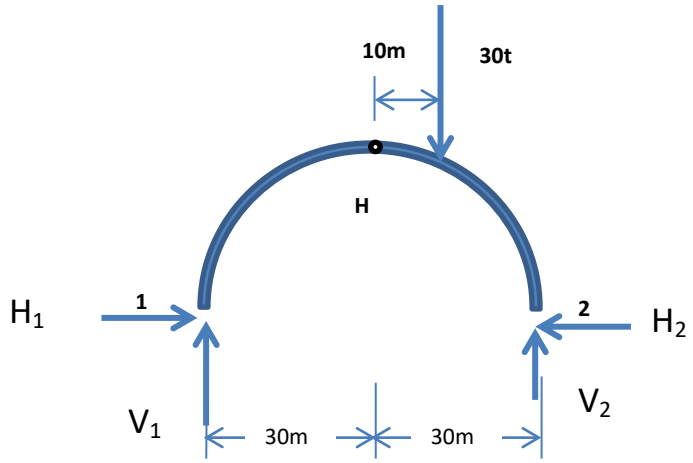


The resultant reaction for the hinge at the side of no force must pass through the interior hinge to give B.M.=0 (for R_1). Then draw R_2 to meet at one point with R_1 and the resultant of the external loads.

Example:

Find the external reactions and then find S.F. and B.M. at the load. The arch is semicircular.





Solution :

1ST Method : (Using B.M.=0 at hinges)

Take the left side and use B.M.=0 at H .

V1X30 –H1X30=0

V1=H1 ……..(1)

Use ΣM=0 at (2),

V1X60-30X20 +H1X0=0…….(2) , Then **V1=10 t** (Up),So **H1=10t** (to the right)

Use ΣFx=0, then **H2=H1=10t** (to the left)

Use ΣFy=0, then **V2=30-V1 =20 t** (Up)

Check by ΣM=0 at end (1)

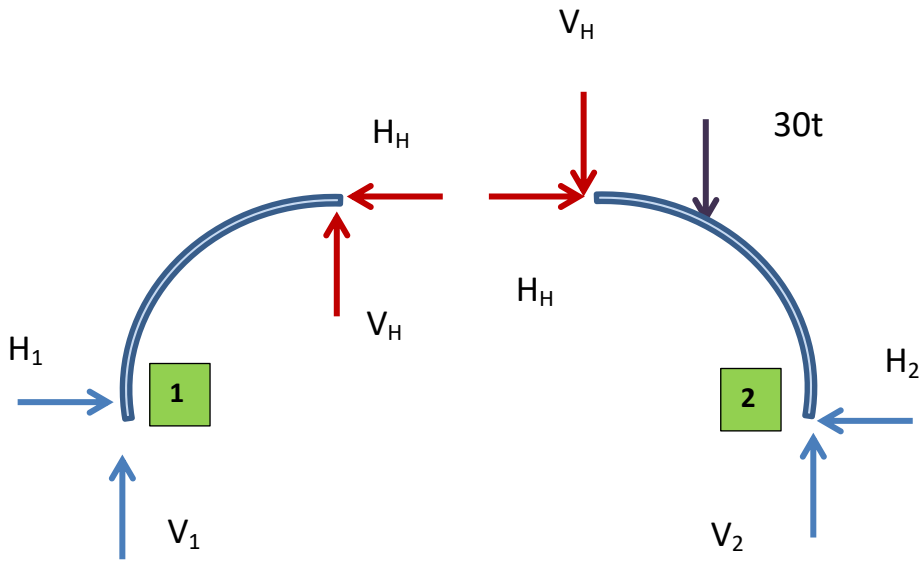
V2x60 +H2x0 -30x40……..20x60+0 -1200 =0 (o.k.)

The B.M. at the load is M= H2 *y –V2x20 …….Here y is found from $x^2+y^2=r^2$

$10^2+ y^2=30^2$ …… y=√(900-100=28.28

Then M=10x28.28-20x20=117.2 t.m (0pening the curve).

2ND Method (Separating into segments)



Use ΣM=0 at end (1),

V_H .30-H_H .30=0……. $V_H = H_H$ ……(1)

Use ΣM=0 at end (2),

30x20- V_H .30- H_H .30=0…….0r 20- V_H - H_H =0……(2)

Solve (1)&(2), $V_H = H_H$ =10 t.

Take left side and use ΣFx=0 and ΣFy=0,

H_1= H_H =10 t (to the right)

V_1=V_H=10 t (UP)

Take the right side , H_2= H_H =10 t (to the left), V_2=30 - V_H =20 t (UP)

Check by any equation for the whole arch.

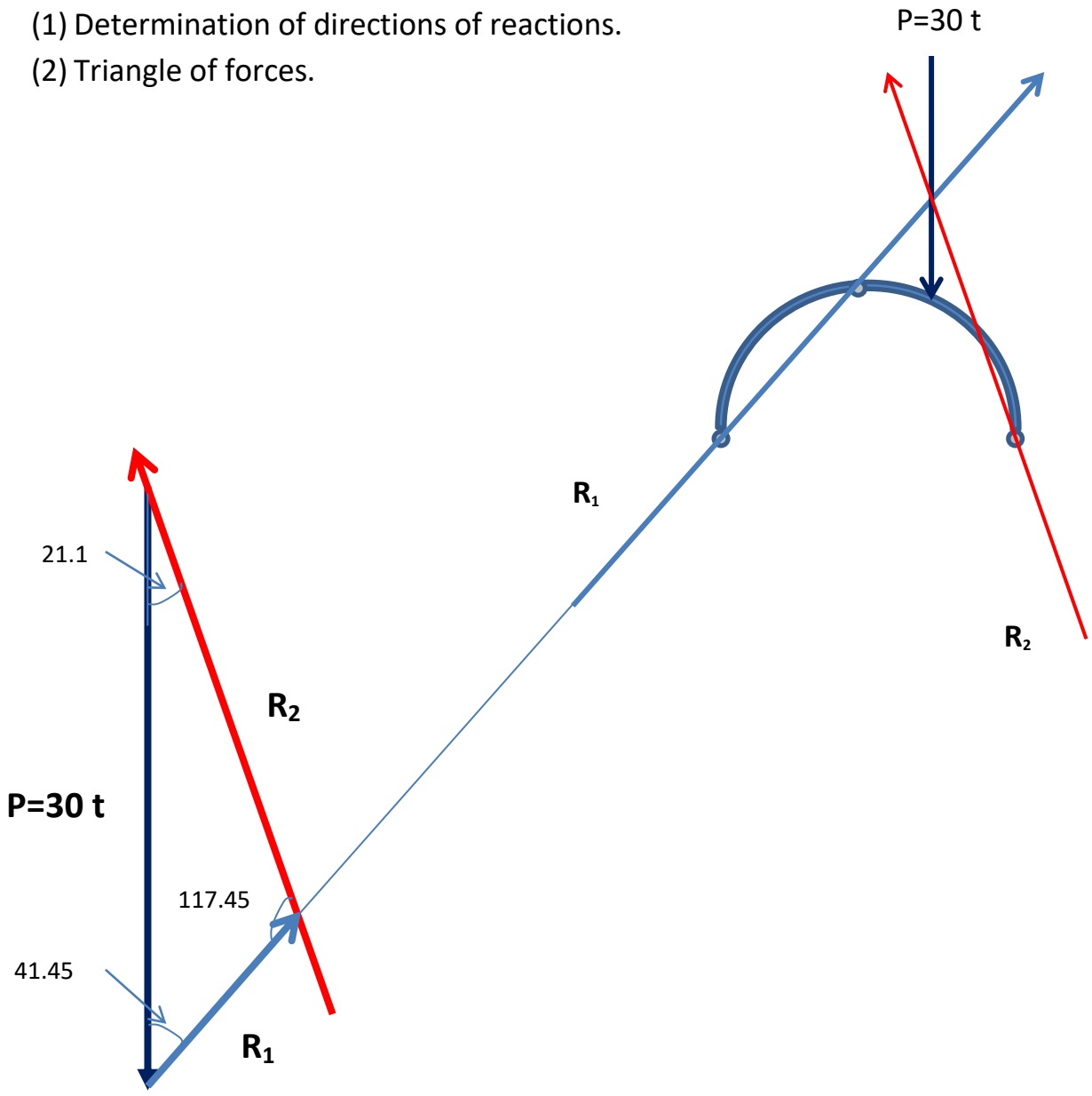
The B.M. at the load 30t is :

$$M = H_2 x\sqrt{30^2 - 10^2} - V_2 x20 = -117.15\ t.m(opening\ the\ curvature)$$

3RD Method (Graphical)

(1) Determination of directions of reactions.

(2) Triangle of forces.

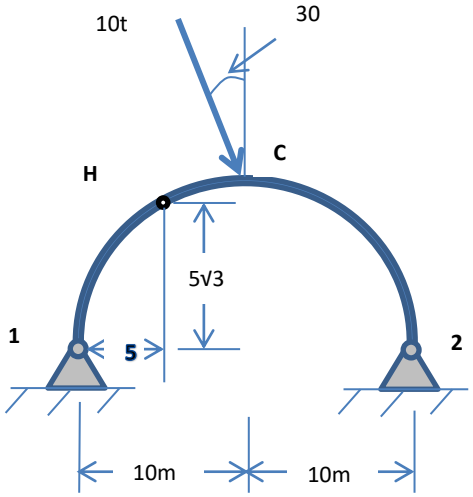


$$\frac{R_1}{\sin(21.1)} = \frac{30}{\sin(117.45)} \dots\dots\dots\dots\dots\dots\dots\dots\dots R_1 = 12.2\ t$$

$$\frac{R_2}{\sin(41.45)} = \frac{30}{\sin(117.45)} \dots\dots\dots\dots\dots\dots\dots\dots\dots R_2 = 22.36\ t$$

Problems to be solved by the students:

1. Find the external reactions , the axial force ,shear force and bending moment at the crown C .



Answer:

External reactions:

V1=1.83t(Up) ,H1=1.055 t (to the right),V2=6.83 t (Up) ,H2=6.055t (to the left)

Axial force at C :

N=H1 =1.055t (compression),just to the left of C.

N=H2 =6.055t (compression),just to the right of C.

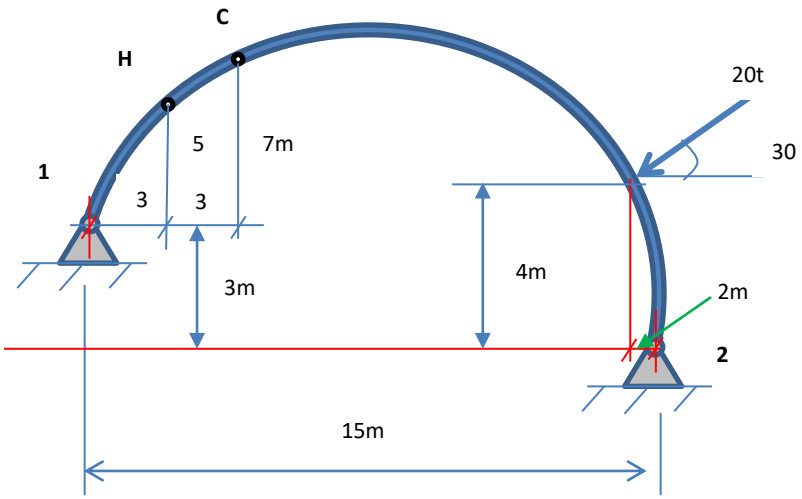
Shearing force at C:

V=V1=1.53t (just to the left of C) , V=V2=6.83t (just to the right of C)

Bending moment at C:

M=H1x10-V1x10 =-7.75 t.m (opening the curvature)

2. Find the external reactions , the axial force ,shear force and bending moment at C .



Answer:

External reactions:

V1=5.32 t(Up) ,H1=3.19 t (to the right),V2=4.68 t (Up) ,H2=-14.13 t (to the right)

Axial force at C :

N=H1 =3.19 t (compression) , or N=H2+20cos 30=3.19 t.

Shearing force at C:

V=V1=5.32 t , Or V= 20sin30-V2 = 5.32 t

Bending moment at C:

M=H1x7-V1x6 =-9.59 t.m (opening the curvature)